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**Westinghouse Electric Corporation
Fusion Power Systems Department**

MASTER

TFTR MAGNETIC FIELD DESIGN ANALYSES

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TFTR MAGNETIC FIELD DESIGN ANALYSES

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ABSTRACT

The three main magnetic field windings for the TFTR are the toroidal field (TF) windings, the ohmic heating (OH) winding, and the equilibrium field (EF) winding. The following information is provided in this report for these windings:

- descriptions
- functions
- magnetic designs, e.g., number and location of turns
- design methods
- descriptions of resulting magnetic fields

This report does not deal with the thermal, mechanical support, or construction details of the windings.

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	i
1.0 INTRODUCTION	1-1
2.0 TF COILS	2-1
2.1 MAGNETIC FIELD ANALYSIS AND COIL DESIGN	2-1
2.2 RESULTS OF ANALYSES	2-5
3.0 EF/OH WINDINGS	3-1
3.1 DESIGN CONSIDERATIONS	3-1
3.2 PROCEDURE	3-4
3.3 RESULTS & DISCUSSION	3-5
3.4 REFERENCES	3-28
APPENDIX I	
LINEAR PROGRAM DESIGN CODES FOR EQUILIBRIUM FIELD WINDINGS	A-1
APPENDIX II	
A LINEAR PROGRAMMING CODE FOR DESIGN OF OHMIC HEATING WINDINGS	A-7

LIST OF FIGURES

<u>Figure</u>	<u>Title</u>	<u>Page</u>
1.1	TFTR Device	1-2
2.1	TF Coil Geometry	2-2
2.2	Computational Model	2-3
2.3	Magnetic Field Strength in Horizontal Symmetry Plane	2-7
2.4	Ripple as a Function of Radius in the Horizontal Symmetry Plane	2-8
2.5	Ripple Contours	2-9
3.1	Arrangement of Poloidal Field Windings	3-9
3.2	Contours of EF Index of Curvature $n = 0.05 \pm 0.04$	3-10
3.3	Contours of EF Index of Curvature $n = 0.21 \pm 0.03$	3-11
3.4	Contours of EF Index of Curvature $n = 0.38 \pm 0.07$	3-12
3.5	Midplane Variations of EF Index of Curvature	3-13
3.6	Near Field Contours of EF Magnitude (Tesla) $n = 0.05$	3-14
3.7	Near Field Contours of EF Magnitude (Tesla) $n = 0.21$	3-15
3.8	Near Field Contours of EF Magnitude (Tesla) $n = 0.38$	3-16
3.9	Near Field Contours of Field Magnitude (Tesla) From Curvature Change Turns	3-17
3.10	Far Field Contours of EF Magnitude (Tesla) $n = 0.05$	3-18

LIST OF FIGURES

<u>Figure</u>	<u>Title</u>	<u>Page</u>
3.11	Far Field Contours of EF Magnitude (Tesla) $n = 0.21$	3-19
3.12	Far Field Contours of EF Magnitude (Tesla) $n = 0.38$	3-20
3.13	Near Field Contours of OH Field Magnitude (Tesla)	3-21
3.14	Far Field Contours of OH Field Magnitude (Tesla)	3-22

LIST OF TABLES

<u>Table</u>	<u>Title</u>	<u>Page</u>
2.1	TOROIDAL MAGNETIC FIELD IN HORIZONTAL SYMMETRY PLANE	2-6
2.2	TOROIDAL FIELD COIL SPECIFICATIONS	2-11
3.1	POSITION COORDINATES AND NUMBER OF TURNS FOR DESIGN H 10	3-7
3.2	RESISTANCE VALUES FOR H 10	3-24
3.3	INDUCTANCE VALUES FOR H 10	3-25
3.4	OHMIC HEATING COIL SPECIFICATIONS	3-26
3.5	EQUILIBRIUM FIELD COIL SPECIFICATIONS	3-27

1.0 INTRODUCTION

The circular TF coils which encircle the vacuum vessel produce a toroidal magnetic field that is the principle field for confining the plasma. The combination of the toroidal magnetic field plus the poloidal magnetic field associated with the plasma current yields a helical field with shear. This is a desirable field in that it is magnetohydrodynamically stable. Ideally, one would produce a pure toroidal field with a continuous winding. However, in order to have access to the plasma the toroidal field is produced by a set of discrete coils. As a result some ripple is introduced into the field. A description of the TF coils, the method of analysis, and results of the analysis including the amount of ripple are presented in Section 2. The location of the TF coils is shown in Figure 1.1. The location of the EF and OH windings are also shown.

The equilibrium field (EF) is mainly a vertical field which interacts with the toroidal plasma current to balance the magnetic "hoop" and the thermodynamic forces which tend to move the hot plasma outward in a radial direction. By controlling the strength of field the plasma may be forced inward in a radial direction to achieve adiabatic compression. A small curvature to this field provides stability against vertical movement of the plasma.

The ohmic heating (OH) field winding provides a large magnetic flux linking the toroidal plasma current. A rapid change in this flux induces a toroidal current in the plasma and leads to an increase in the plasma temperature.

The design of these poloidal field windings (EF/OH) for the TFTR can be considered in terms of a single system. Both the EF and OH field are produced by coaxial toroidal currents which lie outside the TF coils. The design description is facilitated by considering the device in a right-hand (R, θ, Z) cylindrical coordinate system. The poloidal fields ideally have rotational symmetry about the Z axis and are symmetrical with respect to positive and negative

Figure 1.1 TFTR Device

Z. The field parameters which vary only with R and Z are termed poloidal and the winding currents which have only a θ component are called toroidal. A description of the EF/OH winding system, the method of analyses, and the results of the analyses are given in Section 3.

