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**Comparison of Finite-Element
Stress Analysis with Experimental
Copper Sphere Impacts**

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COMPARISON OF FINITE-ELEMENT STRESS ANALYSIS WITH EXPERIMENTAL COPPER SPHERE IMPACTS

by

C. E. Frantz, S. S. Hecker, M. G. Stout, and R. V. Browning

ABSTRACT

Three copper spheres were impacted on targets of varying surface finishes at 100 m/s. Impact face friction was varied for each test and the impact was photographed with a high-speed camera. Postimpact strains and deformation were measured.

A finite-element computer code, NONSAP, was used to model the impact. The best agreement between computer prediction and experiment was obtained using isoparametric elements, a graded mesh, and actual high-strain-rate copper stress-strain data. Frictional conditions at the impact face were also modeled by altering the standard NONSAP code.

The most critical test of NONSAP was accurate prediction of experimental impact strains. The best agreement we could obtain had a maximum point-to-point error of 20%, although in general, the comparison was much better. Results of this research indicate that we must know more about material and impact interface friction in order to obtain reliable numerical predictions.

I. INTRODUCTION

As part of our study at Los Alamos Scientific Laboratory (LASL) of the safety aspects of radioisotope thermoelectric generators (RTG), which are used in space nuclear power systems, we have investigated a possible accident involving the high-velocity impact of RTG heat source components onto rigid surfaces. As a safety requirement, the cladding containing the $^{238}\text{PuO}_2$ fuel must not rupture on impact. However, it is not possible to predict cladding deformations or potential fracture conditions because the heat source has multicomponent systems that use materials whose behavior is unknown under impact conditions. We simulated

actual impact conditions by impacting solid copper spheres onto rigid targets.

Although such experiments provide some information about overall shape changes to be expected, the actual time sequence of the impact, and the approximate strain rates to be expected, the most important aspect was to test the predictions of computer stress analyses. We used copper because we can readily measure the important materials properties input to the stress analysis program, and hence, check the numerical techniques used in the NONSAP finite-element stress analysis code.

Because failure in thin sheets is controlled by strain states and strain levels, these are the most important predictions to be made about the cladding

material in RTG heat source components. However, these parameters are the most difficult to predict analytically because they are most sensitive to specific numerical techniques and accurate materials properties.¹ The results of our experiments should provide a good test for any computer code used to solve dynamic impact problems.

II. EXPERIMENTAL PROGRAM

A. Materials

All impact and materials properties test specimens were machined from a 44.5-mm-diam ETP (electrolytic tough pitch) copper rod. The copper impact spheres were machined to a diameter of 40.64 mm. Tensile and compressive test specimens varied in diameter and gage length. All specimens and spheres were annealed for 1 h at 650°C after machining.

B. Impact Gun

We used the LASL Group CMB-5 isotope fuel impact tester (IFIT) shown in Figs. 1a and b. A more detailed description is available in Ref. 2. The IFIT is a vertically firing gas gun with a nominal 4-m projectile travel and a smooth 178-mm-diam bore, which can achieve projectile velocities between 30 and 1000 m/s. A single copper sphere is simply supported on the projectile permitting free flight of the sphere after the projectile is stopped. The sphere strikes a rigid target plate of fully hardened Rc 52-54 H-13 steel.

We photographed the impact with a Hycam camera that has a framing rate of about 14 000 half-frames per second.

We measured exact impact velocities by determining the projectile velocity immediately before it entered the catch tube that initiated free flight of the copper sphere. This was done with a series of carefully spaced, radially directed, electrically conducting wire pins protruding into the projectile's path. When the projectile struck a pin, an electrical pulse was recorded on an oscilloscope along with calibration time marks. These distance and time data points were reduced to projectile velocity data. All copper spheres were impacted at 94 ± 1 m/s.

C. Impact Target Surface

We used targets of different surface finishes to test various frictional coefficients as boundary conditions in the finite-element calculations. Three degrees of surface friction created for this test series were (1) a low-friction surface obtained by polishing the steel target to a mirror finish and then applying a thin coat of Lubriplate 630 AA grease to the polished surface, (2) a high-friction surface obtained by turning a spiral groove 0.18 mm deep into the target (to prevent any sliding of the copper sphere at impact), and (3) an intermediate surface obtained by grinding a third target plate to a 0.40- μ m finish.

D. Strain Measurements

Overall deformations of the copper spheres were determined from the profiles of the deformed spheres taken at 10x on a shadowgraph, which allowed determination of the maximum hoop strain and height shrinkage. Figure 2 is a typical shadowgraph trace (reduced) of an impacted sphere.

