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THE EFFECT OF
EXPANSION-RATIO LIMITATIONS ON
POSITIVE-DISPLACEMENT, TOTAL-FLOW
GEOTHERMAL POWER SYSTEMS

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The Effect of Expansion-Ratio Limitations on
Positive-Displacement, Total-Flow Geothermal Power Systems

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1. Introduction

A number of so-called "total-flow" expanders have been proposed and/or developed for geothermal power applications. These include a rotary separating expander (or "Biphase Engine" [1]), an axial-flow impulse turbine [2,3], a Lysholm engine (or helical-screw expander [4,5]), a multi-lobe expander [6], and a Wankel-type engine [7]. In some cases the proponents of such devices have advocated their use in a stand-alone mode [2], but usually the total-flow machine is envisaged as a topping device in combination with a more conventional steam turbine.

In the case of a positive-displacement (PD) total-flow device (e.g., Lysholm or Wankel engines), the most severe practical constraint on the design is the expansion ratio, i.e., the ratio of the specific volume of the working fluid at the engine exhaust to the value at the engine inlet.

This problem was encountered in the design of early reciprocating steam engines and led to the design of triple-expansion engines in which the expansion ratio for each cylinder could be kept to reasonable limits and still generate a meaningful amount of power. Typical expansion ratios ranged from 4-7, with the higher values being for steam engines of the Corliss type [8]. Of course steam engines were replaced by steam turbines which are rotodynamic machines and not constrained by expansion ratio in the way reciprocating engines are. It has been estimated that expansion ratios for a helical screw expander may range from 8-15. The upper limit appears constrained to about 15 because of high steam pressures associated with higher volume ratios, inlet pocket porting problems and attendant leakage problems [5].

When positive-displacement engines (PDE's) are considered for geothermal

applications, their value stems from replacing a totally lossy process (i.e., an isenthalpic throttling process) with one that generates power and simultaneously accomplishes essentially the same objective as the throttling process, namely the generation of steam for use in a steam turbine. A proper thermodynamic comparison between a power plant consisting of a PDE topping unit with a steam turbine and one consisting of a throttling separator and a steam turbine (i.e., a conventional flash-steam geothermal plant) must take into account the expansion-ratio limitations on PDE's. If one were free to choose the separated steam conditions (i.e., the outlet conditions of the PDE) without regard to the expansion ratio, then a combined PDE/steam turbine system would always hold a thermodynamic advantage over a basic flash-steam plant, regardless of the efficiency of the PDE engine. This conclusion does not hold when practical limits are placed on the expansion ratio of the PDE.

2. Source of the Difficulty

The root of the problem associated with all total-flow machines stems from the nature of the working fluid and in particular with the relationship between the enthalpy drop and the volume ratio for expansion processes that begin on or near the saturated liquid state. This can be seen from Fig. 1, a schematic temperature-entropy diagram (drawn for water substance). Near the saturated liquid line, isochores are closely packed and have a slope close to that of the saturation line, whereas isenthalpics are nearly vertical. Thus an expansion process (shown as isentropic) from 1 to 2 undergoes very little change in enthalpy and a very large change in volume. The situation is reversed for expansions starting at the saturated vapor

state. The isochores are widely spaced with a mild slope whereas the isenthalpics have steep slopes.

A quantitative measure of this effect is shown in Fig. 2 where a comparison is given of the volume ratio $VR = v_f/v_i$ as a function of the isentropic enthalpy drop $\Delta h_s = h_i - h_f$ for expansions of the type 1-2 and 3-4 shown in Fig. 1. The starting points for the expansions correspond to points $i=1$ and $i=3$ (Fig. 1) for $T = 175^\circ\text{C}$; the curves result from varying the temperature of the final state points 2 and 4 along lines of constant entropy. The contrast between the two cases is striking. For a volume ratio of, say, $VR = 4.5$, one finds that the $\Delta h_{s,3-4}$ is about 750 times larger than $\Delta h_{s,1-2}$.

This characteristic is recognized in the refrigeration industry where a simple throttling device is used between the condenser and evaporator instead of an expansion engine in vapor-compression refrigerators. The amount of work one could receive is too small to justify the cost of such an engine in this case.

3. Reference Flash-Steam Geothermal Plants

In the following sections we shall develop comparisons between plants incorporating a PDE together with a steam turbine, and basic flash-steam plants. We distinguish two cases: (1) the geothermal wells are self-flowing and produce a mixture of liquid and vapor at the wellhead, and (2) the wells are pumped, the product being a pressurized liquid at the wellhead. In the first case, we use a "double-flash" geothermal plant as the reference; in the second case, we use a "single-flash" plant. Each plant will be designed (or optimized) to produce the maximum work output

per unit mass of geofluid, i.e., to produce the highest efficiency. The combined PDE/steam turbine plants will then be compared to the thermodynamically optimum flash-steam plants in each case. For various design options on the PDE we shall seek the minimum PDE efficiency necessary to render the combined plant equal to the reference plant in overall efficiency.