Linear program design codes for the EF winding are described in Appendix I and a linear programming code for the design of the OH winding is described in Appendix II.

2.0 TF COILS

2.1 MAGNETIC FIELD ANALYSIS AND COIL DESIGN

In the TFTR conceptual design, 20 symmetrically located TF coils are centered at 280 cm from the major axis of the torus with an inner and outer copper bore radius of 145 and 189 cm, respectively. Each coil has 44 turns and is supported by a box beam structure. The maximum TF coil current is 73.2 kA. Figure 2.1 shows the TF coil geometry and Figure 2.2 shows the corresponding computational model used in calculating the magnetic field strength and ripple.

To perform the detailed computer analysis of the magnetic field strength of the TF as a function of location in space, the SAWA computer code developed at Princeton was employed.

SAWA calculates the three field components in cartesian space due to a system of coaxial coils distributed symmetrically about a torus. The coils were defined with rectangular cross-section although the code is designed to handle ideal solenoids, ideal discs, and circular filaments. The field contribution at a point due to remote coils is calculated with a series of Legendre functions; for space points near or in a coil, the field contribution is evaluated by first replacing the rectangular cross-section by a set of nested solenoids and then integrating the fields due to these solenoids by Gaussian quadrature.

Figure 2.1 TF COIL GEOMETRY

Figure 2.2 COMPUTATIONAL MODEL

In choosing the space points for which field solutions are required, SAWA provides for two possibilities: (1) a cartesian grid of points and/or (2) points along a pair of radius vectors specified at two angles in the coordinate system. In the second case, ripple values are calculated using the pairs of points along the radius vectors. This method was used in the calculations discussed below.

Note that the SAWA model does not account for the coil taper on the inside portion where the coils wedge together. However, this is not a significant limitation since the field values are only slightly affected (as shown later in Table 2.1), and only near the coils where they are wedged together. Each coil consists of two "pancakes" supported by a box beam structure. In the computational model, the insulation between the pancakes has been treated as part of the copper, i.e., as part of the 44 turns.

If an integration is carried out around a closed path,

$$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{10} \int_S \vec{j} \cdot d\vec{s}$$

where B is the magnetic field intensity in gauss, $d\vec{l}$ is an increment of path length in cm, $\vec{j}$ is the current density in amps/cm² and $d\vec{s}$ is an increment of enclosed area in cm². For a closed path that lies in the horizontal symmetry plane and inside all the TF coils,

$$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{10} n (NI)$$

where NI are the ampere turns per coil and n are the number of coils. If the path is a circle of radius R with center at the major axis, and if the toroidal magnetic field at any point on the circle is tangent to the circle, i.e., the lines of force are circular and the ripple is zero,

$$2\pi R B_T = \frac{4\pi}{10} n (NI)$$

and the magnitude of the toroidal field is,

$$B_T = \frac{n (NI)}{5R}$$

For 20 TF coils each having 44 turns and for a current of 73.2 kA (this represents the maximum current for the present design which has two motor generator sets each capable of supplying 330 MW and 2250 MJ):

$$B_T = 0.01760 \text{ I(A)/R(cm) Tesla} \quad (1)$$

2.2 RESULTS OF ANALYSES

Values of B_T obtained using Equation (1) are shown in Table 2.1 as a function of R and are compared with SAWA computer values obtained along these radii: (a) in a plane midway between coils, and (b) in the central plane of a coil. It can be seen that the ripple is sufficiently small ($\leq 7\%$) throughout most of the vacuum chamber (out to the beam protection-plates) that the analytic expression agrees quite well with the computer results. The percent ripple is defined as:

$$\% \text{ Ripple} = 200 \left[\frac{B_T^c(R) - B_T^m(R)}{B_T^c(R) + B_T^m(R)} \right]$$

where $B_T^c(R)$ and $B_T^m(R)$ are the magnetic field strengths on the same circle of radius R which is centered on the major axis at the plane of a coil and in the plane midway between coils, respectively.

Figure 2.3 is a plot of the magnetic field strength as a function of radius in the horizontal symmetry plane.

TABLE 2.1

TOROIDAL MAGNETIC FIELD IN HORIZONTAL SYMMETRY PLANE

R (in)	R (cm)	Analytic B_T (T)	Computer B_T (T)		
		0.01760 I(A)/R (cm)	Plane of Coil	Midway Between Coils	Ripple Percent
56	142.24	9.057	5.078	9.011	-
64	162.56	7.925	7.927	7.923	-
72	182.88	7.045	7.045	7.044	-
80	203.20	6.340	6.340	6.340	-
88	223.52	5.764	5.764	5.764	0.0017
96	243.84	5.283	5.283	5.283	0.0045
104	264.16	4.877	4.877	4.877	0.0157
112	284.48	4.529	4.530	4.527	0.0557
120	304.8	4.227	4.231	4.223	0.1892
128	325.12	3.963	3.975	3.951	0.6062
136	345.44	3.730	3.764	3.696	1.832
144	365.76	3.522	3.616	3.432	5.24
152	386.08	-	3.587	3.111	14.2
160	406.40	-	-	-	-
168	426.72	-	-	-	-
176	447.04	-	-	-	-
184	467.36	-	-	-	-

Figure 2.3 MAGNETIC FIELD STRENGTH IN HORIZONTAL SYMMETRY PLANE

Figure 2.4 RIPPLE AS A FUNCTION OF RADIUS
IN THE HORIZONTAL SYMMETRY PLANE

Figure 2.5 RIPPLE CONTOURS

The percent ripple is given in Figure 2.4 as a function of radius in the horizontal symmetry plane where the largest ripple values occur. The contours of constant ripple in a vertical plane are shown in Figure 2.5.

