NUCLEAR STRUCTURE STUDIES
WITH
PIONS AND HEAVY IONS

Progress Report
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Abstract

The elastic and inelastic scattering of $\pi^+$ and $\pi^-$ by $^{13}\text{C}$, $^{16}\text{O}$ and $^{17}\text{O}$ was studied at pion energies close to the [3,3] resonance. Data were taken at the Los Alamos Meson Physics Facility. Large asymmetries were observed, two of which are consistent with pure neutron and pure proton excitations. Transitions to strongly excited states of $^{13}\text{C}$ are in strikingly good agreement with theoretical predictions of Lee and Kurath. In sharp contrast, strong disagreements are found for the weakly excited states. The asymmetries for $^{16}\text{O} + \pi^\pm$ were interpreted as due to isospin mixing between the excited states.

High resolution data for ($t,t'$) and ($^3\text{He},^3\text{He}'$) on $^{13}\text{C}$ were taken to supplement the pion work. Asymmetries were found for the relative cross sections for some states.

In studies of the heavy-ion-nucleus potential the effect of potential resonances on the elastic scattering of $^{16}\text{O} + ^{28}\text{Si}$ was found to be small. A close similarity between the elastic exchange amplitude at $180^\circ$ and the effect of the parity dependence on the elastic amplitude was found. Fits to the new data at 40 and 41.226 MeV required the use of a composite absorptive potential. The surface derivative part of this potential can be deduced from a coupled channels calculation.
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I. Introduction

This report summarizes the work done during the period from June 1, 1979 to May 31, 1980 under contract No. DEAC02-79ER10423 between the University of Minnesota and the United States Department of Energy.

The research activities involved the use of the Energetic Pion Channel and Spectrometer, EPICS, at the Los Alamos Meson Physics Facility. Further data were taken at the Los Alamos Van De Graaff Laboratory. The work at EPICS was done in collaboration with C. L. Morris, R. L. Boudrie, J. Piffaretti and H. A. Thiessen from Los Alamos and members of the Univ. of Texas users group. The work at the Van De Graaff was done together with R. E. Brown, J. A. Cizewski, E. R. Flynn, C. L. Morris from Los Alamos and R. J. Peterson from the Univ. of Colorado. The personnel involved were the Principal Investigator, one full time Research Associate (Dr. D. B. Holtkamp, since Jan. 1, 1980) and one Graduate Student (Susan J. Tripp), both at LAMPF, a part-time Research Associate (Dr. V. Shkolnik) at the University of Minnesota and a part-time undergraduate helper (G. Idzorek).

The pion work and the supplementary \((t,t')\) and \((^3\text{He},^3\text{He}')\) comparison are presented in sections II and III. The heavy ion studies are summarized in section IV and some development work at EPICS and at the University of Minnesota PDP 11/60 system is discussed in section V. Section VI gives a list of Publications, Invited Talks and Abstracts.
II. Nuclear Structure Studies with Pions

1. Elastic and Inelastic Scattering of $\pi^\pm$ from $^{13}$C at 162 MeV
   (LAMPF-Experiment #452).

It is agreed upon by the workers in the field that our $^{13}$C($\pi^-,\pi^\pm$)
experiment at 162 MeV has yielded one of the most interesting results
in nuclear structure physics with pions. Surprisingly large asymmetries
$A = (\sigma(\pi^-) - \sigma(\pi^+))/\sigma(\pi^-) + \sigma(\pi^+)$ were observed (Fig. 1). For two
transitions the results are consistent with the values for $\pi^\pm$ scattering
from free neutrons or protons. Thus evidence was found for pure neutron
particle-hole and pure proton particle-hole transitions. A brief
account of this work has been published\(^1\) and a short report was included\(^2\)
in the booklet "Physics News in 1979". Physics News is published
annually by the American Institute of Physics "to call attention to
some interesting and newsworthy developments in physics and related
fields".

a. Present Status of the Experiment

Data taking for Exp. 452 is completed. During the past months we
have therefore concentrated on a careful analysis of the pion spectra
(Fig. 2) by peak fitting and a determination of absolute cross sections.
The finalized elastic scattering cross sections are presented in Fig. 3
together with preliminary optical model calculations to be discussed
below. The inelastic data are still being worked on but publishable
results will be obtained before the end of the current contract period.

Preliminary inelastic cross sections are shown in Fig. 4-7. Some
experimental details and qualitative arguments for the interpretation
of the data were presented in ref. 1 and need not to be repeated here.
Fortunately we are now able to compare our inelastic data with predictions of DWIA calculations of T.-S. Lee and D. Kurath. For strongly excited states the agreement between theoretical prediction and experimental data is often striking. In marked contrast large discrepancies are found for the weak transitions. We will compare experiment and theory for individual inelastic transitions below after a brief discussion of the elastic data and some preliminary optical model calculations.

b. $^{13}$C + $^\pi^\pm$ Elastic Scattering

Similar to $^{12}$C + $^\pi^\pm$ (ref. 3) we find the first diffraction minimum at a smaller scattering angle for $^{13}$C + $^\pi^-$ than for $^{13}$C + $^\pi^+$ (Fig. 3). The shift is only about 0.5° for $^{12}$C but about 2° for $^{13}$C. We also note that the experimental cross sections for the maximum at $\theta_{c.m.} \approx 70^\circ$ are significantly larger for $^{13}$C + $^\pi^-$ than for $^{13}$C + $^\pi^+$ data at 162 MeV for which the ratio is only about 1.1. In addition we find that the peak to valley ratio ($\sigma_{\text{max}} (\approx 70^\circ)/\sigma_{\text{min}} (\approx 52^\circ)$) is larger for $^{13}$C + $^\pi^-$ than for $^{12}$C + $^\pi^-$ but very similar for $^{13}$C and $^{12}$C + $^\pi^+$. We are now doing optical model calculations with the code PIPIT (ref. 5) in an attempt to understand these large differences.

A p-shell gaussian density distribution is assumed (radius parameters $c_p$ and $c_n$ for the proton and neutron densities, respectively) and different approximations are applied for the off-shell extrapolation of the t-matrix (Fig. 3 dashed lines (a): model of ref. 6 and dash-dotted lines (b): model of ref. 7). The positions of the first minima are reproduced nicely in both cases, (a) and (b), and both for $^\pi^+$ and $^\pi^-$ when $c_p = c_n = 1.56$ fm is used. Thus the large shift in the first
minimum does not necessarily imply strongly different neutron and proton distributions. It seems that Coulomb effects (ref. 3) and the neutron excess of one cause most of the observed shift. As frequently observed in pion optical model calculations the radius parameter $c_p = 1.56$ fm is smaller than expected from the electron scattering results. (A different parameterization was used in the electron work which prevents a straightforward comparison of the radius parameters.) But the value of 1.56 fm is not as small as the $c_p = c_n = 1.4$ fm used for the DWIA calculation of ref. 9 and those presented below (ref. 4). The inelastic calculations are however not affected as much as the elastic scattering predictions by the choice of $c_p$ and $c_n$.