4. Basic Positive-Displacement Expander Strategy

To overcome to some degree the volume-ratio limitation imposed on PDE's working near the saturated liquid line, the basic technique is to admit the working fluid to the PDE with a certain fraction of vapor already present. This serves to increase the inlet volume, and hence to mitigate the volume-ratio limitation by allowing a "deeper" expansion process into the two-phase region and a greater enthalpy drop. We will examine two ways of achieving this in a geothermal plant: (1) by allowing a controlled pre-flash of saturated liquid prior to admission to the PDE, and (2) by pre-mixing a controlled amount of separated steam with the saturated liquid before admission. The latter option is clearly not available in the case of pumped wells since no steam is available at the wellhead.

5. Combined PDE/Steam Turbine Configurations

There are three configurations that we will study: Case A: Self-flowing Wells/Pre-flashing Mode; Case B: Self-flowing Wells/Pre-mixing Mode; and Case C: Pumped Wells/Pre-flashing Mode. Schematic plant diagrams for each configuration, together with their respective temperature-entropy diagram, are shown in Figs. 3-8.

It is easy to see from Figs. 3 and 7 that the reference double-flash and single-flash plants are obtained by replacing the PDE with a by-pass.

The steam turbine (ST1-ST2) is shown as a dual-pressure (or pass-in) machine in Figs. 3 and 5, but this is only illustrative; two separate turbines may be used. The condensers shown in Figs. 3, 5 and 7 are surface-type, but jet condensers may be appropriate for certain applications. These details do not affect the analyses presented here.

The exact nature of the waste fluid disposal system has not been specified because this will be site-specific. Since the analysis given here is essentially comparative in nature, each system, i.e., both the combined PDE/steam turbine and the reference plant will be burdened with the same fluid disposal problems. Similarly this analysis does not include pumping power for the wells for Case C. Thus the efficiencies and performance figures quoted in this study apply to the "naked" energy conversion system and may be described as "gross" rather than "net".

6. Thermodynamic Analysis

The thermodynamic analysis of the three configurations is based on the processes shown in Figs. 4, 6 and 8. The plants are assumed to be in steady operation, all components are perfectly insulated (i.e., adiabatic), and pressure losses in pipes and heat exchangers are neglected. The working fluid is pure water with properties taken from Keenan et al. [9]. The governing equations derive from a systematic application of the First Law of thermodynamics for open systems operating in steady state.

The ultimate measure of performance is the utilization efficiency, η_u , which is based on the Second Law and is defined as

$$\eta_u = \dot{W} / \dot{E}_0 , \quad (1)$$

where $\dot{W}$ is the power output of the plant (for unit mass flow rate of geo-fluid) and $\dot{E}_0$ is the rate of exergy (available work) associated with the flow

of geofluid at wellhead conditions [10]. The exergy rate is taken as

$$\dot{E}_0 = \dot{h}_0 - h^\circ - T^\circ (s_0 - s^\circ) , \quad (2)$$

where the dead-state temperature $T^\circ = 323.15 \text{ K}$ (50°C), and h° and s° are found at T° for a saturated liquid [11]. With reference to Figs. 3-6, i.e., Cases A and B,

$$\dot{W} = \dot{m}_4(h_4 - h_5) + \dot{m}_8(h_8 - h_9) + \dot{m}_2(h_2 - h_3) ; \quad (3)$$

with reference to Figs. 7 and 8, i.e., Case C,

$$\dot{W} = \dot{m}_6(h_6 - h_9) + \dot{m}_2(h_2 - h_3) . \quad (4)$$

For convenience, we set $\dot{m}_w = \dot{m}_0 \equiv 1$, and it then follows that:

$$\dot{m}_2 = 1 - x_0 \quad \left. \begin{array}{l} \dot{m}_4 = x_0 \\ \dot{m}_8 = x_0 - x_0 x_3 + x_3 \end{array} \right\} \text{For Figs. 3 and 4} \quad (5)$$

$$\dot{m}_4 = x_0 \quad (6)$$

$$\dot{m}_8 = x_0 - x_0 x_3 + x_3 \quad (7)$$

$$\dot{m}_2 = (1 - x_0)/(1 - x_2) \quad (8)$$

$$\dot{m}_4 = (x_0 - x_2)/(1 - x_2) \quad \left. \begin{array}{l} \dot{m}_8 = x_0 - x_0 x_3 + x_3 \end{array} \right\} \text{For Figs. 5 and 6} \quad (9)$$