The maximum field in the copper occurs at the inner bore of the copper in the horizontal symmetry plane. For the present design $B_{\max} = 9.54$ Tesla (73.2 kA) and this occurs at $R = 135$ cm.

The TF coil specifications are given in Table 2.2.

TABLE 2.2
TOROIDAL FIELD COIL SPECIFICATIONS

<u>Parameter</u>	<u>Units</u>	<u>Value</u>
Ampere	MA _t	64.4
Number of coils		20
Turns/coil		44
Ampere turns/coil	MA _t	3.22
Coil Current	kA	73.2
Total turns		880
Coil major radius, R _o	cm	280
Inside radius of copper	cm	145
Outside radius of copper	cm	189
Field at coil center	T	4.6
Field at vacuum vessel center	T	4.86
Maxium field in coil	T	9.54
I ² R	MW	340
1/2 LI ²	MJ	1366
Equivalent square wave pulse length	s	7.47
Total dissipated energy per pulse	MJ	2538
Inductance of T.F. Coil System	H	.510
Resistance of T.F. Coil System @ 328 K (55°C)	Ω	.0634
Coil time constant	s	7.49
Coil centering force per coil	MN	26.2
Mass of copper (one coil)	kg	12800
Mass of "Nitronic 33" stainless steel	kg	4900
Total mass of finished coil (one)	kg	17700
Max. current density	kA/cm ²	3.32
Insulation test voltage	kV	30

3.0 EF/OH WINDINGS

3.1 DESIGN CONSIDERATIONS

There are considerations which are common to both parts of the EF/OH winding system. These are:

- a. Robustness: The placement of the coils during construction will deviate from the design positions by reasonable tolerances. Forces on the windings during operation will produce elastic deformations of the conductors carrying the field-producing currents. The fields should be relatively insensitive to these displacements.
- b. Competition for space: Ideally the current densities in the conductors should be low to minimize the power supply requirements. However, this consideration is in conflict with other functional parts of the device which require space otherwise available for large conductors. Some of these competing functions are: mechanical support, cooling, diagnostics, and remote maintenance. In addition, the space required for the neutral beam injection has a very large effect on the design and performance of the EF winding.

- c. The OH and EF windings compete with each other for the same space. However, at the locations where this competition is most severe, some of the conductors can carry the algebraic sum of the OH and EF currents and thereby conserve space. Such turns have been called hybrid.
- d. There has been a trend toward using larger conductors, fewer turns, and higher currents. This utilizes a degree of design freedom since, to a first approximation, the fields result from a distribution of ampere turns and are relatively insensitive to a variation in turns or amperes as long as the product is preserved. Of course, in the limit, a one or two turn winding will not produce the desired quality of the field. However, of these two parameters, the limitation appears to be the maximum current that can practically or economically be generated and transmitted to the device. The design with fewer turns has a somewhat better utilization of space, is mechanically stronger, is simpler to disassemble, and the cooling connections are simpler.
- e. The volume occupied by the winding, for a given current density, should be minimal to minimize the cost of the winding conductor.

In addition to these considerations which are common to both elements of the system, there are other properties which are unique to either the EF or OH winding.

EF Windings

The additional design considerations for the EF winding are:

- a. The field within the plasma should be sufficiently large to resist the forces tending to move the plasma outward radially.

- b. The mutual inductance between the EF and OH windings should be nominally zero in order to minimize the effect of pulsing one field current on the other. A consequence of this constraint is that there are as many positive turns as there are negative in the EF winding.
- c. The field curvature over the cross section of the plasma should be uniform and of a magnitude which provides vertical plasma stability. The curvature at the point (R, Z) is defined as

$$n(R, Z) = - \frac{R}{B_Z(R, Z)} \left. \frac{\partial B_Z}{\partial R} \right|_{R, Z}$$

where

B_Z is the Z component of the magnetic field strength at the point (R, Z)

- d. The winding should be arranged to provide a choice of the index of curvature by changing the winding connections. Nominal values for the index of 0.0, 0.2, or 0.4 represent a reasonably broad spread for experimental studies. The proposed design is based on a winding which will provide a nominal curvature of 0.2. Additional "curvature change" turns are provided such that if these turns are connected in series with positive polarity the curvature will be nominally zero. If the polarity of these "curvature change" turns is reversed, the nominal curvature is 0.4.

The field curvature has been the subject of design compromise. A high value of curvature has the advantage that a small power input will accomplish a large movement of the plasma in adiabatic compression. This also provides a large degree of vertical stability. The disadvantage of large curvature is that the equilibrium position of the plasma will be sensitive to uncontrolled variations in plasma temperature, plasma current, and EF current. A dynamic

feedback system to correct these errors would necessitate rapid changes of large amounts of power.

The advantage of a low curvature is that it provides a high degree of radial plasma stability. While there will also be decreased vertical stability, the uncontrolled sources of vertical movement are relatively ineffective and a sufficient feedback control could be provided at a relatively low cost. The circular plasma is definitely vertically unstable for a negative curvature.

OH Windings

The additional design considerations for the OH winding are

- a. The flux linking the plasma should be greater than a prescribed minimum value.
- b. The flux density within the plasma cross-section should be less than a given magnitude so as not to interfere with the EF function.
- c. The torque interaction with the toroidal field coils should be less than a given value.

