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DEVELOPMENT REPORT

ANALYSIS OF THE BORON
PILE MEASUREMENT OF THE
AVERAGE NEUTRON YIELD
PER FISSION OF ^{252}CF

(AWBA DEVELOPMENT PROGRAM)

AUGUST 1977

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BETTIS ATOMIC POWER LABORATORY, WEST MIFFLIN, PA.,
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ANALYSIS OF THE BORON PILE MEASUREMENT
OF THE AVERAGE NEUTRON YIELD PER FISSION OF ^{252}Cf
(AWBA DEVELOPMENT PROGRAM)

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FOREWORD

Subsequent to development and successful operation of the Pressurized Water Reactor in the AEC-owned reactor plant at the Shippingport Atomic Power Station, the Atomic Energy Commission in 1965 undertook a research and development program on a breeder reactor for operation in the Shippingport Station. With fabrication of the Light Water Breeder Reactor (LWBR) nearing completion in 1976, the Energy Research and Development Administration established the Advanced Water Breeder Applications program (AWBA) to develop and disseminate information which would assist U.S. industry in evaluating the LWBR concept. All three of these reactor development projects have been administered by the Division of Naval Reactors with the goal of developing practical improvements in the generation of electrical energy using water-cooled nuclear reactors.

The Shippingport Atomic Power Station located in Shippingport, Pennsylvania was the first large-scale, central-station nuclear power plant in the United States and the first plant of such size in the world operated solely to produce electric power. This project was started in 1953 to confirm the practical application of nuclear power for large-scale electric power generation. It has provided much of the technology being used for design and operation of the commercial, central-station nuclear power plants now in use.

The objective of the Light Water Breeder Reactor project has been to develop a technology that would significantly improve the utilization of the nation's nuclear fuel resources employing the well-established water reactor technology. To achieve this objective, work has been directed toward analysis, design, component tests, and fabrication of a water-cooled, thorium oxide cycle breeder reactor to install and operate at the Shippingport Station. Operation of the LWBR core in the Shippingport Station is expected to start in 1977 and be completed in about 3 to 4 years. Then the fissionable fuel inventory of the core will be measured. This effort, when completed approximately 2 years after completion of LWBR core operation, is expected to confirm that breeding actually took place.

The Advanced Water Breeder Applications (AWBA) project was initiated to develop and disseminate technical information that will assist U.S. industry in evaluating the LWBR concept and applying it to commercial-scale applications. The project will explore some of the problems that would be faced by industry in adapting technology confirmed in the LWBR program. Information to be developed includes concepts for commercial-scale prebreeder cores which will efficiently produce uranium-233 for light water breeder cores while producing electric power, improvements for breeder cores based on the technology developed to fabricate and operate the Shippingport LWBR core, and other information and technology to aid evaluating commercial-scale application of the LWBR concept.

As in the previous work, technical information developed under the AWBA project will be published in technical memoranda, one of which is this present report.

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ABSTRACT

The Harwell Boron Pile measurement of the average number of prompt neutrons emitted per fission $\bar{\nu}_p$ of ^{252}Cf has been analyzed in detail using a Monte Carlo method. From the calculated energy dependence of the neutron detection efficiency a value of $\bar{\nu}_p = 3.733 \pm 0.022$ was obtained. This value is 0.76% higher than the original reported value of 3.705 ± 0.015 . Possible causes for this increase are discussed.

ANALYSIS OF THE BORON PILE MEASUREMENT
OF THE AVERAGE NEUTRON YIELD PER FISSION OF ^{252}Cf
(AWBA Development Program)

J. J. Ulio

I. INTRODUCTION

This report describes a detailed Monte Carlo analysis of the boron pile⁽¹⁾ measurement of the average prompt neutron yield per fission of ^{252}Cf , $\bar{\nu}_p(^{252}\text{Cf})$. The interest in $\bar{\nu}_p(^{252}\text{Cf})$ has been prompted for two major reasons:

1) There is a lack of agreement between the existing measured values of $\bar{\nu}_p(^{252}\text{Cf})$. To measure $\bar{\nu}_p(^{252}\text{Cf})$ three different methods of neutron detection have been employed. These are the liquid scintillator, manganese sulfate bath, and boron pile methods. The liquid scintillator measurements tend to be systematically higher than those yielded by the boron pile and manganese sulfate bath methods⁽²⁾. A great deal of discussion has taken place over possible causes with the result that no single effect seems to lie as the root cause of the difference. It is possible that it is a combination of fortuitous circumstances that is producing a systematic difference. Perhaps, errors assigned to systematic effects have been insufficient in the various experiments.