$$\dot{m}_8 = x_0 - x_0 x_3 + x_3 \quad (10)$$

$$\dot{m}_2 = 1 \quad \left. \begin{array}{l} \dot{m}_6 = x_3 \end{array} \right\} \text{For Figs. 7 and 8} \quad (11)$$

$$\dot{m}_6 = x_3 \quad (12)$$

The PDE is described by its isentropic efficiency, η_{PDE} :

$$\eta_{\text{PDE}} = (h_2 - h_3)/(h_2 - h_{2s}) . \quad (13)$$

The steam turbines are similarly described:

$$\eta_{\text{ST1}} = (h_4 - h_5)/(h_4 - h_{4s}) \quad (14)$$

$$\eta_{\text{ST2}} = (h_8 - h_9)/(h_8 - h_{8s}) \quad (15)$$

$$\eta_{\text{ST}} = (h_6 - h_9)/(h_6 - h_{8s}) . \quad (16)$$

In the computations, η_{PDE} played the role of an assigned parameter with values ranging from $\eta_{\text{PDE}} = 0$ (pure flashing process) to $\eta_{\text{PDE}} = 1$ (ideal isen-

tropic expansion). The steam turbine efficiencies were found from a modified Baumann rule [12,13], i.e.,

$$\eta = \eta_d(1 - \alpha y) , \quad (17)$$

where η_d is the dry turbine efficiency ($\eta_d = 0.85$), α is a wetness loss coefficient ($\alpha = 1.2$), and y is the wetness at the turbine exhaust ($y = 1 - x$).

7. Calculation Methodology

The calculations were performed parametrically in the following manner. Three temperatures were selected at the wellhead: $T_0 = 150, 175$ and 200°C . The condenser temperature was fixed at $T_{11} = 50^\circ\text{C}$. For Cases A and B, i.e., Figs. 3-6, three dryness fractions at the wellhead were chosen: $x_0 = 0.10, 0.15$ and 0.20 . Four volume ratios across the PDE were selected: $v_3/v_2 = 5, 10, 15$ and 20 . For Case A, i.e., Figs. 3 and 4, five pressure drops were selected for the pre-flash process 1-2: $0, 10, 20, 30$ and 40% , $\Delta P/P_1$. For Case B, i.e., Figs. 5 and 6, the dryness fraction x_2 was chosen as $0.05, 0.10, 0.15$ up to 0.20 , but not greater than x_0 . In all cases the exhaust state from the PDE (state 3) fixed the intermediate temperature level T_7 .

In each case, the optimum reference flash-steam plant was determined using the same wellhead conditions (and other assumptions) as for the combined system. The combined systems, Cases A, B and C, were analyzed, all state points calculated, and the work output and utilization efficiency determined. The thermodynamic properties of water substance were calculated by means of an accurate FORTRAN subroutine developed in our laboratory and described in detail elsewhere [14]. It was then possible to determine (by interpolation) the minimum PDE efficiency, η_{PDE}^* , required in order for the combined system to match the reference plant in overall performance.

8. Results

Tables 1.1-1.3 give η_{PDE}^* for Case A; Tables 2.1-2.3 give the same for Case B; and Table 3 gives the results for Case C.

With regard to the case of self-flowing wells and a PDE operating in the pre-flashing mode, the results are similar for the three selected wellhead temperatures, 150, 175 and 200°C. The combined system is not thermodynamically competitive (at least for volume ratios up to 20) without a pre-flash (i.e., $\Delta P/P_1 = 0\%$). Also for low volume ratios (say, $VR = 5$), rather high η_{PDE} are needed unless one is prepared to take very large pre-flash processes (say, $\Delta P/P_1 = 40\%$). Combined systems compete favorably under conditions of high volume ratio (say, $VR > 15$) together with large pressure drop prior to admission (say, $\Delta P/P_1 > 20\%$). As long as one restricts one's attention to these limits, then combined systems will be superior to optimum reference double-flash plants if $\eta_{PDE} > 27\%$ (for the range of conditions studied here). An engine efficiency of this magnitude should not be difficult to obtain. However, whether it can be achieved along with an expansion ratio of at least 15 is a different matter.

A sample calculation is shown in Table 4 for a hypothetical system having a PDE with $\eta_{PDE} = 0.40$, $VR = 15$, and $\Delta P/P_1 = 20\%$, and operating with a geothermal fluid having a wellhead temperature of 200°C and quality of 15%. The system produces 120.65 kJ/kg of geofluid, only 1.2% more than an optimum double-flash plant. Within the system, the two steam turbines together contribute 94.2% of the total plant output, with only 5.8% coming from the PDE.