We measured local surface deformations with the aid of a circle grid scribed into the sphere before impact. The circles, nominally 2.54 mm in diameter,* were cut lightly into the sphere surface to form a great circle (Fig. 3). In most spheres, three great circles were cut to improve the probability of alignment of one great circle with the impact axis. To cut the circles, the copper sphere was held between pressure pads mounted to an indexing head and tail stock, and the assembly was fastened on a rotary table. A scribe was mounted on a vertical mill head and the rotary table was positioned on the mill bed below the scribe. By offsetting the axis of the rotary table from the scribe by the desired circle radius, a measurement circle could be cut by rotating the rotary table. Rotating the indexing head through 41 positions and repeating the scribing procedure produced a great circle of individual overlapping measurement circles. Second and third great circles were obtained by rotating the sphere 120° and reclamping with the pressure pads. The diameter of each circle was

*In later work, 1.27 and 1.90-mm measurement circles were also used; the 1.90-mm circles gave the best results. We obtained excellent resolution of local strain by using 41 measurement circles for each great circle, which caused considerable overlap of adjacent measurement circles.

Fig. 1a.
IFIT gas gun impact chamber.

Fig. 1b.
IFIT gas gun breech.

Fig. 2.
Cross section of sphere Cu-B1 before and after impact, taken from a shadowgraph. Impact velocity = 93.5 m/s at 22°C.

Fig. 3.
Sphere Cu-B1 after impact showing circle pattern used to measure local strains.

measured parallel and normal to the great circle pattern with a toolmaker's microscope and micrometer stage before and after impact to obtain the greatest possible accuracy in hoop and radial strain measurements on the impact face ($\pm 2\%$ engineering strain).

In addition to the overall deformations and local surface strains, we attempted to measure the strains in the sphere interior. Direct measurement of such strains is impossible, so we used an indirect microhardness technique. The impacted copper spheres were sectioned along the great circle that aligned with the impact axis and microhardness measurements were made along two lines, as shown in Fig. 4. One traverse was vertical along the impact axis, and the other was normal to the impact axis at the point of maximum diameter. To convert the microhardness readings to strain levels, we calibrated microhardness against known amounts of tensile and compressive deformation. The resulting conversion curve is shown in Fig. 5. The accuracy of this method suffers at locations of large strains because there is little change in microhardness due to strain because of reduced work hardening of the copper. This calibration was used to convert microhardness readings to approximate strain distributions along both traverses.

Fig. 4.

Cross section of impacted copper sphere showing microhardness traverses used to estimate interior strains.

Fig. 5.

Correlation of measured effective strain to DPH hardness number for ETP copper.

E. Copper Stress-Strain Properties

The stress-strain properties of copper were determined primarily by tensile tests. Two compression tests were conducted to check for agreement with the tensile data. Tensile specimens were machined from different radial locations within the rod and in different orientations with respect to the rod axis to check for directionality in material properties. We found no significant variation in strength properties with orientation or direction, as shown in Table I. The total elongation was lower in transverse specimens than that in longitudinal specimens because of stringers of nonmetallic inclusions resulting from the rod-processing operations. We do not expect this to influence the stress-strain behavior at the strain levels of interest.

Complete stress-strain curves were determined in tension at strain rates of 10^{-3} to 10^{-2} /s with an Instron testing machine. A typical curve of true stress and true plastic strain is shown in Fig. 6a for a 10^{-3} /s strain rate. Two compression specimens tested at equivalent strain rates showed that stress-strain behavior in tension and compression is identical.

Strain rates of 30 to 275/s were achieved in a specially designed high-rate tensile apparatus

TABLE I
ROOM TEMPERATURE TENSILE PROPERTIES OF ANNEALED ETP COPPER (650°C - 1 h)

Specimen	Orientation	Strain Rate (s ⁻¹)	0.2% Yield Strength (MPa)	Ultimate Tensile Strength (MPa)	Uniform Elongation (%)	Total Elongation (%)	Reduction in Area (%)	n ^c	m ^d
Cu-L-2-2	Axial, midpoint	1.3 x 10 ⁻³	25.4	198.2	39.4	50.9 ^a	81.4	0.570 0.330	---
Cu-L-5-2	Axial, midpoint	3.3 x 10 ⁻³	25.2	215.6	40	54.3 ^a	81.3	0.590 0.390	---
Cu-L-6-2	Axial, midpoint	1.3 x 10 ⁻²	26.6	220.5	42	53.5 ^a	82.0	0.579 0.360	---
Cu-L-4-2	Axial, midpoint	30	20-30	---	---	47.5 ^a	81.0	---	---
Cu-L-1-1	Axial, midpoint	60	~32	271	---	67.2 ^b	71.3	---	---
Cu-L-2-1	Axial, midpoint	110	27.6	---	---	73.4 ^b	79.9	---	---
Cu-L-3-1	Axial, midpoint	250	27.6	276	---	64.0 ^b	81.3	---	---
Cu-L-4-1	Axial, midpoint	275	31.7	274	---	65.6 ^b	81.3	---	---
Cu-L-1	Axial, midpoint	3.3 x 10 ⁻³	24.5	214.5	48.5	60.7 ^b	82	---	---
Cu-L-2	Axial, midpoint	3.3 x 10 ⁻³	27.1	214.3	46	57.7 ^b	88	---	---
Cu-C-1	Axial- center line	3.3 x 10 ⁻³	Preloaded	223.6	49	60.8 ^b	79	---	0.006
Cu-C-2	Axial- center line	3.3 x 10 ⁻³	28.9	222.4	42	53.8 ^b	75	---	---
Cu-X-1	Transverse	3.3 x 10 ⁻³	22.5	200.3	31.6	31.6 ^b	34	---	---
Cu-X-2	Transverse	3.3 x 10 ⁻³	27.3	211.8	46.8	51.2 ^b	57	---	0.006
Cu-X-3	Transverse	3.3 x 10 ⁻³	25.9	209.0	48.0	56.7 ^b	68	---	---
Cu-LR-5	Axial, midpoint	70	---	268.0	---	63.0 ^b	76.5	---	---
Cu-LR-6		25	24.8	256.5	---	61.0 ^b	76.5	---	---
Cu-LR-7		2.2 x 10 ⁻⁴	27.9	not tested to failure	---	---	---	---	---
Cu-LR-8		2.2 x 10 ⁻³ 2.2 x 10 ⁻⁴	27.0	204.3	42	61 ^b	81.3	---	---