The large ratio $\sigma(\pi^-)/\sigma(\pi^+)$ for elastic scattering around 70° is reproduced only in part by the above mentioned calculations. Further, the predictions (Fig. 3, curves a and b) depend strongly on the model used for the off-shell extrapolation of the $t$-matrix. Thus it is clear that a reliable procedure for the off-shell extrapolation is needed. Although the optical model calculations with $c_p = c_n = 1.56$ fm go already fairly far in explaining the differences between $^{13}$C $+$ $\pi^+$ and $^{13}$C $+$ $\pi^-$ the remaining differences are large enough to warrant a careful study which we are now starting. Possible reasons for the asymmetries which we consider to investigate are discussed in the renewal proposal.

c. Inelastic Transition to the 9.5 MeV State

The most striking result of the $^{13}$C($\pi^\pm$, $\pi^\mp$) experiment was the observation of $\pi^+/\pi^-$ asymmetries that are consistent with pure neutron
and pure proton particle-hole excitations. For a state at 9.5 MeV, which had not been mentioned in the original proposal, we found a ratio $R = \sigma(\pi^-)/\sigma(\pi^+) \approx 9$ or, equivalently, an asymmetry $A = 0.8 \pm 0.2$ as for pion scattering from free neutrons. This result was particularly surprising since the transitions to the $1/2^+$ (3.09 MeV) and $5/2^+$ (3.85 MeV) "single-neutron states" were found only moderately $\pi^-$ enhanced (see section 1e). We concluded that the large $\pi^-$ enhancement is in agreement with expectations for a transition to a state of $(^{12}\text{C}(2^+, T=0) \times 1d_{5/2}^n)$ weak-coupling structure. Such a structure had been proposed for a $9/2^+$ state at about 9 MeV. It was then learned that a very large overlap exists between the wave function of the lowest $9/2^+$ state of a shell model calculation and the wave function of the $(2^+ \times 1d_{5/2})9/2^+$ weak-coupling state. Consequently it was shown by Lee and Kurath by microscopic DWIA calculations using the wave functions of ref. 11 that the ratio $\sigma(\pi^-)/\sigma(\pi^+)$ was indeed 9 for the lowest predicted $9/2^+$ state. In these calculations it is assumed that the $9/2^+$ state is excited in a one-step process which involves the promotion of a $1p_{3/2}$ neutron to the $1d_{5/2}$ shell with an orbital angular momentum transfer $\Delta L = 3$ and a spin transfer $\Delta S = 1$ to the struck neutron. Only $\Delta S = 1$ is allowed since the minimum total angular momentum transfer for exciting a $9/2^+$ state from a $1/2^-$ ground state is $\Delta J = 4$.

The data for the 9.5 MeV state and the theoretical predictions are shown in Fig. 4. The quality of the fit both for relative ($\sigma(\pi^-)/\sigma(\pi^+)$) and absolute cross sections is very good. No collective enhancement factor was needed as expected for a pure single-particle single-hole excitation.
Our $9/2^+$ assignment for the 9.5 MeV level which previously was thought to be $3/2^-$ (ref. 12) is strongly supported by recent electron scattering experiments.

d. Transitions to the Lowest $3/2^-$ and $5/2^-$ States.

The qualitative arguments presented in the original proposal for the $\pi^-/\pi^+$ asymmetries to be expected for the $[2^+ \times 1p_{1/2}]_{3/2^-},5/2^-$ weak-coupling states were confirmed by the data and the DWIA calculations (ref. 9).

We note that the $5/2^-$ state at 7.55 MeV was not resolved from the $7/2^+$ state at 7.49 MeV and the $3/2^+$ state at 7.68 MeV. Thus the cross sections presented for the 7.5 MeV group (Fig. 5) are the summed cross sections for the three states. The transitions to the $3/2^-$ (3.68 MeV) and $5/2^+$ (3.85 MeV) states yielded an unresolved peak which was significantly wider than single-state peaks. With the aid of a peak fitting routine we "resolved" the two states. In the fitting process it was important to use a known peak (e.g. the elastic peak) as energy reference point in addition to the EPICS energy calibration data to predict the expected peak positions of the 3.68 and 3.85 MeV states which were then kept fixed during the search. We estimate the additional error in the cross sections introduced by the peak fitting: $\approx 20\%$ for the 3.68 MeV and $\approx 30\%$ for the 3.85 MeV state. The $(\pi^+,\pi^+)$ cross sections for the 3.68 MeV state are shown in Fig. 6 together with the results of the DWIA calculations (ref. 9) using the Cohen-Kurath (CK) wave functions.
Collective effects are included in these calculations by enhancement factors ("effective charges") based on a comparison of experimental B(E2) values and predictions using the same model wave functions as in the DWIA calculations. For the proton excitations the enhancement factors are from the B(E2)'s of $^{13}$C, for the neutron excitations they are estimated from the B(E2)'s of $^{14}$N since it has the same number of protons as $^{13}$C has neutrons.

The agreement between the $\pi^+$ data for the 3.68 MeV state and the 7.5 MeV group of states and the DWIA curves for the $3/2^-$ and $5/2^-$ states, respectively, is excellent (Fig. 5+6). Contributions to the 7.5 MeV group from $\pi^+$ scattering to the 7.59 (7/2$^+$) and 7.68 (3/2$^+$) MeV levels are predicted to be very small. The $\pi^-$ data for the 3.68 MeV state (not shown) are below the predictions indicating the need for a smaller neutron enhancement factor. The $\pi^-$ data for the 7.5 MeV peak are considerably larger ($\approx 50\%$) than the predictions. However, the transitions to the 7/2$^+$ and 3/2$^+$ states which could be neglected for $\pi^+$ scattering are predicted to be strongly $\pi^-$ enhanced and the calculated cross sections are not negligible. In Fig. 5 the predicted $\sigma(\pi^-)$ is shown for the 5/2$^-$ level alone and for the sum of the 5/2$^-$, 7/2$^+$, and 3/2$^+$ levels. The remaining difference between the summed DWIA curves and the data might be due to an underestimate of the neutron enhancement factor for the collective 5/2$^-$ state.

e. Transitions to the "Single Neutron" States at 3.09 (1/2$^+$) and 3.85 (5/2$^+$) MeV.