Turning to Case B, i.e., combined plants that use the pre-mixing mode of operation, we see that such plants are more challenging to design in

order to compete with reference flash-steam plants. The most competitive designs appear to be those operating at high volume ratios and low (but not zero) inlet qualities. The case of $x_{PDE} = 0$ is the same as Case A with $\Delta P/P_1 = 0\%$ (see Figs. 4 and 6.), and from Tables 1.1-1.3 we learned that such plants cannot compete thermodynamically with the reference plants.

The pre-mixing mode of operation suffers from an inherent handicap. Geothermal steam that would otherwise be used in a relatively high-efficiency steam turbine is used instead, through a mixing process, in a relatively low-efficiency PDE. Thus we see, from Tables 2.1-2.3, that the requirements on η_{PDE} generally increase as x_{PDE} increases. This trend does not hold for very low volume ratios and very low x_{PDE} for the reason alluded to earlier.

Table 5 lists a set of typical results of the analysis of Case B with geofluid at 200°C and 15% quality and a PDE having a volume ratio of 15, an efficiency of 40% and an admission quality of 5%. The PDE generates 13.60 kJ/kg of geofluid (11.3% of the plant output), and the combined system shows a 1.67 kJ/kg advantage (1.4%) over the reference double-flash plant.

For Case C, Table 3, the reference plant is a single-flash plant. We chose a flash plant as the reference over a binary plant on the grounds that the former is commercial whereas the latter is still developmental. Since the wells are pumped, the presumption is that the resource is not particularly outstanding. For the conditions studied, the utilization efficiency of the reference plant ranged from 25-30%. With regard to the combined system, the same trends appear in this case as with Case A. A combined system with a PDE having $\eta_{PDE} > 20\%$ will be superior to the reference plant provided $VR > 15$ and $\Delta P/P_1 > 20\%$.

A typical analysis is shown in Table 6 for wellhead conditions of 200°C, saturated liquid, a PDE with a volume ratio of 15, a pressure drop of 20%, and an efficiency of 40%. The combined system generates 7.8% more output

than the reference plant. The steam turbine produces 84.1% of the output; the PDE 15.9%.

It is noteworthy that combined systems using the pre-flash mode with pumped wells appear thermodynamically most competitive for resources of lower temperatures where one might expect most applications of this type to occur. From Table 3, for $T_0 = 150^\circ\text{C}$, there is a wide range of reasonable conditions over which the combined system needs only a moderately efficient PDE to gain an advantage over the reference plant.

The effect of η_{PDE} on overall system performance is shown in Table 7 for typical wellhead conditions of 175°C and 20% quality. The plant is a Case B-configuration, the volume ratio is 15, and the results are calculated for three values of inlet quality at the PDE: (a) $x_2 = 0$ (i.e., all steam available at the wellhead is sent directly to the high-pressure steam turbine); (b) $x_2 = 0.10$; and (c) $x_2 = 0.20$ (i.e., no steam is sent to the hp steam turbine). For case (a), there is very little effect from η_{PDE} because the volume ratio limitation is most severe since saturated liquid is admitted to the PDE. The combined system is always inferior to the reference flash plant because the split between the two steam turbines is controlled by the exhaust from the PDE, and is badly off-optimum. For case (b), the η_{PDE} plays a more important role; a gain of 20 percentage points in η_{PDE} produces about 10 kJ/kg additional output and an increase of about 4 percentage points in η_u . For case (c), only one steam turbine is needed (the low-pressure unit) since the total flow from the well is delivered directly to the PDE. For a given η_{PDE} , the performance here is inferior to case (b) until η_{PDE} reaches a value of roughly 60%. The effect of η_{PDE} in this case is about twice as strong as for case (b).

Finally one may ask which type plant, Case A or B, is superior for a particular set of wellhead conditions and a particular PDE. Table 8 shows such a comparison for wellhead conditions of 175°C and 15% quality, and a PDE with a volume ratio of 15 and an efficiency of 60%. For Case B, the combined system will be superior to the reference plant so long as $x_2 > 0.04$, but reaches a maximum in performance when $x_2 = x_0 = 0.15$. For Case A, it exceeds the reference plant so long as the pre-flash involves at least a 20% pressure drop. This plant will be superior to Case B for pre-flash pressure drops greater than 35%.

9. Economic Considerations

No attempt will be made here to perform a detailed economic analysis of systems incorporating a PDE; there are too many technical variables and economic unknowns. However, a few general remarks may be in order.

A combined system with a PDE will be selected over a conventional flash-steam plant only if the combined system shows an economic advantage over an optimized flash plant. The ultimate criterion will be the cost of electricity generated at the busbar. This cost is determined by many factors including the capital cost of the power plant, C_c , the cost of "fuel", C_f , and the cost of operating and maintaining the plant, C_{o-m} . Since the combined system includes the PDE and an additional electric generator plus auxiliaries, relative to the conventional plant, it is clear that

$$C_c \text{ (combined system)} > C_c \text{ (conventional plant)}. \quad (18)$$

Furthermore, it is likely that a PDE will require more maintenance than a throttling device with no moving parts; i.e.,

$$C_{o-m} \text{ (combined system)} > C_{o-m} \text{ (conventional plant)}. \quad (19)$$

Therefore, there must be significant cost savings in the area of "fuel" costs to offset shortcomings in the other two areas.