^a50 8-mm gauge length.

^b12 7-mm gauge length.

^cn designates strain hardening exponent from $\sigma = K\epsilon^n$, where first number is for $\epsilon_p < 15\%$ and second for $\epsilon_p > 15\%$.

^dm designates strain rate sensitivity from $\sigma = K\dot{\epsilon}^m$

described by Hecker and Morgan.³ Strain gages were used to measure strains, but because the strain gage adhesive failed, the maximum measurable strain levels were limited to 4%. Therefore, the stress-strain curves show only the first 4% strain and the ultimate tensile strength. Uniform and total elongations were measured from scribe marks on the test specimens. The strength and elongation values of all strain rates are plotted in Figs. 6a, 6b, and 7.

The average strain rate in a 100 m/s copper sphere impact is $\sim 2000/s$. At present, it is not possible to measure stress-strain curves at large plastic strains at such high rates, so we used the data in Figs. 6 and 7 to extrapolate a stress-strain curve to a strain rate of 2000/s, as shown in Fig. 6a.

Fig. 6a.

Constructed 2000/s strain rate stress-strain curve for ETP copper shown with measured $3.3 \times 10^{-3}/s$ strain rate stress-strain curve.

Fig. 6b.

Ultimate tensile strength and 0.2% yield strength for ETP copper as a function of strain rate.

Fig. 7.

Elongation and reduction in area as a function of strain rate for ETP copper.

III. RESULTS OF IMPACT EXPERIMENTS

A. Surface Strains

The results of three copper sphere impacts into targets with different coefficients of friction are summarized in Table II. Figures 2 and 3 show the deformity produced in sphere Cu-B1. As shown in Table II, the dimensions of the three copper spheres were essentially identical, in spite of the greatly differing frictional conditions. Hence, the overall strain levels were independent of impact face friction.

The local strain distributions, measured with the aid of the scribed circles, are shown in Figs. 8 and 9. The meridional strains were measured along the length of the great circle and the hoop strains were measured perpendicular to the meridional strains. The centerline of the strain distribution denotes the center of the impact face. The transition from the flat impact face to the curved region of the deformed sphere is shown as impact face and the maximum diameter location is also noted.

The meridional strains are principally compressive, except for a region near the center of the impact face, which is reasonable because these strains are in the direction of impact. Maximum compressive strains occur near the impact face transition. The hoop strains are tensile throughout the sphere because the impact promotes a hoop expansion. At least an intermediate strain maximum is reached

near the impact face transition. The strain distribution at the impact face depends on the target friction. The best lubrication (lowest target friction) in Cu-B2 showed the highest strains (20 x 20%) at the center of the impact face. The grooved target, which represents very high friction, does not permit lateral stretching at the impact face and, hence, results in very low strains (2 x 2%). Sphere Cu-B1 with intermediate friction lies in between at 8 x 8%. In all cases the meridional strains must equal the hoop strains at the impact face center because the definitions for both strains coincide at this point. Impact target friction has no effect on the strain distributions from the impact face transition to the back end of the sphere. The strains near the back end are almost zero, which indicates that this part was undisturbed by the impact.

The difference in strain distributions at the impact face with different friction is remarkable considering that the overall shapes of the impacted spheres were identical—and it proves the futility of attempting to predict failure at the impact face from overall shape measurements. It also shows that analytical predictions of the overall shape are not a severe test of the analytical and numerical techniques used in computer codes.