If the 3.09 and 3.85 MeV states could be reached only by the
promotion of the valence neutron from the $1p_{1/2}$ shell to the $2s_{1/2}$ and $1d_{5/2}$ shells, respectively, the cross sections should be relatively small, the $\sigma(\pi^-)/\sigma(\pi^+)$ ratio should be 9 and the asymmetry should be 0.8 as for pion scattering from free neutrons. Experimentally we find $R \approx 2$ or $A = 0.3 \pm 0.1$, i.e. only a very moderate enhancement as, e.g. for the collectively enhanced transition in $^{18}\text{O}(\pi^-,\pi^+)\text{ }^{18}\text{O}(2^+, 1.98\text{ MeV})$ (ref. 15). The DWIA calculations with the MK wavefunctions (ref. 11) underpredict the already small differential cross sections. For the $5/2^+$ state the observed ratio $R \approx 2$ is predicted quite well but the calculated cross sections are a factor of two smaller than the data. For the $1/2^+$ state a preliminary DWIA calculation yields about 20 for the $\pi^-/\pi^+$ ratio and cross sections that are significantly smaller than the data. It appears that the assumptions made for the DWIA calculations are not valid for the weakly excited states.

f. Transitions to Other Weakly Excited States

Similar problems are encountered for the weak transitions to the 2nd $5/2^+$ state at 6.86 MeV, the first excited $1/2^-$ state at 8.86 MeV, and others. The model wavefunctions of ref. 11 for the 2nd $5/2^+$ state have a large overlap with a mixture of $(2^+ \times (2s_{1/2})$ and $(2^+ \times 1d_{5/2}$ weak-coupling structures. The predicted cross sections are much smaller than the data and practically no enhancement is observed in contrast to a prediction of $R \approx 17$. (Ratios larger than 9 can be obtained in the DWIA as a result of interfering amplitudes.) For the 8.86 MeV $1/2^-$ state a ratio of $R = 1/9$ (asymmetry $A = -0.8$) as for a pure proton particle-hole transition is predicted. Only a small $\pi^+$ enhancement is observed ($R \approx 0.7$, $A = -0.2$) and the measured cross sections are larger
than the predictions. We believe that the large discrepancies contain rather valuable information on the pion-nucleus interaction and nuclear structure. The weak transitions might involve both one-step and two-step reaction amplitudes of comparable magnitude. The g.s. \((1/2^-)\) to \(8.86\) MeV \((1/2^-)\) state transition is of particular interest because of its predicted very simple structure. In the one-step process only a \(p_{3/2} \rightarrow p_{1/2}\) proton particle-hole transition should be involved in clear contradiction to the data. Coupled channels calculations which include transitions through intermediate excited states can now be done and some of the questions raised by the disagreements between the DWIA and the data will be answered in the near future.

g. Transitions to Strongly Excited States Above 10 MeV.

The data for the group of states close to \(11.8\) MeV are shown in Fig. 7. The cross sections are too large to be attributed solely to the well known \(2p-1h\) state of spin \(3/2^-\) observed in \(^{14}\)N(\(d,^3\)He). Most of the \(\pi^-\) cross sections can be explained by summing the predictions for the 4th \(5/2^+\) and the 3nd \(7/2^+\) states. For the \(\pi^+\) data a 60% discrepancy with the predictions suggests a transition to a \(\pi^+\) enhanced state still to be identified or some "collectivity" in the transitions to the \((5/2^+)_4\) and \((7/2^+)_2\) states to be accounted for by effective charges that can differ for neutrons and protons. The transition to the strongly \(\pi^+\) enhanced group of states at \(16.1\) MeV has features similar to the strongly \(\pi^-\) enhanced transition to the \(9.5\) MeV state (sec. 1-C). The model of ref. 11 predicts such a state at a slightly \((1\) MeV) higher excitation energy with spin/parity \(J^\pi = 9/2^+\). The strongly excited
group near 21.5 MeV is still being analyzed. The shift in the centroid energies of the group between $\pi^+$ and $\pi^-$ suggests isospin mixing effects as for $^{12}\text{C} + \pi^\pm$ (ref. 17).

h. Summary

There is excellent agreement between experiment and theory for the $\pi^+$ data for the 3.68 and 7.55 MeV states. The agreement with the $\pi^-$ data for the same states is not quite as good. Part of this problem is due to contributions from unresolved states which seem to be more strongly excited by $\pi^-$ than by $\pi^+$, as predicted. Another possible reason is the need for slightly different "effective charges" for the neutron components. Further, the agreement between the calculations and the data for the $9/2^+(9.5 \text{ MeV})$ state is surprisingly good. However, the weakly excited states are generally much more strongly excited than predicted. Some two-step calculations should be done to find out their importance for the weak transitions. Further, a two-step calculation for the $9/2^+$ state would be useful. Perhaps the excellent fits to the 9.5 MeV data are due to the neglect of CC effects?

2. Energy Dependence of $^{13}\text{C}(\pi^\pm,\pi^\pm')$ Between 116 and 240 MeV at a Large, Constant Momentum Transfer. (LAMPF Experiments 452 and 510.)

To support the spin/parity assignment $J^\pi = 9/2^+$ for the 9.5 MeV state in $^{13}\text{C}$ (see sec. 1c) we took spectra at 4 additional pion energies, 116, 180, 200, and 240 MeV. The momentum transfer was kept fixed at a value of $\approx 300 \text{ MeV/c}$. Since a $9/2^+$ state can only be reached by a spin-transfer $\Delta S=1$ a distinctly different energy-dependence is expected for this transition in comparison with others that can be reached by
ΔS = 0 (or ΔS = 0 and ΔS = 1). This expectation was confirmed and a report on the results has been submitted to the recent LAMPF workshop\textsuperscript{18}. A copy of the report is attached.

To be able to extend these measurements to other momentum transfers and more pion energies, a proposal (#510) was written and approved. Phase 1 of experiment #510 will consist of a completion of the large momentum transfer data. It has been scheduled for the end of March 1980. Phase 2, comprising the larger fraction of the approved beam time, will be run later in the year.

3. Inelastic Scattering of $\pi^+$ on $^{17}$O and $^{16}$O.

(LAMPF Experiment #369 and #229.)

a. $^{17}$O($\pi^+,\pi^+$') at 164 MeV.