For a geothermal plant, the "fuel" cost translates into cost of geofluid. Recent contracts between resource developers and utilities have set the price of the geofluid on the basis of a unit mass flow at specified thermodynamic conditions (often with a fixed surcharge for disposal of the waste fluid). This means that the "fuel" cost will vary inversely with the utilization efficiency, i.e.,

$$C_f \sim 1/\eta_u . \quad (20)$$

From the examples cited in the previous section, it was shown that only modest gains could be achieved by combined systems operating at the assumed upper practical limit of $VR = 15$. Thus, unless PDE's can be manufactured and sold fairly inexpensively, and can be made to run reliably and efficiently, it seems doubtful that they will be able to compete with conventional plants when they are used as a "topping" unit in conjunction with steam turbines in geothermal electric power applications.

10. Concluding Remarks

We have examined the use of a positive-displacement engine in combination with a steam turbine in which the PDE operates in a total-flow mode as a "topping" unit. The upper limit on the expansion ratio for the PDE places constraints on the applicability of this type of combined system. Three cases, representing typical geothermal configurations, were studied. Among the several design variables, regions were marked out where combined PDE/steam turbine systems have the best chance to compete thermodynamically with conventional flash-steam plants.

As a final note, it is of interest to observe that a helical rotor expansion engine has been tested at the Cerro Prieto geothermal field recently. The engine was used in a stand-alone mode and generated between 100-930 kWe; the highest efficiency for the engine was reported to be 68% [15], but the full operating conditions under which this peak was achieved were not cited. During a 1000-hour endurance run, the engine malfunctioned eight times. Proponents of the engine were encouraged by these results, and development and further testing will continue.

It is interesting to note that a helical rotor engine should in principle be able to cope with moderately saline brines owing to the self-cleaning nature of its operation.

The need for small, simple, reliable and inexpensive wellhead power units for geothermal applications is now recognized. Through testing of the type carried out at Cerro Prieto (and elsewhere), positive-displacement expanders may be developed to compete thermodynamically and economically with conventional flash-steam plants.

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List of Symbols

Letters:

- C ... cost
- $\dot{E}$... exergy rate
- h ... specific enthalpy
- Δh_s ... isentropic enthalpy drop
- $\dot{m}$... mass flow rate
- ΔP ... pressure drop across throttle valve
- s ... specific entropy
- T ... temperature
- v ... specific volume
- VR ... volume ratio across PDE
- $\dot{W}$... power
- w ... specific work
- x ... dryness fraction
- y ... wetness fraction
- α ... wetness loss coefficient
- η_d ... turbine efficiency for dry expansion
- η_{PDE} ... isentropic efficiency of PDE
- η_{PDE}^* ... minimum η_{PDE} needed for combined system to match optimum flash plant
- η_u ... utilization efficiency

Subscripts:

c ... refers to capital costs
f ... refers to fuel costs
o-m ... refers to operating and maintenance costs
PDE ... refers to positive-displacement expander
ST1,ST2,ST ... refer to steam turbines
w ... refers to wellbottom conditions
0 ... refers to wellhead conditions
1,2,... refers to state points; see Figs. 3-8
1F ... refers to optimized single-flash plant
2F ... refers to optimized double-flash plant

Superscript:

° ... refers to thermodynamic dead state

TABLE 1.1

Minimum η_{PDE} needed for combined system for Case A, Fig. 3,
to match optimum double-flash plant: wellhead temperature = 150°C.

VR	$\Delta P/P_1, \%$				
	0	10	20	30	40
(a): $x_0 = 0.10$, $\eta_{2F} = 0.410$, $w_{2F} = 61.5$ kJ/kg					
5	(*)	(*)	(*)	60.0	13.5
10	(*)	(*)	33.1	3.3	0.8
15	(*)	71.8	6.4	0.4	6.8
20	(*)	32.0	0.6	3.7	13.3
(b): $x_0 = 0.15$, $\eta_{2F} = 0.447$, $w_{2F} = 81.0$ kJ/kg					
5	(*)	(*)	(*)	65.3	14.8
10	(*)	(*)	36.1	3.7	0.7
15	(*)	77.9	6.9	0.3	7.3
20	(*)	35.0	0.6	3.8	14.3
(c): $x_0 = 0.20$, $\eta_{2F} = 0.473$, $w_{2F} = 100.4$ kJ/kg					
5	(*)	(*)	(*)	71.3	16.5
10	(*)	(*)	39.6	4.1	0.7
15	(*)	85.1	7.8	0.4	7.9
20	(*)	38.3	0.6	4.0	15.4

(*) - Optimum double-flash plant always superior, i.e., $\eta_{PDE}^* > 100\%$.