3.2 PROCEDURE

A desirable property of the equilibrium field winding for TFTR is that the curvature of the field in the plasma region have more than one possible average value for a given field strength. The feasibility of attaining such a design using a multiple winding has been investigated previously.¹ A modification to the previous computational routine (see Appendix I) for optimizing the EF coil locations now enables a multiple EF winding to be designed which has different predetermined values of curvature at constant field strength. This modified code together with the routine for optimizing the OH coil locations has been used to obtain a compatible design for the OH and EF windings.

The design is consistent with all present constraints placed on the location of individual coils by other components of the device. These constraints include a 100 cm diameter aperture on the outboard side of the device for the neutral beam injection port, eleven line-of-sight

diagnostic ports for viewing the plasma region, water cooling jackets for the magnetic field coils, and restraining brackets for the TF coils. Also taken into account in the present design is the practicality of supporting and electrically insulating the individual turns of the OH and EF windings and also the ease of wiring the individual turns, particularly of the EF multiple winding.

The locations of both the EF and OH windings were optimized numerically using the linear programming routines. First, the multiple EF winding was designed for nominal curvature values of 0, 0.2, and 0.4. The winding for a nominal curvature of 0.2 was determined with all inboard turns prescribed as hybrid turns. This solution was then fed back into the linear programming routine as a discrete number of prescribed locations having fixed currents together with a number of other possible locations and a new solution obtained having a curvature as close to zero as possible. In this case, it was necessary to obtain a solution having a small positive value of average curvature because of the finite variation in curvature about the average value over the plasma region and the requirement that there be no negative curvature over the main plasma volume. In principle, the tolerance on the curvature may be made negligibly small by including a sufficiently large number of locations or alternatively by programming the current through individual turns. However, practical considerations such as power supply capability and availability of location space make a judgment necessary as to the lower limit of average curvature that may be attained. Thus, a solution was obtained which comprised the set of turns necessary to give the central curvature value together with an additional set of turns at different locations. By reversing the current path through this additional set of turns, the solution for the high curvature value was automatically obtained.

The OH winding was then determined using all possible locations except for those occupied by EF turns (with the exception of the hybrid turns).

3.3 RESULTS & DISCUSSION

This section describes a practical design (H 10) for the placement of OH and EF coils that produces the prescribed magnetic field configurations for TFTR and which is included in Vol. I

of the Conceptual Design Study. The equilibrium field is generated using a multiple winding which allows the average curvature of the field in the main plasma volume to have a value 0.05, 0.21, or 0.38 depending on the current path through the winding. The complete design is consistent with all present constraints placed on the location of individual coils due to, for example, the neutral beam port, diagnostic line-of-sight access ports to the plasma, water cooling jackets, and TF coil supports. The design is also compatible with present arrangements for supporting, insulating, and wiring the individual coils of the OH and EF windings. The locations of the coils are given in this section together with field plots and inductance and resistance values of the windings in their three possible configurations. Finally, the characteristics of the windings are summarized in the Systems Description format.

The locations of the individual turns for the overall design (H 10) are given in Table 3.1 and Figure 3.1. Figures 3.2 - 3.4 show the EF curvature plots for the three possible average values $0.05 + 0.04$, $0.21 + 0.03$, $0.38 + 0.07$. Thus, the additional set of turns in the EF multiple winding changes the curvature by approximately the same amount about the central value. Figure 3.5 shows the midplane variation of curvature for the three cases; in each case the curvature is relatively constant over the extent of the main plasma region.

The near field due to the EF winding is shown in Figures 3.6 - 3.8 in the three different configurations, while Figure 3-9 shows the near field due only to the additional set of coils in the EF winding which vary the curvature about the central value. These fields are those resulting for a current of 80 kA in the winding, and for all three cases the average field in the main plasma region is 0.3 Tesla. The far field due to the EF winding is shown in Figure 3.10-3.12 which show that the variations in the field remote from the device in the three different configurations are minor.

The near field due to the OH winding is shown in Figure 3.13 for a current of 40 kA, from which it may be seen that over the main plasma volume, the OH field is less than 0.0037 Tesla. Figure 3.14 shows the far field due to the OH winding for the same value of current. This field is approximately 1/3 of that due to the EF winding.

TABLE 3.1

POSITION COORDINATES AND NUMBER OF TURNS FOR DESIGN H 10

	<u>R(m)</u>	<u>Z(m)</u>	<u>Turns</u>
OH	4.9400	0.8685	1
	3.9270	1.8460	1
	3.7111	2.0055	1
	2.9834	2.2600	2
	2.8000	2.2409	1
	2.2498	1.9850	2
	2.2498	2.0422	1
	2.0664	1.9100	2
	1.5900	1.7250	4
	1.4230	1.5960	4
	1.2500	1.4300	4
	1.2500	1.4900	2
	1.0900	1.2700	3
	0.9097	0.9270	6
	0.9097	1.0030	6
	0.9097	1.0600	3
	0.7520	0.0191	2
	0.7329	0.0571	3
	0.7329	0.1143	6
	0.7138	0.2476	2
	0.7329	0.3810	6
	0.7329	0.4572	6
	0.7329	0.5334	6
	0.7329	0.6096	6

TABLE 3.1 (continued)

	<u>R(m)</u>	<u>Z(m)</u>	<u>Turns</u>
OH	0.7329	0.6858	6
	0.7329	0.7430	3
HY	1.0900	1.2320	2
	0.9097	0.8300	3
	0.9097	0.8680	2
	0.7329	0.1905	6
	0.7710	0.2476	1
EF*	4.9400	0.5540	-2
	4.9400	0.6620	-2
	4.9400	0.7430	-1
	4.5600	1.1000	-2
	4.5600	1.1800	-1
	3.9270	1.7740	-1
	3.7111	1.9070	-2
	2.9834	2.1450	-2
	2.8000	2.1170	-1
	<hr/>		
	4.9400	0.7970	-1
	2.8000	2.1720	1
	1.8831	1.8750	1
	0.7329	0.3048	-1

* The turns below the dashed line represent the additional set of coils for varying the curvature of the EF field. For current of polarity shown $n = 0.05$; for current of reverse polarity $n = 0.38$; for zero current $n = 0.21$.