After the completion of the run on $^{13}$C (experiment #452) data taking was started on pion scattering from $^{17}$O (experiment #369). Only a limited number of spectra was taken. An example is presented in Fig. 8. We succeeded in resolving the 0.871 MeV ($1/2^+$) "single-particle" state from the tail of the elastic peak. As expected, a $\pi^-$ enhancement was observed for this state, however the ratio $R = 2.6 \pm 0.9$ was considerably smaller than the free neutron value. Several other states display $\pi^+$ enhancements of about 2. These measurements were however not continued at that time because it was felt that improved energy resolution would greatly increase the nuclear structure information to be obtained. Therefore we decided to postpone the completion of the $^{17}$O($\pi,\pi'$) experiment until after the installation of the new
scattering chamber. We presume that the $^{170}$ experiment will continue in the fall of 1980.

The relative isotopic abundances of the oxygen isotopes in the target used for the $^{170}$ run are 34.63% of $^{160}$, 54.75% of $^{170}$, and 10.63% of $^{180}$. Thus $^{160}$ constitutes a major contaminant and consequently some $^{160} + \pi^+$ data were taken using a natural oxygen (99.8% of $^{160}$) target to allow a subtraction of the $^{160}$ contaminants from the spectra taken with the $^{170}$ target.

b. $^{160}$ ($\pi^+,\pi^+$) at 164 MeV. (Reported by D. B. Holtkamp)

The $^{160}$ data taken for the contaminant subtraction mentioned above turned out to be extremely interesting by themselves and permission was obtained to use some of the beam time allocated to experiment 229 (Univ. of Texas) to extend the data taking on $^{160}$. A Ph.D. Thesis was written by D. Holtkamp on these data and the interpretation of some large $\pi^-/\pi^+$ asymmetries for three $4^-$ states around 19 MeV in terms of isospin mixing. A manuscript is now being prepared for publication also and some of the results will be discussed below.

Ice targets of 160 and 320 mg/cm$^2$ thickness were used. The incident pion energy was 164 MeV, and the angular range was from 53° to 89° (lab). Typical energy resolution was 350 keV (FWHM). The absolute normalization is believed to be accurate to approximately 10% while the relative $\pi^+/\pi^-$ uncertainty is about 5%.

Spectra obtained from $\pi^+$ and $\pi^-$ scattering at a lab angle of 77° are shown in Fig. 9. The ($4^-, T=0$) states at 17.79 and 19.80 MeV show large $\pi^+/\pi^-$ asymmetries but the ($4^-, T=1$) state at 18.98 MeV
does not. From the angular distributions shown in Fig. 10, one may obtain the following ratios of the summed yields for $\pi^+$ vs. $\pi^-$, averaged over the angular range measured: (1) 17.79 MeV ($4^-$, $T=0$), $R(\pi^+ / \pi^-) = 1.59 \pm 0.12$; (2) 18.98 MeV ($4^-$, $T=1$), $R(\pi^+ / \pi^-) = 0.96 \pm 0.08$; and (3) 19.80 MeV ($4^-$, $T=0$), $R(\pi^+ / \pi^-) = 0.61 \pm 0.05$. (Errors quoted are statistical.) Because of charge symmetry, $\pi^+/\pi^-$ scattering from states of good isospin in a self conjugate nucleus such as $^{16}O$ is expected to be symmetric.

These states in $^{16}O$, which are believed to be of a $(p_{3/2}^{-1} \cdot d_{5/2})$ configuration, have been seen previously in one-nucleon transfer reactions\textsuperscript{19} and in inelastic proton scattering at medium energies\textsuperscript{20}. The 18.98 MeV ($4^-$, $T=1$) state has also been observed in high energy electron scattering\textsuperscript{21,22}. Continuum effects in the reaction channels have been investigated by Siciliano and Weiss\textsuperscript{23} and cannot explain the asymmetries observed. Our interpretation of the asymmetries for pion excitation of these states is that they are substantially mixed in isospin.

In previously observed cases of strong isospin mixing\textsuperscript{24-28} in $^6$Be, $^{12}$C, and $^{16}$O, only two states were believed to be involved. A comparative study with $\pi^+$ and $\pi^-$ scattering to mixed states enjoys the advantage that the two probes differ only in their charge (i.e., their $T_z$ component), so that direct comparisons of the two probes to a given state (or states) are quite sensitive to the isospin composition of that state (or states). Pions were also used by Moore, et al.\textsuperscript{29} to excite $4^-$ states in $^{12}$C, which are of a $(p_{3/2}^{-1} \cdot d_{5/2})$ configuration and which appear to be strongly isospin mixed\textsuperscript{17}.\textsuperscript{17}
In the previous cases of two-state isospin mixing$^{17,24-28}$, the $T_<$ state becomes mostly a proton state ($\pi^+$ enhanced) and the $T_>$ state is predominantly a neutron state ($\pi^-$ enhanced). In the case of $^{16}O$ three $4^-$ states are known within 2 MeV so that three-state mixing has to be considered. The qualitative picture of the mixing in $^{16}O$ is that the two $T=0$ states mix through the $T=1$ state. The $T=1$ state apparently remains nearly pure in isospin as indicated by the $\pi^+/$$\pi^-$ ratio of unity, however the lower level becomes mostly a proton state and the upper level becomes mostly a neutron state.

A quantitative analysis of the data is almost completed. Charge dependent matrix elements $<4^-, T=0, 17.79 \text{ MeV}|\psi_{c,d}\rangle|4^-, T=1, 18.98 \text{ MeV}> = (-147 \pm 25) \text{ keV}$ and $<4^-, T=1, 18.98 |\psi_{c,d}\rangle|4^-, T=0, 19.80 \text{ MeV}> = (-99 \pm 17) \text{ keV}$ were extracted. A complete account of this work is now being prepared for publication.
III. Comparative Study of \((t,t')\) and \((^3\text{He},^3\text{He}')\) on \(^{13}\text{C}\) to Supplement the \(^{13}\text{C}(\pi^\pm,\pi^\pm')\) Experiments.

a. Motivation for the Experiment

Two reasons prompted us to start a comparative study of inelastic scattering of tritons and \(^3\text{He}\) from \(^{13}\text{C}\).

(i) For several groups of states that were not resolved in the pion experiment information was needed on how strongly conventional probes excite the individual states. However, no high resolution (\(\Delta E \approx 10\text{-}20\text{ keV}\)) work had been done previously on inelastic scattering of \(p, d, \alpha\), etc. The most complete work on \(^{13}\text{C}(^3\text{He},^3\text{He}')\) and \(^{13}\text{C}(^3\text{He},t)\) (ref. 30) dates back to 1969. (Energy resolution \(\Delta E \approx 150\text{-}175\text{ keV}\).)