TABLE 1.2

Minimum η_{PDE} needed for combined system for Case A, Fig. 3,
to match optimum double-flash plant: wellhead temperature = 175°C.

VR	$\Delta P/P_1, \%$				
	0	10	20	30	40
(a): $x_0 = 0.10$, $\eta_{2F} = 0.409$, $w_{2F} = 77.8$ kJ/kg					
5	(*)	(*)	(*)	96.6	34.1
10	(*)	(*)	55.6	13.5	1.9
15	(*)	90.6	17.5	2.1	2.2
20	(*)	46.2	5.7	1.3	5.9
(b): $x_0 = 0.15$, $\eta_{2F} = 0.444$, $w_{2F} = 99.6$ kJ/kg					
5	(*)	(*)	(*)	(*)	34.5
10	(*)	(*)	57.6	12.8	0.6
15	(*)	94.6	17.1	0.8	1.2
20	(*)	47.5	4.7	0.1	5.2
(c): $x_0 = 0.20$, $\eta_{2F} = 0.470$, $w_{2F} = 121.4$ kJ/kg					
5	(*)	(*)	(*)	(*)	37.8
10	(*)	(*)	62.8	14.3	0.7
15	(*)	(*)	18.8	1.0	1.2
20	(*)	51.9	5.1	0.0 ⁽¹⁾	5.5

(*) - See note Table 1.1.

(1) - Conditions correspond essentially to those of optimum double-flash plant.

TABLE 1.3

Minimum η_{PDE} needed for combined system for Case A, Fig. 3,
to match optimum double-flash plant: wellhead temperature = 200°C.

VR	$\Delta P/P_1, \%$				
	0	10	20	30	40
(a): $x_0 = 0.10$, $\eta_{2F} = 0.405$, $w_{2F} = 94.8$ kJ/kg					
5	(*)	(*)	(*)	(*)	47.8
10	(*)	(*)	64.8	20.0	3.6
15	(*)	88.5	22.7	3.7	0.0
20	(*)	46.6	8.3	0.0	1.6
(b): $x_0 = 0.15$, $\eta_{2F} = 0.438$, $w_{2F} = 118.3$ kJ/kg					
5	(*)	(*)	(*)	(*)	51.0
10	(*)	(*)	68.0	21.0	4.0
15	(*)	95.0	25.0	4.0	0.0
20	(*)	48.0	5.0	0.1	1.0
(c): $x_0 = 0.20$, $\eta_{2F} = 0.464$, $w_{2F} = 141.7$ kJ/kg					
5	(*)	(*)	(*)	(*)	56.6
10	(*)	(*)	76.0	24.2	4.6
15	(*)	(*)	27.3	4.8	0.0
20	(*)	54.8	10.3	0.4	1.6

(*) - See note Table 1.1.

TABLE 2.1

Minimum η_{PDE} needed for combined system for Case B, Fig. 5,
to match optimum double-flash plant: wellhead temperature = 150°C.

VR	$x_{PDE}, \%$			
	5	10	15	20
	(a): $x_0 = 0.10$, $\eta_{2F} = 0.410$, $w_{2F} = 61.5$ kJ/kg			
5	80.6	58.9	na	na
10	38.8	46.0	na	na
15	33.4	45.7	na	na
20	33.4	47.1	na	na
	(b): $x_0 = 0.15$, $\eta_{2F} = 0.447$, $w_{2F} = 81.0$ kJ/kg			
5	80.4	59.8	58.7	na
10	39.2	46.1	52.1	na
15	33.5	46.0	52.9	na
20	33.6	47.5	54.3	na
	(c): $x_0 = 0.20$, $\eta_{2F} = 0.473$, $w_{2F} = 100.4$ kJ/kg			
5	87.8	60.6	59.1	60.2
10	39.5	46.1	52.2	56.3
15	33.5	46.2	53.2	57.4
20	33.8	48.0	55.0	58.8

na - Not applicable.

TABLE 2.2

Minimum η_{PDE} needed for combined system for Case B, Fig. 5,
to match optimum double-flash plant: wellhead temperature = 175°C.