The results of local strain distribution shown in Figs. 8 and 9 give some insight into the problem of heat source impacts. As mentioned above, the tendency of the cladding to fracture depends on the

TABLE II
COPPER SPHERE IMPACT TEST DATA

Specimen	Temp. (°C)	Impact Velocity (m/s)	Rebound Velocity (m/s)	Target	Max Diam after Impact (mm)	Height after Impact (mm)	Impact Face Diam (mm)	Max Hoop Strain (%)	Strain at Center of Impact Face
Cu-B1	25	93.6		Ground tool steel	43.23	32.66	31.24	6.4	8 x 8%
Cu-B2	25	93.6	5.7	Polished tool steel with lubrication	43.24	32.66	31.24	6.4	20 x 20%
Cu-B3	25	94.0	6.5	Tool steel circumferential grooves; V-shaped with 60° angle; 0.18 mm deep x 0.56 mm center-to-center spacing	43.14	32.61	~30.75	6.2	2 x 2%

Fig. 8.

Measured meridional strain distribution for the three impacts, each at 93.6 m/s, of different impact face frictions.

Fig. 9.

Comparison of hoop strain distribution from centerline to back pole for three copper spheres impacted at 93.6 m/s and 22°C.

strain state (combination of hoop and meridional strain). Figure 10 shows anticipated failure loci for cladding materials in strain space. Strain states actually measured at several locations on the sphere surface are superimposed to show their relative severity. The strains at the center of the impact face are most severe in spheres with the best and intermediate lubrication, whereas this point is moved away from the center for poor lubrication. The strain state in all cases is near balanced biaxial tension. The locations of the impact face transition and maximum diameter have essentially the same severity,

because the meridional strain is more negative at the impact face transition where the hoop strain is more positive. If the fracture limit curve (Fig. 10) for the cladding is very sensitive to biaxiality as shown for a typical brittle material, then fracture would be expected to occur at the impact face center.

The results of these tests suggest that friction between the cladding and target in heat source components should be maximized. Also, for brittle cladding materials, the impact face is the location of

Fig. 10.

Strain space diagram showing strain combinations at three locations for three impacted copper spheres. Failure lines are hypothetical for typical ductile and brittle cladding materials.

most probable failure, followed by hoop cracking at the maximum diameter. To predict failure analytically, we must predict accurately the strain distributions shown in Figs. 8 and 9 and we must know the failure limit curves for the cladding material at the temperature and strain rate of impact.*

B. Interior Strains

For impacts of actual heat source components, the sphere interior was $^{238}\text{PuO}_2$ ceramic fuel. We do not expect the copper spheres to model the deformation of this ceramic, but measurement of the interior strains will permit an additional test of the computer stress analysis. The microhardness traverses taken across the section of sphere Cu-B1 in the two directions shown in Fig. 4 are plotted in Figs. 11a and b. They are converted to strain values with the aid of the conversion curve in Fig. 5, which are indicated on the right axes of Figs. 11a and b. The outer envelopes represent an approximate 95% confidence limit. These data show that strains in the

*The strain rate of the cladding at the impact face and at the maximum diameter may differ because of the dynamics of the impact.

sphere interior reach levels of $\sim 60\%$, much higher than the surface strains.

C. Dynamic Measurements

The Hycam high-speed camera photographs permitted an approximate determination of diametrical strain during impact as shown in Fig. 12. We were also able to determine an approximate timing of the impact, including the rebound velocity, which can be compared with computer predictions. The maximum strain levels were reached at 75 to 100 μs and rebound occurred by 230 μs . The rebound velocity (Table II) was ~ 6 m/s, or 6% of the impact velocity. There are uncertainties in the dimension measurements and in the exact timing of the impact. An error limit of $\pm 1\%$ strain is reasonable in dimension measurements, and the error in exact timing could be 25 μs , the time between successive frames taken by the Hycam camera. These error bands are indicated in Fig. 12.

D. Finite-Element Calculations

Several programs exist for computer modeling of copper sphere impacts, but we will restrict our discussion to NONSAP, which produced our results. Although basic NONSAP, a nonlinear finite-element program,⁴ handles static and dynamic large deformation problems with a variety of standard material models, we made extensive modifications to produce more accurate calculations of the copper sphere response. These modifications and the standard NONSAP features are described below, followed by the results obtained for various yield functions, meshes, and frictional modeling.

E. Modifications of Standard NONSAP

1. **Standard Options.** Originally, calculations used a linear four-node element with a lumped-mass approximation and the standard von Mises elastic plastic element. Later calculations used parabolic eight-node elements combined with consistent-mass matrices, which gave more accurate results. Newmark's time integration with default parameters was used with a 0.5- μs time step for reasonable time

Fig. 11a.

Vertical hardness scan of sphere Cu-B1 along the axis of symmetry (50-g load).

Fig. 11b.

Horizontal hardness scan of sphere Cu-B1 at its maximum diameter (50-g load).

Fig. 12.

