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MODELING TWO-PHASE FLOW IN A SWIRL
COMBUSTOR

Final Report.

A.R.A.P. Report No. 310

By
W. S. Lewellen
Harvey Segur
Ashok K. Varma

MASTER

June 1977

Work Performed Under Contract No. EY-76-C-02-4062

Aeronautical Research Associates of Princeton, Inc.
Princeton, New Jersey

U. S. DEPARTMENT OF ENERGY

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Contract No. EY-76-C-02-4062

Prepared for
Energy Research and Development Administration

Aeronautical Research Associates of Princeton, Inc.
50 Washington Road, Princeton, New Jersey, 08540

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ABSTRACT

A phenomenological model of flow in a cyclone coal combustor-gasifier is presented. The model predicts how four performance parameters which measure the carbon conversion efficiency, ash separation, pressure drop, and heat losses depend on such design variables as chamber pressure, size and shape, flow rates, and stoichiometry, along with coal particle size and composition. Correct prediction of the performance parameters requires accurate analysis of the coal particle combustion, particle mechanics and fluid mechanics, all of which are coupled in this confined vortex flow. The model is based on assimilating simple approximations to the most important mechanisms governing flow and combustion in the chamber and thus necessarily involves empirical coefficients. Model verification by comparison with laboratory results from Pittsburgh Energy Research Center show reasonable agreement, but due to the empiricism required in the model further tests are warranted. Model derivation and sensitivity analysis pinpoint the phenomena which are most critical to the performance of a cyclone combustor. Methods for reducing the empirical requirements of the model are discussed and recommendations made for model improvement.

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1. INTRODUCTION

Over the last year, a relatively simple computer model of two-phase flow in a cyclone combustor has been developed at A.R.A.P. The principal purpose of the model has been to implement design of the combustor to be built at the Montana CDIF (Component Development and Integration Facility); in addition, the model has been of value in interpreting experimental data obtained in the smaller scale atmospheric vertical cyclone combustor at PERC (Pittsburgh Energy Research Center). The purpose of this final report is to provide an integrated description of the entire model.

The basic configuration of a cyclone combustor is shown in Fig. 1. We have also shown there the separate regions of flow, each of which is characterized by its own balance of forces. These different balances suggest the different approximations used in the model in these different regions. Before giving a detailed discussion of the model, we first describe qualitatively the flow pattern that occurs in such a combustor. Much of the motivation for the specific analyses in the separate regions can be found directly within such a qualitative picture.

In a cyclone combustor, fluid is injected tangentially at the outer wall of the combustor at a high (subsonic) velocity as shown in Fig. 1. A substantial fraction of the injected momentum (often more than half) is lost to drag forces at the cylindrical outer wall. As they swirl around the combustor, individual fluid particles must drift inward toward the exhaust port, because new fluid is injected continuously at the outer wall. In the absence of external forces (in region III, away from any walls), these particles maintain their angular momentum, and their tangential velocities increase as they drift inward. Because of mass conservation, the radial velocity also increases as the radius decreases. The ratio of tangential to radial velocity of the fluid in region III (which depends

weakly on radius) is called the swirl parameter, ψ , and measures the strength of the swirling flow.

A strongly swirling flow creates a strong radial pressure gradient, which is nearly balanced by the centrifugal forces on the rotating fluid particles in region III. In the end wall boundary layers (in region II), however, when the tangential velocity decreases to zero, this radial pressure gradient is unbalanced and strongly accelerates the flow inward. Even for strongly swirling flows ($\psi \gg 1$), the local swirl parameter in region II is typically of order unity. Thus, a significant difference between the flows in regions II and III is that (for $\psi \gg 1$) the ratio of radial velocity to tangential velocity is an order of magnitude larger in region II than in region III.

Small coal particles (with diameters on the order of 100 μ) are also injected into the outer wall layer. The single dominant force on these particles in the tangential direction is the drag from the swirling flow, and they rapidly acquire the tangential velocity of the fluid. In the radial direction, the slow radial inflow of the fluid drags the particles inward, but (because of their larger density) the centrifugal forces dominate the pressure gradient and propel them out. Which effect dominates depends on the density and size of the particle, and on the swirl parameter. In a well-designed cyclone combustor, the swirl parameter is high enough to prevent the ash particles from escaping with the exhaust gas, which is therefore relatively clean. Because an unburned coal particle is heavier than the corresponding ash particle (by a factor of about 10.), it follows that if the ash particles are too heavy to be carried out of the chamber by the fluid, the coal particles certainly are.

Because of the argument given above, a flow with high swirl tends to confine unburned coal particles to the cylindrical wall layer (region I). The slow radial velocity of the fluid from region I into region III is insufficient to overcome the centrifugal force on the coal particles, and they remain in region I, unable to enter region III. Consequently, particle

dynamics are quite important in determining the flow in region I, but entirely unimportant in region III. As mentioned above, the radial velocity of the fluid in region II is much larger than in region III, and coal particles do flow inward through region II. In fact, the two thin end-wall boundary layers (i.e., region II) provide the only paths by which coal particles can leave region I before they have burned.

Confinement of the coal particles to region I (with some loss into region II) has some important consequences on the flow pattern, which we now enumerate.

(i) Even though the mass ratio of injected coal to injected fluid is typically less than one (to obtain the desired stoichiometry), the local mass ratio (called "loading") in region I can be much larger. Sample calculations show that the loading in region I is often as high as 10 in the PERC atmospheric unit. For the same reason, the loading in regions III or IV is quite small.

(ii) The two-phase nature of the flow in region I (as seen by the large loading there) alters the fluid dynamics significantly from what would occur in the absence of the coal particles. For example, we will show in §2 that an increased loading decreases the swirl in the chamber. This in turn decreases the effectiveness of the cyclone as a particle separator, and also decreases the total pressure drop. The importance of these effects suggests that it is particularly important to model the flow in region I correctly.

(iii) Virtually all of the coal burns in region I, so that the entire increase in fluid temperature can be considered to occur in this region. The fluid temperature in the rest of the chamber can be treated as constant to a good approximation. Thus, the most important coupling between the combustion and the fluid dynamics occurs in region I.

(iv) Most of the wall surface area is contiguous to region I, so that most of the heat loss occurs there. The accuracy of any heat transfer analysis depends on an accurate description of the fluid flow in the boundary layer. Again, the importance of an accurate model of region I emerges.

In the next section, we analyze each of these regions in detail. The computer model, listed in Appendix A, synthesizes these separate analyses. It should be emphasized at the start that high swirl is a design objective in a cyclone combustor; therefore, the model described here assumes throughout that the swirl is high. The model need not accurately predict flows in which the swirl is low.

Since the boundary layer next to the cylindrical wall turns out to be so critical in determining the efficiency of the combustor, a separate analysis of this region including heat transfer is given in §3. The two separate analyses of region I give the model user the choice between specifying wall roughness as an input or specifying c_f and δ directly. A semi-empirical model like this one should not be considered to be a working design tool until it has been substantiated by comparison with experimental data. This verification of the model is effected in §4. The model can then be used in a number of ways, as is done in §5. At the simplest level, without calculating any specific cases, some conclusions can be drawn about the dependence of performance parameters (overall pressure drop, carbon conversion efficiency, etc.) on the input parameters (chamber geometry, particle size, etc.). Some of this dependence is too complicated to predict without actually exercising the model, and in §5 we also calculate the performance parameters for a variety of configurations related to the proposed CDIF design.

2. DERIVATION OF THE COMPUTER MODEL

The purpose of this section is to show that the qualitative arguments given in §1, when formalized, lead to the computer model which is listed in Appendix A. Whenever feasible, the derivation begins with first principles, in order to make the report as self-contained as possible.

2.1 Region I

We consider region I to be a thin layer with an average thickness δ which covers the entire cylindrical wall. Both fluid and coal particles are injected tangentially, possibly at different speeds and at different (regularly spaced) locations. Fluid flows directly through this region (primarily into region III), but the more dense coal particles remain in this region until they burn or reach region II. The accumulation of coal in the region increases the effective density of the two-phase mixture, and alters the fluid flow. It is essential that the fluid flow, particle dynamics and combustion be coupled in this region. In this section we analyze the flow in this region with a simple model that follows the individual coal particles. This model readily incorporates the effects of a range of particle sizes. Alternatively, one could treat the coal particles as a continuum and determine the coupled flows of the two fluids. However, if there are n particle sizes, one needs $n+1$ fluids. An alternate analysis of the flow in this region, which obviates some of the assumptions used here, and includes heat transfer is presented in §3.

Let us consider first the combustion of a particle of radius a . The particle is composed of ash (mass fraction = p_{10}), volatiles (p_{11}) and carbon ($1-p_{10}-p_{11}$). (An inconvenience of this problem is the large number of parameters that effect the flow. Appendix B provides a complete glossary of the symbols used in this report.) The volatiles vaporize very rapidly (in a few milliseconds), and we assume that this vaporization occurs immediately upon entering the chamber. More precisely, therefore, p_{11} is the mass fraction of the coal that vaporizes almost immediately; it may or may not account for all of the

volatiles in the coal. If the combustion of the carbon is diffusion dominated and occurs at pressure p , the combustion time may be represented by (see Appendix C)

$$\tau_c = \frac{C_1(2a)^2}{p}, \quad (2.1.1)$$

where C_1 is an empirical constant. In many of our calculations, C_1 was chosen so that a 100-micron particle burns at atmospheric pressure in 0.2 seconds, based on considerations given in Appendix C. We assume that this combustion occurs at constant radius, and that the carbon mass decreases linearly to zero during this time. Moreover, we assume that only pure ash particles enter the molten slag layer, and that they do so as soon as the combustion process is completed if the particle is heavy enough to be separated from the flow.

If $\dot{m}_f$ is the entering fluid flow rate, and K_o is the ratio of coal to fluid flow rates, then because of the particle combustion in region I, the effective fluid flow rate within the chamber becomes

$$\dot{m} = \dot{m}_f[1 + K_o\{p_{11} + (1-p_{10}-p_{11})*(1-\eta)\}] \quad (2.1.2)$$

where η represents the mass fraction of carbon that leaves the chamber unburned. The parameter η is an unknown of the problem, but for a well-designed cyclone, it is small and often can be neglected. The volume of region I is $2\pi RL\delta$, and the average time spent by a fluid particle in this region is

$$\tau_f = \frac{2\pi RL\delta\rho}{\dot{m}} \quad (2.1.3)$$

where ρ is the fluid density in the chamber.

Calculation of the time spent in region I by a coal particle is more involved. We will show in §2.2 that the fraction of fluid that enters region II directly from region I may be approximated by

$$FCTR = (p_5)\left(\frac{\rho}{\rho_i}\right)(r_f)(C_T+C_B/\cos \phi)\left(\frac{\delta}{R}\right)\left(\frac{2\pi R^2}{A}\right)\left(\frac{\dot{m}_f}{\dot{m}}\right) \quad (2.1.4)$$

where p_5 is an empirical constant, ρ_i is the inlet fluid density, r_f is the (unknown) ratio of chamber to inlet tangential velocity (called the recovery factor), C_T and

C_B are drag coefficients, ϕ is the angle between the two end walls, and A is the inlet area. An order-of-magnitude analysis of the forces on a coal particle in region I shows that the velocity of the particle can differ from the velocity of the fluid only in the radial direction, where the centrifugal force is unbalanced. (The equations required to show this are presented in §2.2.) Therefore, coal particles move in the axial direction with the fluid. Equation (2.1.4) gives the probability of entering region II directly from region I for a fluid particle. Thus, combining (2.1.3) and (2.1.4) yields the average time an unburned coal particle resides in region I before entering region II:

$$\tau_r = \frac{\rho_i A L}{p_s m_f r_f (C_T + C_B / \cos \phi)} \quad (2.1.5)$$

Note that:

- (i) τ_r is independent of δ , the assumed boundary layer thickness, and independent of η , the fraction of unburned carbon;
- (ii) if $\tau_c < \tau_r$, only (2.1.1) is relevant;
- (iii) if $\tau_r < \tau_c$, the particle leaves region I partially burned, after which it may either leave the chamber entirely or return to region I to continue burning;
- (iv) even for a fixed fluid flow, the comparison of τ_c and τ_r depends on particle size, so that two particles of different size will experience the same flow differently.

The mass of coal in region I, and the loading there, depend on the combustion as well as the trajectory of typical coal particles. As stated above, these particles consist of carbon and ash (no volatiles remain). For subsequent calculations, it will be helpful to compute separately the accumulated mass of carbon and that of ash. Coal particles sizes are distributed over some range, but we assume that only a finite set of particle radii, $\{a_i\}_{i=1}^N$, exists. (At best, the distribution of sizes is known only by a mesh analysis; the assumed

discretization is adequate to represent all of the information available from a mesh analysis.) Let f_i be the mass fraction of coal particles with radius a_i , with

$$\sum_{i=1}^N f_i = 1 \quad (2.1.6)$$

(i) Given a coal particle of radius a_i , for which $\tau_c < \tau_r$, its mass-averaged time spent in region I is

$$\tau_i = (1-p_{11}-p_{10})\tau_c/2 + (p_{10})\tau_c. \quad (2.1.7)$$

(ii) If $\tau_c > \tau_r$, let M be the greatest integer below (τ_c/τ_r) , so that the particle could go from region I to region II M times, before completely combusting on the $(M+1)$ th visit. On the first visit, the mass-averaged time spent in region I is

$$\tau_{i,1} = (1-p_{11}-p_{10})\left(1-\frac{\tau_r}{2\tau_c}\right) + (p_{10})\tau_r \quad (2.1.8)$$

A particle entering region II returns to region I with probability (TERM); we will derive an expression for (TERM) in §2.2. We assume the time required for the excursion out of region I can be neglected. On the second visit (if $M \geq 2$), the mass-averaged time spent in region I is

$$\tau_{i,2} = (\text{TERM}) \left[(1-p_{11}-p_{10})\left(1-\frac{3\tau_r}{2\tau_c}\right) + (p_{10}) \right] \tau_r, \quad (2.1.9)$$

and on the j th visit (if $M \geq j$)

$$\tau_{i,j} = (\text{TERM})^{j-1} [(1-p_{11}-p_{10})(1-(2j-1)\tau_r/2\tau_c) + (p_{10})] \tau_r. \quad (2.1.10)$$

On the last visit

$$\tau_{i,M+1} = (\text{TERM})^M [(1-p_{11}-p_{10})(1-M\tau_r/\tau_c)(\tau_c-M\tau_r)/2 + (p_{10})(\tau_c-M\tau_r)] \quad (2.1.11)$$

Summing (2.1.8)-(2.1.11) yields the total mass-averaged time spent in region I for a particle of radius a_i , with $\tau_c > \tau_r$:

$$\begin{aligned} \tau_i = (1-p_{11}-p_{10}) & \left[\tau_r \sum_{j=1}^M (1-(2j-1)\tau_r/2\tau_c)(\text{TERM})^{j-1} \right. \\ & \left. + (\tau_c/2)(\text{TERM})^M(1-M\tau_r/\tau_c)^2 \right] \\ & + (p_{10}) \left[\tau_r \sum_{j=1}^M (\text{TERM})^{j-1} + (\text{TERM})^M(\tau_c-M\tau_r) \right] \end{aligned} \quad (2.1.12)$$

Summing again over all particle size yields the average time spent in region I by all particles:

$$\tau_m = \sum_{i=1}^N f_i \tau_i, \quad (2.1.13)$$

where τ_i is given by (2.1.7) or (2.1.12), depending on whether $\tau_c < \tau_r$ or $\tau_c > \tau_r$. The total mass of coal in region I is then

$$m = \dot{m}_c \sum_{i=1}^N f_i \tau_i. \quad (2.1.14)$$

The loading, K , is the ratio of coal mass to fluid mass in region I:

$$K = \dot{m}_c \sum_{i=1}^N f_i \tau_i / 2\pi R L \delta \rho \quad (2.1.15)$$

The loading of ash can be extracted easily:

$$\begin{aligned} K_a = \dot{m}_c p_{10} & \left[\sum_{i=1}^{N'} f_i \tau_r \left\{ \sum_{j=1}^{M_i} (\text{TERM})^{j-1} + (\text{TERM})^{M_i} (\tau_{ci}/\tau_r - M_i) \right\} \right. \\ & \left. + \sum_{i=N'+1}^N f_i \tau_{ci} \right] / 2\pi R L \delta \rho \end{aligned} \quad (2.1.16)$$

where $\tau_r < \tau_{ci}$ if $i \leq N'$, $\tau_r > \tau_{ci}$ if $i > N'$. The difference between (2.1.15) and (2.1.16) gives the carbon loading. In these formulae, everything is presumed known except for τ_r , which depends on the unknown r_f via (2.1.5).

An expression for the recovery factor, r_f , can be found from a simple tangential-momentum balance in region I, following the approach of Lewellen and Stickler (1972). We assume here that only one inlet exists, and that it extends the entire length of the chamber. If there are several inlets, the analysis requires simple modifications, and the final result is unchanged. Let (ξ) represent the total tangential momentum flux at the inlet:

$$\xi = \dot{m}_f V_i + (\dot{m}_f K_o)(V_i p_g) \quad , \quad (2.1.17)$$

where V_i is the inlet velocity of the fluid ($=\dot{m}_f/\rho_i A$), and $(V_i p_g)$ is the inlet coal velocity. After one revolution, some of this momentum flux is lost to shear stress on the wall, while the rest appears as momentum flux at the free stream velocity, V_o :

$$\xi = \dot{m}_f(1+K_o)V_o + L \int_0^{2\pi R} \tau_w ds \quad . \quad (2.1.18)$$

The biggest assumption in this formulation regards τ_w , the shear stress along the cylindrical wall. We assume

$$\tau_w = 1/2 C_f[\rho_i(1+K)]V_m^2 = C_w \rho_i(1+K)V_m^2 \quad , \quad (2.1.19)$$

where V_m is a mean velocity within the wall layer, and $\rho_i(1+K)$ represents the effective density near the wall. Lewellen and Stickler (ref. 3) recommend

$$V_m = (V_i V_o)^{1/2} \quad . \quad (2.1.20)$$

Combining (2.1.17) to (2.1.20) yields

$$r_f = \frac{V_o}{V_i} = \frac{1 + K_o P_g}{1 + K_o + \left(\frac{2\pi R^2}{A}\right)\left(\frac{L}{R}\right)C_w(1+K)} \quad (2.1.21)$$

We now have a closed system of equations. For a given distribution of particle sizes, the combustion times are fixed by (2.1.1). If we assume a value of r_f , the residence time is determined from (2.1.5). With these two times for each particle size, one uses (2.1.7), (2.1.12) and (2.1.15) to calculate the loading, K , in this region. This value of loading is then used in (2.1.21) to obtain a revised estimate of r_f . When implemented numerically, the process converges rapidly to a solution. Sections I, II and III of the program listed in Appendix A simply implement this process.

2.2 Region II

The analysis of the flow in region I focused on the trajectories of particles of different sizes, primarily because the fluid mechanics, the particle mechanics, and the combustion are strongly coupled in this region. We will show that combustion is unimportant in the remainder of the chamber (regions II, III, and IV) and we can derive the relevant approximations in a more systematic manner. [More precisely, we will assume

that combustion can be ignored in these regions, calculate the resulting flow, and then show a posteriori that the combusting coal particles visit these regions too briefly to affect thermodynamically the fluid density there.] Thus, for these regions, we begin with the equations of motion of the fluid and of the particles, and our approximations in the various regions will be specializations of this general set of equations.

Because combustion can be ignored outside of region I, the relevant equations are those of a nonreacting dusty gas at constant density. From Marble (1970) the equations of motion in cylindrical coordinates for a steady flow, with axial symmetry, of a nonreacting dusty gas are

$$\frac{\partial}{\partial r} (\rho u r) + \frac{\partial}{\partial z} (\rho w r) = 0 \quad (2.2.1)$$

$$\frac{\partial}{\partial r} (\rho K u_p r) + \frac{\partial}{\partial z} (\rho K w_p r) = 0 \quad (2.2.2)$$

$$u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{K}{\tau_p} (u_p - u) + \frac{\epsilon}{\rho} \left[\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial^2 u}{\partial z^2} \right] \quad (2.2.3)$$

$$u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{K}{\tau_p} (v_p - v) + \frac{\epsilon}{\rho} \left[\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} (rv) + \frac{\partial^2 v}{\partial z^2} \right] \quad (2.2.4)$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \frac{K}{\tau_p} (w_p - w) + \frac{\epsilon}{\rho} \left[\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial w}{\partial r}) + \frac{\partial^2 w}{\partial z^2} \right] \quad (2.2.5)$$

$$u_p \frac{\partial u_p}{\partial r} + w_p \frac{\partial u_p}{\partial z} - \frac{v_p^2}{r} = \frac{1}{\tau_p} (u - u_p) \quad (2.2.6)$$

$$u_p \frac{\partial v_p}{\partial r} + w_p \frac{\partial v_p}{\partial z} + \frac{v_p u_p}{r} = \frac{1}{\tau_p} (v - v_p) \quad (2.2.7)$$

$$u_p \frac{\partial w_p}{\partial r} + w_p \frac{\partial w_p}{\partial z} = \frac{1}{\tau_p} (w - w_p) \quad (2.2.8)$$

Here (u, v, w) and (u_p, v_p, w_p) are the components of fluid and particle velocities, respectively; ϵ is the eddy viscosity of the fluid; and

$$\tau_p = \frac{4/3 \pi a^3 \rho_s}{6 \pi a \mu} \quad (2.2.9)$$

is a characteristic time for acceleration of a particle of radius a and solid density ρ_s in a Stokes flow. Our analysis of these equations parallels that of Rott and Lewellen (1966).

A fundamental assumption in region II is that the time in (2.2.9) is smaller than any relevant time scale of the motion; for example,

$$\tau_p \ll \frac{2\pi r}{v} . \quad (2.2.10)$$

This implies that the particles must follow the fluid in this region, because (2.2.6) - (2.2.9) reduce to

$$u_p = u , \quad v_p = v , \quad w_p = w . \quad (2.2.11)$$

Order-of-magnitude estimates seem to confirm (2.2.10) for the ash particles, and for all but the large coal particles. However, it does limit the range of validity of this analysis, which may not predict the dynamics of the largest coal particles very well.