2) The measurements of $\bar{\nu}_p(^{252}\text{Cf})$ have yielded a nominal value which is much too low to reproduce the directly measured values of η , neutron yield per absorption. The values of η and $\bar{\nu}_p(^{252}\text{Cf})$ are related by

$$\eta^x = \frac{\bar{\nu}^x}{1 + \alpha^x} = \frac{\left(\frac{\bar{\nu}^x}{\bar{\nu}^{252}} \right) \times \bar{\nu}^{252}}{1 + \alpha^x}, \quad (1)$$

where x represents any one of the fissile nuclei (i.e., ^{233}U , ^{235}U , and ^{239}Pu), $(\bar{\nu}^x / \bar{\nu}^{252})$ as a directly measured $\bar{\nu}$ -ratio corrected for delayed neutrons, α^x is the ratio of capture to fission cross sections for nuclide x, and $\bar{\nu}^{252}$ is the $\bar{\nu}$ for ^{252}Cf also corrected for delayed neutrons. In the case of ^{233}U , Eq. (1) becomes

$$\eta(^{233}\text{U}) = \frac{\left(\frac{\bar{\nu}_p(^{233}\text{U}) + \bar{\nu}_d(^{233}\text{U})}{\bar{\nu}_p(^{252}\text{Cf}) + \bar{\nu}_d(^{252}\text{Cf})} \right) \times \left(\bar{\nu}_p(^{252}\text{Cf}) + \bar{\nu}_d(^{252}\text{Cf}) \right)}{(1 + \alpha(^{233}\text{U}))} \quad (2)$$

$$= \frac{R(^{233}\text{U}) \bar{\nu}^{252}(\text{Cf})}{1 + \alpha(^{233}\text{U})}, \quad (2a)$$

where $\bar{\nu} = \bar{\nu}_p + \bar{\nu}_d$ and $\bar{\nu}_d$ is the delayed neutron component. From Ref. (2) the values of $R(^{233}\text{U})$ and $\alpha(^{233}\text{U})$ can be taken to be

$$R(^{233}\text{U}) = 0.6606 \pm 0.0013$$

$$\alpha(^{233}\text{U}) = 0.0874 \pm 0.0005.$$

Assuming a nominal measured value of $\bar{\nu}^{252} = 3.73 \pm 0.02$, Eq. (2a) yields

$$\eta(^{233}\text{U}) = 2.277 \pm 0.013.$$

This is to be compared with the directly measured values of $\eta(^{233}\text{U})$ which yield^(3,4)

$$\eta(^{233}\text{U}) = 2.298 \pm 0.008.$$

The difference between the values of $\eta(^{233}\text{U})$ measured directly and derived from $\bar{\nu}^{252}(\text{Cf})$ amounts to about 1%. This lack of consistency between determinations of $\eta(^{233}\text{U})$ has caused serious problems in choosing a "best" value for $\eta(^{233}\text{U})$. Such inconsistency increases the uncertainty of breeding performance predictions and furnishes the prime motivation

for the study reported here. The above example concerned ^{233}U , but essentially the same situation exists for ^{235}U .

The direct measurements of $\eta(^{233}\text{U})$ and $\eta(^{235}\text{U})$ have been reanalyzed in detail, and the experimental values have been confirmed^(3,4). Since the errors in the measured values of the $\bar{\nu}$ -ratios and α are small, it has been suspected that the measured values of $\bar{\nu}_p(^{252}\text{Cf})$ are systematically low. Therefore a detailed reanalysis of several key measurements of $\bar{\nu}_p(^{252}\text{Cf})$ has been attempted. A reanalysis⁽⁵⁾ of the most recent liquid scintillator determination of $\bar{\nu}_p(^{252}\text{Cf})$ due to Boldeman⁽⁶⁾ has resulted in an increase of his measured value for $\bar{\nu}_p$ from 3.738 ± 0.015 to 3.747 ± 0.018 ($\bar{\nu} = 3.756 \pm 0.018$).

As mentioned previously in this introduction, this report details a similar reanalysis of another key $\bar{\nu}_p(^{252}\text{Cf})$ measurement, namely, the Harwell boron pile measurement due to Colvin and Sowerby⁽¹⁾. This measurement is of particular interest for several reasons:

- 1) The value of $\bar{\nu}_p(^{252}\text{Cf}) = 3.705 \pm 0.015$ ($\bar{\nu} = 3.713 \pm 0.015$) was one of lowest reported.
- 2) The relatively small uncertainty gave the measurement a large weight in any weighted average.
- 3) The boron pile experiment was fundamentally different from the liquid scintillator experiments, and the differences could be due to systematic errors that differ between the two types of measurement.
- 4) The neutron detection efficiency of the boron pile has never been checked independently using modern calculational techniques such as Monte Carlo in which the geometry of the pile was accurately represented.