(ii) As for \((\pi^-,\pi^-')\) and \((\pi^+,\pi^+')\) comparisons of \((t,t')\) and \((^3\text{He},^3\text{He}')\) are sensitive to the isospin (or neutron/proton) structure of inelastic transitions. The large asymmetries between \(\pi^+\) and \(\pi^-\) scattering on \(^{13}\text{C}\) suggest significant differences between \((t,t')\) and \((^3\text{He},^3\text{He}')\) for the transitions to the same states even though the \(^3\text{He}+p\) and \(t+n\) interactions are not as different from the \(^3\text{He}+n\) and \(t+p\) interactions as \(\pi^+p\) and \(\pi^-n\) are from \(\pi^+n\) and \(\pi^-p\).

Of course, differences are expected and observed between \((n,n')\) and \((p,p')\), but such comparison are experimentally more cumbersome than comparisons.
of \((t, t')\) and \(({}^3\text{He}, {}^3\text{He})\) although the reaction mechanism in the latter pair of reactions is probably more complex than in the former.

Before the details of the experiment are reported we will discuss the complementary nature of inelastic scattering of pions and mass-3 projectiles.

b. The Complementary Nature of \(\pi^-/\pi^+\) and \(t/{}^3\text{He}\) Inelastic Studies

The large differences between the \(\pi^+ p\), \(\pi^- n\), and the \(\pi^+ n\), \(\pi^- p\) interactions is usually represented by the pion-nucleon scattering amplitudes \(A(\pi^+ N)\) with \(A(\pi^+ p) = A(\pi^- n) \approx 3\) at the \([3, 3]\) resonance. Ignoring distortion effects the inelastic pion-nucleus amplitudes are approximately proportional to the elastic pion-nucleon amplitude and a nuclear structure factor which contains the p-h matrix elements \(M_n\) and/or \(M_p\). \(M_n \neq 0\) and \(M_p = 0\) for a pure neutron particle-hole (p-h) excitation, and \(M_p \neq 0\) and \(M_n = 0\) for a pure neutron p-h excitation. Usually both neutron and proton p-h excitations contribute so that the cross section ratios are given by

\[
R = \frac{\sigma(\pi^-, \pi^{-\prime})}{\sigma(\pi^+, \pi^{\prime+})} \propto \frac{3M_n + M_p}{M_n + 3M_p}^2
\]

which equals 9 for \(M_p = 0\), i.e. pure neutron, and 1/9 for \(M_n = 0\), i.e. pure proton excitations.

Completely equivalent to and often more convenient than the description of the transitions in terms of mixed proton and neutron p-h excitations is the description in terms of a mixture of isospin transfers \(\Delta T=0\) and \(\Delta T=1\) (isotensor terms are small). The associated p-h matrix-elements \(M_0\) and \(M_1\) can be written in terms of \(M_n\) and \(M_p\):
\[ M_0 = \frac{1}{\sqrt{2}} (M_n + M_p) \quad \text{and} \quad M_1 = \frac{1}{\sqrt{2}} (M_n - M_p) \]
so that \[ R \approx \left| \frac{2M_0 - M_1}{2M_0 + M_1} \right|^2. \]
The factor 2 represents the ratio of the \( \Delta T=0 \) and \( \Delta T=1 \) components of the \( \pi \)-nucleon amplitudes. For a pure neutron transition \( M_0 = -M_1 \) and for a pure proton transition \( M_0 = M_1 \).

In a comparison of \( (t,t') \) and \( (^{3}\text{He},^{3}\text{He}') \) the cross section ratio cannot be written as simply as for \( (\pi^+,\pi^+) \) for \( [3,3] \) resonance dominance. Several interaction terms contribute, i.e. central forces without and with spin, spin-orbit forces, etc. In addition distortion effects are very strong and the reaction mechanism might not be a simple one-step mechanism. But to point out the analogy to \( (\pi^+,\pi^+) \) we discuss a simple case that proceeds by a one-step direct process with a spin transfer \( \Delta S=1 \) due to the \( \sigma_1 \cdot \sigma_2 \) and \( (\sigma_1 \cdot \sigma_2) (\tau_1, \tau_2) \) terms of the force. Here \( \sigma_1 \) \( (\tau_1) \), \( \sigma_2 \) \( (\tau_2) \) are the spins (isospins) of projectile and target-nucleon, respectively. Distortion effects may be taken care of by a simple DWBA calculation with a force of reasonable radial behavior. With \( r = \sigma_{\text{exp}} / \sigma_{\text{DWBA}} \) we may then write
\[ \rho = \frac{r(t,t')}{r(^3\text{He},^{3}\text{He}')} = \left| \frac{\alpha M_0 - M_1}{\alpha M_0 + M_1} \right|^2 \]
where \( \alpha = \frac{V_0}{V_1} \) is the ratio of the strengths of the \( \sigma_1 \cdot \sigma_2 \) and \( (\sigma_1 \cdot \sigma_2) (\tau_1, \tau_2) \) parts of the projectile/target-nucleon interaction. If \( \alpha \) is known \( \beta = \frac{M_0}{M_1} \) can be extracted from the experimental ratio \( \rho \).

In a more rigorous approach the interactions and the nuclear structure matrix elements (if known) have to be inserted explicitly into the DWBA calculations. Then a comparison with individual experimental transitions can be made.
For a number of states the pion data showed very good agreement with theoretical predictions when the values of $M_n$ and $M_p$ (or $M_0$ and $M_1$) obtained from the CK (ref. 9) and MK (ref. 11) wave functions were used. Specifically the pion data for the 9.5 MeV state required an interpretation in terms of a pure neutron transition ($M_0 = -M_1$) and a spin transfer $\Delta S = 1$. In this case the $(t,t')$ and ($^3\text{He},^3\text{He}')$ data should give a ratio $\rho = \left| \frac{\alpha_{1}+1}{\alpha_{1}} \right|^2$ which can be solved for $\alpha$. Note that for pion scattering the spin transfer is due to the $L\sigma_2$ term of the potential ($\sigma_1 = 0$ for the pion), but for mass 3 scattering it is probably mainly due to the $\sigma_1\sigma_2$ term. We mention in passing that a further test of the $(t,t')$ and ($^3\text{He},^3\text{He}')$ results can be obtained from ($^3\text{He},t$). Only the $\Delta T = 1$ parts of the force can contribute to ($^3\text{He},t$), if the reaction is one-step direct.