[The main distinction between regions I and II on the particle dynamics now becomes apparent. Equation (2.2.10) applies equally well in region I, but the swirl parameter, $\psi (= \frac{v}{u})$, is large in region I but not in region II. Consequently, in region I (2.2.6) - (2.2.9) reduce to

$$v_p = v , \quad w_p = w , \quad (2.2.12)$$

$$-\frac{v^2}{r} = \frac{1}{\tau_p} (u - u_p) , \quad (2.2.13)$$

rather than to (2.2.11). For the same reasons (2.2.12), (2.2.13) also apply in region III.]

Because of (2.2.11), the two continuity equations combine to

$$\frac{\partial}{\partial r} (\rho(1+K)ur) + \frac{\partial}{\partial z} (\rho(1+K)wr) = 0 , \quad (2.2.14)$$

and (2.2.4), (2.2.7), and (2.2.14) yield

$$\begin{aligned} & \frac{\partial}{\partial r} (\rho(1+k)ruv) + \frac{\partial}{\partial z} (\rho(1+K)rwv) + \rho(1+K)uv = \\ & \epsilon r \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (rv) \right) + \frac{\partial^2 v}{\partial z^2} \right] . \end{aligned} \quad (2.2.15)$$

These two equations show that the effective density in this region is $\rho(1+K)$. This is also true in region I, which fact is implicit in (2.1.19).

The flow in region II is boundary layer flow, which permits further simplification of the equations, and momentum-integral analyses in this region. For definiteness, we concentrate first on the top boundary layer, using the coordinate system shown in Fig. 2. We assume

$$w \ll u = O(v) \quad \frac{\partial}{\partial r} \ll \frac{\partial}{\partial z}, \quad (2.2.16)$$

and that (2.2.14) is unchanged. By the usual boundary layer arguments, r -derivatives are neglected in comparison with z -derivatives in all viscous terms, and (2.2.5) reduces to

$$\frac{\partial p}{\partial z} = 0; \quad (2.2.17)$$

i.e., the pressure distribution from region III is maintained across the boundary layer. Integrating (2.2.14) and (2.2.15) across the boundary layer (of thickness $\bar{\Delta}(r)$) yields

$$\frac{d}{dr} \int_{-\bar{\Delta}}^0 \rho(1+K)(ru)(rv-\Gamma)dz + \int_{-\bar{\Delta}}^0 \rho(1+K)(ru)dz \frac{d\Gamma}{dr} = r^2 \tau_w \quad (2.2.18)$$

where

$$\Gamma(r) = rV_o(r) \quad (2.2.19)$$

is the circulation in region III, and τ_w is the shear stress on the end wall.

The total radial mass flow through this boundary layer is

$$2\pi Q_T(r) = 2\pi \int_{-\bar{\Delta}}^0 \rho(1+K)(ru)dz \quad (\leq 0), \quad (2.2.20)$$

which identifies the second term in (2.2.18). If we assume that the particle loading, K , is uniform across the layer at each r , then

$$F_T(r) = \int_{-\bar{\Delta}}^0 ru \, dz \quad (2.2.21)$$

is proportional to the volume flow rate of the fluid, and

$$Q_T(r) = \rho(1+K(r))F_T(r) . \quad (2.2.22)$$

Mass conservation in region II requires

$$\frac{d}{dt} (2\pi Q_T) = 2 \pi r \rho (1+K_\infty) w_T , \quad (2.2.23)$$

where K_∞ is the loading outside the boundary layer, and w_T is the local vertical velocity.

To proceed further, it is necessary to assume parameterized velocity profiles in the layer. The choice of appropriate velocity profiles for single-phase flows was studied in some depth by Rott and Lewellen (1966). Using their results (and assuming that the particles are uniformly distributed across the layer), we assume

$$Q_T(r)\Gamma(r) = -p_5 \int_{-\Delta}^0 \rho(1+K)(ru)(rv-\Gamma)dz , \quad (2.2.24)$$

where p_5 depends on the profile shapes; from Ref. 5, (see Fig. 2)

$$p_5 = 4.93 . \quad (2.2.25)$$

[There is some evidence that the distribution of particles in this layer depends on particle size, and that under some circumstances this distribution can have important dynamic effects. It is not clear how to include this possibility without abandoning the simple momentum-integral method altogether.]

We assume

$$\tau_w = -\rho(1+K)\left(\frac{\Gamma}{r}\right)^2 \left(\frac{c_f}{2}\right) = -\rho(1+K)\left(\frac{\Gamma}{r}\right)^2 C_T , \quad (2.2.26)$$

which is appropriate for a rough wall. [If there is blowing at the wall, either from outgassing of carbon burning in the slag layer, or from externally injected fluid, this should be replaced by

$$\tau_w = -\rho(1+K)\left(\frac{\Gamma}{r}\right)^2 C_T + \rho \frac{\Gamma}{r} w_0 , \quad (2.2.26a)$$

where w_0 is the blowing velocity at the wall. This velocity also should appear in (2.2.22). In the program listed in Appendix A, we have assumed that w_0 vanishes.]

The simplest situation is that of constant circulation in region III, which corresponds to a potential vortex there and has some practical importance. If

$$\frac{d\Gamma}{dr} = 0 , \quad (2.2.27)$$

then (2.2.18) simplifies to

$$\frac{d Q_T}{dr} = p_5 \rho(1+K)C_T \Gamma . \quad (2.2.28)$$

The mechanics of rotating boundary layers are somewhat different from those in ordinary viscous boundary layers, and it might be worthwhile to interpret (2.2.18), (2.2.28) and (2.2.23) before proceeding further. The right side of (2.2.28) represents angular momentum lost from the boundary layer to shear stress on the wall. It is proportional to the right side of (2.2.23) which represents the flux of an equal amount of angular momentum into the boundary layer from the mainstream. Thus, the boundary layer flow (region II) actually helps to determine the mainstream flow (region III), contrary to what one might expect by analogy with nonrotating boundary layers. This qualitative conclusion applies even if (2.2.27) fails.

If the upper and lower end-walls are parallel these results apply to the lower boundary layer as well. If not, the same method applies but in a coordinate system rotated by the angle between the two walls. Thus, the counterpart of (2.2.18) is

$$\frac{d}{dr} \int_0^{\bar{\Delta}} \rho(1+K)(r\tilde{u})(rv-\Gamma)dn + \int_0^{\bar{\Delta}} \rho(1+K)(r\tilde{u})dn = -r^2\tau_w/\cos \phi , \quad (2.2.29)$$

where $\tilde{u}$ is the velocity parallel to the sloping wall:

$$\tilde{u} = u \cos \phi + w \sin \phi . \quad (2.2.30)$$

We define

$$Q_B(r) = \int_0^{\bar{\Delta}} \rho(1+K)(r\tilde{u})dn , \quad (2.2.31)$$

and

$$F_B(r) = \int_0^{\bar{\Delta}} r\tilde{u} dn . \quad (2.2.32)$$

Conservation of fluid mass yields

$$\frac{dF_B}{dr} = ru \tan \phi - r w \quad , \quad (2.2.33)$$

where the right side is evaluated at the outer edge of the layer. Conservation of particles yields

$$\frac{d}{dr} (K_B F_B) = K_\infty \left[\left(ru + \tau_p \left(\frac{r}{r} \right)^2 \right) \tan \phi - r w \right] \quad , \quad (2.2.34)$$

where we have used (2.2.12) and (2.2.13) to eliminate the particle velocities in region III. Equations (2.2.33) and (2.2.34) correspond to (2.2.23), and reduce to it if ϕ vanishes. We assume that the same velocity profiles apply in this lower layer so that (2.2.24) - (2.2.26) apply directly. If (2.2.27) holds, (2.2.29) reduces to

$$\frac{dQ_B}{dr} = p_5 \rho(1+K)C_B r / \cos \phi \quad , \quad (2.2.35)$$

corresponding to (2.2.28).

Comparing the equations for the upper and lower boundary layers, one notices two effects of the sloping walls.

- (i) For the same radial increment, Δr , the sloping wall has more surface area, and therefore more shear stress. Thus, (C_T) in (2.2.28) is replaced by $(CB)/\cos \phi$ in (2.2.35), and the fluid flow rate in the lower layer is larger than in the upper layer by this same factor. This effect appears even if there are no particles present.
- (ii) Some of the recirculating particles, which have traversed region II and are headed back to region I, will land instead in the lower (but not in the upper) boundary layer. This additional source of particles for the lower layer is represented by the term involving τ_p in (2.2.34). If there is no recirculation, ($\tau_c \ll \tau_r$ for all particles), this effect vanishes.

The expression given in (2.1.4) for the fraction of fluid that enters region II directly from region I can now be derived. Strictly speaking, the equations (2.2.18) - (2.2.35) apply after the flow in the boundary layer has attained a parabolic

character, but not to its initial stages. The beginning of any boundary layer is a difficult problem; the layers comprising region II begin in corners, which makes their analysis even more difficult. We obtain a crude approximation of the flow in these corners by assuming:

- (i) that the equations apply, even at the leading edge of each layer;
- (ii) that $K = K_{\infty}$ across the width of region I;
- (iii) that (2.2.27) holds across the width of region I.

With these assumptions, (2.2.28) can be integrated trivially to give the mass flow rate in the top boundary layer at the beginning of region III:

$$2\pi Q_T \Big|_{r=R-\delta} = 2\pi p_5 \rho(1+K)C_T \Gamma \delta \quad (2.2.36)$$

Similarly, (2.2.34) gives the corresponding mass flow rate in the bottom layer:

$$2\pi Q_B \Big|_{r=R-\delta} = 2\pi p_5 \rho(1+K)C_B \Gamma \delta / \cos \phi \quad (2.2.37)$$

Adding (2.2.36) and (2.2.37) yields the total mass flow entering region II from region I, and the total fluid flow is obtained by dividing the sum by $(1+K)$. Dividing again by the total fluid flow rate, $\dot{m}$, yields the fraction of fluid that enters region II directly from region I; this expression was given above in (2.1.4).

We consider next the coupling between regions II and III. In the general case, we have a set of equations for each boundary layer, we need a third set for region III, and all three sets must be solved simultaneously. The analysis simplifies considerably whenever (2.2.27) holds, because each boundary layer flow can be calculated by itself, as we now show. Justification of (2.2.27) is deferred to §2.3.

To be explicit, we consider the upper boundary layer, so that the relevant equations are (2.2.23), (2.2.27) and (2.2.28). As discussed in §1, the loading in region III should be small, so we also assume

$$K_{\infty} \ll 1 \quad (2.2.38)$$

(This assumption can be tested explicitly, and the program in Appendix A does so. Consequently, (2.2.38) is never assumed without justification.) It follows that no new particles enter region II from region III, so that

$$\frac{d}{dr} (K F_T) = 0 \quad (2.2.39)$$

At $r = R - \delta$,

$$F_T(r) = -p_5 C_T \Gamma \delta, \quad K(r) = K_I, \quad (2.2.40)$$

where K_I is the loading in region I. Thus

$$K(r)F_T(r) = -p_5 C_T \Gamma \delta K_I. \quad (2.2.41)$$

Using (2.2.39), (2.2.28) becomes

$$\frac{d F_T}{dr} = -\frac{F_T}{K} \frac{dK}{dr} = p_5 C_T (1+K) \Gamma. \quad (2.2.42)$$

Combining this with (2.2.41) yields

$$\delta K_I \frac{dK}{dr} = K^2 (1+K), \quad (2.2.43)$$

which can be integrated; the result is

$$-\frac{1}{K(r)} + \ln \left(\frac{1+K(r)}{1+K_I} \right) + \ln \left(\frac{K_I}{K(r)} \right) = -\frac{R-r}{K_I \delta}, \quad (2.2.44)$$

which defines $K(r)$, or more easily defines the radius at which a given loading appears, $r(K)$. Tabulation of $K(r)$, based on (2.2.44), is effected in §V of the program listed in Appendix A. By plotting $K(r)$, it may be seen that the loading decreases very rapidly near $r = R - \delta$, and decreases more slowly further into the layer. With $K(r)$ known, $F_T(r)$ is found from (2.2.41) and the vertical velocity into the upper layer is found (2.2.23) and (2.2.28):

$$w_T(r) = p_5 (1+K(r)) C_T \frac{\Gamma}{r}. \quad (2.2.45)$$

It is interesting to note from (2.2.44) that the loading in the top layer is independent of both the shear stress (C_T) and the

circulation Γ (except through K_I), despite the fact that the entire layer exists because of these two quantities.

The effective differences between the upper and lower boundary layers were discussed below (2.2.35). The first difference, which can be incorporated into a redefined (c_f), has no effect at all on (2.2.44) and changes (2.2.41) by a factor, $\cos \phi$. The second difference, that the lower layer has an additional source of particles, is more difficult to assimilate. However, if

$$K_\infty \tau_p \frac{\Gamma}{r^2} \sin \phi \ll p_5 C_B(1+K) \quad (2.2.46)$$

in the lower layer, it can be seen from (2.2.34) and (2.2.35) that this second source of particles contributes negligibly and can be ignored. This requirement is tested explicitly in the program listed in Appendix A.

Now that the flow in region II is known, we may determine the quantities that are controlled by it. First, to complete (2.1.9) - (2.1.12), we need an expression for (TERM), the probability that a particle entering region II returns to region I. It follows from the previous analysis that of the particles in region II, the fraction in the upper layer is $C_T/(C_T + C_B/\cos \phi)$. Upon reaching the exhaust radius, we assume that (P_T) of these particles exhaust with the fluid, while $(1-P_T)$ return to region I. Similarly, in the lower layer (P_B) of the particles enter the slag port, while $(1-P_B)$ return to region I. The parameters P_B , P_T are empirical constants ($0 \leq P_B$, $P_T \leq 1$), to be chosen in order to match the results of the program with experimental data. The flow in the regions where these boundary layers separate are extremely complicated. The boundary layer approximations break down, the flow may fluctuate in time, and could entrain pieces of the slag layer as well. Thus, the parameters (P_B , P_T) encompass a variety of effects, not readily amenable to analysis. In any case, the fraction of particles in region II that return to region I is

$$(TERM) = 1 - (C_T P_T + C_B P_B / \cos \phi) / (C_T + C_B / \cos \phi) \quad (2.2.47)$$

Next, we compute blowby which is defined to be the fraction of fixed (i.e., nonvolatile) carbon that enters the slag port unburned. (The reader is warned that other definitions of this term are possible.) From (2.2.37) the mass flow rate of coal into the bottom layer is $2\pi p_5 \rho K \Gamma \delta C_B / \cos \phi$. Hence, the mass flow rate of carbon, using (2.1.15) and (2.1.16), is $2\pi p_5 \rho (K - K_a) \delta C_B / \cos \phi$. The fraction that enters the slag port is P_B , from which it follows that the blowby is

$$BB = \frac{\pi p_5 \rho (K - K_a) \delta C_B P_B}{\dot{m}_f \cdot K_o (1 - p_{10} - p_{11}) \cos \phi} \quad (2.2.48)$$

The corresponding coal flow rate in the upper layer contributes to carryover, the fraction of ash that exhausts with the gases. For this calculation, it is necessary to consider only the flow rate of ash, as the carbon will continue to burn downstream. The contribution to carryover from this source is

$$\frac{2\pi p_5 \rho K_a \delta C_T P_T}{\dot{m}_f K_o p_{10}} \quad (2.2.49)$$

Other sources of carryover are discussed in §2.3.

Finally, we need to estimate the time that a particle spends in region II. From (2.2.11), this is approximately the time that a fluid particle spends in region II, at least for the small particles. Inspection of measured velocity profiles in rotating boundary layers (Savino and Keshock, 1965) suggests that the average radial velocity in the layer is on the order of

$$\bar{u} = \frac{\Gamma}{4r} \quad (2.2.50)$$

Using $dt = \frac{dr}{u}$ and (2.2.27), it follows that

$$\tau_{II} = \frac{2}{\Gamma} r^2 \bigg|_{r_1}^{r_2} \leq \frac{2R^2}{\Gamma} \quad (2.2.51)$$

is a bound on the time spent in region II. But (2.1.5) may be written as

$$\tau_r = \left(\frac{L}{R}\right) R^2 / (p_5 \Gamma (C_T + C_B / \cos \phi)) \quad (2.2.52)$$

so that

$$\frac{\tau_{II}}{\tau_r} = 2p_5\left(\frac{R}{L}\right)(C_T + C_B/\cos \phi) \quad (2.2.53)$$

Thus,

$$\frac{\tau_{II}}{\tau_r} = O(C_T, C_B) \ll 1 \quad (2.2.54)$$

2.3 Region III

The analysis of region III is somewhat simpler than either of the analyses for region I and II, both because the combustion may be neglected (as in region II) and because the loading is fairly small. It will be shown below that only very small coal particles can remain in region III (typically, $a < 10 \mu$ for the PERC atmospheric combustor). The mass fraction of these particles is small enough that their presence can be neglected in this region. Ash particles can be somewhat larger (by a factor of about 3), but even the ash particles tend to be on the small end of the spectrum of sizes. Consequently, in the governing equations (2.2.1) - (2.2.8), we may assume that all of the particles are of one size (e.g., the smallest particle size available in our discrete spectrum, or the average of the smallest two sizes). In addition to these two types of particles in region III, a third source of loading is the much larger coal particles that pass through this region as they recirculate. We will see that the loading from these particles is too small to affect the fluid flow.

The analysis of region III begins with (2.2.1) - (2.2.8) and the following assumptions:

- (i) no combustion occurs, and the fluid temperature is constant;
- (ii) the swirl is large,
 $u, w \ll \frac{\Gamma}{r}$; (2.3.1)
- (iii) the loading can be neglected for all of the particles except those that remain suspended in region III; even for these particles (which lie in a narrow range of sizes), the loading is not large ($K < 1$);

(iv) the fluid flow entering from region I shows no variation in θ or z , especially

$$\frac{\partial \Gamma}{\partial \theta} = \frac{\partial \Gamma}{\partial z} = 0 \quad \text{at} \quad r = R - \delta ; \quad (2.3.2)$$

(v) the total radial Reynolds number is large,

$$\frac{\rho(Ku_p + u)r}{\epsilon} \gg 1 ; \quad (2.3.3)$$

(vi) the particles are small enough that they quickly accelerate to the swirl velocity of the fluid,

$$\frac{\tau_p \Gamma}{r^2} \ll 1 . \quad (2.3.4)$$

Our first objective is to solve for $\Gamma(r, z)$, because of the importance of (2.2.27) in region II. As discussed in §2.2, the particle velocities in this region are given by

$$v_p = v , \quad w_p = w , \quad (2.2.12)$$

$$u_p = u + \tau_p \frac{\Gamma^2}{r^3} \quad (2.2.13)$$

Combining (2.2.3) and (2.2.6), using (2.2.13) and the assumptions (ii), (v) and (vi) above, yields

$$\frac{\partial p}{\partial r} = \rho(1+K)\Gamma^2/r^3 . \quad (2.3.5)$$

As (2.2.8) suggests, in this region as in region II, there is an "effective fluid" with density $\rho(1+K)$. Combining (2.2.5) and (2.2.8) in the same way yields

$$-\frac{1}{\rho} \frac{\partial p}{\partial z} = (Ku_p + u) \frac{\partial w}{\partial r} + (K+1) w \frac{\partial w}{\partial z} . \quad (2.3.6)$$

Comparing (2.3.5) and (2.3.6) in light of (2.3.1) shows that

$$\left| \frac{\partial p}{\partial z} \right| \ll \left| \frac{\partial p}{\partial r} \right| ,$$

so that the pressure may be considered uniform in z throughout this region. Similarly

$$\frac{\partial^2 p}{\partial z \partial r} = \frac{\rho}{r^3} \frac{\partial}{\partial z} \left\{ (1+K)\Gamma^2 \right\} \doteq 0 . \quad (2.3.7)$$

If the loading is small, (2.3.7) shows that Γ is also uniform in z .

Even for moderate loading ($K < 1$), the same conclusion may be reached with some more effort, as follows. Combining (2.2.4) and (2.2.7) and using (2.3.3) yields

$$(Ku_p + u) \frac{\partial \Gamma}{\partial r} + (K+1) w \frac{\partial \Gamma}{\partial z} = 0, \quad (2.3.8)$$

which has a simple physical interpretation. The "effective fluid" is a mass-averaged combination of gas and particles, the velocity of which is $(Ku_p + u, (K+1)w)$. According to (2.3.8) Γ is conserved along the streamlines of the effective fluid. If the loading is small enough, these streamlines may be approximated by the streamlines of the gas.

It is instructive to pursue the case of small loading (in region III) to its conclusion, because it is representative of the general situation. Moreover, a well-designed combustor has $K \ll 1$ in region III, in order to minimize the carryover (i.e., the mass fraction of ash particles in the exhaust gases). All of the streamlines which begin in region I have the same circulation, from (2.3.2). Thus, all of region III which is covered by these streamlines has constant circulation. Depending on the strength of the end wall boundary layers, this zone of constant circulation may or may not completely fill region III, as shown in Fig. 3. If these boundary layers are not strong enough to suck all of the entering fluid into them, then some streamlines cross region III (as in Fig. 3a), and the circulation is constant over the entire region III, which justifies (2.2.27) and the simplified analysis given at the end of the previous section. Thus, the validity (2.2.27) rests on whether the maximum flow rate in the two boundary layers equals the total fluid flow rate. In analyzing the corresponding problem without particles Lewellen (1971) called this ratio of flow rates a "boundary-layer interaction parameter," and gave a simple expression for it. Such a simple expression does not exist in this case, but the ratio of flow rates is available from the analysis of the preceding section. The maximum fluid flow rate in the upper layer is $2\pi r F_T(r)$, evaluated at the radius of the exhaust,

$r = RP_3$. Assuming the validity of (2.2.27), this can be evaluated from (2.2.44) and (2.2.41). This flow rate typically is about 5 to 10 percent of the total flow rate for the PERC atmospheric combustor, and 10 to 20 percent for the CDIF design, with similar values for the flow rate in the lower layer. Thus, for the entire parameter range in question, the end-wall boundary layers are always weak enough that some fluid flow remains outside them, and Γ may be assumed constant throughout region III.

Dropping the assumption that the loading be very small in region III does not significantly change the conclusion that Γ is constant throughout region III. We will see below that any nonnegligible loading that does occur in this region is restricted to an outer zone contiguous to region I. In the inner zone where the last streamlines do or do not enter region II, the loading is negligible and the previous argument applies. The program listed in Appendix A is based on the validity of (2.2.27), subject to a computation of the total fluid flow rates in region II. However, for the sake of completeness, we discuss at the end of this section the required analysis when (2.2.27) fails.