Figure 3.1 ARRANGEMENT OF POLOIDAL FIELD WINDINGS

H10 EF CURVATURE $n = 0.05 \pm 0.04$

Figure 3.2

CONTOURS OF EF INDEX OF CURVATURE $n = 0.05 \pm 0.04$

3-11

Figure 3.3 CONTOURS OF EF INDEX OF CURVATURE, $n = 0.21 \pm 0.03$

H10 EF CURVATURE $n = 0.38 \pm 0.07$

Figure 3.4 CONTOURS OF EF INDEX OF CURVATURE, $n = 0.38 \pm 0.07$

Figure 3.5 MIDPLANE VARIATIONS OF EF INDEX OF CURVATURE

Figure 3.6 NEAR FIELD CONTOURS OF EF MAGNITUDE (TESLA) $n = 0.05$

Figure 3.7 NEAR FIELD CONTOURS OF EF MAGNITUDE (TESLA) $n = 0.21$

Figure 3.8 NEAR FIELD CONTOURS OF EF MAGNITUDE (TESLA) $n = 0.38$

3-17

Figure 3.9 NEAR FIELD CONTOURS OF FIELD MAGNITUDE (TESLA)
FROM CURVATURE CHANGE TURNS

Figure 3.10 FAR FIELD CONTOURS OF EF MAGNITUDE (TESLA)
 $n = 0.05$

H10 EF FAR FIELD ($n = 0.21$)

Figure 3.11 FAR FIELD CONTOURS OF EF MAGNITUDE (TESLA)
 $n = 0.21$

Figure 3.12 FAR FIELD CONTOURS OF EF MAGNITUDE (TESLA)
 $n = 0.38$

Figure 3.13 NEAR FIELD CONTOURS OF OH FIELD MAGNITUDE (TESLA)

Figure 3.14 FAR FIELD CONTOURS OF OH FIELD MAGNITUDE (TESLA)

Values of resistance and inductance of the OH and EF windings have also been obtained for each of the three values of EF curvature. The mutual inductances between each winding and the plasma have been obtained for four different sizes and positions of the plasma. Then inductances and resistances have been calculated for the newly specified geometry of the coils, i.e., for the EF turns a square cross section of 2" x 2" of copper conductor with a 3/8" diameter hole for water cooling, and for the OH and HY turns a square cross section of 1-3/8" x 1-3/8" of copper conductor with a 3/8" diameter hole for water cooling. The coil jumpers have been approximated by a single turn of radius 2.4 m with a reverse turn of the same radius separated by 0.3 m to cancel the field produced by the jumpers. The leads to the bus from each winding have been approximated by two 8.38m lengths of conductor. The cross sectional area of these jumpers and leads have been taken to be the same as for the OH and EF conductors, respectively. In these calculations, the resistivity of copper has been taken corresponding to the value at (55° C) 328 K.

Table 3.2 gives the resistances of the OH and EF windings in the three different configurations while Table 3.3 gives the corresponding inductances. It should be noted that the incremental change in the self inductance of the EF winding when the curvature n is varied from 0.21 to 0.05 is not the same as when n is varied from 0.21 to 0.38. This is because the self inductance of the additional set of turns in the multiple EF winding remains the same regardless of the direction of current. The inductance due to the leads from each winding to the bus is neglected, being $\leq 0.1\%$ of the total inductance.

Finally, Tables 3.4 and 3.5 summarize the characteristics of the OH and EF windings, respectively, for design H 10. These are given in the format normally used for Systems Descriptions.

TABLE 3.2
RESISTANCE VALUES FOR H 10

	Resistance (m Ω)		
	<u>n = 0.05</u>	<u>n = 0.21</u>	<u>n = 0.38</u>
OH *	25.58	25.58	25.58
EF **	9.770	8.742	9.770
OH - EF	2.581	2.581	2.581

* Includes 0.52 m Ω for coil jumpers and reverse lead and .26 m Ω to bus for leads

** Includes 0.238 m Ω for coil jumpers and reverse lead and .119 m Ω to bus for leads

TABLE 3.3
INDUCTANCE VALUES FOR H 10

		Inductance (m H)		
		<u>n = 0.05</u>	<u>n = 0.21</u>	<u>n = 0.38</u>
OH *		27.22	27.22	27.22
EF **		6.878	6.264	5.910
OH - EF		-0.146	-0.144	-0.142

		Inductance (μ H)				
<u>R(m)</u>	<u>R1(m)</u>	<u>PL</u>	<u>OH-PL</u>	<u>n = 0.05</u>	<u>EF-PL</u>	<u>n = 0.38</u>
					<u>n = 0.21</u>	
3.10	0.60	7.755	135.9	-93.85	-91.56	-89.27
2.48	0.85	4.523	135.7	-53.57	-53.29	-53.00
2.48	0.54	5.850	135.7	-52.47	-52.21	-51.95
2.09	0.492	4.734	135.4	-30.94	-31.04	-31.15

* Includes 0.02 m H for coil jumpers and reverse lead

** Includes 0.015 m H for coil jumpers and reverse lead

TABLE 3.4
OHMIC HEATING COIL SPECIFICATIONS

		<u>Design H 10</u>
Ampere turns (2.5 MA plasma current)	MA _t	8.24
Total turns		206 *
Max. design current (2.5 MA plasma current)	kA	40
Max. current density (2.5 MA plasma current)	kA/cm ²	3.48 ⁺
Inductance (including coil jumpers)	H	0.0272
OH Field per unit coil current	mT/kA	0.04
Mutual inductance to plasma (2.5 MA plasma current)	μH	136
Resistance (including coil jumpers and leads to bus)	Ω	.0256
Insulation test voltage	kV	62
Total copper mass	kg	14,750 ⁺⁺

* Includes 28 turns of HY winding

+ 1-3/8" x 1-3/8" minus 3/8" diameter coolant hole

++ Assuming resistivity $1.98 \times 10^{-6} \Omega \text{ cm}$ (55°C, 328 K), density 8.93 g cm^{-3} .