VR	$x_{PDE}, \%$			
	5	10	15	20
	(a): $x_0 = 0.10$, $\eta_{2F} = 0.410$, $w_{2F} = 77.8$ kJ/kg			
5	(*)	68.9	na	na
10	45.6	43.6	na	na
15	32.2	40.1	na	na
20	28.6	39.9	na	na
	(b): $x_0 = 0.15$, $\eta_{2F} = 0.444$, $w_{2F} = 99.6$ kJ/kg			
5	(*)	69.9	61.7	na
10	45.7	43.3	47.7	na
15	31.7	39.7	46.3	na
20	27.9	39.7	46.9	na
	(c): $x_0 = 0.20$, $\eta_{2F} = 0.470$, $w_{2F} = 121.4$ kJ/kg			
5	(*)	72.0	62.7	60.9
10	47.0	43.5	47.8	51.5
15	32.0	39.7	46.5	51.0
20	28.0	39.8	47.1	51.7

(*) - See note Table 1.1; na - Not applicable.

TABLE 2.3

Minimum η_{PDE} needed for combined system for Case B, Fig. 5,
to match optimum double-flash plant: wellhead temperature = 200°C.

VR	$x_{PDE}, \%$			
	5	10	15	20
(a): $x_0 = 0.10$, $\eta_{2F} = 0.405$, $w_{2F} = 94.8$ kJ/kg				
5	(*)	81.5	na	na
10	54.5	43.2	na	na
15	33.6	36.1	na	na
20	26.4	34.3	na	na
(b): $x_0 = 0.15$, $\eta_{2F} = 0.438$, $w_{2F} = 118.3$ kJ/kg				
5	(*)	84.6	67.9	na
10	56.8	43.8	45.3	na
15	34.3	36.2	41.7	na
20	26.7	34.3	41.2	na
(c): $x_0 = 0.20$, $\eta_{2F} = 0.464$, $w_{2F} = 141.7$ kJ/kg				
5	(*)	88.1	69.8	63.9
10	59.4	44.4	45.6	48.3
15	35.3	36.4	41.8	46.1
20	27.0	34.4	41.3	46.0

(*) - See note Table 1.1; na - Not applicable.

TABLE 3

Minimum η_{PDE} needed for combined system for Case C, Fig. 7,
to match optimum single-flash plant.

VR	$\Delta P/P_1, \%$				
	0	10	20	30	40
(a): $T_0 = 150^\circ\text{C}$, $\eta_{1F} = 0.259$, $w_{1F} = 22.7 \text{ kJ/kg}$					
5	(*)	(*)	(*)	50.4	11.0
10	(*)	(*)	27.5	2.5	0.6
15	(*)	60.7	5.0	0.3	5.9
20	(*)	26.7	0.3	3.2	11.5
(b): $T_0 = 175^\circ\text{C}$, $\eta_{1F} = 0.280$, $w_{1F} = 34.3 \text{ kJ/kg}$					
5	(*)	(*)	(*)	60.5	26.8
10	(*)	(*)	45.4	9.7	0.4
15	(*)	75.6	13.1	0.6	1.2
20	(*)	37.3	3.5	0.2	4.6
(c): $T_0 = 200^\circ\text{C}$, $\eta_{1F} = 0.296$, $w_{1F} = 48.0 \text{ kJ/kg}$					
5	(*)	(*)	(*)	(*)	40.8
10	(*)	(*)	55.9	16.8	2.9
15	(*)	76.8	19.2	3.0	0.1
20	(*)	39.9	6.8	0.1	1.6

(*) - See note Table 1.1.

TABLE 4

Performance of combined system Case A with the following conditions:

$$T_0 = 200^\circ\text{C}, x_0 = 0.15, \Delta P/P_1 = 20\%, VR = 15, \eta_{PDE} = 0.40.$$

<u>State point</u>	<u>Pressure</u> kPa	<u>Temperature</u> °C	<u>Enthalpy</u> kJ/kg
1	1553.8	200.0	852.46
2	1243.0	189.6	852.46
4	1553.8	200.0	2793.20
2s	322.0	136.0	831.92
3	322.0	136.0	844.24
6	322.0	136.0	2728.58
7	322.0	136.0	571.88
4s	322.0	136.0	2509.27
5	322.0	136.0	2581.31
8	322.0	136.0	2642.74
8s	12.3	50.0	2166.17
9	12.3	50.0	2324.54
10	12.3	50.0	2592.07
11	12.3	50.0	209.32

Work output, kJ/kg

$$w_{ST1} = 31.78, \quad w_{ST2} = 81.89, \quad w_{PDE} = 6.98$$

$$\Sigma w = 120.65$$

Utilization efficiency

$$\eta_u = 0.448$$

TABLE 5

Performance of combined system Case B with the following conditions:

$$T_0 = 200^\circ\text{C}, x_0 = 0.15, x_2 = 0.05, \text{VR} = 15, \eta_{\text{PDE}} = 0.40.$$