c. The Experiment

To perform the $^{13}\text{C}(t,t')$ and ($^3\text{He},^3\text{He}')$ experiments a collaboration was started with Drs. R. E. Brown, E. Flynn, and J. Clzewski at the Van De Graaff facility at Los Alamos. Both triton and $^3\text{He}$ beams are available at this laboratory in addition to a super-high-resolution Q3D magnetic spectrograph. Due to the limited incident beam energy ($E_t \approx 23$ MeV (lab.) or 14.5 MeV(c.m.)) the study had to be restricted to states below $\sim 11$ MeV. A 30 $\mu$g/cm$^2$ thick target foil enriched to 97% in $^{13}\text{C}$ was used. The 23-MeV triton beam was obtained in the three-stage mode of operation. For a few runs a 23-MeV $^3\text{He}$ beam was obtained from the tandem alone. Scattered particles were momentum analyzed with the Q3D spectrograph and detected by a wire chamber positioned along the focal plane. Three momentum bites were taken to cover the ranges
of excitation energy between 2.8 and 4.8, 6.5 and 8.5, and 8.5 and 10.5 MeV. A few elastic data were measured also for normalization purposes. Spectra for the 2.8-4.8 MeV region are shown in Fig. 11. The energy resolution width for the triton spectra was $\Delta E = 16$ keV. The $^3$He data are not as good because the accelerator was quite unstable and $^3$He spectra were taken at 4 angles only. Another run is scheduled for March 1980. For (t,t') a total of 10 angles for the three regions of excitation were done. The spectra (Fig. 11) show significant differences between (t,t') and ($^3$He,$^3$He') in the relative yields of the states.

The angular distributions (Fig. 12) for the 3.68 and 3.85 MeV levels quite clearly show isospin effects. For the 3.68 MeV state the experimental $(t,t')/($$^3$He,$^3$He$'$) ratio is 1.4. With the aid of DWBA calculations we explained this ratio as simply due to the difference between the Coulomb terms in the optical potentials for the tritons and the $^3$He.

This is in agreement with the pion results which showed the 3.68 MeV transition to be fairly purely isoscalar. In contrast, the ratio for the 3.85 MeV state is 0.80, and if normalized to the 3.68 MeV state ratio - which is equivalent to a normalization to DWBA calculations - the double ratio is found to be $\rho \approx 0.58$ i.e. quite different from 1.

The observed $^3$He enhancement of the 3.85 MeV (5/2$^+$) level is consistent with the $\pi^-$ enhancement; both enhancements can be interpreted with $M_n > M_p$. However we found the $(t,t')/($$^3$He,$^3$He$'$) ratio for the 9.5 MeV level almost equal to the one for the 3.68 MeV level, i.e. consistent with a pure isoscalar ($M_1=0$) transition in contrast to the pion data which show a pure neutron ($M_p=0$) transition. We believe that this problem is due to two-step processes being quite important in t and $^3$He scattering. Further analysis of the data is in progress.
IV. Nuclear Structure Studies with Heavy Ions.

The data taken at the Brookhaven National Laboratory for the $^{16}O + ^{31}p$ system (see previous proposal) in collaboration with F. D. Becchetti and J. Jaenecke of the University of Michigan were somewhat discouraging. The cross sections showed structure in the $180^\circ$ excitation function which we obtained by bombarding a SiO$_2$ target with a $^{31}p$ beam and by detecting the recoiling $^{16}O$ ions at $0^\circ$ at the focal plane of the BNL-Q3D with the Minnesota heavy-ion detector. However, the cross sections were found an order of magnitude smaller than for $^{28}Si + ^{16}O$ and it appeared that an unreasonable amount of beam time would be needed to complete the experiment.

Only a relatively small fraction of the contract has been dedicated to heavy-ion physics and we felt that a continuation of our analysis of the $^{16}O + ^{28}Si$ would make best use of the available funds. We therefore decided to concentrate on an extension and completion of the work on $^{16}O + ^{28}Si$ for which a large body of data already exists instead of continuing with the data accumulation during this contract period.

Studies of the $^{16}O + ^{28}Si$ system have contributed much to our understanding of the interaction between $^{16}O$ and nuclei up to Calcium. Nevertheless, there still are considerable controversies regarding the different interpretations by the large number of groups who presently work on attempts to fit the experimental data. The large number of papers published within the last two years alone shows how widespread the interest in $^{16}O + ^{28}Si$ still is as a good example for studies of the heavy-ion-nucleus interaction.
The progress made in our continuing study of the $^{16}$O + $^{28}$Si is discussed below (reported by V. Shkolnik and D. Dehnhard).

Attempts to fit the $^{16}$O + $^{28}$Si data made use of a whole range of models: from a simple phenomenological optical model\textsuperscript{31,32}, modified optical models ($l$-dependence\textsuperscript{33}, parity-dependence\textsuperscript{34,35} - together with surface transparency), optical model based on folding\textsuperscript{36}, to more exotic models like Regge-poles added to "backgrounds" generated by strongly absorbing potentials\textsuperscript{37,38,39}. We strongly believe that the most promising approach at the present time is to use the phenomenological optical model modified on physical grounds to provide \textbf{systematic close fits to a large set of data over a sufficiently large range of energy}. We are critical of investigators who fit only selected data such as at one or two energies, even when they fit them well, and we are as critical of poor fits. We differ strongly with some investigators as to the propriety of their models and the explanations and conclusions they put forward. Yet we appreciate the break-through achieved by the Stony Brook-BNL collaboration by measuring complete angular distributions and 180° excitation functions.

It seems that the potential needed to fit the "high-energy" data, E\textsubscript{Lab}\textsuperscript{40}, (E\textsubscript{Lab} = 147.5\textsuperscript{40} and 215.2 MeV) is qualitatively different from the potential in the 40-63 MeV region. A future goal is a smooth connection to a potential that fits these higher energies. Between E\textsubscript{Lab} = 66.0 MeV, which we plan to fit shortly, and 142.5 MeV the only published data consist of two featureless angular distributions at E\textsubscript{Lab} = 72.0 and 81.0 MeV\textsuperscript{40}. This is the reason why it is
not presently possible to make the connection to the potential at 142.5 MeV, since data at several intermediate energies would be needed. Theorists also have shown interest in this system and we refer here to the work by Satchler, Takemasa and Tamura, Lipperheide, and Landowne.

A particularly sensitive test of any description is the 180° excitation function measured by Barrette et al. It has been quite tempting to associate the resonance-like structure in the 180° data with single partial wave resonances. Potential shape resonances are expected close to the grazing angular momentum provided the real potential is sufficiently deep to generate a pocket in the total potential energy. However, whether potential shape resonances are contributing significantly to the elastic cross sections depends critically on the absorption.