We consider next the radial fluid velocity, which is required to compute the loading in region III. For our purposes it is sufficient to integrate over the axial length of region III and compute only the average (in z) radial velocity

$$\bar{u}(r) = \int_{III} u(r,z) dz / [L - (R-r)\tan \phi] \quad (2.3.9)$$

One integrates (2.2.1) across region III (in z), using for boundary conditions (2.2.45) and its counterpart in the lower layer. The result is merely conservation of fluid mass,

$$r\bar{u}(r) = -[\dot{m}/2\pi\rho - (1+C_B/C_T \cos \phi)F_T(r)] / (L - (R-r)\tan \phi), \quad (2.3.10)$$

where the right side is evaluated using (2.1.2), (2.2.41) and (2.2.44) and we have neglected the displacement thicknesses of the end-wall boundary layers.

Once $\bar{u}(r)$ is known, it is straightforward to calculate, for each particle, the (z-averaged) radius at which centrifugal and drag forces exactly balance (and u_p vanishes). At this equilibrium radius, (2.2.26) becomes

$$r\bar{u} + \tau_p \frac{\Gamma^2}{r^2} = 0 \quad (2.3.11)$$

Applying this condition at $r = R - \delta$ yields $a_{\max}$, the size of the largest particle that can remain in region III, while at $r = R(1-P_3)$, $a_{\min}$ defines the smallest particle that remains there. Particles smaller than $a_{\min}$ exhaust with the fluid, and contribute to the carryover. Moreover, particles whose radius lies between $a_{\max}$ and $a_{\min}$ remain suspended in region III until they are sucked into region II, so a substantial fraction of these also contribute to carryover. For the purpose of computing the carryover, it is necessary to calculate these critical sizes only for the ash particles, because coal particles residing in region III eventually become ash particles.

Next, we estimate the loading due to these suspended particles. Because the circulation is constant, it follows from (2.3.7) that

$$\frac{\partial K}{\partial z} = 0 \quad (2.3.12)$$

Then using (2.2.1), and (2.2.12) and (2.2.13), (2.2.2) becomes

$$u_p(r) \frac{dK}{dr} = -2\tau_p \Gamma^2 K(r)/r^4 \quad (2.3.13)$$

But u_p vanishes at the equilibrium radius, and (2.3.13) predicts an infinite loading there. This conclusion is unacceptable on several grounds, one of which is that the resulting flow (even for very large local loading) is dynamically unstable. Rayleigh's well-known criterion for the inviscid stability of a rotating flow (cf., Lin, 1955) is

$$\frac{d}{dr} (\rho(1+K)\Gamma^2) \geq 0 ; \quad (2.3.14)$$

i.e., the squared circulation of the effective fluid must increase with radius. Thus, there is a tendency for particles

of a given size to accumulate at their equilibrium radius, as expressed by (2.3.13), but any significant accumulation creates an unstable situation that cannot persist.

Rather than attempting to solve for this possibly unsteady flow, we approximate the situation as follows. For each (ash) particle that can remain in region III, we compute the equilibrium radius, $\hat{r}$, and assume that the loading in region III from these particles is uniformly distributed between region I and the equilibrium radius:

$$\begin{aligned} K(r;a) &= \hat{K} & \hat{r} < r < R - \delta, \\ &= 0 & r < \hat{r}. \end{aligned} \quad (2.3.15)$$

This distribution is intrinsically stable, and satisfies (2.3.14) even if several such loadings (for different particle sizes) are superposed in region III. Moreover, it reflects the fact that the particles primarily originate in region I (at $r = R - \delta$) and cannot progress further than $r = \hat{r}$. It remains to compute $\hat{K}$, so that the mass flux of these particles from region I balances that into region II. The mass flux into the chamber of this size of ash (whose original mass fraction of the coal was f_j) is

$$\dot{m}_f K_o f_j (1-SLG)p_{10} \quad (2.3.16)$$

where SLG is the fraction caught in the slag. If $K_p(r)$ denotes the partial loading from these particles in the upper layer, (2.2.23) becomes

$$\frac{d}{dr} (K_p F_T) = K_a r w_T, \quad (2.3.17)$$

subject to

$$\frac{d F_T}{dr} = r w_T. \quad (2.3.18)$$

Between (2.3.17), (2.3.18) and (2.2.39), one shows that

$$K_p(r) - \hat{K} = \alpha K_T(r) \quad (2.3.19)$$

for some constant α . At $r = R - \delta$, $K_T = K_I$, $K_p = 0$, and

$$\alpha = -\frac{\hat{K}}{K_I} \quad (2.3.20)$$

The total mass flux of these ash particles out of the chamber is $[2\pi\rho K_p(\hat{r}_f)F_T(\hat{r})P_T]$, and balancing this with (2.3.16) yields

$$\hat{K} = \frac{\dot{m}_f f_j K_o(1-SLG)p_{10}}{2\pi\rho P_T |F_T(r)| \left(1 - \frac{K_T(r)}{K_I}\right)} \quad (2.3.21)$$

This computation is performed for each particle that can remain in region III, the total loading from these suspended ash particles being the superposition of the partial loadings. It is evident that the loading from a given particle size can be significant only if its equilibrium radius is only slightly less than $r - \delta$, so that these particles are confined to a narrow zone contiguous to region I. This possibility does not contradict the earlier assumption that $K \ll 1$ over most of region III.

Let us now consider all of the possible sources of carryover. The flow of particles of all sizes directly from region I was discussed above (2.2.49). The other possible mechanisms all tend to select smaller particles, as determined by various solutions of (2.3.11). As mentioned above, every particle with radius smaller than a_{min} contributes to carryover. Particles between a_{min} and a_{max} are trapped in region III until they either enter the slag port via the lower boundary layer or leave the chamber as carryover. The probability that an ash particle in region III exhausts as carryover is

$$\frac{C_T P_T}{C_T P_T + C_B P_B / \cos \phi} \quad (2.3.22)$$

The other possible source of small ash particles for carryover is the particles in region I that are small enough to be completely entrained by the fluctuating turbulent motion there. These particles are prevented from slagging because of the large fluctuations in velocity, and remain suspended in region I, much as smaller ash particles do in region III. This process is actually quite complicated, because it depends not only on

the particle size and the mean fluid motion, but also on the magnitude and character of the turbulent fluctuations, and the proximity of the particle to the wall. We lump all of this uncertainty into a single parameter, p_4 , the ratio of the r.m.s. radial velocity fluctuation to the mean tangential velocity. Thus, a_{crt} , the critical particle size for this mechanism, is obtained by solving

$$\Gamma p_4 + \tau_p \frac{\Gamma^2}{R^2} = 0, \quad (2.3.23)$$

corresponding to (2.3.-1). If $a_{crt} < a_{max}$, these particles have already drifted into region III and there is no additional carryover.

To be consistent, we note that these entrained ash particles that increase the carryover also increase the loading in region I and increase the radiative heat transfer to the walls. The increase in loading usually turns out to be a small fraction of $(1+K)$ in region I, and can be neglected with some justification. (As a precaution, the program listed in Appendix A prints a warning whenever the loading of the entrained particles exceeds 10 percent of $(1+K)$ in region I.) The increase in radiative heat transfer is often significant, and is discussed in §3.

It remains to calculate the trajectories of the recirculating coal particles that are returning to region I. Whereas the particles we have just discussed were among the smallest in the spectrum, only the largest particles can recirculate before they have completely burned. Our previous assumption about the smallness of the particles, (2.3.4), is no longer appropriate, but order of magnitude estimates suggest that

$$\tau_p \frac{\Gamma}{r^2} \gg 1 \quad (2.3.24)$$

may be appropriate instead. (Recall that if the smallest and largest particle diameters differ by a factor of 10, τ_p changes by a factor of about 1000 from the largest coal particle to the smallest ash particle.) For these massive coal particles, (2.2.6) may be approximated by

$$u_p \frac{\partial u_p}{\partial r} - \frac{v_p^2}{r} = 0 \quad (2.3.25)$$

Assuming that (2.2.12) still holds approximately, and that these particles have no radial velocity at $r = R \cdot P_3$, one finds that

$$u_p(r) = \Gamma[(RP_3)^{-2} - r^{-2}]^{1/2}$$

The time spent in region III by these particles is obtained by integrating $dt = dr/u_p$:

$$\tau_{III} = \frac{R^2 P_3}{\Gamma} (1 - P_3^2)^{1/2} \quad (2.3.26)$$

This estimate has two applications. First, we must verify that this time is small in comparison with residence time in region I. But this time does not exceed the bound, (2.2.51), on the time spent in region II; it follows immediately from (2.2.54) that

$$\frac{\tau_{III}}{\tau_r} = O(C_T, C_B) \ll 1 \quad (2.3.27)$$

which finally vindicates the assumption that combustion outside of region I can be neglected. Second, we must verify that these particles contribute negligibly to the loading in region III. The total mass flow rate of these recirculating particles does not exceed

$$2\pi\rho(-K_T F_T)[1 - P_T + (1 - P_B)C_B/(C_T \cos \phi)] ,$$

the volume of region III (neglecting displacement thicknesses) is

$$\pi R^2(1 - p_3)L \left(1 + p_3\right) + \left(\frac{R}{3L}\right) \tan \phi (1 + p_3 - 2p_3^2) ,$$

and the residence time of the particles is given by (2.3.26). Thus, an estimate of the loading in region III from the recirculating particles is

$$\tilde{K} = 2p_5 K_I \left(\frac{S}{R}\right) \left[\frac{p_3}{\frac{L}{R}(1 + p_3) + \frac{1}{3}(1 + p_3 - 2p_3^2)} \right] \left(\frac{1 + p_3}{1 - p_3}\right)^{1/2} [(1 - P_T)C_T + (1 - p_B)C_B/\cos \phi] , \quad (2.3.28)$$

where K_I is the loading in region I, and we have used (2.2.41). From (2.1.15) it follows that

$$K_I\left(\frac{\delta}{R}\right) = 0(1) , \quad (2.3.29)$$

so that

$$\tilde{K} = 0(C_T, C_B) \ll 1 \quad (2.3.30)$$

as anticipated. This concludes our discussion of the particles.

The pressure drop across region III is obtained by integrating (2.3.5). Neglecting the fairly small loading in this region, normalizing with respect to the pressure at the inlet of the chamber (and using the perfect gas law), we obtain the normalized pressure drop across region III:

$$\left(\frac{\Delta p}{p}\right)_{III} = \left(\frac{\rho}{\rho_i}\right) \Gamma^2 [(R p_3)^{-2} - (R - \delta)^{-2}] / 2 \tilde{R} T_i , \quad (2.3.31)$$

where $\tilde{R}$ is the gas constant appropriate to the fluid. Equation (2.3.5) also applies across region I, but the loading there is given by (2.1.15). We approximate

$$\Gamma^2 / r^3 = v^2 / r = \left(\frac{\dot{m}_f}{\rho_i A}\right)^2 r_f / R , \quad (2.3.32)$$

which is consistent with (2.1.20). The result for region I is approximately given by

$$\left(\frac{\Delta p}{p}\right)_I = \left(\frac{\rho}{\rho_i}\right) (1 + K_I) \left(\frac{\dot{m}_f}{\rho_i A}\right)^2 \left(\frac{\delta}{R}\right) r_f / \tilde{R} T_i . \quad (2.3.33)$$

There is also a pressure drop in the inlet nozzle, which is required to accelerate the fluid to the inlet velocity. Assuming that the Mach number in this region is small, the pressure drop in the nozzle is

$$\left(\frac{\Delta p}{p}\right)_O = \left(\frac{\dot{m}_f}{\rho_i A}\right)^2 / 2 \tilde{R} T_i . \quad (2.3.34)$$

After determining the pressure drop across region IV, the total pressure drop is obtained by adding these separate contributions.

We conclude this section with a brief description of the changes in the analysis that are required when (2.2.27) fails, even though the program listed in Appendix A does not allow for this possibility. We assume that (2.2.27) applies for $r > r^*$,

and that all of the fluid has entered region II in the interval $r > r^*$. At $r = r^*$, $\bar{u}$ vanishes and it follows from (2.2.13) that no particles, no matter how small, can drift beyond r^* . (Thus, $a_{\min} = 0$.) For $r < r^*$ it is consistent with (2.3.17), (2.3.18) to assume

$$a) \quad K = 0, \quad \frac{\partial \Gamma}{\partial z} = 0 \quad \text{in region III;} \quad (2.3.35)$$

$$b) \quad \bar{u} = 0 \quad \text{in region III;} \quad (2.3.36)$$

c) no net mass flow exists between regions II and III,

$$\frac{d}{dr} (Q_T + Q_B) = 0 \quad (2.3.37)$$

This last assumption does not preclude the possible growth of one end-wall boundary layer at the expense of the other.

For $r < r^*$, (2.2.18) and (2.2.29) are rewritten as

$$-\frac{1}{p_5} \frac{d}{dr} (Q_T \Gamma) + Q_T \frac{d\Gamma}{dr} = -\rho(1+K_T)\Gamma^2 C_T, \quad (2.3.38)$$

$$-\frac{1}{p_5} \frac{d}{dr} (Q_B \Gamma) + Q_B \frac{d\Gamma}{dr} = -\rho(1+K_B)\Gamma^2 C_B / \cos \phi, \quad (2.3.39)$$

where $K_T(r)$, $K_B(r)$ refer to the local average loadings in each layer. Combining these with (2.3.37) yields

$$(p_5 - 1)(Q_T + Q_B) \frac{d\Gamma}{dr} = p_5 \rho \Gamma^2 [C_T(1+K_T) + C_B(1+K_B)/\cos \phi], \quad (2.3.40)$$

which shows that the circulation diminishes in response to the combined shear stress on both end walls.

If (2.2.46) holds (e.g., if $\phi = 0$) and if

$$C_T = C_B / \cos \phi, \quad (2.3.41)$$

then

$$K_B(r) = K_T(r) = K_T(r^*),$$

$$Q_B(r) = Q_T(r) = Q_T(r^*) = \rho(1+K_T(r^*))F_T(r^*) \quad (2.3.42)$$

$$F_T(r^*) = 1/2 \frac{\dot{m}}{2\pi\rho}$$

and (2.3.42) can be integrated to give the circulation for $r < r^*$:

$$\frac{1}{\Gamma(r)} = \frac{1}{\Gamma(r^*)} + \left(\frac{p_5}{p_5 - 1} \right) \left(\frac{4\rho\pi C_T}{m} \right) (r - r^*) \quad (2.3.43)$$

When region III is matched to region IV in the next section, (2.3.43) is evaluated at $r = RP_3$ to provide the correct circulation for the matching.

2.4 Region IV

Particle loading has been the object of a major part of the analysis up to this point, but can be neglected entirely in region IV. However, two new features appear to complicate the analysis in this region:

- (i) the tangential velocity, which increases with decreasing radius in region III, must decrease to zero on the centerline;
- (ii) the axial velocity, which is small in comparison with the tangential velocity in the other regions, becomes large in this region and admits the possibility of "vortex breakdown."

The first feature creates no major obstacles, and even a crude approximation may be adequate to account for it. Proper handling of vortex breakdown is more difficult, partly because the phenomenon itself is not entirely understood.

Although vortex breakdown has been the subject of a great deal of recent research (e.g., Hall, 1966; or Bossel, 1969), a definitive a priori treatment of the problem is still lacking. On the other hand, a very simple analysis is adequate to demonstrate the basic feature of "critical flow." Thus, for the purpose of illustration, we consider the axial flow of a rapidly rotating homogeneous fluid in a circular streamtube of slowly-varying radius $R(z)$, as shown in Fig. 4. We assume the fluid to be in rigid-body rotation (angular velocity Ω), with uniform pressure along the centerline, and we neglect the small radial velocity. Equations (2.2.1), (2.2.3), and (2.2.5) may be written approximately as

$$\frac{d}{dz} (\pi R^2 w) = 0, \quad (2.4.1)$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial r} = -\frac{v^2}{r} = -\Omega^2 r, \quad (2.4.2)$$

$$w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z}. \quad (2.4.3)$$

The pressure along the surface $R(z)$ is obtained by integrating (2.4.2):

$$p \Big|_{r=R(z)} = p_0 + \frac{1}{2} \rho (\Omega R(z))^2 \quad (2.4.4)$$

Equations (2.4.1), (2.4.3) and (2.4.4) can be combined into

$$\left[w^2 - \frac{(\Omega R)^2}{2} \right] \frac{dR^2}{dz} = 0 \quad (2.4.5)$$

which requires either:

- (i) uniform flow, $\frac{dR^2}{dz} = 0$; or
- (ii) "critical flow,"

$$w^2 = \frac{1}{2} (\Omega R)^2 = \frac{1}{2} v^2. \quad (2.4.6)$$

Thus, there is a critical axial speed, which is roughly the same size as the tangential speed, and which is preferred in the sense that no other nonuniform flow is allowed. This critical speed for the axial flow of a rotating fluid is analogous to sonic speed in one-dimensional gasdynamics. The phenomenon of vortex breakdown is a discontinuous change in a rotating flow between two regions of uniform flow, analogous to a shock wave in gasdynamics. As illustrated in Fig. 5, breakdown is signaled by the occurrence of a stagnant core (or "bubble") along the centerline, which the axial flow does not penetrate. For a complete review of the problem of vortex breakdown, the reader is referred to Hall (1972).

The major effect of vortex breakdown on the efficiency of the combustor is that it changes the pressure drop: there is no axial pressure drop along the centerline if there is a stagnant core, and conversely. Two requirements for the model in region IV, therefore, are that it contain a criterion to decide whether or not a stagnant core exists along the centerline, and that the pressure drop across region IV reflect this possibility.

Let us consider first the case in which a stagnant core exists along the centerline. Specifically, we assume that there is a radius, $\tilde{r}$ such that for all z ,

$$u = v = w = 0 \quad \text{if } r < \tilde{r} . \quad (2.4.7)$$

The requirement that $\tilde{r}$ be positive will eventually provide the necessary criterion for the existence of the stagnant core. Both u and v are independent of z in region III ($r > R_{p3}$) and within the core ($r < \tilde{r}$), and we assume this to be true in region IV as well:

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0 . \quad (2.4.8)$$

It then follows from (2.2.1) that w can depend only linearly on z . The simplest continuous tangential and axial velocity profiles consistent with (2.4.8) and (2.4.7) are of the form (for $r > \tilde{r}$)

$$v = c_0(r - \tilde{r}) , \quad (2.4.9)$$

$$w = (c_1 + c_2 z)(r - \tilde{r}) . \quad (2.9.10)$$

Bossel (1969) presented a semi-empirical theory of vortex breakdown that predicted well the observed shapes of the stagnant bubbles. In his solutions, the ratio of axial to tangential velocity tended to a constant (in r) far downstream of the bubble; the constant was approximately 1. If we replace (2.4.9) with

$$v = c_1(r - \tilde{r}) \quad (2.4.11)$$

then this condition is achieved at the throat of the exhaust, at $z = 0$. If "critical flow" is defined by $w = v$, then this assumption also gives critical flow at the exhaust throat, consistent with the one-dimensional analysis given earlier.

At the exhaust radius, $r = R_{p3}$, we require that the tangential velocities in regions III and IV match. (It is not uncommon to observe a complicated set of thin layers between regions III and IV, which prevent us from also matching the axial velocities there.) This matching condition is

$$c_1(R_{p3} - \tilde{r}) = \frac{\Gamma}{R_{p3}} , \quad (2.4.12)$$

which can be solved for c_1 .

At the exhaust (more precisely, just below the top boundary layer in region II), we require fluid mass conservation. The total fluid flow out of the chamber (except for that in the top layer) is

$$2\pi \int_{r_4}^{r_3} r w dr \Big|_{z=0} = \frac{\dot{m}}{\rho} - 2\pi P_T |F_T(RP_3)| \quad (2.4.13)$$

Substituting (2.4.11) and (2.4.12) into (2.4.13) eventually gives an equation for $\tilde{r}$:

$$\frac{\tilde{r}}{R} = \frac{1}{2} P_3 \left[\left\{ 9 - \frac{12}{\pi \Gamma R P_3} \left(\frac{\dot{m}}{\rho} - 2\pi P_T |F_T(RP_3)| \right) \right\}^{1/2} - 1 \right] \quad (2.4.14)$$

This whole analysis presumes that $\tilde{r}$ be nonnegative. Thus, it follows from (2.4.14) that a stagnant core exists only if

$$\frac{\dot{m}}{\rho} - 2\pi P_T |F_T(r_c)| < \left(\frac{2\pi}{3}\right) \Gamma R P_3 ; \quad (2.4.15)$$

i.e., the existence of the core depends on the circulation being large enough, relative to the volume flow rate.

The radial velocity is not required to compute the pressure, but it can be calculated by substituting (2.4.10) into the continuity equation, (2.2.1), integrating and matching to (2.4.7). The result for $r > \tilde{r}$ is

$$ru = -\frac{c_2}{6} (r-\tilde{r})^2 (2r+\tilde{r}) \quad (2.4.16)$$

The constant c_2 can be found from a fluid mass balance either along $r = RP_3$ or on $z = -[L + R(1-P_3)\tan \phi]$. Either method gives the same result.

Because the core is stagnant, it sustains no pressure gradients within it. If the exhaust gases empty into a large settling chamber, in which the velocities vanish, the pressure in that chamber is the pressure in the stagnant core. Consequently, the only pressure gradient across region IV is the radial gradient outside the core. Using the large swirl approximation and (2.4.11), (2.2.3) becomes

$$-\frac{1}{\rho} \frac{\partial p}{\partial r} = -[c_1(r-\tilde{r})]^2/r \quad (2.4.17)$$

Integrating (2.4.17) between RP_3 and $\tilde{r}$ gives the pressure gradient across region IV in the presence of a stagnant core.