TABLE 3.5

EQUILIBRIUM FIELD COIL SPECIFICATIONS

<u>Parameter</u>	<u>Units</u>	<u>Design H 10</u>		
		<u>n = 0.05</u>	<u>n = 0.21</u>	<u>n = 0.38</u>
Ampere turns (2.5 MA plasma current)	MA _t	5.12	4.48	5.12
Total turns		64**	56*	64**
Max. design current (2.5 MA plasma current)	kA	80	80	80
Max. current density (2.5 MA plasma current)	kA/cm ²	3.19 ⁺	3.19 ⁺	3.19 ⁺
Inductance (including coil jumpers)	mH	6.88	6.26	5.91
Field per unit coil current	mT/kA	3.75	3.75	3.75
Mutual inductance to plasma (2.5 MA plasma current)	μH	-53.6	-53.3	-53.0
Ohmic to equilibrium field - mutual inductance	μH	-146	-144	-142
Resistance (including coil jumpers and leads to bus)	Ω	.00977	.00874	.00977
Peak transient coil voltage	kV	20	20	20
Insulation test voltage	kV	40	40	40
Total copper mass	kg	26,750 ⁺⁺	23,850 ⁺⁺	26,750 ⁺⁺

* Includes 23 turns of HY winding

** Includes 8 turns of VC winding

+2" x 2" minus 3/8" diameter coolant hole

++ Assuming resistivity $1.98 \times 10^{-6} \Omega \text{ cm}$ (55°C, 328K), density 8.93 g cm^{-3}

3.4 REFERENCES

1. G. V. Sheffield, private communication.

APPENDIX I

LINEAR PROGRAM (LP) DESIGN CODES FOR EQUILIBRIUM FIELD WINDINGS

I. Standard Design Approach Code

Model: The design variables are the filamentary currents of unknown magnitude, $X(j)$. These are assumed at a large number of locations in the upper half plane, e.g. 98; the L P solution typically contains a smaller set of non-zero values and thus selects the winding locations from the larger set. Each turn should carry the same current and thus feasible $X(j)$ must be integral multiples of the same unspecified current. This discrete constraint is not difficult to meet in practice, however. Because linear programming variables can only take on positive values, $X(j)$ is represented as the difference between its positive and negative components, $XP(j)$ and $XM(j)$.

$$X(j) = XP(j) - XM(j) .$$

Minimization of Cost: The cost of the winding is assumed to be proportional to its volume. If we assume a given constant winding current density throughout the winding, the volume is also proportional to ohmic loss. Thus the power supply cost is also proportional to the winding volume. The conductor cross-section is proportional to the absolute value of $X(j)$ at any location which is given by

$$X(j) = XP(j) + XM(j)$$

The length of a turn at location j is proportional to the radial position $R(j)$. Therefore the total volume is proportional to

$$\sum_j R(j) * (XP(j) + XM(j))$$

which is to be minimized.

Within the scope of linear programming, it is easy to include min-max (Tschebychev) terms or to approximate other non-linear terms. Integer programming further expands the alternatives in this respect. These alternatives have not proven necessary in these applications, however.

Magnetic Field, Field Curvature, O.H. Coupling Definitions and Constraints:

At any point, i , in space, the magnetic field, B_{zi} , is a linear function of the $X(j)$,

$$B_{zi} = \sum_j B_{zij} * X(j) = \sum_j B_{zij} * (XP(j) - XM(j)),$$

where the transfer coefficients B_{zij} are the magnetic field at point i caused by a unit positive current at point j . Since this function is linear, upper, lower, and equality constraints can be placed on it in the linear programme. In the design process B_z is constrained at a mid-point to equal a specified value $\bar{B}_{z0}$, viz

$$B_{z0} = \sum_j B_{zij} * (XP(j) - XM(j)) = \bar{B}_{z0}$$

Also, upper limits on $XP(j)$ and $XM(j)$ are used to express positionwise space limitations. These limits are in effect bounding the Tschebychev norm.

The curvature of the magnetic field, n , is given by

$$n = - \frac{R}{B_z} \frac{\partial B_z}{\partial R} = - \frac{R_i}{\sum_j B_{zij}} X(j) \sum_j \frac{\partial B_{zij}}{\partial R} X(j)$$

which is a non-linear function of the $X(j)$. In order to place lower and upper limits on n , say

$$n_o - \Delta_n \leq n \leq n_o + \Delta_n$$

it is necessary to transform the individual non-linear inequalities

$$n_o - \Delta_n \leq - \frac{R_i}{\sum_i B_{zij}} X(j) + \sum_i \frac{\partial B_{zij}}{\partial R} X(j) \leq n_o + \Delta_n$$

into the two linear inequalities

$$0 \leq \sum_i \left\{ (n_o + \Delta_n) B_{zij} + R_i \frac{\partial B_{zij}}{\partial R} \right\} X(j)$$

$$0 \leq - \sum_i \left\{ (n_o - \Delta_n) B_{zij} + R_i \frac{\partial B_{zij}}{\partial R} \right\} X(j)$$

In practice a finite difference approximation to $\frac{\partial B_z}{\partial R}$ was used in the above inequalities.