We have tried in the past to interpret the available data by a smoothly energy-dependent optical potential. We succeeded in fitting very closely angular distributions between 45 and 63 MeV and the 180° excitation function with a surface transparent potential that has a very small "parity-dependent" term, i.e. the depth of the potential is modified by the factor $(1 + (-1)^L C)$. We have now investigated the properties of this potential in some detail and report here on the results. Figure 13 shows the real phase shifts as a function of the energy for the parity dependent potential. We note that above 48 MeV all resonances are of the "dispersion" type due to strong absorption, i.e. the resonances decay preferentially into nonelastic channels.
(For this best fit potential the resonating partial wave \( L \) is three units below grazing \( L \) and thus strongly absorbed.) The strongly absorbed resonating partial wave contributes to the elastic scattering (like neighboring nonresonating waves with \( L < L \) grazing) as part of diffraction scattering. It seems clear that any resonance modification of a strongly absorbed wave has only small effects on the elastic scattering. It also seems clear that the peaks in the 180° excitation function are not correlated with the positions of the potential shape resonances. (See Fig. 1 of ref. 34.)

The phase shifts calculated without the \((-1)^L\) term in the potential differ only very little but in a very systematic way from the phase shifts calculated with the \((-1)^L\) term. Since the parity-dependence is probably related to exchange terms we have searched for a connection between our phenomenological \((-1)^L\) term and the \(^{12}\text{C}\) transfer amplitude\(^{44}\).

Fig. 14 shows on the r.h.s. the amplitude \( f_{\text{exch}}(180°) \) and on the l.h.s. the amplitude at the same angle obtained from the difference of two scattering two amplitudes: one calculated with the potential with the parity terms present, the other amplitude calculated without the parity terms. The resemblance of the two curves obtained by two entirely different methods is striking.

We have investigated our parity-dependent potential for possible ambiguities in the real potential depth. Fig. 15 shows the results of this investigation at \( E_{\text{Lab}} = 55.0 \text{ MeV} \) obtained by searching on the complete angular distributions (top) and at forward angles only (bottom). Clearly, \( V_0 \) does not have the continuous Igo ambiguity as seen from the
parabolas. During the search the parameter $C$ of the parity dependence was kept constant, the real well depth $V_o$ was gridded on and all other parameters of the WS form were searched until convergence. There appears to be a definite preference for the depth $V_o \sim 24.5$ MeV. Further, there is no "parabola" for a smaller $V_o$. We also note that $V_o = 24.5$ MeV is the first real depth that is deep enough to generate a "pocket" in the total potential energy $V_{\text{tot}} = V_{\text{nucl}} + V_{\text{coul}} + V_{\text{centrif}}$.

A potential extrapolated from the parameters of ref. 34 did not yield satisfactory fits to recent angular distribution data at 41.226 MeV (ref. 38). However, we found a modification needed for a good fit that also removes an objection raised in the past to the rather small absorptive strength at low energies, below the region for which it was derived. We obtained an excellent fit to the data (shown for 41.226 MeV in Fig. 16) by use of the old real potential. However, the imaginary part had to be composite, i.e. an interior part of WS form (about 40 MeV deep, with radius parameter $r_i = 0.890$ fm and diffuseness $a_i = 0.485$ fm) and an additional WS derivative term (WD) centered at $R = 7.2$ fm. The strength of the volume absorption is about 2.2 times that of the real potential $V_o$ and it shares the energy dependence of $V_o$. The strength $W_D$ of the surface term increases faster than linearly with energy, e.g. $W_D \sim 1.1$ MeV at 41.226 MeV and $W_D \sim 3$ MeV at 55 MeV. The fall-off of the composite potential at large radii corresponds to that of a diffuseness of about 0.31 fm, in good agreement with the previous potential of simple WS form for the 45-63 MeV region. The additional interior absorption also improves the fits at other energies, especially in the "interference region" between 100° and 140° at 50
and 55 MeV. The combined volume/surface potential is still surface-transparent: Its interior absorption is mainly confined to radii less than 6 fm - a region barred to the partial waves of $L > L_{\text{grazing}}$ by the centrifugal barrier. Partial waves with $L < L_{\text{grazing}}$ are more strongly absorbed in comparison with the old potential (ref. 34) in particular for the low energies. To illustrate these points Fig. 17 shows the total potential energy for partial waves $L = 16$, 17, and 18, the shape of the absorptive potential and the absorption coefficient $\eta_L = 1 - |S_L|^2$. We emphasize that the surface derivative term was not imposed ad-hoc but that it was required by the fit to the data.

We are now investigating the connection between the surface derivative term in the composite absorption potential and the coupling of the elastic and inelastic channels, specifically the first excited $2^+$ state of $^{28}\text{Si}$ (1.78 MeV). The new potential (with the surface term left out) was used in a coupled channels calculation with $\beta_2 = -0.32$ as quadrupole deformation. The coupled channels wave functions were then used to calculate the effective absorption due to the coupling with the $2^+$ state. The strength of the effective absorption was found in agreement with the phenomenological $W_0$ within about 20%. This work is still in progress.
V. Development Work

1. EPICS Development

Graduate student Susan J. Tripp participated in the EPICS Development run during cycle 24. The procedures used and the results obtained are discussed below. (Reported by S. J. Tripp.)

A chopped $H^-$-beam was used to measure the relative intensities of pions, muons, and electrons at the EPICS target as a function of pion energy. The spectrometer was set at $0^\circ$ and timing information from the low intensity 40 ns chopped $H^-$-beam was used to measure the time of flight of the particles through the channel and spectrometer. The time-of-flight spectrum was gated by the elastic group in the missing mass spectrum, which helped eliminate the counting of muons from the pion decay. The $\pi/\mu/e$ fractions were determined by peak fitting each particle type in the time-of-flight spectrum and correcting for pion decay along the 14 meter flight path from the target to the focal plane of the spectrometer.

A beam steering exercise was conducted in which the primary beam was moved from the center of the Al production target to 6 mm on either side of the center. $\pi/\mu/e$ fractions were measured for 110 MeV $\pi^-$ at each beam position, as was the centroid of the "$\phi$-target" distribution. From this data it was concluded that if the beam stays within 1 mm of the center of the Al target, the pion fraction changes by less than 1.5% and that the angle of the beam coming out of the channel changes by less than 0.5 mrad.