If (2.4.15) fails, there is no stagnant core, and the velocity profiles we assumed above are not appropriate. In this case the flow is not critical at the exhaust, and we assume, for $r < RP_3$,

$$v = c_3 r, \quad (2.4.16)$$

$$w = (c_3 + c_4 a)r + (c_5 + c_6 z). \quad (2.4.17)$$

As above, we match the tangential velocity at the exhaust radius to determine c_3 :

$$c_3 RP_3 = \Gamma / RP_3. \quad (2.4.18)$$

The parameter c_5 , which corresponds to $\tilde{r}$, is obtained from substituting (2.4.17) into (2.4.13), to assure fluid mass conservation. The result is

$$c_5 = 2 \left[\frac{\dot{m}}{2\pi\rho} - P_T |F_T(RP_3)| - \frac{\Gamma RP_3}{3} \right] / (RP_3)^2 \quad (2.4.19)$$

The parameter c_6 has no counterpart when there is a stagnant core. In order to determine it, we require that the axial velocity vanish on the centerline at the bottom of the chamber.

$$w(r = 0, z) = c_5 + c_6 z = 0 \quad \text{at} \quad z = -[L + R(1 - P_3)\tan\phi]. \quad (2.4.20)$$

The total axial flow at the bottom of the chamber equals the fluid flow in the bottom boundary layer. Its radial distribution is not known with any certainty, but we observe that:

- (i) if the swirl, ψ , is very high, a stagnant core forms on the centerline and the vertical velocity vanishes there;
- (ii) if ψ is very small, the flow in the bottom boundary layer is very small and the vertical velocity vanishes along the bottom of region IV.

The requirement (2.4.20) is consistent with these two limiting cases. With c_5 known from (2.4.19), (2.4.20) can be solved for c_6 . The other parameter, c_4 , is not required to compute the pressure, but can be obtained by requiring fluid mass conservation, as was done below (2.4.16).

When no stagnant core exists, the total pressure drop across region IV is obtained in two steps. First, the pressure drop from region III to the centerline in region IV, using the large swirl approximation, is obtained from integrating

$$-\frac{1}{\rho} \frac{\partial p}{\partial r} = -\frac{v^2}{r} = -\frac{\Gamma^2}{(RP_3)^4} r \quad (2.4.21)$$

Next, the pressure drop along the centerline, where u and v both vanish, is obtained by integrating

$$-\frac{1}{\rho} \frac{\partial p}{\partial z} = w \frac{\partial w}{\partial z} = (c_5 + c_6 z)c_6 \quad (2.4.22)$$

over the length of the chamber. The pressure drop across region IV is the sum of these two parts.

3. ANALYSIS OF THE CYLINDRICAL WALL LAYER

3.1 Introduction

The modeling discussed in the previous section has enabled us to isolate the regions of critical importance in determining the performance of the cyclone combustor. The assumptions that have been made in the simplified analysis for these regions should, therefore, be examined in detail and a more comprehensive analysis of the flowfield should be carried out to reduce the empiricism that was introduced in the initial analysis.

Region I, the cylindrical wall boundary layer appears to be the most important region in these cyclone combustors as has been discussed in the previous section. Almost all the combustion takes place in this region. It is a region of high particle loading and the dynamics of the wall layer are important in determining the flowfield in the rest of the cyclone. The ash separation and slagging which are crucial in the successful operation of these combustors, also occur in this region. The slag has to be maintained at an optimal temperature for proper removal, and this temperature is determined by the heat transfer from region I. These processes underline the importance of this region.

The current analysis of this region does not include all of these effects--in fact, it is only a first step towards the proper analysis. However, it enables us to reduce somewhat the empiricism in the selection of the model parameters used in the simplified analysis. In particular, the analysis of the cylindrical wall layer enables us to calculate the total boundary layer thickness, δ and the skin friction, c_f as a function of a single parameter, a wall roughness scale, y_0 . In the simple analysis, δ and c_f are independently assumed parameters (in our sample calculations, we used $\delta = 0.1 R_0$ and $c_f = .005$). In actuality, δ and c_f are intimately tied together and for a rough wall boundary layer depend on the roughness scale y_0 , and other geometrical and flow parameters of the combustor.

3.2 Basic Equations

We present a two-phase boundary layer analysis of the cylindrical wall flow. The analysis follows generally the procedure outlined in Lewellen & Stickler (1972), with some modifications and improvements.

The problem basically involves the solution of a periodic boundary layer flow. The structure of the flow is governed by the inlet stream (wall jet) and the periodic boundary conditions at the stations $x = 0$ and $x = \ell$. The flowfield is sketched in Fig. 5. Previous studies (Lewellen and Stickler, 1972) have demonstrated that the lower layer of the wall jet (the region below the point of maximum velocity of the jet profile) plays a decisive role in determining the wall skin friction c_f and the recovery factor r_f of the vortex flow. A reasonable analysis of this layer should enable us to obtain a satisfactory estimate of the total boundary layer thickness δ and the recovery factor r_f .

The equations for the two-phase boundary layer are (2.3.1) - (2.3.8). The integral momentum equation for the wall layer can be derived to be,

$$(\rho + \rho_p) \left[\frac{d}{dx} \int_0^{\delta_1(x)} U^2 dy - U_m \frac{d}{dx} \int_0^{\delta_1(x)} U dy \right] = -\tau_w \quad (3.2.1)$$

The derivation of this equation involves a number of minor assumptions that are discussed by Lewellen and Stickler (1972). There are also two important assumptions that are discussed below.

The current analysis assumes that ρ_p is constant inside the boundary layer region δ ($\frac{\delta}{R_0} \ll 1$). In the actual flow, due to the centrifugal force, the particle density will probably be highest near the wall and will decrease towards the edge of the boundary layer. However, the exact variation of the particle density across the boundary layer is not easy to predict. It is complicated by the interaction of the particles with the turbulent flowfield, and the unstable mixing processes that occur

in the region immediately downstream of the inlet stream. In this region we have a high particle density main boundary layer flow on top of the low particle density inlet flow. This is an unstable flow configuration and rapid turbulent mixing of the two streams will take place leading to a fairly uniform particle loading for some distance downstream. The centrifugal forces due to the swirling flow and the turbulent entrainment processes in the boundary layer will then re-establish an equilibrium particle density ρ_p distribution. Calculation of this variation of ρ_p in both x and y directions can best be done using a finite difference solution of the partial differential equations. The use of a constant ρ_p simplifies the current analysis significantly. It is recognized that the effect of the variation of ρ_p needs further investigation.

The other major assumption concerns the free stream velocity $U_m(x)$ for the wall layer analysis. In a detailed two-dimensional analysis of the boundary layer one would have $U_m(x) = U_i$, the injection velocity at $x = 0$, and $U_m(x) = U_c$, the free stream recovery velocity at $x = \ell$. However, for the current simplified analysis, the initial wall jet velocity corresponds to the velocity $\hat{U}$ that results from the initial rapid mixing of the two streams. The calculation of $\hat{U}$ is illustrated later on in this section, and is somewhat more rigorous than the procedure used by Lewellen and Stickler. We further assume that the spatial variation of $U_m(x)$ is adequately accounted for in the analysis by taking U_m to be a constant equal to the geometric mean value between $\hat{U}$ and U_c , that is, $U_m^2 = \hat{U} \cdot U_c$. The use of a constant U_m results in significant simplifications in the present integral momentum equation analysis of the cylindrical wall boundary layer region.

3.3 Calculation of the Initial Wall Jet Velocity $\hat{U}$

Consider the mass and momentum balance for the simple control volume shown in the sketch. We assume that a certain mass of particles from the mean boundary layer flow mix immediately with the inlet jet.

Mass balance

$$\rho_i U_i \Delta + \rho_{p_i} U_{c_i} \Delta + \rho_p VX = (\rho + \rho_p) \hat{U} \Delta$$

$$\text{defining } K_o = \frac{\rho_{p_i} U_{c_i}}{\rho_i U_i}, \quad K = \frac{\rho_p}{\rho} \quad \text{and} \quad p_9 = \frac{U_{c_i}}{U_i}$$

$$\rho_i U_i \Delta (1 + K_o) + \rho_p VX = (\rho + \rho_p) \hat{U} \Delta$$

x-momentum balance

$$\rho_i U_i^2 (1 + K_o \cdot p_9) + \rho_p VX U_c = (\rho + \rho_p) \hat{U}^2 \Delta$$

These two equations can be solved to calculate $\hat{U}$.

$$\hat{U} = \frac{1}{2} U_c \pm \frac{1}{2} \sqrt{U_c^2 - 4 \frac{\rho_i}{\rho} \frac{1 + K_o}{1 + K} \left(U_i U_c - \left(\frac{1 + K_o}{1 + K_o} p_9 \right) U_i^2 \right)} \quad (3.3.1)$$

$$U_m = \sqrt{\hat{U} \cdot U_c} \quad (3.3.2)$$

3.4 Calculation of the Boundary Layer Thickness, δ

We now assume logarithmic velocity profiles for a rough wall turbulent boundary layer (Schlichting, 1968). If y_o is the roughness scale for the wall,

$$U = \frac{U^*}{\kappa} \ln\left(\frac{y}{y_o}\right), \quad \kappa \text{ von Karman constant} = 0.39$$

$$U^{*2} = \frac{\tau_w}{(\rho + \rho_p)}$$

$$\text{At } y = \delta_1(x), \quad U = U_m$$

$$U = U_m \cdot \ln\left(\frac{y}{y_o}\right) / \ln\left(\frac{\delta_1}{y_o}\right) \quad (3.4.1)$$

$$\tau_w = \kappa^2 (\rho + \rho_p) U_m^2 / \ln^2\left(\frac{\delta_1}{y_o}\right) \quad (3.4.2)$$

substituting (3.4.1) and (3.4.2) in (3.2.1), and completing the integration, one obtains the following differential equation

for $\delta_1(x)$.

$$\left[\frac{3\delta_1 + y_0}{\delta_1} - \ln\left(\frac{\delta_1}{y_0}\right) - \frac{4}{\ln\left(\frac{\delta_1}{y_0}\right)} + \frac{4y_0}{\delta_1 \ln\left(\frac{\delta_1}{y_0}\right)} \right] \frac{d\delta_1}{dx} = -\kappa^2 \quad (3.4.3)$$

The boundary conditions are,

$$\begin{aligned} \text{At } x = 0 \quad \delta_1 &= \frac{\Delta}{2} \\ x = \ell \quad \delta_1 &= \delta \end{aligned} \quad (3.4.4)$$

Equation (3.4.3) can be integrated subject to the boundary conditions in Eq. (3.4.4) to obtain an expression for δ . Define the nondimensional variables,

$$\hat{\delta} = \frac{\delta}{y_0}, \quad \hat{\Delta} = \frac{\Delta}{y_0}, \quad \hat{\ell} = \frac{\ell}{y_0}$$

Then,

$$\begin{aligned} 4\left(\frac{\hat{\Delta}}{2} - \hat{\delta}\right) + \left(\ln \frac{\hat{\Delta}}{2} - \ln \hat{\delta}\right) - \left(\frac{\hat{\Delta}}{2} \ln \frac{\hat{\Delta}}{2} - \hat{\delta} \ln \hat{\delta}\right) \\ - 4 \int_{\frac{\hat{\Delta}}{2}}^{\hat{\delta}} \frac{d\hat{\delta}_1}{\ln \hat{\delta}_1} + 4 \left(\ln \left| \ln \frac{\hat{\Delta}}{2} \right| - \ln \left| \ln \hat{\delta}_1 \right| \right) = \kappa^2 \hat{\ell} \end{aligned} \quad (3.4.5)$$

Equation (3.4.5) is solved in the program subroutine DELTA to determine the total boundary layer thickness for a given roughness scale y_0 and the geometrical parameters of the cyclone. The integral of the $1/\ln \hat{\delta}_1$ term is evaluated by an asymptotic series for the exponential integral as given in Bateman (1953).

3.5 Calculation of the Skin Friction c_f and the Recovery Factor r_f

The overall x-momentum balance condition for the boundary layer region is used to calculate the skin friction and the recovery factor,

$$\text{mass inflow } \dot{m}_t = \rho_i (1 + K_0) \Delta U_i$$

$$\text{momentum inflow} = \rho_i (1 + K_0 \cdot p_9) \Delta U_i^2$$

$$\text{momentum outflow} = \dot{m}_t U_c$$

The overall x-momentum balance is,

$$\rho_i(1+K_o \cdot p_9) \Delta U_i^2 = \rho_i(1+K_o) \Delta U_i U_c + \int_0^{\ell} \tau_w dx$$

$$\rho_i(1+K_o) \Delta \left(U_i^2 \frac{1+K_o \cdot p_9}{1+K_o} - U_i U_c \right) = \int_0^{\ell} \tau_w dx \quad (3.5.1)$$

From (3.2.1)

$$\int_0^{\ell} \tau_w dx = -\rho(1+K) U_m^2 y_o \left[\left(\frac{\hat{\Delta}}{2} + 1 \right) \frac{1}{\ln \frac{\hat{\Delta}}{2}} - \frac{\hat{\delta}+1}{\ln \hat{\delta}} \right]$$

$$+ 2 \left(\frac{\hat{\delta}-1}{\ln^2 \hat{\delta}} - \frac{\frac{\hat{\Delta}}{2}-1}{\ln^2 \frac{\hat{\Delta}}{2}} \right) \quad (3.5.2)$$

Consistent with (2.1.19) we define

$$\frac{c_f}{2} = \frac{\int_0^{\ell} \tau_w dx}{\rho_i(1+K) U_c U_i \ell} \quad (3.5.3)$$

The program subroutine SKNFR calculates the skin friction $CW = C_f/2$ using (3.5.2) and (3.5.3), for an assumed value of the recovery factor $r_f = U_c/U_i$ and the particle loading K . The subroutine SKNFR is part of the iteration loop for the determination of K and r_f . The program calculates CW for the cylindrical wall layer. If the skin friction for the top and bottom wall layers, CT and CB are input as negative numbers, SKNFR also sets these values equal to CW .

The expression for the recovery factor can now be obtained by using (3.5.3) in (3.5.1).

$$r_f = \frac{U_c}{U_i} = \frac{\left(\frac{1+K_o \cdot p_9}{1+K_o} \right)}{1 + \frac{c_f}{2} \frac{(1+K)}{(1+K_o)} \frac{\ell}{\Delta}} \quad (3.5.4)$$

which is consistent with (2.1.21).

3.6 Calculation of the Heat Losses

The fluid and particle energy equations have been partially decoupled from the momentum equations by the assumption that the chamber temperature is known. An overall energy balance equation including heat losses and combustion inefficiencies could be used to determine the chamber temperature.

However, due to the fuel-rich-gasifier mode in which the combustor operates, a detailed equilibrium chemistry program is required as a part of such an energy balance. In lieu of this, the present program is to be run iteratively with the equilibrium chemistry program currently operational at PERC.

The current A.R.A.P. program requires the chamber temperature T_c (first guess, say from the equilibrium chemistry program) to be a specified input quantity. The subroutine HTRAN will be used to calculate the convective heat transfer losses to the slag wall and the radiative heat transfer losses from the particles in the flame zone. The radiation from the gases can be calculated following present procedures used at PERC. The total energy losses can be used to improve the estimate of the chamber temperature, which can then be used as a new input to the cyclone combustor program. The process should converge in a few iterations.

The procedure used for the calculation of the convective heat transfer and the particle radiation from the flame is now described.

3.7 Convective Heat Transfer

In the absence of a detailed analysis of the thermal boundary layer, we assume Reynolds analogy to be applicable and compute

$$\frac{Q_{\text{conv}}}{A_w} = \frac{c_f}{2} \text{Pr} \rho (1+K) U_c C_p (T_c - T_w) \quad (3.7.1)$$

In the above formula, Pr is the turbulent Prandtl number. We take $\text{Pr} = 0.75$ on the basis of our second-order closure models for turbulent flows. The slag wall temperature T_w is an input variable and has been taken to be 1533°K (2300°F) in the present calculations.

A number of comments on the above formula are in order. It should be noted that Reynolds analogy is not strictly valid for heat transfer on rough walls and on curved surfaces. Schlichting (1968) points out that the percent increase in the heat transfer coefficient may be smaller than the increase

in skin friction on rough surfaces. The effect of stream-line curvature on skin friction has been shown (Meroney and Bradshaw, 1975) to be quite important, for example, a 1 percent curvature could result in a 10 percent change in skin friction, and similar magnitude changes in heat transfer would be expected. These effects of roughness and curvature on the heat transfer coefficient and skin friction could be determined using A.R.A.P.'s complete second-order closure analysis of turbulent flows, but such an analysis falls outside the scope of the present simple model.

The other uncertainty in the prediction of the convective heat transfer is in the effect of the high particle loading on the heat losses. The calculation of the convective heat transfer uses the full particle loading to determine the gas density in the neighborhood of the wall. A simple calculation shows that the thermal equilibration time of the particles, $\tau_T = m_p c_p / 4\pi a k$ is of the same order of magnitude as the velocity equilibration time $\tau_v = m_p / 6\pi a \mu$. This supports the use of the full particle loading in the calculation. However, the details of the flow in region I, for example, particle impact on the walls, actual particle temperatures etc. are expected to affect the heat transfer. In the actual flow, the particle loading is expected to be highest near the wall and will decrease near the edge of the boundary layer. Both the present analyses consider uniform particle loading throughout the boundary layer. The boundary layer analysis predicts a larger boundary layer thickness than was assumed in the simplified analysis and this leads to a significant difference in the particle loading K predicted by the two analyses. The consequence is that the convective heat transfer calculated by the two procedures is different as discussed in Section 4. A variable loading analysis will probably predict more comparable values of the convective heat transfer.

3.8 Radiative Heat Transfer from Particles

Due to high particle loading in the flame region of a cyclone combustor it is expected that radiative heat transfer from the solid particles will be a major source of heat loss in these devices.

The net radiative loss from particles in the flame to an enclosing wall is given by

$$\frac{Q_{\text{radp}}}{A_w} = \sigma(T_c^4 - T_w^4)\epsilon_F\epsilon_w \quad (3.7.2)$$

$$\sigma = 1.355 \times 10^{-11} \text{ kcal/m}^2\text{sec } ^\circ\text{K}^4$$

In all the current calculations we have assumed,

$$T_c = 2200^\circ\text{K} (3500^\circ\text{F})$$

$$\epsilon_w = 0.8$$

$$T_w = 1533^\circ\text{K} (2300^\circ\text{F})$$

$$A_w = 2\pi R_o L, \text{ area of the flame envelope.}$$

These quantities are input parameters in the subroutine HTRAN.

The calculation of the emissivity of the flame, ϵ_F is one of the critical parameters in the computation of the radiative heat transfer. We have used two procedures to estimate ϵ_F . The first is the use of an empirical correlation given by Brown and Marco (1951). The second is a simple model developed at A.R.A.P. which calculates the emissivity based on the ratio of the total surface area of the coal and ash particles in the combustor to the wall area. The second calculation assumes that there is no self-shielding by the coal and ash particles and, therefore, is expected to overestimate the emissivity of the flame when the emissivity is not small. The simple model developed by us is discussed in Appendix D.

Brown and Marco (1951) propose that the emissivity of a powdered coal flame is given by the expression,

$$\epsilon_F = 1 - \exp(-x) \quad (3.7.3)$$

where

$$x = \frac{238 \cdot L \cdot \left[(1-V) \frac{\rho_o}{\rho_l} \right]^{2/3}}{g_B \rho_o d_o T_f} \quad (3.7.4)$$

- L = effective flame thickness, feet
 V = volatile + moisture % in coal
 ρ_0, ρ_1 = initial densities of coal and coked coal, respectively
 lbm/ft^3
 ρ_0/ρ_1 = 1.15 assumed in all calculations
 g_B = lbs. of combustion products per lb. of coal in the original formulation. However, for a cyclone combustor where the particle loading becomes large, g_B should be considered to be a measure of the total mass of particles in the flame, $1/Kp$, where p is the pressure in atmospheres.
 d_0 = initial average particle diameter, inches
 T_f = average flame temperature, $^{\circ}\text{R}$.

The empirical formula given above is for a conventional pulverised coal flame. The simplest procedure to apply it to a thin annular flame zone as exists in the cyclone combustor is to define an effective loading as if the coal and ash particles were uniformly distributed over the entire combustor and then use the cylinder diameter as the effective flame thickness. This procedure accomplishes the desired objective of taking into account all the coal particles present in the combustion chamber.

$$\begin{aligned}
 K_{\text{eff}} R_o^2 L &= K [R_o^2 - (R_o - \delta)^2] L \\
 K_{\text{eff}} &= K \left[1 - \left(1 - \frac{\delta}{R_o} \right)^2 \right]
 \end{aligned}
 \tag{3.7.5}$$

In the program calculations, d_0 is taken to be the mass-averaged particle diameter in the coal feed. It must be noted that this empirical correlation is valid for pulverised coal flames where $d_0 \sim 70 \mu\text{m}$. The validity of the extension of the empirical correlation to the larger size coal particles has to be examined. Further, our simple model for the calculation of the emissivity of the flame indicates that ash particle loading in regions I and III may contribute significantly to the particle radiation in the cyclone combustor. The calculations with the Brown and Marco correlation have only used the coal

loading in region I. How well the empirical correlation accounts for the radiation from the small ash particles in a pulverized coal flame is not known. The simple model (see Appendix D) suggests that the flame emissivity should be considerably higher than that predicted by the empirical correlation. The actual numbers predicted by the simple model cannot be considered exact because for $\epsilon_F > 0.3$ we expect some interference to be present among the particles. The Brown-Marco empirical correlation has been used in the program in the absence of anything more appropriate at this time, and should not be considered as a definitive radiation model for cyclone combustor applications.

The heat transfer computations are done in the subroutine HTRAN. The convective and radiative heat transfer losses are expressed as fractions of the flux of sensible enthalpy of the chamber gases, Q_c

$$Q_c = \dot{m}_t \cdot C_p \cdot T_c \quad (3.7.6)$$

The results of these calculations are discussed in Section 4. The radiative heat transfer from the particles is seen to be the dominant heat loss mechanism for the atmospheric cyclone combustor in most of the test cases, even when the emissivity values used are estimated conservatively. The radiative transfer from the combustion gases could be added to obtain total heat transfer loss (the procedures being used by PERC appear to be adequate for this contribution). This gas radiation contribution has not been included in the present program for two reasons. First, it is expected to be smaller than the uncertainty in the particle radiation; and second, it can best be accomplished after the precise gas composition is determined by the equilibrium chemistry program. The total heat transfer loss has to be compared to the total energy released in the combustor to obtain a better estimate of the final chamber temperature for use in a subsequent iteration step.