The mutual inductance between the OH winding and the EF winding should be small, and there appears to be little penalty in making it zero. Thus the constraint

$$\sum_j \phi(j) X(j) = 0$$

is added. The flux $\phi(j)$ is that flux from the OH winding which links the j th EF winding. Since $X(j)$ is proportional to the number of turns at location j , this summation is proportional to the mutual inductance. The values of $\phi(j)$ are predetermined from an analysis of the OH field.

Constraints were also included which permit fixing the solutions for a subset of the $X(j)$, leaving the remainder free to be determined by the code.

The sensitivity of the minimum with respect to each constraint is contained in the L P solution. This sensitivity information was used in determining which constraints to modify or add.

II. ADJUSTABLE CURVATURE EQUILIBRIUM FIELD DESIGN CODE

Introduction: The linear programming (L P) design code for the design of equilibrium field windings as described in Part I has been modified to design windings which will produce fields whose curvature can be adjusted by means of changing some electrical connections.

Background: The standard approach code has been used for designing equilibrium field (EF) windings to produce any desired index of curvature. This design is subjected to a number of constraints specifying performance and tolerance in measures of performance. Mechanical constraints on coil locations and the maximum ampere turns at each location are included.

Reasons for designing for a low EF field index of curvature are contrasted with those for a high index. A low index (near zero) will give better radial stability which is relatively difficult to obtain by other means. Disadvantages of a low index are (a) lesser vertical stability and (b) greater equilibrium field power input to quickly move the plasma from one radial position to another. The vertical stability is presently not a serious problem, but the cost of the power supply is serious.

George Sheffield of PPPL has suggested a design scheme for a winding where several curvatures can be selected by means of making some winding reconnections. Specifically, a winding is to be designed with a curvature at 0.2. The location of additional turns, in series with the main EF winding, are to be found which will produce a curvature of 0.0. These same turns when connected in series but with reversed polarity will necessarily produce a curvature of about 0.4.

Design Logic: The previously existing EF design code has been modified to permit the following design procedure:

1. Using the old features of the code, design a winding which will produce a curvature of 0.2. (The values of curvature can be changed but specific values are given here as an example for clarity.)
2. The design code is instructed by means of input data that the ampere turns designed in (1) are to be kept the same but additional turn locations are to be found which together with the fixed turns will produce a curvature of zero.

APPENDIX II

A LINEAR PROGRAMMING CODE FOR DESIGN OF OHMIC HEATING WINDINGS

Objective: The following are considerations in the design of an ohmic heating winding for a Tokamak fusion reactor:

1. The flux linking the plasma must be greater than or equal to a given value.
2. Within the several plasma regions the field strength from the ohmic heating winding must be small compared to other field strengths such as multipole nulls and the equilibrium field.
3. The flux lines crossing the toroidal field coils will cause a torque on these coils. This torque must be limited. If the TF should have a superconducting winding, the field strength in the TF coils must be limited for other reasons.
4. The turns of the winding are competing for space with other components' space. Restrictions which result must be observed.
5. Consideration must be given to the trade-offs affected by the conductor current density.
6. The size of the conductor as influenced by the amperes per turn has mechanical ramifications such as connections of cooling hoses, mechanical strength, and remote disassembly.
7. From the standpoints of cost of the winding and its power supply and the space required for other components, volume of the winding should be minimized.

Background: A design code utilizing linear programming (L P) has previously been used to design equilibrium field windings. The use of this approach enables the routine design of these windings to a given specification.

These windings have previously been designed by Princeton Plasma Physics Laboratory by setting up field points and current source points then designing the current magnitudes so that the field points, which surround the plasma, all link, in a least squares sense, the same value of flux. In the limiting case, when the field points become a line and that line is a flux line, there will be zero flux density within the plasma. Since the winding performance will necessarily be less than ideal, the linear programming approach has the advantage of a more direct statement of the design requirements.

Program Logic: The following are elements of the program logic:

1. A number of candidate current locations are selected. An upper limit to the current can be specified individually for each of these locations.
2. The flux at the plasma center position is constrained to be greater than or equal to a required value.
3. The magnitude of the component of flux density normal to the TF current is limited at several points in the TF coil (in the R-Z plane).
4. The R and Z components of the flux density are limited in magnitude at several locations. The upper limit on the magnitude of the components can both be individually specified at each of the field points. These field points are intended to be points which bound the plasma locations.

5. Within the limitations of the above constraints, a weighted sum of the current magnitudes is to be minimized. If the weight is the radial position of the current and if we assume a constant current density in the winding conductors, the minimization is of the conductor volume.

LP Formulation: There are NL possible current locations, each specified by coordinates RL(L) and ZL(L). Each of these locations has a weighting factor WL(L) and a limitation of the magnitude of the solution BNDCUR(L). Since the linear programming variables may only take on positive values the solution value X(L) is the difference of a positive quantity XP(L) and a negative quantity XM(L), i.e.

$$X(L) = XP(L) - XM(L).$$

- (A) The flux is to be constrained at a position with coordinates (RJ0, 0, 0) to a value greater than or equal to PKRHS. Thus we have the constraint

$$\sum_{L=1}^{NL} FL(0,L) (XP(L) - XM(L)) \geq PKRHS \quad (1)$$

where FL(J,L) is the flux at location J due to unit currents at two locations, one with coordinates (RL(L), ZL(L)) and the other with coordinates (RL(L) - ZL(L)). The code is in terms of the MKS system and solutions are in units of 10^6 amperes.