Dynamic strain measurements from sample Cu-B2 using the Hycam high-speed camera. An error band is given instead of specific numbers.

resolution and iteration convergence to the default accuracy parameters in a small number of iterations. When larger time steps were tried, convergence difficulties quickly stopped the solution. Equilibrium iteration at each time step was used, modified as described below to reformulate the stiffness matrix during equilibrium iteration. No explicit damping of any kind was used.

2. Equilibrium Iteration. With standard NON-SAP, reformulation during equilibrium iteration is impossible. That is, the initial stiffness matrix is used for all steps of the equilibrium iteration. This can cause the iteration to be unstable at any significant stiffening of the system because large imbalanced forces are generated without correspondingly large restraining stiffness. This effect arises because of elastic-plastic unloading, or more commonly, because of the stiffening of the truss elements used to enforce boundary conditions at the impact face. To avoid these difficulties, the program was modified to allow for stiffness reformulation during the equilibrium iteration process. Although expensive in terms of computer time, it greatly improves convergence in many cases. However, it is not a complete cure for instabilities, and in general, the time step was limited by slow convergence in the equilibrium iteration.

3. Large Strain. Standard NONSAP provides two ways to handle large deformation problems: one uses the second Piola-Kirchhoff stress and Green's strain, and the other uses Cauchy stress and Almansi's strain measure. Unfortunately, it is difficult to define a physically realistic constitutive relation in terms of these measures. For instance, the usual small strain plasticity models separate the strains into bulk and deviatoric components, where the bulk strain is the first invariant. However, when Green's or Almansi's strain measures are used, the first invariant is a function of shearing deformations as well as volumetric deformations. When elastic or linear stress-strain relations are used with either of these pairs, the resulting load deflection curves become very soft and even unload at relatively small compressive strains. Figure 13 shows a normalized load deformation curve resulting from the assumption that Green's strain is linearly related to second Piola-Kirchhoff stress.

Fig. 13.
Load deflection curve resulting from a linear relation between Green's strain and the second Piola-Kirchhoff stress.

In principle, any constitutive relation expressed in terms of one set of variables can be transformed for use with another pair, however, it was very difficult to determine appropriate explicit relations. Therefore, we performed the required transformations numerically. The constitutive relations were expressed in terms of a logarithmic strain measure and Cauchy stress rotated to a material coordinate frame, as defined below.

The use of a logarithmic strain measure offers several practical advantages. Experimental data are often reported in terms of "true stress" and "true strain," which (within the assumption of constant volume plastic deformation) are identical with the

logarithmic strain and Cauchy stress used here. These measures can be used with simple small-deformation plasticity relations to produce a reasonable large-strain-plasticity theory. Finally, explicit stress and strain measures preserve the logic of a finite-element program like NONSAP, including the equilibrium iteration method, without imposing restrictions on load step size, which can arise when using stress and strain rate as the constitutive variables and numerically integrating as a function of time or load.

For a precise definition of logarithmic strain, we use the polar decomposition theorem on the deformation gradient

$$\underline{F} = \underline{R} \underline{U}$$

where $\underline{R}$ is an orthogonal transformation ($\underline{R}^T = \underline{R}^{-1}$), and $\underline{U}$ is a positive definite symmetric matrix. Thus, $\underline{U}$ may be decomposed to $\underline{Q}^T \underline{\lambda} \underline{Q}$, where $\underline{Q}$ is also orthogonal and $\underline{\lambda}$ is a diagonal tensor and, in fact, the diagonal components are the principal stresses. Green's strain can be expressed as

$$\underline{E} = 1/2 (\underline{F}^T \underline{F} - \underline{1}) = 1/2 (\underline{Q}^T \underline{\lambda}^2 \underline{Q} - \underline{1})$$

Almansi's strain is

$$\underline{\epsilon} = 1/2 \{ \underline{1} - (\underline{F}^{-1})^T (\underline{F}^{-1}) \} = 1/2 (\underline{1} - \underline{R} \underline{Q}^T \underline{\lambda}^{-2} \underline{Q} \underline{R}^T)$$

As in Ref. 5, we define the material logarithmic strain as

$$\underline{H} = \underline{Q}^T \log \underline{\lambda} \underline{Q}$$

where

$$\log \underline{\lambda} = \begin{Bmatrix} \log \lambda_{11} & 0 & 0 \\ 0 & \log \lambda_{22} & 0 \\ 0 & 0 & \log \lambda_{33} \end{Bmatrix}$$

and the corresponding spatial logarithmic strain as

$$\underline{h} = \underline{R} \underline{Q}^T \log \underline{\lambda} \underline{Q} \underline{R}^T$$

Note that the rotation tensor $\underline{R}$ is used to transform from the spatial to material frame, and $\underline{Q}$ relates the material frame to the principal frame, that is, principal coordinates for a material strain measure.