An attempt was also made during this development to optimize the spectrometer resolution by adjusting the polynomials used to calculate
the percent momentum change. A value of $\Delta p/p = 4.2 \times 10^{-4}$ at 290 MeV was obtained. During these measurements, the primary beam spot size may have been as large as 4 mm at the Al target, which could account for a large part of the measured resolution. Two sets of runs were taken, at 190 and 290 MeV pion energy, to determine whether the shunts on the channel magnet power supplies are necessary. One run was taken with the shunts set by the normal procedure, and a second run was taken with all four shunts set to zero and the main supply readjusted to give the proper field in BM04. After adjusting the fm magnets and retuning the polynomials it was found that the resolution was the same as it had been when the shunts were used. Yields were measured and it was found that no flux was lost when the shunts were turned off.

2. Data Reduction Facility at the University of Minnesota.

The PDP 11/60 computer of the High Energy Group at the University of Minnesota is now being used for reduction of LAMPF data. This was made possible by a grant to Professor N. M. Hintz designated to purchase auxiliary equipment for the 11/60 system to satisfy the needs of the two U. of M. LAMPF programs. Undergraduate student G. Idzorek, supported part-time by this contract, was instrumental in the installation of this equipment and the start-up of the HRS and EPICS programs. He is presently writing a report on the system and its application to a special case for future users.
References

4. T.-S. Lee and D. Kurath, private communication. We are grateful to Drs. Lee and Kurath for making their DWIA calculations available to us.
14. L. E. Smith, computer program LOAF.


VI. Publication, Invited Talks, Abstracts

A. Publication

B. Invited Talks
D. Dehnhard

Nuclear Seminars at:
- Lawrence Livermore Laboratory
- Indiana University Cyclotron Facility
- Los Alamos Van De Graaff Laboratory
- Oak Ridge National Laboratory
- Physics Colloquium at University of Minnesota

V. Shkolnik:
Nuclear Seminar at Oak Ridge National Laboratory (May 1980)

D. G. Holtkamp:
Nuclear Seminar at University of Minnesota

S. J. Tripp:
Nuclear Seminar at University of Minnesota (April 1980)
Abstracts


7. "Comparison of ($^3$He,t) and ($^3$He,$^3$He) Yields to Analog States in A=13", R. J. Peterson, J. R. Shepard, R. A. Emigh, M. Rayman, T. G. Masterson and D. Dehnhard, Bull. Am. Phys. Soc., to be published.


VII. Distribution of Effort of Principal Investigator

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Fig. 1. Asymmetries \( A = (\sigma_{\pi^-} - \sigma_{\pi^+})/(\sigma_{\pi^-} + \sigma_{\pi^+}) \) from \(^{13}\text{C}(\pi^\pm, \pi^\pm')\) at \( T_\pi = 162\) MeV for selected excited states.
Fig. 2. Normalized yield spectra $Y_{\pi^\pm}$ for $^{13}$C($\pi^\pm,\pi'$) summed over

$\theta_{\text{Lab}} = 62^\circ, 68^\circ, 74^\circ, 80^\circ, 86^\circ$ and the difference between

$Y_{\pi^-}$ and $Y_{\pi^+}$. 
Fig. 3. Differential cross sections $\sigma(\theta)$ for elastic scattering of $^{13}\text{C} + \pi^+$ at 162 MeV. Dashed Line (a): "Off-shell model of ref. 6, Dash-dotted line (b): Off-shell model of ref. 7."
Fig. 4. $\sigma(\theta)$ inelastic scattering of $\pi^+$ and $\pi^-$ to the 9.5 MeV state in $^{13}\text{C}$. Broken line: DWIA calculations of Lee and Kurath (ref. 4) for $1/2^- \rightarrow 9/2^+$ transition.
Fig. 5. $\sigma(0)$ for inelastic $\pi^+$ (left) and $\pi^-$ (right) scattering to the group of $^{13}\text{C}$ at 7.5 MeV. Solid lines: DWIA calculations of ref. 9 for the $5/2^-$ member of the triplet. Broken line (for $\pi^-$ only): Sum of DWIA predictions ref. 9 and 4 for all three states of the triplet.
Fig. 6. $\sigma(\theta)$ inelastic $\pi^+$ scattering to the $3/2^-$ state at 3.68 MeV.

Solid line, DWIA pred. of ref. 9.
Fig. 7. $\sigma(\theta)$ for inelastic $\pi^+$ (left) and $\pi^-$ (right) scattering to
the group of $^{13}$C states at 11.8 MeV. Broken lines: Sum
of the DWIA predictions of ref. 4 for the 4th $5/2^+$ state
and the 2nd $7/2^+$ state of ref. 11.
Fig. 8. Spectra for $^{17}$O($\pi^+,$ $\pi^-$) at 164 MeV and $\theta_{\text{Lab}} = 45^\circ$. 
Figure 9: Energy loss spectra for $\pi^+$ and $\pi^-$ at an energy of 164 MeV and a laboratory angle of 77°.
Figure 10: Center of mass differential cross sections for \( \pi^- \) scattering at 164 MeV.
Fig. 11 SPECTRA FOR ONE MOMENTUM BITE FROM \( ^{13}\text{C}(t,t') \) (BOTTOM) AND \( ^{13}\text{C}(\text{He},\text{He}') \) (TOP) AT 23\,\text{MeV} AND \( \theta_{\text{LAB}} = 22.5^\circ \)
Fig. 12. $\sigma(\theta)$ for $^{13}$C$(t,t')$ and ($^3\text{He},^3\text{He}')$ to the $3/2^-$ (3.68 MeV) and $5/2^+$ (3.85 MeV) states.
Fig. 13. Real phase shifts for the parity-dependent potential from ref. 34.
Fig. 14

a) Difference between scattering amplitudes at 180° for elastic scattering of 16O on 28Si calculated with the parity term present and with the parity term absent.

b) Exchange amplitude at 180° for 16O core exchange in elastic scattering of 16O on 28Si.
Fig. 15. $\chi^2/N$ for the angular distribution for elastic scattering of $^{16}O$ on $^{28}Si$ at $E_{Lab} = 55.0$ MeV, plotted as function of real potential strength $V_0$. 
Fig. 16. Fit to the angular distribution for elastic scattering of $^{16}$O on $^{28}$Si at $E_{\text{Lab}} = 41.226$ MeV.
Fig. 17

a) Real (v) and imaginary (w) potential for elastic scattering of $^{16}$O on $^{28}$Si at $E_{Lab} = 41.225$ MeV.

b) The absorption coefficient $\eta_L$ and the total potential $V_{tot}$ for partial waves $L = 16, 17, 18$ in the elastic scattering of $^{16}$O on $^{28}$Si.