4. MODEL VERIFICATION

The complete A.R.A.P. computer program for the modeling of the cyclone combustor has been used to perform a series of computations to: (1) demonstrate the sensitivity of the model to variation of certain parameters (the parameters selected for these studies were the ones about which there was some uncertainty as to the correct value to be used or the parameters for which the model assumptions were the least certain); (2) compare the model predictions for key performance parameters such as blowby, carryover, pressure loss and heat transfer to experimental measurements on the atmospheric cyclone combustor as performed at the Pittsburgh Energy Research Center. In general, the model predictions are in reasonable agreement with the data. The model predictions also track the expected (and measured) trends in the cyclone combustor performance for a variety of changes in the input conditions.

4.1 Input Parameters for Atmospheric Cyclone Computations

The various parameters are listed in the order that they appear in a typical input deck for the cyclone combustor program.

Card 1

LFDEL These logical flags are set at 1 for a computation
 LFSKN that uses the subroutines for the integral momentum
 LFHTR equation analysis for the calculation of δ , c_f and
 the heat transfer. Otherwise, they are set at 0,0,1.

Card 2

RO = .1524

Run	126	137	148	149	151
EMF $\frac{\text{kg}}{\text{sec}}$	.202	.091	.090	.0623	.0597
RHOI $\frac{\text{kg}}{\text{m}^3}$	.281	.287	.320	.292	.301
TI °K	1261	1232	1105	1210	1175

ROCOL = 1344 kg/m³

VIS = .6548 x 10⁻⁴ kg/m sec

TBAR = .200 sec

.100 sec (test run 151)

Cards 3, 4 (2 cards each required)

The coal mesh analysis has typically been divided into seven groups (six for run 151).

particle radii, μm		500	300	150	80	60	45	25
mass frac- tions	126	.080	.170	.140	.120	.100	.100	.290
	137	.090	.290	.320	.060	.050	.050	.140
	148	.135	.210	.273	.053	.069	.080	.180
	149	.115	.330	.314	.048	.031	.068	.094
particle radii, μm		300	170	80	64	45	25	
mass fraction	151	.010	.623	.056	.069	.075	.167	

Card 5

$$P1 = .04468$$

A value of .03351 was also used in run 151 to simulate a 25 percent area reduction due to slag blockage.

$$P2 = 3.166$$

$$P3 = 0.5$$

$$P4 = 0.1$$

$$P5 = 4.93$$

$$P6 = 0.1666$$

$$P7 = 0.1$$

$P7 = \delta/R_0 = 0.1$ has been used for all the test calculations with the simplified analysis. When $LFDEL = 1$, $P7$ is calculated from the subroutine DELTA using the analysis of Section 3. The empiricism then shifts from a choice of δ and c_f to the choice of the roughness scale y_0 , which itself is not easy to estimate at present. Further, in many cases the inlet geometry will have a significant influence on δ , and all these features may not be included in the current theoretical analysis for the calculation of δ . It may, therefore, be easier in many cases to estimate directly a suitable δ for the calculations.

	Run	126	137	148	149	151
P8		.573	.560	.502	.550	.534

Card 6, 7

$$P9 = 1. , 0.$$

The initial coal velocity depends on the inlet configuration and would be between these limiting values.

$$P10 = 0.11$$

$$P11 = 0.38 , 0.00$$

The amount of volatiles that evolve from the coal particle at entry to the cyclone will range between zero and the total amount of volatiles in the coal. The two limiting cases were examined.

$$\left. \begin{array}{l} CT \\ CB \\ CW \end{array} \right\} = .0025$$

If LFSKN = 1 , CW is calculated in the subroutine SKNFR.

If CT and CB are input as negative numbers, they are set equal to CW in SKNFR.

	Run	126	137	148	149	151
K_o		.139	.132	.181	.257	.275

$$PT = 0.5 \text{ (0.4 in Run 148 as a test)}$$

$$PB = 0.3$$

PT and PB are empirical parameters. They have been set at these values to match the coal blowby and ash carryover data in the atmospheric cyclone experiments. This matching was done with the predictions obtained with the choice of $c_f = .005$ and $\delta = 0.1R$. The results obtained from the cylindrical boundary layer analysis (for a reasonable guess of $y_o = 25 \mu m$) are somewhat different and would require a somewhat lower value of both PT and PB to match the data.

$$PHI = 0 .$$

Card 8

$$Y0 = 2.5 \times 10^{-5} , 5.0 \times 10^{-5}$$

Computations have been made with roughness scales of 25 and 50 μm which corresponds to a physical surface roughness of

750 to 1500 μm . The former figure corresponds to the size of the largest coal particles in the feed.

NOP = 1, 2

The number of ports is 1 for the experiments with the scroll inlet (148, 149, 151) and 2 for runs with two coal inlets (126, 137).

Card 9

DELS 0.0
DELINC 0.1

These inputs are merely required to start the iteration procedure to calculate δ .

Card 10

PR 0.75
CPC kcal/kg 0.30
TW $^{\circ}\text{K}$ 1533.
TC $^{\circ}\text{K}$ 2200.
EMSW 0.8

4.2 Sensitivity to Model Assumptions

A number of the input parameters are not accurately known in these test calculations and we have carried out a sensitivity analysis for the major performance parameters of the cyclone combustor to the possible range of values of these input parameters.

Effect of the initial coal velocity. The input parameter $P9 = U_{ci}/U_i$ is expected to be between 0. and 1. In the experimental runs 126 and 137 the coal inlet is separate from the main air inlet and P9 should be near 0 . In the scroll inlet runs 148, 149 and 151, P9 depends on the time available in the inlet ducts prior to entry into the cyclone for the coal to accelerate up to the air stream velocity, and is expected to be closer to 1 . P9 is not a very crucial parameter as the total mass of coal is typically below 25 percent of the total fluid flow. The table below shows the sensitivity of the predictions to P9 for a number of computations. The conclusion

is that P9 has a very minor effect on the results and can be selected equal to the appropriate limiting value in the absence of other detailed information.

		Blowby	Carryover	$\Delta p/p$
126	P9 = 0	.134	.47	.0337
	= 1.	.142	.31	.0389
137	= 0	.149	.443	.0064
	= 1.	.161	.464	.0072
148	= 0	.117	.45	.0051
	= 1.	.131	.42	.0056
149	= 0	.10	.391	.0026
	= 1.	.12	.416	.0029
151	= 0	.042	.286	.0027
	= 1.	.053	.274	.0035

The apparent large change in the carryover for run 126 is really due to the breakup of the input coal feed into a finite number of groups. The critical particle size for carryover due to the turbulent entrainment mechanism, ACRT just changes enough (from 23.5 μ to 25.8 μ) to include a large fraction of the smaller particle size group in the carryover loss. A more detailed feed analysis would be expected to predict a change in the carryover of the same order of magnitude as in the other tests. The three critical particle sizes AMX, AMN and ACRT should always be examined and compared to the average particle sizes in the input feed groups to uncover similar problems in other computations.

Effect of the fraction of the volatiles evolved. The input parameter P11 reflects the mass fraction of the volatiles in the coal that have evolved in the inlet ducts or are instantaneously released at entry into the cyclone combustor. The model is further discussed in Appendix C. P11 ranges from 0 to 0.38, the total amount of volatiles present in the coal. The results of this sensitivity analysis are shown in the following table. The calculations are a little more sensitive

to the choice of P11 than they were to the choice of P9, but again the sensitivity is small compared to the other uncertainties in the model assumptions and in the experimental measurements.

		Blowby	Carryover	$\Delta p/p$
126	P11 = 0 P11 = 0.38	.127 .142	.46 .31	.0330 .0389
137	P11 = 0 P11 = 0.38	.137 .161	.421 .464	.0063 .0072
148	P11 = 0 P11 = 0.38	.105 .131	.43 .42	.0053 .0056
149	P11 = 0 P11 = 0.38	.09 .12	.402 .416	.0029 .0029
151	P11 = 0 P11 = 0.38	.045 .053	.251 .274	.0031 .0035

Effect of roughness scale Y_0 . The roughness scale Y_0 appears in the logarithmic rough wall velocity profile and a factor of 2 increase in Y_0 will not be expected to lead to significant changes in the predictions. The table below compares the results of these tests. The boundary layer thickness δ and the skin friction CW do change to some extent, but the blowby, carryover and pressure drops are virtually unchanged.

P9 = 1. P11 = 0.38 NoP = 1 CB = CT = .0025						
Run	Y_0	δ	CW	Blowby	Carryover	$\Delta p/p$
148	25 μ	.291	.00176	.169	.482	.0084
	50 μ	.333	.00219	.167	.481	.0081
149	25 μ	.291	.0018	.182	.474	.0047
	50 μ	.333	.00222	.179	.470	.0046
151	25 μ	.291	.00188	.112	.412	.0041
	50 μ	.333	.00234	.109	.408	.0039

In the above runs, CB and CT are not changed from the values used in the simplified analysis. If CB and CT are set equal to the value of CW obtained from the boundary layer analysis, the results are:

Run	Y_0	δ	CW=CB=CT	Blowby	Carryover	$\Delta p/p$
148	25 μ	.29	.00168	.145	.444	.0087
151	25 μ	.29	.00185	.089	.365	.0042

Thus, setting the skin friction on the top and bottom walls to be the same as that on the cylindrical wall layer slightly reduces the blowby and the carryover, because for the assumed roughness, the boundary layer analysis predicts a lower skin friction than was assumed in the simple analysis.

The roughness scale of 25 μ m based on the largest particle sizes in the coal feed appears to be the more reasonable estimate in the absence of a detailed analysis of the dynamics of the slag layer surface.

Comments on the boundary layer analysis. The simplified analysis assumes $DEL = \delta/R_0 = 0.1$ and $CW = .0025$. Calculations using a roughness scale $Y_0 = 25$ m in the detailed boundary layer analysis predict the following results.

Run	NOP	DEL	CW
126	2	.167	.00204
137	2	.167	.00195
148	1	.291	.00176
149	1	.291	.00179
151	1	.291	.00188

The detailed boundary layer analysis predicts values of DEL and CW that are intimately coupled together and cannot be selected independently. The DEL and CW values in the runs with two inlet ports are quite comparable to the values assumed in the simplified analysis. However, the boundary layer analysis for the scroll inlet configuration predicts $\delta \sim 0.3 R_0$, and lower CW values than used in the simplified analysis. The predicted δ values are not too surprising for

this case as the scroll inlet width is quite large with the initial size of the wall jet, $\Delta = .09 R_o$. However, when the boundary layer thickness is of this size, the assumption of uniform loading across the entire thickness of the boundary layer is highly suspect. In the actual flow, the loading will be higher at the wall and this will result in the prediction of smaller δ . The results for the blowby and carryover predicted by the current boundary layer analysis are somewhat higher than the data. The simplified analysis for the scroll inlet tests still assumes $\delta = 0.1 R_o$. The geometry of the inlet in this case, that is, $\Delta \sim 0.09 R_o$ suggests that δ should perhaps be chosen somewhat larger for these calculations ($\sim 0.15 R_o$) and the other empirical parameters (for example, P_T , P_B) adjusted to match the experimental data. The results for the specific test runs are discussed later on in this section.

Effect of combustion time. The combustion time for a 100 μm coal particle at atmospheric pressure has been taken to be 200 m sec in most of the calculations. The discussion in Appendix C suggests that due to oxygen enrichment and the high temperature conditions in the cyclone combustor, the combustion time may be somewhat lower. The sensitivity of the model predictions to a smaller combustion time has been tested in calculations corresponding to the experimental run 151.

Run 151	P9 = 1.0	P11 = 0.38	
TBAR	Blowby	Carryover	$\Delta p/p$
200 m sec	.078	.378	.0030
100 m sec	.053	.272	.0035

4.3 Comparison of Model Predictions with Experimental Measurements

Predictions for four basic cyclone combustor performance parameters are compared to the results of the experimental measurements on the atmospheric cyclone combustor test facility at the Pittsburgh Energy Research Center (Private Communication, 1976, 1977).

Carbon Loss in Blowby. The computer program computes the percentage of the fixed carbon that is lost from the combustor

as blowby through the slag port at the bottom of the cyclone.

The experimental measurement reports the total mass of coal collected through the slag tap and in some cases, the percentage of carbon in the blowby. We have calculated the percentage of fixed carbon in the blowby from the experimental data to compare to the model predictions. We assume the percentage of fixed carbon in the coal to be 0.495 for all the runs and carbon in blowby to be 0.83 unless a detailed analysis is available. The results are tabulated below.

Experimental Run Numbers	Percentage of Fixed Carbon in Blowby			Comments
	Measured	Calculated		
		$\frac{c_f}{2}=.0025, \delta=0.1R_o$	$y_o=25\mu m$	
126,127	.221, .102	.135	.152	P9=0,P11=0.38
137,138,141	.26,.24, .088	.149	.176	P9=0,P11=0.38
148	.114	.131	.169	P9=1,P11=0.38
149	.134	.122	.182	P9=1,P11=0.38
151	.022	.078	.112	P9=1,P11=0.38
151(TBAR= 100 m sec)	.022	.053	.0695	P9=1,P11=0.38

The initial experimental runs have problems of repeatability as shown in the variation of blowby for apparently identical run conditions. Except for test run 151, the agreement of the experiments and the model predictions is quite satisfactory with the assumed empirical parameters. Run 151 has a much finer coal feed and the model correctly predicts the trend towards lower blowby. However, the measured value of the blowby is extremely low. The model predictions for other parameters of run 151 are also not in good agreement and at the present time there is no clear explanation of this anomaly. If we reduce the combustion time to 100 m sec, the predicted blowby for run 151 is still .053, that is, higher than the measurements. The

predictions from the boundary layer analysis are usually higher than the predictions from the simplified analysis and it appears likely that empirical parameters like P_T and P_B will have to be modified to secure better agreement with data, if the boundary layer analysis is used.

Ash carryover. The experimental results for the ash carryover are taken to be 1 - fraction rejected in slag port, as the actual ash carryover measurements in the exhaust ducts appear to account for only a small fraction of the total carryover.

The model predictions and the experimental results are tabulated below. The agreement between the experimental data and the model predictions is quite satisfactory. The low value for the calculation for $Y_O = 25 \mu\text{m}$ for run 126 is artificial and simply due to the ACRT being just above the $25 \mu\text{m}$ particle group, as has been discussed before. The predictions using the specified roughness are normally higher than those for $c_f/2 = 0.0025$, $\delta = 0.1 R_O$, similarly to the results for the blowby.

Experimental Run Numbers	Carryover			Comments
	Measured	Calculated		
		$\frac{c_f}{2}=.0025, \delta=0.1R_o \quad y_o=25\mu m$		
126,127	.462, .521	.472	.33	P9=0,P11=0.38
137,138,141	.411,.52, .47	.442	.484	P9=0,P11=0.38
148	.37	.418	.482	P9=1,P11=0.38
149	.30	.415	.474	P9=1,P11=0.38
151	.30	.378	.412	P9=1,P11=0.38
151(TBAR= 100 m sec	.30	.272	.310	P9=1,P11=0.38

A test calculation was made for run 148 to evaluate the effect of decreasing P_T to 0.4 . The results are tabulated below.

Ash Carryover				$P_B = 0.3$	
Run 148	Experimental Value	Analysis $\frac{c_f}{2} = .0025 \quad \delta = 0.1R_o$		Analysis $y_o = 25\mu m$	
		$P_T = 0.5$	$P_T = 0.4$	$P_T = 0.5$	$P_T = 0.4$
	.37	.418	.363	.482	.427

As expected, reducing P_T reduces the carryover, but the carryover reduction is smaller than the relative change in P_T .

Heat losses to the walls. The heat losses to the wall also have been calculated using either an assumed wall roughness or assumed values of c_f and δ . The calculations have only been made for the convective heat transfer and the radiation due to the particles using the Brown-Marco empirical correlation for the flame emissivity. The radiation losses due to the gas species should still be added to the present calculations to obtain the total heat loss to the wall.

The calculation of the convective heat transfer has some uncertainty as regards to the proper treatment of the heat loss due to the particle impact on the walls, and due to the assumption of uniform particle loading in both the analyses, as was discussed in Section 3.

The particle radiation loss most likely is underpredicted by the use of the Brown-Marco empirical correlation as was discussed in Section 3. Even so, it is the dominant source of heat transfer losses in the atmospheric cyclone combustor.

The calculated results in the program are expressed as a fraction of the sensible enthalpy of the chamber gases (3.7.6). This is different from the normalization used in the experimental data and the calculated results have been modified to be on the same basis as the measurements in the second table. The results are tabulated below.

Calculated Results (fraction of Q_c)						
Run	$\frac{c_f}{2} = .0025, \delta = 0.1R_o$			$y_o = 25\mu m$		
	Q_{conv}	Q_{radp}	Q_{tot}	Q_{conv}	Q_{radp}	Q_{tot}
148	.0714	.0922	.1636	.0291	.0540	.0831
149	.0881	.1694	.2575	.0383	.0958	.1341
151	.0657	.1420	.2077	.0306	.1117	.1423

Run	Experimental Results	Calculated Results (fraction of inlet sensible enthalpy + fuel heating value)	
		Q_{tot}	
		$\frac{c_f}{2} = .0025, \delta = 0.1R_o$	$y_o = 25\mu m$
148	.172, .159	.0896	.0455
149	.1488	.1125	.0586
151	.1494	.0872	.0597

The radiation heat transfer from the gases is estimated to be of the order of 7000 btu/ft²hr (from PERC calculations for runs 125 and 128). If this figure is used for the other test runs, we estimate that this adds about 2 percent to the heat loss. The total heat loss calculations are still somewhat lower than the measurements. This is probably due to the underestimation of the radiation from the particles. The results from the boundary layer analysis are believed to be low because of the low Q_{conv} estimate resulting from the uniform particle loading assumption, as well as the underestimation of the particle radiation. In conclusion, reducing the uncertainty in the particle emissivity calculation is the most critical factor needed to enable a more reliable estimate of the heat transfer from the PERC cyclone combustor. The heat transfer due to convection and gas radiation together appear to be smaller than the radiation from the solid particles in this combustor.

Pressure drop. The total pressure drop across the chamber is primarily a measure of the fluid flow because it depends only weakly on either the particle dynamics or the combustion. A comparison of measured and calculated pressure drops, therefore, tests primarily the assumptions within the model concerning the fluid flow. In this sense, such a comparison complements the previous comparisons of blowby and carryover, which focused on the particle dynamics.

Out of the experiments conducted at PERC that were discussed above, pressure drops were measured for runs 126, 127, 138, 139, 141, 151. Our comparison of measured and predicted pressure drops is restricted to these experiments. Calculation of the pressure drop in the model is based on (2.3.31), (2.3.33), (2.3.34) and either (2.4.17) or (2.4.21) and (2.4.22). In each of these formulae, $(\Delta p/p)$ is proportional to the square of the fluid flow rate, so the total pressure drop should also be expected to depend on the square of the fluid flow rate.

The result of our comparison is shown in Fig. 6, where we have plotted both measured and predicted pressure drops as functions of the fluid flow rate. The experimental data are numbered, and more than two points for one experiment indicates the variation in pressure drop that was recorded during a "steady-state" experiment. The vertical bars mark the range of results predicted by the model, depending on our choices of P7, P9, and P11, which are not well-known. The curve shown is

$$\Delta p/p = C(\dot{m}_f)^2 \quad (4.3.1)$$

with C chosen to intersect the first set of predictions.

The results show that the predicted pressure drops are consistent with (4.3.1), as are most of the data. Given the scatter in the experimental data (especially, 126 and 127), the general agreement between theory and experiment seems to be quite acceptable. This is not true for 141 or 151, which require special comment. There seems to be no significant difference between the running conditions of experiments 137, 138, and 141, but the pressure drop (and blowby) in 141 disagree with both the

predictions of the model and the measured values in the other two (apparently similar) experiments. Thus, all of the available evidence suggests that the data taken in 141 may not be reliable.

In experiment 151, the incoming fluid was enriched with oxygen, and the pressure drop for this experiment need not correlate with those in the other experiments shown in Fig. 6 (i.e., the factor C in (4.3.1) depends on the chemical composition of the incoming fluid). However, because the oxygen fraction is larger, the same stoichiometry is achieved by increasing K_O , the rate of mass flow rates at the inlet. This change could be expected to increase K , the loading in region I (according to (2.1.15)), to decrease r_f (according to (2.1.21)), and to decrease the pressure drop in region III. Thus, it appears that the effect of oxygen enrichment should be to decrease the pressure drop, whereas the observed pressure drop for 151 is much higher than that for 137 and 138.

Another possible candidate for this discrepancy is the variability of the inlet area, which could decrease if the slag covered part of the inlet port. The predicted range of pressure drops for 151 shown in Fig. 6 includes one computation in which the inlet area was decreased by 25 percent. This change increases the predicted pressure drop, as anticipated, but even this increase does not account for the large pressure drop measured in 151. We are unable to explain the large discrepancy between the predicted and measured pressure drops in 151 at this time.

In conclusion, the model seems to predict pressure drops in agreement with measured values in most of the experiments in the PERC program, although the data shows a great deal of scatter.

5. CYCLONE SCALING

The prime purpose of our model is to provide a tool for proper scaling of cyclone-combustors. In this chapter we consider the sensitivity of the model to a number of variables, but before presenting numerical results we derive some general scaling trends from the equations presented in the previous sections.

5.1 General Scaling Trends

The major parameters measuring the performance of a cyclone combustor-gasifier are the carbon conversion efficiency η_c , slagging efficiency η_s , thermal efficiency η_t , and pressure drop required to drive the unit, i.e., a flow efficiency η_p . The parameters appearing in the model are the losses $(1-\eta)$ for the different efficiencies. It is clear that tradeoffs between these different measures of efficiency are required for a successful design. We present here a rough estimate of how chamber geometry and input flow variables influence these efficiencies.

The blowby, which represents the carbon losses, is principally determined by the ratio of the combustion time of the particle to its residence time in region I. When there is a large spectrum of particle sizes present in the coal inflow, the major parameter affecting blowby is the fraction of input coal larger than the particle size which has its combustion time exactly equal to the average residence time of the particles. From Equations (2.1.1) and (2.1.5) this radius is given by

$$a_c = \left(\frac{p\tau_r}{2C_1} \right)^{1/2} = \left[\frac{pp_1AL}{2C_1p_5m_f r_f (C_T + C_B / \cos \phi)} \right] \quad (5.1.1)$$

This critical particle size is given as one of the model outputs. If we ignore variations in recovery factor which can only be determined by running the complete model and assuming that chamber geometry and inlet temperature are fixed then

$$a_c \sim p(AL/\dot{m})^{1/2} \sim p(L^3/\dot{m})^{1/2} \quad (5.1.2)$$

Blowby may be reduced either by increasing a_c or reducing the fraction of coal particles which are larger than a_c .