- (B) The upper bounds on the current magnitudes are implemented by constraints

$$\left. \begin{array}{l} XP(L) \leq BNDCUR(L) \\ XM(L) \leq BNDCUR(L) \end{array} \right\} L = 1, NL \quad (2)$$

- (C) There are NJ points with coordinates RJ(J) and ZT(J). Each of these points is to have a limitation on the magnitude of the radial component of B of DELBR(J) and on the axial component of DELBZ(J). This is expressed as four sets of constraints

$$\left. \begin{aligned}
 \sum_{L=1}^{NL} BR(J,L) (XP(L) - XM(L)) - BR(J) &= 0 \\
 - DELBR(J) &\leq BR(J) \leq DELBR(J) \\
 \sum_{L=1}^{NL} BZ(J,L) (XP(L) - XM(L)) - BZ(J) &= 0 \\
 - DELBZ(J) &\leq BZ(J) \leq DELBZ(J)
 \end{aligned} \right\} \text{For } J = 1, NJ \quad (3)$$

Where

BR(J,L) is the radial field at J location due to a unit current at L location. This infers a unit current at (RL(L), ZL(L)) and at (RL(L), -ZL(L)).

BZ(J,L) is the axial field defined similarly to BR(J,L).

BR(J) is the total radial field at J.

BZ(J) is the total axial field at J.

The constraints (3) have the defect that BR(J) and BZ(J) are variables which may take on negative values. Therefore we define variables

$$DELBR(J) + BR(J) = BR^*(J) \quad (4)$$

$$DELBZ(J) + BZ(J) = BZ^*(J)$$

Then constraints (3) become

$$\begin{aligned}
& \sum_{L=1}^{NL} BR(J,L) (XP(L) - XM(L)) - BR^*(J) = - DELBR(J) \\
& 0 \leq BR^*(J) \leq 2 DELBR(J)
\end{aligned} \tag{5}$$

$$\begin{aligned}
& \sum_{L=1}^{NL} BZ(J,L) (XP(L) - XM(L)) - BZ^*(J) = - DELBZ(J) \\
& 0 \leq BZ^*(J) \leq 2 DELBZ(J)
\end{aligned}$$

- (D) The component of the field normal to the TF current is to be constrained at NK locations to values BNMAX(K). These locations have coordinates (RK(K), ZK(K)). For purposes of establishing minor radial directions, the center of the circular TF coils is given with coordinates (R0, 0.0).

At point K, the unit vector normal to the TF coil is

$$\frac{i(R(K)-R0) + j(Z)}{((R(K)-R0)^2 + Z^2)^{1/2}} = iNR(K) + jNZ(K) \tag{6}$$

where i and j are unit vectors in the R and Z directions.

The normal component of B, BN(K) is defined by the expression

$$NR(K) BR(K) + NZ(K) BZ(K) - BN(K) = 0 \tag{7}$$

where BR(K) is the R component of B at the point K,

BZ(K) is the Z component,

BR(K) and BZ(K) are functions of all of the currents and (7) becomes

$$\begin{aligned}
& \sum_{L=1}^{NL} (NR(K) BR(K,L) + NZ(K) BZ(K,L)) (XP(L) - XM(L)) - \\
& BN(K) = 0
\end{aligned} \tag{8}$$

where $BR(K,L)$ is the R component of B at K due to a unit current at L

$BZ(K,L)$ is a similar coefficient for the Z component.

$BN(K)$ is subject to the constraint

$$-BNMAX(K) \leq BN(K) \leq BNMAX(K) \quad (9)$$

Since $BN(K)$ may take on negative values we define the variables

$$BN(K)^* = BN(K) + BNMAX(K) \quad (10)$$

and constraints 8 and 9 become

$$\sum_{L=1}^{NL} (NR(K) BR(K,L) + NZ(K) BZ(K,L)) (XP(L) - XM(L)) \quad (11)$$

$$-BN(K)^* = BNMAX(K)$$

$$0 \leq BN(K)^* \leq ZBNMAX(K) \quad (12)$$

(E) The objective function which is to be minimized is the weighted sum of the current magnitudes or

$$\text{Minimize } \sum_{L=1}^{WL} WL(L) (XP(L) + XM(L))$$

Summary of LP Statement:

$$\text{Minimize } \sum_{L=1}^{NL} WL(L) (XP(L) + XM(L))$$

Subject to constraints

$$\sum_{L=1}^{NL} FL(0,L) (XP(L) - XM(L)) \geq RKRHS$$

$$\left. \begin{array}{l} XP(L) \leq BNDCUR(L) \\ XM(L) \leq BNDCUR(L) \end{array} \right\} L = 1, NL$$

$$\sum_{L=1}^{NL} BR(J,L) (XP(L) - XM(L)) - BR^*(J) = - DELBR(J)$$

$$0 \leq BR^*(J) \leq 2.0 DELBR(J)$$

$$\sum_{L=1}^{NL} BZ(J,L) (XP(L) - XM(L)) - BZ^*(J) = - DELBZ(J)$$

$$0 \leq BZ^*(J) \leq 2.0 DELBZ(J)$$

$$\sum_{L=1}^{NL} (NR(K) BR(K,L) + NZ(K) BZ(K,L)) (XP(L) - XM(L)) - BN^*(K) = BNMAX(K)$$

$$0 \leq BN^*(K) \leq 2.0 BNMAX(K)$$

$J = 1, NJ$

$K = 1, NK$