It is simple to numerically decompose $\underline{F}$ into $\underline{R}$, $\underline{Q}$, and $\underline{\lambda}$, and then to calculate the logarithmic strains as required. The first modification to NONSAP, then, is to perform this decomposition and calculate $\underline{H}$, which is given to the constitutive subroutines. These routines calculate and return a stress and a tangent stiffness tensor. We assume the stress to be Cauchy's stress rotated to a material frame $\underline{\Sigma}$. To calculate the spatial stress $\underline{\sigma}$ as required for the updated Lagrangian method in NONSAP, we use

$$\underline{\sigma} = \underline{R} \underline{\Sigma} \underline{R}^T$$

Similarly, the tangent stiffness must be rotated

$$C_{ijkl} = C_{IJKL} R_{Ii} R_{Jj} R_{Kk} R_{Ll}$$

where we have used a Cartesian tensor notation for clarity.

Finally, we must modify the method of calculating the element stiffness matrix since, in the standard updated Lagrangian formulation, the tangent stiffness matrix is assumed to relate Cauchy stress and Almansi strain increments. The element stiffness matrix $\underline{K}$ is calculated as the sum of linear and nonlinear terms

$$\underline{K} = \underline{K}_L + \underline{K}_{NL}$$

where $\underline{K}_{NL}$ contains the correction terms generated from the use of Almansi's strain measure, while $\underline{K}_L$ contains the terms that come from the basic principle of virtual displacements using variation in the small strain tensor calculated from the current state

$$\int_V \underline{\sigma} : \delta \underline{\epsilon} \, dv = \underline{Q}$$

Now, $\delta \underline{h} \cong \delta \underline{\epsilon}$, so the correct stiffness matrix is obtained by deleting the $\underline{K}_{NL}$ terms.

These changes are all contained in the kinematic portions of the program and can be used with any of the material models.

4. Nonlinear Strain Hardening. The plasticity model used for the copper sphere calculations uses a von Mises yield condition with linear isotropic hardening and the associated flow rule. This is a rate-independent formulation. For copper at large strain levels, that is, several per cent, this is a reasonable approximation, except for the linear hardening rule. The annealed copper used in the experiments had a very low initial yield stress and then exhibited a very nonlinear strain hardening behavior. At first, linear hardening approximations were used in the calculations, but later, we simply incorporated nonlinear hardening in the program.

The basic elastic plastic theory used in NONSAP needs a yield stress and derivative as a function of plastic strain. This is provided by a piecewise linear fit to the yield curve and by the use of differences to obtain the required derivative. In general, we use a table of 10 to 30 values defining the yield curve.

The only difficulty was that most of the impact calculations involved strains higher than those in the tables, since it is very difficult to achieve tensile strains in excess of 0.5 using standard testing methods. The yield curve was arbitrarily extended at its last slope. Note that this is done in a logarithmic strain-Cauchy stress space. This is almost certainly in error as the curvature is significant. Improved extensions are being considered.

5. Gap Elements. Initially, we tried to enforce boundary conditions at the impact face by using the standard truss elements. The idea was to place a truss element between each node on the impacting face of the sphere and a set of fixed nodes describing the rigid target. If the standard NONSAP truss element is used, the strain in the truss is calculated as the current length of the truss divided by the initial length less one. With this definition, a node on the sphere can move around its corresponding target node without producing any force. The correction is redefinition of the strain measure in terms of displacements projected onto the original direction of the truss element. Thus, whenever a surface node of the sphere crosses the plane normal to the initial truss direction, that is the target plane, the strain in the truss will be -1. A piecewise linear stress-strain definition is used with the truss elements that gives zero stresses for all strains greater than -1, but becomes very stiff for strains less than -1. Figure 14 shows the curve used in the copper sphere calculations.

Fig. 14.
Stress-strain curve used for the truss elements in the copper sphere calculations.

tions. The stiffness of the truss elements is chosen to produce no significant penetration of the target. However, too large a stiffness causes numerical problems. The stiffness we used limited penetration to $\sim 3 \times 10^{-2}$ mm.

6. Friction Elements. To simulate friction at the impact face, we modified the truss elements to provide lateral forces as well as normal forces. These lateral loads are related to both the normal load and the lateral motion of the node on the sphere. The basic procedure is to calculate a frictional force as some percentage of the normal compressive force and apply this in a direction that opposes the current relative motion of the nodes. Unfortunately, this method can produce two instabilities in the calculation.

One situation occurs when the normal force first arises, that is, on first contact. There is no normal contact force in the calculation, so the nodes cross to a considerable extent. In the equilibrium iteration process, this results in a very large force that tends

to push the nodes back to their proper contact position; the large force is controlled by the correspondingly large stiffness of the truss element. The frictional force during the iteration is proportionately as large as the normal force, but it is not balanced by stiffness, which causes large displacements of the nodes. As an expedient, we simply limited the magnitude of the frictional force to a reasonable value, which we chose in accordance with the calculated normal forces.

The second instability occurs when the nodes have such a small relative velocity, that the frictional force can change the direction of the relative velocity during equilibrium iteration. This causes oscillation of the force direction and very slow convergence, which is corrected by making the frictional force proportional to the relative velocity for sufficiently small velocities.