In order to determine slagging efficiency, the model estimates the fraction of ash particles that are carried out of the chamber with the exhaust gas and assumes this carryover accounts for the prime slag inefficiencies. The ash carryover has contributions from several mechanisms, not all of which scale in the same way. As discussed in §2.3, one of these contributions correlates strongly with blowby (and with the mass fraction of large particles), while the other three depend on the fraction of particles too small to be separated out of the flow and form slag on the walls.

A simple scaling law can be derived for these last three contributions, but it should be noted that there is another contribution to carryover that scales as blowby.

The small ash particles that contribute to carryover may flow out directly, or may leak from region III or from region I. If the swirl parameter, ψ , exceeds $1/p_4$ (≈ 10), the critical particle size is that which is too small to be effectively separated out of the flow in region I. This a_{crt} , by equations (2.3.23) and (2.2.9), is

$$a_{crt} = \left(\frac{9\mu}{2\rho_s} \frac{RA\rho_1 p_4}{\dot{m} r_f} \right)^{1/2} \quad (5.1.3)$$

Thus under the same scaling conditions as given for equation (5.1.2) this critical particle size will scale as

$$a_{crt} \sim \left(\frac{pAR}{\dot{m}} \right)^{1/2} \sim \left(\frac{pL^3}{\dot{m}} \right)^{1/2} \quad (5.1.4)$$

Carryover may be reduced either by decreasing a_{crt} or by reducing the fraction of coal particles which are smaller than a_{crt} .

By comparing equations (5.1.2) and (5.1.4) it is evident that the spread between a_c and a_{crt} can be increased by increasing the pressure. Thus increasing chamber pressure should

permit a combustor-gasifier to operate with good carbon conversion and slagging over a somewhat broader range of input coal particle sizes.

The pressure drop across the chamber is composed of four separate contributions, as discussed in §2.3 and §2.4. Each of these contributions can be related to the inlet dynamic pressure.

$$\frac{\Delta p}{p} = \frac{\rho_i v_i^2}{2p} \left\{ 1 + r_f^2 \frac{\rho_c}{\rho_i} \left[P_3^{-2} - \left(1 - \frac{\delta}{R}\right)^{-2} \right] + r_f^2 \frac{\rho_c}{\rho_i} (1+K_I) \frac{\delta}{R} + f_e(\psi, P_3, P_1) \right\} \quad (5.1.5)$$

where f_e represents the normalized contributions from equations (2.4.21) and (2.4.22). Thus the pressure drop will, at least roughly, scale as

$$\frac{\Delta p}{p} \sim \left(\frac{\dot{m}}{AP} \right)^2 \sim \left(\frac{\dot{m}}{L^2 P} \right)^2 \quad (5.1.6)$$

It is interesting to note that increasing c_f , which decreases r_f , thus generally decreases pressure drop according to equation (5.1.5). This is in sharp contrast to the normal role of viscous losses in nonrotating flows. However, this should not be interpreted as a recommendation for increasing c_f as a means of improving design. The pressure drop can be reduced more efficiently by increasing the inlet area.

Heat is lost to the chamber walls by convection and by radiation, as discussed in §3 and in Appendix D. There does not appear to be any simple scaling formula that covers both the convection and the radiation, and we are forced to consider each separately. The ratio of convective heat loss to flux of enthalpy, from (3.7.1) is

$$\frac{Q_{\text{conv}}}{\dot{m}_t c_p T_c} = c_w r_f (Pr) \left(\frac{\rho}{\rho_i} \right) \left(\frac{A_w}{A} \right) \left(\frac{1+K}{1+K_o} \right) \left(1 - \frac{T_w}{T_c} \right) \quad (5.1.7)$$

It is important to note that for fixed chamber shape, the effects of changing chamber pressure and chamber size are weak, entering (5.1.7) only through their effects on r_f and K . One might anticipate (incorrectly) that increasing the wall-area of the

chamber would increase the convective heat loss; this reasoning neglects the fact that for similar geometries, increasing the inlet area also reduces the tangential velocity, so that the two effects tend to cancel.

As discussed in §3, radiative heat loss depends strongly on the small ash particles suspended in the chamber. In this sense, improving ash separation, according to (5.1.4), also decreases the radiative heat loss.

Not only do the two mechanisms for heat loss scale differently, but also neither one always dominates the other. In our model simulations, we have found that radiation dominates in the PERC tests, while convection dominates in the CDIF runs.

Some simple conclusions about design changes and scaling can now be made, based on (5.1.2), (5.1.4), (5.1.6) and (5.1.7).

(i) While holding other parameters fixed, one can reduce the blowby by increasing the chamber pressure or chamber size, or by reducing the flow rate. Any of these changes will also decrease the pressure drop, but will increase the fraction of small ash particles in the exhaust gases, and the radiative heat transfer. The total carryover could either increase or decrease, because it has contributions from competing mechanisms, as could the total heat transfer.

(ii) Increasing the angle, ϕ , tends to strengthen the bottom boundary layer, and could reduce the swirl somewhat. The largest effect of this change should be to increase the blowby with a somewhat smaller decrease in the carryover. Small changes in ϕ do not affect the pressure drop or the heat loss.

(iii) The dominant effect of reducing the mass fraction of large coal particles in the inlet flow is to reduce blowby. The dominant effects of reducing the mass fraction of small coal particles in the inlet flow is to reduce carryover, and to reduce the radiative heat loss.

Comparison of equations (5.1.2), (5.1.4), (5.1.6) and (5.1.7) suggests that for a fixed input coal mesh analysis, fixed stoichiometry, fixed inlet temperature and similar chamber

geometry the four efficiencies will remain constant if $pL^{3/2}/\dot{m}^{1/2}$, $pL^3/\dot{m}$, and $\dot{m}/L^2p$ are all constants. These can not all be held constant and permit the mass flow to increase. In fact, an examination of these constraints reveals it is not possible to scale up from a model in such a way that the larger unit will have efficiencies equal to or better than the laboratory model. Either the coal mesh analysis must be shifted, the chamber geometry improved, or some sacrifice in at least one of the efficiencies must be accepted.

5.2 Analysis of Sensitivity to Pressure and Chamber Geometry

In this section we present the sensitivity of the performance parameters to changes in pressure and in the principal geometrical variables. Since a running copy of the program has been supplied to PERC and it is relatively easy to run, we will not attempt to make an exhaustive sensitivity analysis of all possible input variable combinations. Rather we seek to provide an indication of the trends to be expected for possible variations in the configuration of the CDIF first-stage combustor.

For Figs. 7 - 12, we chose a basic configuration that represents a possible candidate for the final CDIF design. The configurations whose performance is predicted in these figures were obtained by varying individually the following design parameters around this basic configuration:

- (a) chamber pressure;
- (b) chamber radius, R ;
- (c) inlet area of the secondary air, normalized with respect to $2\pi R^2$ ($P1 = A_1/2\pi R^2$);
- (d) chamber length-to-radius ratio ($P2 = L/R$);
- (e) ratio of exhaust radius to chamber radius ($P3$);
- (f) angle of bottom wall away from horizontal, ϕ .

For completeness, the values used for the other parameters are listed in Table 5.1.

Figure 7 shows that all four performance parameters improve as the pressure is increased through the range tested. The decreases in blowby and pressure drop were anticipated by (5.1.2) and (5.1.6); the decrease in carryover is small, and

Table 7

Model parameters held fixed in the
sensitivity studies of Figs. 7-12.

EMF = 6.62 kg/sec of air

TI = 1867°K

P4 = 0.10

P5 = 4.9

P6 = 0.167

P7 = .1

P8 = .7713

P9 = .0.

P10 = .16

P11 = 0.38

PT = 0.5

PB = 0.3

CT = 0.0025

CB = 0.0025

K-0 = .246

TC = 2420°K

TW = 1533°K

probably should not be considered significant. This is the only parameter for which all four performance parameters improved simultaneously. The optimal chamber pressure probably should be chosen on the basis of other requirements such as compressor power and/or vessel strength.

Figure 8 shows the expected conflict between performance parameters with carryover increasing as the other three losses decrease with increasing radius. We noted earlier that heat loss does not necessarily increase with increasing chamber size; these runs show that it actually can decrease.

Figure 9 also shows carryover behaving in an opposite manner from the other three performance parameters as inlet area is increased. The trends in this figure are all consistent with what should be expected from the decrease in swirl ratio as the inlet area is reduced.

In Fig. 10 it is the heat transfer trend that is counter to the other three. Apparently the decreased particle loading resulting from the increased chamber length is sufficient to reduce carryover, blowby, and pressure drop, but not sufficient to overcome the increased wall area subject to heat transfer.

Figure 11 shows little or no variation of carryover, blowby, or heat transfer with exhaust radius variation, but as should be expected the pressure drop required to drive the swirling flow through a reduced exhaust radius is significantly increased.

Figure 12 shows that little is to be gained by inclining the bottom end wall of the combustor.

6. CONCLUSIONS AND RECOMMENDATIONS

Appendix A presents the program listing for our phenomenological model of a cyclone combustor-gasifier. The model has been designed for use in estimating how the various measures of performance will vary with changes in size, pressure, mass flow, input coal mesh analysis, and various geometrical shape parameters. We believe this model represents an improvement over previously available empirical scaling laws. Unfortunately, our model still relies on several empirical factors which are only roughly determined. Further attempts to improve the model should work both towards supplying the additional empirical information needed to reduce uncertainty in these factors, and towards supplying the more fundamental analyses required to reduce the required empiricism.

In the previous discussion of the derivation of the model we have indicated where critical assumptions have been made to complete the model. The most critical of these fall into two categories: (a) those surrounding the nature of how the coal particle burns and forms slag on the wall, and (b) the turbulent mixing process which plays a dominant role in determining the detailed particle distributions next to the walls.

The first of these will most likely be resolved empirically, but the second is amenable to an analysis of the turbulent mixing process in the wall layers. A.R.A.P. has been developing a second-order-closure approach to turbulent-transport modeling (Lewellen and Teske, 1975) which could be applied to this problem. This should permit the elimination of the assumption of no loading variation across the boundary layers, the elimination of the arbitrariness in the entrainment mechanism for ash carryover, the inclusion of the time permitted for the coal particle to burn in the end-wall boundary layers, as well as a means of estimating the separation factors P_T and P_B at the exhaust and slag port.

The inclusion of turbulent transport modeling will necessarily greatly increase the complexity of our model. Simultaneously with accepting this complication it would appear consistent to seek a full finite difference solution of the governing partial

differential equations in the critical wall layers and greatly reduce the uncertainties associated with the choice of boundary-layer profiles.

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Fig. 1. Flow regions in combustion chamber

(a) Coordinate system

(b) Normalized velocity profiles

Fig. 2. The upper wall boundary layer

(a) Weak end wall boundary layers

(b) Strong end wall boundary layers

Fig. 3. Fluid flow in region III

Fig. 4. Streamtube of rotating flow

Fig. 5. Schematic design of cylindrical wall layer

Fig. 6. Pressure drop as a function of fluid flow rate

Fig. 7. Sensitivity of design performance parameters to chamber pressure, p

Fig. 8. Sensitivity of design performance indicators to combustor radius, R

Fig. 9. Sensitivity of design performance indicators to variation of inlet area parameter, $P1$

Fig. 10. Sensitivity of design performance indicators to variation of combustor length-to-radius ratio, P_2

$R = .483 \text{ m}$
 $P1 = .023$
 $P2 = 3$
 $\phi = 0$
 $p_i = 6.7 \text{ atm}$

Fig. 11. Sensitivity of design performance indicators to variation of exhaust radius parameter, $P3$

$R = .483 \text{ m}$ $P3 = .5$
 $P1 = .023$ $p_i = 6.7 \text{ atm}$
 $P2 = 3$

Fig. 12. Sensitivity of design performance indicators to angle of inclination combustor bottom wall, ϕ