<u>State point</u>	<u>Pressure</u> kPa	<u>Temperature</u> °C	<u>Enthalpy</u> kJ/kg
1	1553.8	200.0	852.46
2	1553.8	200.0	949.50
4	1553.8	200.0	2793.20
2s	285.7	131.9	911.50
3	285.7	131.9	934.30
6	285.7	131.9	2723.06
7	285.7	131.9	554.37
4s	285.7	131.9	2489.77
5	285.7	131.9	2568.58
8	285.7	131.9	2661.00
8s	12.3	50.0	2197.36
9	12.3	50.0	2345.24
10	12.3	50.0	2592.07
11	12.3	50.0	209.32

Work output, kJ/kg

$$w_{\text{ST1}} = 23.64, \quad w_{\text{ST2}} = 82.73, \quad w_{\text{PDE}} = 13.60$$

$$\Sigma w = 119.97$$

Utilization efficiency

$$\eta_u = 0.445$$

TABLE 6

Performance of combined system Case C with the following conditions:

$$T_0 = 200^\circ\text{C}, x_0 = 0.0, \Delta P/P_1 = 20\%, VR = 15, \eta_{\text{PDE}} = 0.40.$$

<u>State point</u>	<u>Pressure</u> kPa	<u>Temperature</u> °C	<u>Enthalpy</u> kJ/kg
1	1553.8	200.0	852.46
2	1243.0	189.6	852.46
2s	322.0	136.0	831.92
3	322.0	136.0	844.24
6	322.0	136.0	2728.58
7	322.0	136.0	571.88
8s	12.3	50.0	2233.98
9	12.3	50.0	2383.99
10	12.3	50.0	2592.07
11	12.3	50.0	209.32

Work output, kJ/kg

$$w_{\text{ST}} = 43.52, w_{\text{PDE}} = 8.22$$

Utilization efficiency

$$\eta_u = 0.319$$

TABLE 7

Effect of η_{PDE} on Case B combined system performance:

$T_0=175^\circ\text{C}$, $x_0=0.20$, and $VR=15$.

($\eta_{2F}=0.470$, $w_{2F}=121.4$ kJ/kg)

η_{PDE}	w_{ST1}	w_{ST2}	w_{PDE}	Σw	η_u
	kJ/kg	kJ/kg	kJ/kg	kJ/kg	
(a) $x_2 = 0.00$					
0.00	16.09	86.40	0.00	102.49	0.397
0.20	16.15	86.32	0.42	102.89	0.398
0.40	16.21	86.24	0.85	103.30	0.400
0.60	16.27	86.16	1.29	103.72	0.402
0.80	16.34	86.08	1.73	104.15	0.403
1.00	16.40	86.00	2.18	104.58	0.405
(b) $x_2 = 0.10$					
0.00	28.60	74.11	0.00	102.71	0.398
0.20	28.82	72.51	10.68	112.01	0.434
0.40	29.05	70.88	21.62	121.55	0.471
0.60	29.28	69.22	32.85	131.35	0.509
0.80	29.53	67.53	44.39	141.45	0.548
1.00	29.78	65.79	56.25	151.82	0.588
(c) $x_2 = 0.20$					
0.00	0.00	72.22	0.00	72.22	0.280
0.20	0.00	69.47	21.61	91.08	0.353
0.40	0.00	66.69	43.78	110.47	0.428
0.60	0.00	63.87	66.53	130.40	0.505
0.80	0.00	61.00	89.93	150.93	0.584
1.00	0.00	58.10	114.01	172.11	0.666

TABLE 8

Comparison between Cases A and B for
 $T_0=175^\circ\text{C}$, $x_0=0.15$, $VR=10$, and $\eta_{PDE}=0.60$.
 $(\eta_{2F}=0.444, w_{2F}=99.6 \text{ kJ/kg})$

<u>Case A</u>		
ΔP	Σw	η_u
%	kJ/kg	
0	77.58	0.346
10	91.58	0.408
20	99.86	0.445
30	105.43	0.470
40	109.24	0.487
<u>Case B</u>		
x_2	Σw	η_u
	kJ/kg	
0.00	77.58	0.346
0.05	102.69	0.458
0.10	106.53	0.475
0.15	107.40	0.479

Fig. 1. Comparison between isentropic expansion processes starting from saturated liquid and from saturated vapor.

Fig. 2. Isentropic enthalpy drop as a function of expansion ratio for processes of the type 1-2 and 3-4 in Fig. 1.

Fig. 3. Plant schematic for Case A, self-flowing wells and pre-flashing mode.

Fig. 5. Plant schematic for Case B, self-flowing wells and pre-mixing mode.

Fig. 6. Thermodynamic diagram for Case B; see Fig. 5.

Fig. 7. Plant schematic for Case C, pumped wells and pre-flashing mode.

Fig. 8. Thermodynamic diagram for Case C; see Fig. 7.