Thus, these two problems lead to two problem-dependent constants—a limiting force and a minimum relative velocity. If these constants are chosen carefully, they should not affect the results, but only the rate of convergence in equilibrium iteration. Proper selection and verification of these constants requires experience with a given problem.

IV. CALCULATIONAL RESULTS

As noted earlier, experiments were performed to investigate the accuracy of NONSAP in calculating the relatively large strains (20-200%) that occur in impacting bodies at ~ 100 -m/s velocities. The emphasis on accurate strain measurements is important because the goal is to use the program to predict rupture or failure in cladding materials. The solid sphere of annealed copper was selected because the material could be accurately characterized, the final surface strains directly measured, and impact interface conditions easily varied.

The calculations were made as separate studies of the influence of yield functions, finite-element mesh geometry, and interface frictional behavior, in each case, using the best method for subsequent studies. Thus, we have not examined the combined influence of various yield functions and interface frictional models, but always used the best nonlinear hardening yield curve for all frictional studies. Different conclusions may be reached if the order of the investigation is changed, but we believe that all the

improvements discussed here are important if accurate calculations are to be made

Results for Different Yield Functions. The shape of the yield surface, as specified by a uniaxial stress-strain curve, has a strong influence on the overall predicted shape of the sphere, as well as on the surface strains. We examined both common approximations to the yield curve [(1) an elastic-perfectly plastic model with a 300-MPa yield, and (2) a linear hardening model with 70-MPa initial yield and a hardening modulus of 850 MPa], as well as high- and low-rate curves specified by measured data points. Figure 15 shows the curves for all four models, and the calculated deformations are shown in Fig. 16 as a plot of the outlines of the spheres.

Agreement between the predicted shapes and the experimental results is surprisingly good, especially for the very coarse elastic-perfectly plastic approximation. The high rate model gave the best fit, so it was used for subsequent calculations of strains and mesh refinement.

A. Mesh Refinement

We developed several mesh geometries for these tests to determine the influence of the finite-element numerics on the calculated results. Figures 17-19 show a fine, evenly distributed mesh, fine graded

Fig. 15.
Stress-strain curves for ETP copper.

Fig. 16.

Experimental and calculated outlines of copper specimen using mesh M-1 for various yield conditions.

mesh, and coarse graded mesh. Computations with these meshes were run with linear elements and with parabolic elements.

The calculated shapes were substantially the same as those in Table III. However, Fig. 20 shows a significant difference between the calculated and measured strains. Measurements taken outside the impact face region agree, however, measurements taken within the impact region differ markedly. There are significant differences between the uniform and graded meshes in addition to large discrepancies between linear and parabolic elements. Note that for these calculations, a piecewise linear, high-strain-rate curve was used to describe the copper yield behavior. The graded mesh with parabolic elements gave the calculated results that most closely matched the empirical data from Cu-B2. Mesh M-2C-P was selected for the frictional calculations because of short running time and match with the Cu-B2 data.

In addition to surface strains, interior effective strains were obtained from the M-2C-P mesh.

Fig. 17.

Even mesh (M-1) used for the impact calculations. Only half the sphere is needed from symmetry.

Figures 21a and b compare the calculated results with those experimentally estimated from vertical and horizontal traverses. Because the copper sphere interior is subject to a multiaxial state of strain, an effective strain criterion must be used to relate the calculational results to the uniaxial compression tests and hardness measurements. A von Mises criterion

$$\frac{\sqrt{2}}{3} ((\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2)^{1/2}$$

Fig. 18.

Fine graded mesh (M-2). Note large number of elements on the impact face.

Fig. 19.

Coarse graded mesh (M-2C).

was used for this purpose. Considering the uncertainties in the hardness estimation of strain, agreement between "hardness" strain values and calculated effective strain is quite good. The calculated trend of highest strains in the center of the sphere is proved.

B. Frictional Model

As discussed previously, the frictional model is based on the standard concept of a force opposing the sliding motion, but limited in magnitude by some fraction of the normal load. This is modified slightly to improve stability, which will provide an

upper limit to the frictional force and a reduction in the frictional force that is proportional to the sliding velocity for very small velocities. These modifications enable or enhance convergence during the equilibrium iteration process.

Calculations were made with coefficients of friction of 0, 0.25, 0.5, and 1.0; the surface hoop strains are plotted in Fig. 22a. Figure 22b is a replot of the experimentally determined hoop strains for these target conditions. Agreement between Cu-B2 results and the calculations for a frictional coefficient $\mu = 0$ is fairly good. Higher μ values produce results similar to those of the Cu-B1 and Cu-B3 experiments. Characteristic differences in the results might be due to the difference between the actual dry friction and the simple calculational model.