APPENDIX A

Computer Model Listing

C ERDA CYCLONE COMBUSTOR
 C 1 PARAMETERIZE THE PROBLEM
 C THERE ARE SEVEN DIMENSIONAL QUANTITIES IN THE PROBLEM. ALL
 C OTHER VARIABLES ARE MADE DIMENSIONLESS RELATIVE TO THESE SEVEN.
 C A) R0 IS THE OUTER RADIUS OF THE CHAMBER, IN METERS.
 C B) MASS FLOW RATE OF FLUID, EMF, IN KILOGRAMS/SECOND.
 C C) INLET FLUID DENSITY, RHOI, IN KILOGRAMS/(METER)**3.
 C D) INLET TEMPERATURE OF FLUID (KELVIN), TI
 C E) R0COL IS THE SOLID DENSITY OF THE COAL, IN KG/(M)**3.
 C F) VIS IS THE DYNAMIC VISCOSITY OF THE FLUID AT CHAMBER TEMPERATURE,
 C IN KG/(M-SEC)
 C G) TBAR IS THE TIME REQUIRED FOR COMPLETE COMBUSTION OF A 100 MICRON
 C COAL PARTICLE AT ATMOSPHERIC PRESSURE, IN SEC.
 C
 C 1) DIMENSIONLESS INLET AREA
 C $P1 = (\text{TOTAL INLET AREA}) / (2 * \pi * (\text{OUTER RADIUS})^2)$
 C P1 MUST BE SMALL FOR THE LARGE SWIRL APPROXIMATION.
 C
 C 2) CHAMBER LENGTH TO RADIUS RATIO.
 C $P2 = (\text{OUTER LENGTH}) / (\text{OUTER RADIUS})$
 C P2 SHOULD BE ROUGHLY ORDER ONE.
 C
 C 3) RATIO OF EXHAUST RADIUS TO OUTER RADIUS.
 C $P3 = (\text{EXHAUST RADIUS}) / (\text{OUTER RADIUS})$
 C P3 IS LESS THAN ONE.
 C
 C 4) RATIO OF FLUCTUATING TURBULENT VELOCITY TO SWIRL VELOCITY, P4
 C
 C 5) VELOCITY PROFILE FACTOR IN END WALL BOUNDARY LAYERS, P5.
 C P5 IS 'LAMBDA 1' IN ROTT AND LEWELLEN.
 C
 C 6) RATIO OF SLAG PORT RADIUS TO OUTER RADIUS, P6.
 C P6 IS LESS THAN ONE.
 C
 C 7) RATIO OF THICKNESS OF CYLINDRICAL WALL BOUNDARY LAYER TO OUTER RADIUS, P7
 C $P7 = \Delta / (\text{OUTER RADIUS})$
 C IF LFDL = 1, P7 IS OVERRIDDEN BY SUBROUTINE DELTA.
 C
 C 8) RATIO OF CHAMBER FLUID DENSITY TO INLET FLUID DENSITY, P8.
 C NOTE - P8 INCLUDES BOTH THE CHANGE DUE TO TEMPERATURE AND THE
 C CHANGE DUE TO CHEMICAL COMPOSITION
 C
 C 9) RATIO OF COAL INLET VELOCITY TO FLUID VELOCITY, P9.
 C
 C 10) FRACTION OF COAL MASS THAT BECOMES ASH, P10 (ABOUT 0.10 - 0.20)
 C
 C 11) FRACTION OF COAL MASS IN VOLATILES, P11
 C
 C 12) SKIN FRICTION COEFFICIENT ON CYLINDRICAL WALL, CW.
 C IF LFSKN = 1, CW IS OVERRIDDEN BY SUBROUTINE SKNFR..
 C
 C 13) SKIN FRICTION COEFFICIENT ON THE TOP WALL, CT.
 C $CT = CF/2$
 C IF LFSKN = 1 AND CT = -1., CT = CW FROM SUBROUTINE SKNFR.
 C
 C 14) SKIN FRICTION COEFFICIENT ON THE BOTTOM WALL, CB.
 C IF NO BURNING OCCURS ON BOTTOM WALL, CB = CT.
 C IF BURNING OCCURS,
 C $CB = CT - (\text{FLUID DENSITY}) * (\text{INLET AREA}) * (\text{BLOWING VELOCITY}) / (\text{MASS FLOW RATE OF FLUID})$
 C IF LFSKN = 1 AND CB = -1., CB = CW FROM SUBROUTINE SKNFR.

```

C 15) FUEL TO AIR RATIO, CAYO.
C   CAYO = (MASS FLOW OF COAL)/(MASS FLOW OF FLUID)
C
C 16) THE DISTRIBUTION OF PARTICLE SIZES PLAYS AN IMPORTANT ROLE IN THE
C   CYLINDRICAL WALL LAYER.
C   IN THIS PROGRAM, TEN (10) DISTINCT PARTICLE SIZES ARE PERMITTED.
C   A(I) IS A TEN-COMPONENT VECTOR OF PARTICLE RADII, IN MICRONS.
C   A(1) IS THE LARGEST PARTICLE RADIUS, ETC.
C   IF LESS THAN TEN SIZES ARE DESIRED, SET A(I)=0.0 FOR THE OTHER
C   COMPONENTS.
C   F(I) IS A TEN-COMPONENT VECTOR OF MASS FRACTIONS.
C   F(1) IS THE FRACTION OF COAL PARTICLES WITH RADIUS A(1), ETC.
C   SUM (F(I))=1.0
C
C 17) LEAKAGE PARAMETERS, PT, PB.
C   PT IS THE PERCENTAGE OF FLOW IN THE TOP BOUNDARY LAYER THAT
C   EXHAUSTS DIRECTLY OUT THE TOP.
C   PB IS THE PERCENTAGE OF FLOW IN THE BOTTOM BOUNDARY LAYER THAT
C   FLOWS DIRECTLY INTO THE SLAG PORT.
C
C 18) ANGLE ON THE BOTTOM WALL (IN RADIANS), PHI.
C   PHI IS ACUTE. IF TOP AND BOTTOM WALLS ARE PARALLEL, PHI IS ZERO.
C
C
C   DIMENSION A(10), F(10)
C   COMMON PI,R0,CAYO,II,RHOI,DEL,AIN,DELN,RF,CAY,SLN,DNOT,ROCOL,UC,
1  UI,CW,CB,CT,CHI,CFI,EMF,GAMA0,SLNN,D2
C   COMMON P1,P2,P7,P8,P9,P11
C   PI = 3.14159265
C   READ (5,1) LFDL, LFSKN, LFHTR
1  FORMAT (3I4)
C   READ (5,2) R0, EMF, RHOI, II, ROCOL, VIS, TBAR
2  FORMAT (7E10.0)
C   READ (5,5) A
C   READ (5,5) F
5  FORMAT (8F10.6)
6  FORMAT ('1 PARTICLE RADII (MICRONS) ',10F8.2)
7  FORMAT ('/ MASS DISTRIBUTION ',10F8.3)
8  FORMAT ('0      R0      EMF      RHOI      II      ROCOL
1  VIS      TBAR' /' ',3F10.4,2F10.1,E12.3,F10.3/)
C   READ(5,10) P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11.
1  CT,CB,CA,CAYO,PT,PB,PHI
10 FORMAT ( ( 8 F10.0) )
C   WRITE (6,6) A
C   WRITE (6,7) F
C   WRITE(6,8) R0, EMF, RHOI, II, ROCOL, VIS, TBAR
C   WRITE(6,12) (K,K=1,11)
12 FORMAT ('0',4X,11(1HP,12.7X))
C   WRITE(6,14) P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11
14 FORMAT (/ 11F10.5)
C   WRITE(6,16)
16 FORMAT ('/' '      CT      CB      CW      PHI
1  '      K=0      PT      PB      ')
C   WRITE(6,18) CT,CB,CA,PHI, CAYO,PT,PB
18 FORMAT(/ 4F10.5, 5F10.5/ 1H0)
C   DEL = P7
C   IF (LFDL .EQ. 1) CALL DELTA
C   IF((P3+P7) .LT. 1.0) GO TO 20
C   WRITE(6,19)
19 FORMAT ('/ CYLINDRICAL BOUNDARY LAYER FILLS CHAMBER')
C   GO TO 300
20 CONTINUE
C

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C  II CALCULATE THE FLOW RATE PARAMETERS AT THE INLET.
    C = COS(PHI)
    T = TAN(PHI)
    CB1 = CB
    CT1 = CT
C  1) AVAILABLE CIRCULATION IN THE FLUID, GAMAO.
    GAMAO = EMF/(RHO1*P1*2.*PI*R0)
C  2) RADIAL FLUID FLUX, RU.
    RU = GAMAO * P1/(P2*P8)
C  3) DN0T IS THE MASS-AVERAGED PARTICLE DIAMETER AT THE INLET.
    SM = 0.0
    DO 22 I = 1, 10
22  SM = F(I)*A(I) + SM
    DN0T = 2.0*SM
C
C  III ITERATE TO FIND FLOW IN THE CYLINDRICAL WALL LAYER (WITH RECIRCULATION)
C  FLUID RESIDENCE TIME IN WALL LAYER
    TF = P7*R0*R0/RU
C  RF IS THE RECOVERY FACTOR, INITIALLY ASSUMED AT 0.5
    RF = 0.5
C  COAL RESIDENCE TIME IN WALL LAYER
25  DO 50, K = 1, 100
    FCTR = P5*P7*P8*ABS( CT + CB/C)* RF/P1
    TR = TF/FCTR
    IF (CT .LT. 0.0) GO TO 31
    IF (TR .GT. TF) GO TO 31
    WRITE (6,26)
26  FORMAT (' INCORRECT SPECIFICATION OF WALL LAYER ')
    GO TO 300
31  CONTINUE
C  FIND THE PARTICLE SIZE BELOW WHICH COMBUSTION DOMINATES.
    CONST = TBAR* 1.013E 5/ (8314.3/28.83)
C  28.83 IS THE MOLECULAR WEIGHT OF AIR
    CMP = 50.*SQRT(TR*RHO1*TI/CONST)
    DO 40 I = 1, 10
    IF (A(I) .LT. CMP) GO TO 42
40  CONTINUE
C  FIND THE WEIGHTED AVERAGE RESIDENCE TIME OF A COAL PARTICLE,
C  INCLUDING RECIRCULATION.
42  TM1 = 0.0
    TM2 = 0.0
C  D1 IS THE MASS-AVERAGED PARTICLE DIAMETER IN REGION I.
    D1 = 0.0
C  D2 IS THE SURFACE AREA-AVERAGED PARTICLE DIAMETER IN REGION I.
    E2 = 0.0
    I = I - 1
    IF (I .LT. 1) GO TO 46
43  DO 45 J = 1, I
C  SUM1 ACCOUNTS FOR COMBUSTING MATERIALS, SUM2 ACCOUNTS FOR ASH.
    SUM1 = 0.0
    SUM2 = 0.0
    TC = ((A(J)/50.)*2.)*CONST/(RHO1*TI)
    M = TC/TR
    TERM = 1.0 - (CT*PI + CB*PB/C)/(CT + CB/C)
C  TERM IS THE FRACTION OF DEPARTING COAL (FROM REGION I) THAT
C  RETURNS TO IT (RECIRCULATES).
    DO 44 M1 = 1, M
    SUM1 = SUM1 + (1.0 - (2.0*M1-1.)*TR/(2.*TC))* ((TERM)**(M1-1.))
44  SUM2 = SUM2 + TERM**((M1-1.))
    SUM1 = SUM1 + (1.0 - M*TR/TC)*(TC/TR-M)*((TERM)**(M))/2.0
    SUM2 = SUM2 + (TC/TR - M)* ((TERM)**(M))
    D1 = D1 + (SUM1*(1.-P10-P11) + SUM2*P10) *TR* F(J)* A(J)* 2.0
    E2 = E2 + (SUM1*(1.-P10-P11) + SUM2*P10) *TR* F(J)/A(J)

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      TM1 = TM1 + F(J)*SUM1*TR
45  IM2 = IM2 + F(J) *SUM2*TR
46  IP1 = I + 1
      DO 48 J=IP1, 10
      TM1= (F(J)*(2.*A(J)/100.)**2.)*CONST/(2.*RHOI*TI) + TM1
      D1 =((F(J)*(2.*A(J)/100.)**2.)*CONST/(2.*RHOI*TI))*(1.0+ P10-P11)
      1 *A(J) *2.0 + D1
      IF (A(J) .LT. 1. E-6) GO TO 48
      E2 =((F(J)*(2.*A(J)/100.)**2.)*CONST/(2.*RHOI*TI))*(1.0+ P10-P11)
      1 /A(J) + E2
48  TM2= (F(J)*(2.*A(J)/100.)**2.)*CONST/(RHOI*TI) + TM2
      TM1 = TM1*(1.0 -P10-P11)
      TM2 = TM2*P10
      IM = IM1 + TM2
      D1 = D1/IM
      D2 = 2.0*TM/E2
      EM = EMF*CAY0*TM
C    LOADING, CAY
      CAY = EM/(2.*PI*P2*P7*P8*RHOI*(R0**3))
C    IF (LFSKN .EQ. 1) CALL SKNFR
C    RECOVERY FACTOR.
      RFI = (1.0 + CAY0*P9)/(1.0 + CAY0 + C*P2*(1.0 +CAY)/P1)
      IF (ABS (RFI - RF) .LT. 0.001) GO TO 54
50  RF = RFI
      WRITE (6,52)
52  FORMAT (/ ' CONVERGENCE IN FIRST ITERATION LOOP NOT ACHIEVED ' )
54  RF = RFI
      GAMA = GAMA0 + RF
C
C    TRNST IS THE TIME SPENT IN THE LOWER WALL LAYER BY A FLUID PARTICLE.
      TRNST = 2.0 *R0*R0*((1.0 -P7)**2. - (P6)**2.)/GAMA
      TRY = TRNST/TR
      IF (TRY .LT. 0.10) GO TO 57
55  WRITE (6,56)
56  FORMAT (/ ' PROBLEM - APPRECIABLE TIME SPENT IN LOWER WALL LAYER ' )
57  WRITE (6, 58) RF, CAY, TR, TRNST, GAMA
58  FORMAT (/ ' RECOVERY FACTOR = ', F8.4, ' LOADING = ', F8.4, ' TR = ',
1  F8.4, ' SECONDS, TRNST = ', F8.4, ' SECONDS, GAMA = ', F8.4,
2  ' M.**2./SEC. ' )
      A1 = D1/2.
      A2 = D2/2.
      WRITE (6,59) SM, A1, A2, CMP
59  FORMAT (/ ' THE AVERAGE PARTICLE RADIUS AT THE INLET IS ', F8.2,
1  ' MICRONS. IN REGION I, IT IS ', F8.2, ' MICRONS. ' /
2  ' THE AREA-AVERAGED RADIUS IN REGION I IS ', F8.2, ' MICRONS. ' /
3  ' THE RADIUS OF THE LARGEST PARTICLE THAT IS ',
4  ' COMPLETELY COMBUSTED WITHOUT RECIRCULATING IS ', F8.2)
      IF (LFHTR .EQ. 1) CALL HTRAN
C
C    IV CALCULATE THE FLOW IN THE END WALL BOUNDARY LAYERS.
C    EMB IS THE COAL FLOW RATE IN THE BOTTOM WALL LAYER.
      EMB = GAMA*RHOI*R0*P5*P7*P8*CAY*CB*2.*PI/C
C    BB IS THE COAL FLOW RATE INTO THE SLAG PORT
      BB = EMB *PH
C    TEST THE LEGITIMACY OF THE ASSUMPTIONS
      TST1 = EMB*(1.0 + C*CT/CB)/CAY -EMF
      IF (TST1) 62,60,60
60  WRITE (6,61)
61  FORMAT (/ ' ALL OF THE FLUID ENTERS THE END WALL LAYERS WITHIN THE
1  CYLINDRICAL LAYER ' )
62  IST2 = EMB*(1.0 + C*CT/CB) - EMF*CAY0
      IF (IST2) 80, 64, 64
64  WRITE (6,65)

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65 FORMAT (/ ' TOO MUCH COAL IN THE WALL LAYERS.ERROR IN FORMULATION' )
GO TO 300
80 RATIO = 88*(TM1/TM)/(EMF*CAY0*(1.0 -P10 -P11))
WRITE (6,82) RATIO
82 FORMAT (10HF ' FRACTION OF CARBON THAT ENTERS THE SLAG PORT '
1, ' UNBURNED IS ' F8.4)
C INCREASE THE FLUID MASS FLOW TO ACCOUNT FOR THE COAL GASSIFICATION
EMF1= EMF*(1.0+ CAY0*P11 + CAY0*(1.0 -P10-P11)*(1.0 -RATIO*
1 (1.0 +CT*PT*C/(CH*PR))))
C
C V LOADING IN THE TOP WALL LAYER.
C THE LOADING IN THIS LAYER IS A MAXIMUM AT THE OUTSIDE AND
C DECREASES AS R DECREASES. WE WANT KT AT THE EXHAUST RADIUS.
C DEFINE R1 = R/R0
CAY2 = CAY
DO 90 M = 1,5
DO 88 J = 1,10
CAYT = CAY2 -(J/(10.**M))*CAY
R1 = 1.0 + P7*CAY*(ALOG((1.0 + CAYT)/(1.0 + CAY)) +ALOG(CAY/CAYT)
1 -1.0/CAYT)
IF ( R1 - P3) 90, 92, 85
85 IF (M .GT. 1) GO TO 88
86 IF (CH .GT. 0.) GO TO 88
WRITE (6, 87) R1, CAYT
87 FORMAT (/ ' AT R/R0 = ', F8.4, ' KT = ', F10.4)
88 CONTINUE
90 CAY2 = CAYT + CAY/(10.**M)
IF (CH .GT. 0.) GO TO 93
92 WRITE (6, 87) R1, CAYT
93 FT = P5*GAMA*CT*R0*P7*CAY/CAYT
FRT = 2.*PI*FT* RHOI*P8/EMF1
FRB = FRT*CB/(CT*C)
CMPR = (FRT + FRB) - 1.0
IF (CMPR) 98, 98, 94
94 WRITE (6,95)
95 FORMAT (/ ' ALL FLUID ENTERS END WALL LAYERS BEFORE R = R-EXHAUST.
1 REANALYZE' )
GO TO 300
98 WRITE (6,99) FRT, FRB
99 FORMAT (/ ' AT R = R-EXHAUST, ', F8.4, ' OF THE FLUID FLOW IS IN '
1 ' THE TOP WALL LAYER',/, 17X, F8.4, ' OF THE FLUID FLOW IS IN THE '
2 ' BOTTOM WALL LAYER. ' )
C
C VI CHECK LOADING IN MAIN CHAMBER (THIS ANALYSIS ASSUMES IT IS SMALL).
C A) FROM RECIRCULATING COAL PARTICLES
ELAV = (1.0 -P3)*(P2*(1.0 + P3) + 1*(1.0 + P3 -2.0*P3*P3)/3.)
CAYM = CAY*P5 *P7*P3* SQRT(1.0 - P3*P3)*(CT*(1.0 - PT) + CH*(1.0-
1 PR)/C)/(0.5*ELAV)
WRITE (6,103) CAYM
103 FORMAT (/ ' THE LOADING IN THE MAIN CHAMBER FROM THE RECIRCULATION
1 IS ', F8.4)
C B) EFFECT OF CENTRIFUGING PARTICLES INTO SLOPING BOTTOM WALL LAYER.
TAU = R0COL/(VIS* 18.E8)
FOR A CHAMBER TEMPERATURE OF 2200 (KEL), TAU IS ABOUT 0.01.
TAU IS CHARACTERISTIC OF THE TIME REQUIRED FOR A 100 MICRON
PARTICLE TO ACCELERATE TO THE FLUID VELOCITY, IN A STOKES-FLOW.
FOR A COAL PARTICLE OF RADIUS A, THIS CHANGES TO TAU*(A/50.)**2.
FOR AN ASH PARTICLE, CHANGE TO TAU*P10.
TEST = SIN(PHI)* (TAU *(A(1)/50.)**2.)*GAMA*P7/(R0*P3*P3*(1.0-P3)
1 *(P2*(1.0 +P3) +1*(1.0 +P3- 2.*P3*P3)/3.))
IF (TEST .LT. 0.01) GO TO 110
WRITE (6,107)
107 FORMAT (/ ' LOADING OF THE TOP AND BOTTOM BOUNDARY LAYERS IS',

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1' DYNAMICALLY DIFFERENT. ')
110 CONTINUE
C C) ASH LOADING IN THE MAIN CHAMBER
C 1) MAXIMUM ASH PARTICLE SIZE (AT R = 1. - P7)
RU = (EMF1/(2.*PI*RHOI*PB) - P5*GAMA*R0*P7*(CT + CB/C))/(R0*(P2 +
1 T*(P7)))
SW = GAMA/RU
WRITE (6,111) SW
111 FORMAT (/,' THE SWIRL PARAMETER AT THE EDGE OF THE CYLINDRICAL ',
1 'WALL LAYER = ', F8.3)
AMX = 50.*R0*(1.-P7)*SQRT(RU/(TAU*P10))/GAMA
DO 113 J = 1,10
IF (AMX - A(J)) 113, 114, 114
113 CONTINUE
114 IF (J .LE. 10) GO TO 117
J = 10
J1 = J
115 WRITE (6, 116)
116 FORMAT (/,' NO ASH PARTICLES SUSPENDED IN MAIN CHAMBER ')
RU = (EMF1/(2.*PI*RHOI*PB) - FRT - FRB)/(R0*(P2 + T*(1. - P3)))
AMN = 50.*R0*(P3)*SQRT(RU/(TAU*P10))/GAMA
GO TO 150
C 2) MINIMUM ASH PARTICLE SIZE (AT R = 1. - P3)
117 RU = (EMF1/(2.*PI*RHOI*PB) - FRT - FRB)/(R0*(P2 + T*(1. - P3)))
AMN = 50.*R0*(P3)*SQRT(RU/(TAU*P10))/GAMA
DO 118 J1 = J,10
IF (A(J1) - AMN) 119, 118, 118
118 CONTINUE
119 IF (J1 .EQ. 1) GO TO 115
IF (J1 .GT. 10) J1 = 10
C FOR EACH ASH PARTICLE SIZE SUSPENDED IN THIS REGION, FIND THE
C RADIUS AT WHICH IT IS STATIONARY. THEN SPREAD THESE PARTICLES
C BETWEEN THIS RADIUS AND THE OUTER WALL LAYER (FOR ROTATIONAL STAB)
CAY3 = CAY
CASH = 0.0
C SLG IS THE FRACTION OF PARTICLES THAT MIGHT REMAIN IN REGION III
C THAT ARE CAPTURED BY THE SLAG LAYER INSTEAD. (= 0 FOR NOW)
SLG = 0.0
DO 131 L = J, J1
DO 125 M = 1, 10
CATR = CAY3 - (CAY3 - CAYT)*M/10.
R2 = 1.0 - P7*CAY*((1.0/CATR) - ALOG((1.0 + CATR)/(1.0 + CAY))) -
1 ALOG(CAY/CATR))
RU = -(EMF1/(2.0*PI*RHOI*PB) - P5*P7*R0*GAMA*(CT + CB/C)*CAY/CATR)/
1 ((P2 + T*(1.0 - R2))*R0)
RUP = RU + TAU*P10*((A(L)/50.)*2.)*(GAMA/(R0*R2))*2.
IF (RUP) 125, 130, 130
125 CONTINUE
130 CAASH = (EMF*CAY0*F(L)*(1.0 - SLG)*P10)/(2.0*PI*RHOI*PB*PT*
1 P5*P7*R0*GAMA*CT*CAY/CATR*(1.0 - CATR/CAY))
CAY3 = CATR
CASH = CASH + CAASH
330 WRITE (6, 132) L, CAASH
131 CONTINUE
132 FORMAT (/,' THE INCREMENTAL LOADING IN III FROM PARTICLE # ',
1 I4, ' IS KIII = ', F8.4)
WRITE (6, 135) CASH
135 FORMAT (//,' THE MAXIMUM LOADING IN REGION III FROM SUSPENDED ASH
1 PARTICLES IS KIII = ', F8.4)
C
C VII COMPUTE THE CARRY-OVER, FROM FOUR DIFFERENT SOURCES.
C A) SMALL PARTICLES DRIFTING THROUGH REGION III.
150 SUM3 = 0.0

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      IF (A(10).GT. AMN) GO TO 153
      DO 152 L = J1, 10
152  SUM3 = F(L) + SUM3
153  CARY = SUM3*(1.0 - SLG - FRT*PT - FRB*PB)
      CAR1 = CARY
C   B) THROUGH THE TOP BOUNDARY LAYER (CARRY-OVER = BLOWBY)
156  CAR2 = BB*(TM2/TM)* C*CT*PT/(PB*CB*EMF*CAYO*P10)
159  CARY = CARY + CAR2
C   C) TURBULENT ENTRAINMENT
      ACRT = 50.0* R0/(SQRT (TAU *P10 *GAMA/(P4)))
      IF (ACRT -AMX) 160, 160, 164
160  WRITE (6, 161)
161  FORMAT (/, ' NO TURBULENT ENTRAINMENT. SWIRL TOO SMALL. ')
      DO 163 N1 = 1, 10
      IF (A(N1) .LT. AMX) GO TO 170
163  CONTINUE
      GO TO 170
164  CAR3 = 0.0
      CAYS = 0.0
      PROT = CAYO* P1* PB* P10/(P5* P7* RF* (PT*CT + PB*CB/C))
      DO 167 N1 = 1, 10
      IF (A(N1) .GT. ACRT) GO TO 167
      IF (A(N1) .LT. AMX) GO TO 168
      CAR3 = CAR3 + F(N1)*(1.0-FRT*PT -FRB*PB)*CT*PT/(CT*PT +CB*PB/C)
      CAYP = PROT*F(N1)
      WRITE (6, 165) N1, CAYP
165  FORMAT (/, ' THE INCREMENTAL ASH LOADING IN REGION I FROM PARTICLE
1  ' , I4, ' IS ', F8.3)
167  CAYS = CAYS + CAYP
168  CARY = CARY + CAR3
      CHECK = CAYS/(1.0 + CAY)
      IF (CHECK .LT. 0.10) GO TO 170
      WRITE (6, 169)
169  FORMAT (/, ' PROBLEM IN REGION I. ASH LOADING EXCEEDS 10% OF ',
1  '(1.0 + K) THERE, AND AFFECTS THE DYNAMICS')
C   D) FROM ASH PARTICLES SUSPENDED IN MAIN CHAMBER
170  CAR4 = 0.0
      IF (A(10).GT. AMX) GO TO 172
      IF (N1 .GT. 10) N1 = 10
      DO 171 N2 = N1, 10
      IF (A(N2) .LT. AMN) GO TO 172
171  CAR4 = CAR4 + F(N2)*(CT*PT/(CT*PT + CB*PB/C))* (1.0- FRT*PT -
1  FRB*PB -SLG)
172  WRITE (6, 173) CAR1
173  FORMAT (/, ' THE FRACTION OF ASH THAT DRIFTS INTO CARRY-OVER IS ',
1  F 6.4)
      WRITE (6, 174) CAR2
174  FORMAT (/, ' THE FRACTION OF ASH CARRIED OUT THE TOP LAYER IS ',
1  F 6.4)
      WRITE (6, 175) CAR3
175  FORMAT (/, ' CARRY-OVER FROM TURBULENT ENTRAINMENT IS ', F6.4)
      WRITE (6, 176) CAR4
176  FORMAT (/, ' CARRY-OVER FROM PARTICLES SUSPENDED IN MAIN ',
1  'CHAMBER IS ', F8.4)
179  CARY = CARY + CAR4
180  WRITE (6, 181) CARY
181  FORMAT (/, ' THE TOTAL MASS FRACTION OF ASH THAT EXITS AS ',
1  'CARRYOVER IS ', F8.5)
      WRITE (6, 185) AMN, AMX, ACRT
185  FORMAT(/, ' AMN = ', F8.4, ' AMX = ', F8.4, ' ACRT = ', F8.4)
C
C VIII FLOW IN THE CORE REGION
C   THE VELOCITY PROFILES USED IN THIS REGION ARE CONSISTENT WITH

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C   THE RESULTS OF BUSSEL (1969).
C   TEST FOR A STAGNANT CORE
      TST = EMF1/(RHOI*P8*2.*PI) - PT*FT - GAMA*R0*P3/3.
      IF (TST) 210, 230, 230
210  R4 = 0.5*P3*( SQRT(9.-12.*(EMF1/(RHOI*P8) -2.*P1*PT* FT)/
      1 (PI*GAMA*R0*P3)) - 1.0)
      WRITE (6, 212) R4, P3
212  FORMAT (// ' THERE IS A STAGNANT CORE AT R = ', F10.6, ' (P3 = ',
      1 F10.6, ' )')
      C1 = GAMA/((P3 -R4)*P3*R0*R0)
213  WRITE (6, 214)
214  FORMAT (/ ' THE SWIRL VELOCITY IN THE EXHAUST IS -')
      WRITE (6, 215) C1, R4
215  FORMAT (' V = ', F10.4, ' * (R = ', F10.4, ' ) M./SEC.')
      GO TO 280
230  WRITE (6, 232)
232  FORMAT (// ' THERE IS NO STAGNANT CORE.')
233  C1 = GAMA/ ((R0*P3)**2.)
      WRITE (6, 234)
234  FORMAT (/ ' THE SWIRL VELOCITY IN THE EXHAUST IS -')
      WRITE (6, 235) C1
235  FORMAT (/ ' V = ', F10.4, ' * (R) M./SEC.')
C
C   IX TOTAL PRESSURE DROP / INLET PRESSURE
C   A) ACROSS INLET
280  DELP = ((EMF / (2.*PI*P1*RHOI*R0*R0))**2.)/(2.0*287.*TI)
      WRITE (6, 281) DELP
281  FORMAT ('0', ' ACROSS INLET, DELP/P = ', F8.4)
C   B) ACROSS CYLINDRICAL LAYER
      DELP1 = DELP*P8*(1.0 + CAY)*RF*P7*2.0
      WRITE (6, 283) DELP1
283  FORMAT (/, ' ACROSS OUTER WALL LAYER, DELP/P = ', F8.4)
      DELP = DELP + DELP1
C   C) ACROSS MAIN CHAMBER
      DELP2 = (P8*(GAMA/R0)**2.0)*(1.0/(P3*P3) - 1.0/((1.0 -P7)**2.))/
      1 (2.0*287.0*TI)
      WRITE (6, 285) DELP2
285  FORMAT(/, ' ACROSS MAIN CHAMBER, DELP/P = ', F8.4)
      DELP = DELP + DELP2
C   D) ACROSS EXHAUST
      IF (TST) 286, 290, 290
286  DELP3 = (P8*(C1*R0)**2.)*((P3 - R4)*(P3- 3.*R4) + 2.*R4*R4* ALOG
      1 (P3/R4))/(2.0*287.0*TI)
288  WRITE(6,289) DELP3
289  FORMAT (/, ' ACROSS EXHAUST, DELP/P = ', F8.4)
      DELP = DELP + DELP3
      WRITE (6, 295) DELP
295  FORMAT (//, ' THE OVERALL PRESSURE DROP, DELP/P = ', F8.4)
      GO TO 300
290  DELP3 = (P8*(C1*R0*P3)**2.)/(2.0*287.0*TI)
      DELP4 = (P8*2.*(EMF1/(2.* PI*RHOI*P8)-PT*FT - GAMA*P3*R0/3.))**2.)/
      1 (287.0*TI*(P3*R0)**4.)
      DELP3= DELP3+ DELP4
      GO TO 288
300  CONTINUE
      END

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SUBROUTINE DELTA

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C
C  CALCULATION OF THE BOUNDARY LAYER THICKNESS FOR A ROUGH WALL
C  USING A INTEGRAL MOMENTUM EQUATION SOLUTION
C
C  THE INPUT PARAMETERS FOR THIS SUBROUTINE ARE
C  Y0 THE ROUGHNESS SCALE FOR THE WALL , METERS
C  NOP NUMBER OF INLET PORTS
C
      IMPLICIT REAL (A-H,K-Z)
      COMMON PI,R0,CAY0,TI,RHOI,DEL,AIN,DELN,RF,CAY,SLN,DNOT,ROCOL,UC,
1    UI,CX,CR,CI,CBI,CTI,EMF,GAMA0,SLNN,D2
      COMMON P1,P2,P7,PA,P9,P11
      DIMENSION QAN(2),LHSV(2)
      EQUIVALENCE (QAN(1),AIN),(QAN(2),DELN)
      READ(5,100) Y0,NOP
100  FORMAT (8F10.0)
      WRITE(6,110)
110  FORMAT (//)
      WRITE(6,105) Y0,NOP
105  FORMAT (9X,'ROUGHNESS SCALE Y0 =',E11.4,/,9X,'NUMBER OF PORTS =',E1
11.4,/)
      L = R0 * P2
      SL = 2. * PI * R0 / NOP
      AI = SL * P1 / P2
C    KAPPA IS THE VON KARMAN CONSTANT FOR THE LOGARITHMIC VELOCITY PROFILE
      KAPPA = 0.39
      RHS = KAPPA*KAPPA*SL/Y0
      AIN = AI*0.5/Y0
      SLN = SL /Y0
      SLNN = SL/AI
C
C  THIS INPUT CARD SELECTS THE START VALUE FOR DEL AND THE INCREMENT
C  THE EQUATION FOR DEL IS SOLVED BY MATCHING THE LHS AND RHS
C
      READ(5,100) DELS,DELINC
      DELT = DELS
150  DEL = DELT+DELINC
      DELN = DEL * R0 / Y0
      LHS1 = 4.*(AIN-DELN)
      LHS2 = ALOG(AIN)-ALOG(DELN)
      LHS3 = -(AIN*ALOG(AIN)-DELN*ALOG(DELN))
C
C    LHS4 IS NOW FOUND USING AN ASYMPTOTIC SERIES EXPANSION PROCEDURE. REF.
C  BATEMAN VOL. 2, PAGE 144.
C
      DO 30 J=1,2
      OTERM = 1.E30
      SUM = 0.0
      M = 1
      MFACT = 1
10    TERM = MFACT/(ALOG(QAN(J))**M)
      IF (TERM .GT. OTERM) GO TO 20
      OTERM = TERM
      SUM = SUM+TERM
      M = M+1
      MFACT = MFACT*M
      GO TO 10
20    CONTINUE
      SUM = SUM-OTERM
      LHSV(J) = EXP(ALOG(QAN(J)))/ALOG(QAN(J))*SUM

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30  CONTINUE
    LHS4 = -4.*(LHSV(1)-LHSV(2))
    LHS5 = 4.*(ALOG(ALOG(AIN))-ALOG(ALOG(DELN)))
    LHS = LHS1+LHS2+LHS3+LHS4+LHS5
    IF ( LHS-RHS)200,220,210
200  CONTINUE
    DELT = DEL
    GO TO 150
210  CONTINUE
    DELINC = DELINC/2.
    IF(DELINC .LT. 0.001) GO TO 220
    GO TO 150
220  CONTINUE
    WRITE(6,160)
160  FORMAT(14X,'DEL',8X,'DELN')
    WRITE(6,170) DEL,DELN
170  FORMAT(10X,2E11.4,/)
    P7 = DEL
    RETURN
    END
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```

SUBROUTINE SKNFR
  IMPLICIT REAL (A-H,K-Z)
  COMMON PI,R0,CAY0,II,RHUI,DEL,AIN,DELA,RF,CAY,SLN,DNOT,ROCOL,UC,
1 UI,CW,CB,CT,CHI,CTI,EMF,GAMA0,SLNN,D2
  COMMON P1,P2,P7,P8,P9,P11
  UI = GAMA0 / R0
  UC = UI * RF
  RHS1 = -((1.+DELN)/ALOG(DELN)) + ((1.+AIN)/ALOG(AIN))
  RHS2 = 2. * (((DELN-1.)/(ALOG(DELN)**2.)) - ((AIN-1.)/(ALOG(AIN)**
12.)))
  RHST = RHS1+RHS2
  FACG1 = (1. + CAY0 ) / ( P8*( 1.+CAY ))
  FACG2 = ( 1.+CAY0*P9 ) / ( 1.+CAY0 )
  UHAT = 0.5 * ( UC + (((UC*UC) - (4.*FACG1*((UC*UI) - (FACG2*UI*UI
1 )))) **0.5))
  UM = (UHAT+UC)**0.5
  CW =-RHST * P8 * (UM+UM/(UC*UI)) / SLN
  IF ( CBI .LT. 0. ) CB = CW
  IF ( CTI .LT. 0. ) CT = CW
  WRITE(6,110) RF,CW,CB,CT
110 FORMAT(9X,'RF =',E11.4,4X,'CW =',E11.4,4X,'CB =',E11.4,4X,'CT =',E
111.4)
  RETURN
  END