TABLE III
CALCULATED AND MEASURED DIMENSIONS
OF COPPER SPHERES
FOR DIFFERENT MESHES

	Final Dimensions of Copper Spheres	
	Overall Height (mm)	Diameter (mm)
Experiment Cu-B2	32.66	43.23
NONSAP M-1	32.642	43.150
NONSAP M-1-P	32.605	43.178
NONSAP M-2C	32.658	43.158
NONSAP M-2C-P	32.812	42.980

Fig. 20.
*Copper sphere surface hoop strain comparison
between different meshes and sphere Cu-B2.*

Fig. 21a.
*Horizontal strain distribution across sphere
Cu-B1 at point of maximum diameter. Strains
are from DPH hardness numbers and Fig. 5.
These are compared with predicted effective
strains from mesh M-2C-P.*

Fig. 21b.
*Vertical strain distribution from the impact
face to the back pole along the axis of sym-
metry of sphere Cu-B1. Strains are from the
DPH hardness traverse and Fig. 5. These are
compared with strains predicted from the M-
2C-P calculations.*

Fig. 22a.

Calculated surface hoop strains for different friction coefficients.

Fig. 23.

Comparison of the dynamic strain measurements from sphere Cu-B2 with calculations using mesh M-2C-P.

Fig. 22b.

Replot of engineering surface hoop strains measured for different frictional conditions.

C. Dynamic Strain Comparison

Dynamic predictions of NONSAP were compared with experimental observations in addition to the final state measurements discussed previously. Figure 23 is a replot of Fig. 13 showing the calculated hoop strain at the maximum sphere diameter, along with photographic measurements. The calculations

are within the error bars of the dynamic measurement between 0 and 100 μ s, where most deformation occurs. The only apparent discrepancy is in the prediction of a lift-off time at 140 μ s after impact. At 230 μ s, the sphere was calculated to be ~ 0.5 mm from the impact surface. It is quite likely that the resolution of the high-speed Hycam camera is similar to the 0.5-mm distance; thus, the sphere could have left the impact surface undetected well before 230 μ s. Finally, the calculated lift-off velocity taken with reference to a band of elements at the maximum sphere diameter was ~ 7 m/s as opposed to a measured velocity of ~ 6 m/s. This comparison shows that the calculated dynamic response is as accurate as can be measured experimentally. Possible errors in the calculation are too fine to discern with present techniques.

SUMMARY AND CONCLUSIONS

Three impact experiments were conducted with copper spheres and different impact target surfaces. Interior and surface strains and overall shape were measured from the impacted spheres, but only impact surface strain varied significantly with target surface. Behind the impact face, strains were very similar and the deformed shapes were within 0.1% of each other.

We used NONSAP to make predictions of sphere shape after impact with varied mesh geometry, yield conditions, and frictional coefficients. In agreement with results of the experiments, the predicted deformed shape was insensitive to all changes. The elastic-perfectly plastic yield condition showed the same deformations as those of the much more exact high-strain-rate nonlinear hardening data. However, impact face strain predictions were very sensitive to the variations. In fact, large differences in strain distributions occurred when only the type of finite element (linear or parabolic) was varied. Good agreement between experiment and theoretical results was obtained using a graded mesh with many elements on the impact face, parabolic isoparametric elements, and high-strain-rate hardening data for copper. We also matched experimental measurements of impact strains produced from the ground and grooved targets by adding frictional elements at the sphere target interface. However, this involved assuming a range of frictional coefficients because there is no way to measure the frictional coefficient from the impact event. Better agreement between experiment and calculations would require measurements of actual coefficients of friction. Improvements in material modeling, such as including rate-dependent constitutive relations and allowing for the possibility of plastic flow on unloading, may lead to improved agreement.

These results demonstrate that the most critical test of a computational stress analysis code is strain prediction. The shape prediction will probably be accurate in spite of a poor mesh design and an improper yield and hardening condition, but this does not indicate that the solution is accurate. On the other hand, predicted strains are greatly affected by slight changes in calculational conditions. Thus, solutions with correct strain predictions will be reliable.

We found that reliable strain predictions required measurement of a high-strain-rate hardening curve for copper. Even with these data and the refined graded mesh, there was a maximum point-to-point error of 20% in strain predictions (general agreement with experiment was much better). Some of this discrepancy may be caused by averaging errors in measuring the local circles, but the most likely cause is improper modeling of the friction between the copper sphere and the target. It is impossible to

know the exact frictional coefficient, and in fact, the coefficient will probably vary across the interface.

These experiments show that the greatest limitation to accurate numerical stress calculations is lack of specific materials data. Exact hardening curves are needed and available data pertain only to the condition of uniaxial tension. On the other hand, stress analysis codes deal with complex states of stress where the material response might be quite different from that predicted from a von Mises hardening and uniaxial stress-strain data. The questions raised by the condition of a sliding interface and unknown frictional response point to the lack of materials behavioral and material properties knowledge. Information does not exist for these situations, but such data are necessary to obtain reliable numerical results.

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