```

```

SUBROUTINE HTRAN
IMPLICIT REAL (A-H,K-Z)
COMMON PI,R0,CAY0,TI,RHOI,DEL,AIN,DELN,RF,CAY,SLN,DNOT,ROCOL,UC,
1 UI,CW,CB,CT,CBI,CTI,EMF,GAMA0,SLNN,D2
COMMON P1,P2,P7,P8,P9,P11
C
C   CALCULATION OF THE CONVECTIVE HEAT TRANSFER USING REYNOLDS ANALOGY
C
  READ (5,100) PR,CPC,TW,TC,EMSW
100 FORMAT (8F10.0)
  RHOC = RHOI * P8
  UI = GAMA0 / R0
  UC = UI * RF
  QCONV = CW * PR * RHOC * (1.+CAY) * UC * CPC * (TC-TW)
C
C   CALCULATION OF RADIATION HEAT TRANSFER FROM THE PARTICLES IN THE FLAME.
C   REF. BRONN AND MARCO.
C
C   THE FORMULA FOR X IS IN BRITISH UNITS. THE VARIOUS TERMS ARE
C   LOCALLY CONVERTED TO THESE UNITS.
C
  VOL = P11
  DB = DNOT / (1.E4 * 2.54)
  ROCOLB = ROCOL * 62.43 / 1000.
  RB = R0 / .3048
  TCB = TC * 1.8
C
C   CALCULATE AN EFFECTIVE K . THE EFFECTIVE LENGTH IS THE CHAMBER
C   DIAMETER.
C
  K = CAY * (1. - (1.-DEL)**2.)
C
C   CALCULATE OPERATING PRESSURE IN ATMOSPHERES
C
  P = RHOI * TI * 8314.3 / (28.83 * 1.013E5)
C
C   DENSITY RATIO OF COAL AND CAKED COAL IS ASSUMED TO BE 1.15
C
  X = 238. * 2. * RB * (((1.-VOL)*1.15)**.67) * K * P / (ROCOLB * DB * TCB)
  EMSFL = 1. - EXP(-X)
  SIGMA = 1.3546E-11
  GRADP = SIGMA * EMSFL * EMSW * ((TC**4.) - (TW**4.))
  WRITE(6,110)
110 FORMAT(//)
  WRITE(6,120)
120 FORMAT(14X,'PR',9X,'CPC',8X,'TC',8X,'RHOC',7X,'ROCOL',6X,'DNOT',8X,
1,'TW',8X,'EMSW',9X,'X',8X,'EMSFL')
  WRITE(6,130) PR,CPC,TC,RHOC,ROCOL,DNOT,TW,EMSW,X,EMSFL
130 FORMAT(10X,10E11,4)
  WRITE(6,110)
  QCHAM = EMF*(1.+CAY0) * CPC * TC / (2*PI*R0*R0*P2)
  QCONV = QCONV / QCHAM
  GRADP = GRADP / QCHAM
  WRITE(6,140) QCONV,GRADP
140 FORMAT(9X,'QCONV / QCHAM' =',E11.4,/,9X,'GRADP / QCHAM' =',
111.4,/)
  RETURN
END

```

APPENDIX B
Glossary of Symbols

Roman symbols

a	particle radius
a_i	average radius of the i^{th} set of particles in the assumed discretization
a_{max}	radius of ash particle suspended at boundary of regions I and III
a_{min}	radius of ash particle suspended at boundary of regions III and IV
a_{crt}	radius of the largest ash particle entrained in the turbulent flow in region I
a_c	radius of the coal particle whose combustion time equals the residence time in region I
A	inlet area for the fluid
A_w	surface area of the chamber
BB	blowby; fraction of fixed carbon that enters the slag port unburned
CO	carryover; mass fraction of ash that exhausts with gases
C_p	specific heat of the chamber gases at constant pressure
C_f	drag coefficient
C_B	$c_f/2$ for the bottom end wall
C_T	$c_f/2$ for the top end wall
C_W	$c_f/2$ for the cylindrical wall
$C, C_1, C_2, C_3, C_3, C_4, C_5, C_6$	arbitrary constants
d	particle diameter
d_o	mass-averaged particle diameter at inlet
d_2	area-averaged particle diameter in region I
D_o	diffusivity of chamber gases at reference temperature $T_o = 273^\circ\text{K}$
f	swelling factor, 1.0 in present analysis
f_i	mass fraction of the i^{th} set of particles in the assumed discretization of the entering coal
F_T	volumetric flow rate of fluid in the top boundary layer
F_B	volumetric flow rate of fluid in the bottom boundary layer

FCTR	fraction of fluid that enters region II directly from region I
g_B	See equation (3.7.4)
k	thermal conductivity of fluid in region I
K	loading; local ratio of coal density to fluid density
K_O	loading in inlet flow; ratio of coal to fluid mass flow rates
K_I	coal loading in region I
K_a	partial loading in region I due to noncarbon part of coal
K_{ashi}	partial loading from suspended ash particles of the i th size in region I
K_B	coal loading in bottom boundary layer
$\hat{K}$	loading in region III from suspended ash particles
K_p	partial loading in top boundary layer from ash particles that had been suspended in region III
$\tilde{K}$	loading in region III from coal particles returning to region I
K_T	coal loading in top boundary layer
K_∞	loading just outside region II (either region I or III)
K_{eff}	effective coal loading for entire combustor volume (see equation (3.7.5))
ℓ	circumferential distance between inlet ports
$\hat{\ell}$	normalized ℓ
L	outer length of chamber
m	total mass of coal in region I
m_p	particle mass
$\dot{m}_f$	mass flow rate of fluid at inlet
$\dot{m}_c$	mass flow rate of coal at inlet
$\dot{m}$	mass flow rate of fluid in chamber
$\dot{m}_t$	total mass flow rate
N	number of coal particles in region I
p	pressure
p_{O_2}	partial pressure of oxygen
$\Delta p/p$	normalized pressure drop
P_1	$A/2\pi R^2$; normalized inlet area
P_2	L/R ; normalized chamber length
P_3	ratio of exhaust radius to outer radius

P_4	ratio of fluctuating turbulent velocity to swirl velocity
P_5	4.93 , from Rott & Lewellen (1966)
P_6	ratio of slag port radius to outer radius
P_7	δ/R ; normalized thickness of cylindrical wall layer
P_8	ρ/ρ_i ; ratio of chamber to inlet fluid densities
P_9	U_{ci}/U_i ; ratio of inlet coal velocity to inlet fluid velocity
P_{10}	mass fraction of ash in original coal particles
P_{11}	mass fraction of volatiles in original coal particles
Pr	turbulent Prandtl number
P_B	fraction of particle flow in bottom boundary layer that exhausts directly into the slag port
P_T	fraction of particle flow in top boundary layer that exhausts directly with gases.
Q_B	total mass flow in top boundary layer
Q_T	total mass flow in bottom boundary layer
Q_{conv}	total convective heat loss
Q_{radp}	radiative heat loss from particles
Q_{tot}	$Q_{conv} + Q_{radp}$
Q_c	flow rate of enthalpy in chamber.
r	radial position in chamber
r^*	equilibrium position of ash particle suspended in region III
$\tilde{r}$	radius of the stagnant core in region IV
R	total radius of chamber
$R_o = R$	
r_f	recovery factor, (2.1.21)
R	gas constant of incoming fluid
Re	Reynolds number
SLG	fraction of ash particles small enough to be suspended in chamber that are caught in slag layer
T_c	chamber temperature
T_f	flame temperature
T_i	temperature of inlet fluid.
T_w	wall temperature
TERM	see (2.2.47)
(u,v,w)	velocity components of fluid in cylindrical coordinates

(u_p, v_p, w_p)	velocity components of particles in cylindrical coordinates
$\bar{u}$	length-averaged radial velocity of fluid in region III; (2.3.9)
$\tilde{u}$	see (2.2.30)
U_i	inlet tangential velocity of fluid
U_{ci}	inlet velocity of coal
U_c	tangential velocity of fluid at boundary of regions I and III
U_m	$(U \cdot U_c)^{1/2}$, see Equation (3.3.2)
$\hat{U}$	initial wall jet velocity after mixing, equation (3.3.1)
U^*	friction velocity in the boundary layer
$V_i = U_i$	
$V_o = U_o$	
$V_m = (V_o V_i)^{1/2}$	, see equation (2.1.20)
V	mixing velocity normal to wall at initial station of periodic boundary layer.
w_o	blowing velocity at wall
w_T	local vertical velocity into top boundary layer
y_o	roughness scale, metres

(reek symbols

α	arbitrary constant in (2.3.19)
Γ	circulation in region III
Γ_o	Γ/r_f
δ	thickness of cylindrical wall layer
δ_l	thickness of wall layer
$\hat{\delta}$	normalized thickness of wall layer = δ_l/y_o
$\bar{\Delta}$	thickness of end wall layers in Section 2.2
Δ	thickness of inlet port in §3
$\hat{\Delta}$	normalized thickness of inlet port = Δ/y_o
ϵ	eddy viscosity
ϵ_F	emissivity of flame
ϵ_w	emissivity of wall
η	mass fraction of coal that leaves the chamber unburned
κ	0.39 ; von Kármán's constant
μ	dynamic viscosity of fluid
ξ	total tangential momentum flux at inlets

ρ	fluid density in chamber
ρ_i	fluid density at inlet
ρ_p	particle density at chamber
ρ_o	density of coal in equation (3.7.4)
	density of chamber gases at reference temperature To = 273°K in equation D-1
ρ_l	density of coked coal in equation (3.7.4)
ρ_s	average density of unburned and partially burnt particles
ρ_{ash}	density of ash
ρ_{coal}	density of coal
ρ_{pi}	particle density at inlet
τ_c	particle combustion time; (2.1.1)
τ_f	fluid residence time in region I; (2.1.3)
τ_i	total residence time in region I of particle with radius a_i
τ_m	mean total residence time of particles; (2.1.13)
τ_p	particle acceleration time; (2.2.9)
τ_r	particle "residence time" in region I; (2.1.5)
τ_w	shear stress on wall
τ_{II}	fluid residence time in region II
τ_{III}	time spent traversing region III by a recirculating coal particle
ϕ	angle between top and bottom end walls
ψ	(v/u); swirl parameter
Ω	angular velocity in region IV

APPENDIX C

Combustion Model

The combustion of coal particles involves a number of complex processes. During combustion coal particles usually first pyrolyse and the volatiles burn in the gas phase with subsequent combustion of the solid char particle. However, there may also be pre-pyrolysis ignition of the coal particle. Further, due to the porous nature of the coal, there are possibilities of varying degrees of reactive gas penetration into the particle and internal burning in the pores. Particle swelling is another possible effect. Opinions are divided as to whether the particles burn at constant diameter or with increasing or reducing diameters. Essenhigh (1976) recently stated that swelling in oxidizing flow conditions is likely to be only 10 percent in diameter. Earlier, Beeston and Essenhigh (1963) used a 50 percent swelling factor in their empirical correlation for single particle burning time. The renewed research interest in the area of coal chemistry and combustion should help to settle some of the current uncertainties in this field.

A very simple combustion model has been used in our current studies of coal combustion in a cyclone combustor. The objective was to take a first step to understand the importance of various parameters. Our studies definitely suggest that the coal combustion model is a key element in the predictions and that an "improved" combustion model in the program could significantly improve the accuracy of the predictions.

The coal combustion model being used at present is the following. The coal particle is assumed to burn at constant diameter. A specified mass fraction of volatiles (P_{11}) are assumed to be evolved instantaneously at entry into the cyclone. The remaining mass of the coal is burnt in a linear fashion over a combustion time τ_c . After time τ_c , the particle is completely combusted and only the ash fraction (P_{10}) remains and is immediately lost to the slag layer if the particle size is above the three characteristic sizes $AMIN$, $AMAX$ and $ACRT$,

which correspond to different mechanisms of ash carryover loss. The combustion model is sketched below.

Some comments on these assumptions are in order. The use of a constant diameter burning model is conservative as far as prediction of ash carryover is concerned. If the diameter decreases, the particle density is higher and it can be shown that the carryover decreases. The mass fraction of the volatiles evolved instantaneously (P11) will range between zero and the total amount of volatiles present in the dry coal. Calculations have been carried out for the two limiting cases. One important factor to be considered in estimating P11 for the cases where the coal is injected with the main air supply is the high initial temperature of the inlet stream. A significant amount of devolatilisation may be completed prior to entry into the cyclone combustor.

The additional important input variable that has to be selected is the combustion time τ_c . We assume that results of single particle coal combustion studies in atmospheric room temperature conditions can be extended to the cyclone combustor conditions. We further assume that the particles are burning in the "flame temperature" region (Thring and Essenhigh, 1963), that is, temperatures greater than 1200°C and that the particle

size is large enough (typically, $d > 10$ microns) for diffusional controlled combustion to be valid. Then, for combustion in stagnant atmosphere (Beeston and Essenhigh, 1963)

$$\tau_c = \left(\frac{1 - P_{10} - P_{11}}{f} \right) \frac{\rho_{\text{coal}}}{3 \rho_o D_o \left(\frac{T}{T_o} \right)^{.75}} \frac{1}{\ln(1 - p_{O_2})} \cdot d^2 \quad (C-1)$$

$$= \frac{861 d^2}{p} \text{ sec } (d \text{ in cm, } p \text{ in atmos.}) \quad (C-2)$$

The surface temperature of the coal particle, T is taken to be 2200°K . The swelling factor $f = 1.0$, consistent with the assumption of a constant diameter burning model.

Equation C-2 predicts $\tau_c \approx 90$ m sec for the burning of a coal particle of diameter 100 microns. A number of corrections have to be made to this burning time because the conditions in the cyclone combustor do not correspond to the assumptions made in the idealized single particle combustion studies.

Limited Oxygen Supply

The formula predicts the dependence of τ_c on the ambient oxygen level to be

$$\tau_c \sim \frac{1}{\ln(1 - p_{O_2})}$$

Under the substoichiometric burning conditions present in the cyclone combustor, the oxygen concentration would decay to rather small values away from the wall region. However, coal particles can "burn" in an atmosphere of CO_2 and Spalding (1951) suggests that one can estimate the burning rate in a mixed $\text{O}_2 + \text{CO}_2$ atmosphere by defining the effective oxygen partial pressure

$$p_{O_2 \text{ eff}} = p_{O_2} + \frac{1}{2} p_{\text{CO}_2}$$

For the device under study, the molar CO_2 concentration is about 6 percent in the exhaust. As $p_{O_2 \text{ eff}}$ changes from 0.21 to 0.03 from the start to the end of the periodic boundary layer flow, the burning time increases by a factor of

$$\frac{\ln(1-0.21)}{\ln(1-0.03)} = 7.74$$

Effect of Convective Flow

The effect of the convective flow on the carbon particle burning time has been estimated by the use of an empirical correlation popularly used for liquid droplet combustion problems (Ranz and Marshall, 1952)

$$\frac{\tau_c \text{ stagnant}}{\tau_c \text{ convective}} = 1 + .276 \text{ Re}^{1/2} \text{ Pr}^{1/3}$$

The Reynolds number is based on the relative velocity between the gases and the particles, and is estimated to be of the order of 10 . The correction factor is then 2 .

Therefore, under cyclone combustor conditions, the burning time of a 100 micron coal particle at atmospheric pressure can be estimated as follows:

- combustion in stagnant ambient atmosphere 90 m sec
- combustion in oxygen depleted atmosphere may increase the burning time to 700 m sec
- convective flow effects may reduce the time by a factor of 2, so that the time of combustion will range from 50 to 350 m sec .

We use an average combustion time of 200 m sec for a 100 micron particle in most of the calculations. This is the input parameter TBAR. The combustion may be somewhat lower because of oxygen enrichment and the high chamber temperatures and some calculations have evaluated the effect of reducing TBAR to 100 m sec.

These combustion times are in general agreement with the recommended times for other practical furnace configurations (Essenhigh, 1976).

The combustion times for other size particles are calculated by the use of a d^2 dependence.

APPENDIX D
Model for Flame Emissivity

The model calculates the total surface area of all the particles in the cyclone combustor and calculates the emissivity assuming no self-shielding or interference between the particles. This is probably a good assumption when the total emissivity of the particles in the flame is low, say below 0.3 .

Consider the total mass of unburnt and partially burnt particles in region I of the combustor,

$$m = K\rho 2\pi R_o L \cdot \delta$$

We define an area-averaged mean particle diameter for steady-state burning conditions in region I to be

$$d_2 = \frac{2 \sum f_i \tau_i}{\sum \frac{f_i \tau_i}{a_i}}$$

Then, the total number of solid particles of average density ρ_s and diameter d_2

$$N = \frac{m}{\frac{\pi}{6} d_2^3 \rho_s}$$

with no self-shielding between the particles, the walls of the combustor see the total surface area of the particles. Then, the emissivity contribution due to the unburnt and partially burnt coal particles is

$$\epsilon_{F_1} = \frac{N \times \pi d_2^2}{2\pi R_o L}$$

or,

$$\epsilon_{F_1} = 6 \frac{K\rho\delta}{\rho_s d_2} \quad (D-1)$$

Region I also contains ash particles, and the program predicts the loading due to ash particles of various sizes. The contribution to the total flame emissivity due to these ash particles can be calculated in a similar manner.

$$\epsilon_{F2} = \sum_i \frac{3 K_{ash_i} \rho \delta}{\rho_{ash} a_i} \quad (D-2)$$

The total emissivity,

$$\epsilon_F = \epsilon_{F1} + \epsilon_{F2}$$

The total emissivity from particles in region I has been calculated by following the above procedure for a number of the test cases. The details of the run conditions for the various test cases have been discussed in Section 4. The emissivity calculations are shown below.

	Simple Shielding Model	Brown-Marco Model
Run 148	0.424	0.072
Run 149	0.598	0.097
Run 151	0.608	0.079

The results from the simple shielding model will be even higher when the small ash particles in region III are included in the analysis. The simple model suggests that the flame emissivity should be considerably higher than that predicted by the Brown-Marco empirical correlation. The actual numbers predicted by the simple model are not to be considered exact because for $\epsilon_F > 0.3$ we expect some interference to exist among the various particles. However, the actual flame emissivity appears to be higher than that predicted by the empirical correlation, and indicates that the correlation may not be applicable to high loading cyclone type combustors.