Relative to item 4 above, it must be stated that for all three experimental methods used to measure $\bar{v}_p(^{252}\text{Cf})$, the final value of $\bar{v}_p(^{252}\text{Cf})$ is inversely proportional to an effective efficiency $\bar{\epsilon}$. If $\epsilon(E, \hat{\Omega})$ is the probability of detecting a neutron of initial energy E and initial direction of emission $\hat{\Omega}$ and if $N(E)$ is the value of the ^{252}Cf prompt fission spectrum at energy E , then

$$\bar{\epsilon} = \int_0^\infty \int_{4\pi} \epsilon'(E, \hat{\Omega}) N(E) dE d\hat{\Omega} \quad (3)$$

$$= \int_0^\infty \epsilon(E) N(E) dE. \quad (3a)$$

$\epsilon(E)$ is the angle averaged efficiency at energy E . Hence, the probability (efficiency) of neutron detection must be determined for a range of energies which spans the ^{252}Cf fission neutron spectrum. When the energy dependence of $\epsilon(E)$ has been determined, Eq. (3a) yields $\bar{\epsilon}$. It is the calculation of this energy dependence of the neutron detection efficiency which the analyses of Ref. (5) and this work specifically focus on.

In Section II the experimental arrangement of the boron pile is described. The procedures used to measure the detection efficiency at several specific neutron energies and to deduce the value of $\bar{v}_p(^{252}\text{Cf})$ reported in Ref. (1) are reviewed. The detailed Monte Carlo calculations carried out in this work of the energy dependence of the neutron detection efficiency of the boron pile appear in Section III. These calculations coupled with the efficiency measurements of Ref. (1), which provided normalization points, were used to deduce revised values of $\bar{\epsilon}$ (depending on assumption) based on an improved calculational model. A discussion and reassessment of the uncertainties in the boron pile measurement of $\bar{v}_p(^{252}\text{Cf})$ are presented in

Section IV followed by the conclusions of study in Section V.

II. DESCRIPTION OF THE BORON PILE EXPERIMENT

In this section a description of the boron pile experimental arrangement and a review of the procedure for deducing $\bar{v}_p(^{252}\text{Cf})$ are presented. For reference, Figure 1 is a cross sectional view of the boron pile. Briefly, the boron pile (which is a misnomer) was a large cubic graphite stack through which penetrated a large number of $^{10}\text{BF}_3$ neutron detectors. Detection of neutrons, which originated at the center of the pile, was achieved through moderation by the graphite and subsequent capture of the thermalized neutrons in the $^{10}\text{BF}_3$ neutron detectors.

More specifically, the boron pile consisted of a graphite stack with a cubic core of side 220 cm. In the core was a 11 x 11 lattice of holes (Fig. 1), the lattice pitch being 20 cm. Each hole diameter was 5.33 cm. Surrounding the core was a 35 cm-thick graphite reflector and a neutron shield consisting of 0.4 mm of cadmium and 61 cm of concrete (not shown). There was no cadmium sheet under the pile; it sat on a $\frac{1}{2}$ -inch-thick aluminum plate. The center hole was 10.1 cm in diameter and was "key-hole" shaped.

Each hole, excluding the center hole, contained two $^{10}\text{BF}_3$ proportional counters fabricated out of high conductivity oxygen-free copper of 0.025-inch wall thickness. The overall length of the counters was 119 cm, and their active length was 107 cm. It was determined that the active region of the counters started one inch from the midplane of the pile. This could cause a dip in the pile neutron detection efficiency along the center midplane

FIGURE I. BORON PILE LATTICE

perpendicular to the lattice direction. A study of the possible anisotropy in the efficiency was made and is discussed in Section IV.C.

The pile efficiency was measured for several neutron energies using the ${}^2\text{H}(\gamma, n){}^1\text{H}$ reaction for which " $\bar{\nu}$ " was one and the energy of the neutron was known from the γ -ray energy. These measurements were accomplished by use of a fast ionization chamber which was placed in the "key" shaped hole at the center of the pile. In the fast ionization chamber, which contained tetra-deuteromethane, recoil protons from the deuteron photodisintegration reaction triggered an electronic gating system that measured $\bar{R}$, the average number of neutrons detected per event. The gating system consisted of two neutron counting gates of equal length (about 4 ms) separated by a 15 ms time interval. In the first (prompt) gate the recoil neutron could be detected and counted. In the second (background) gate a neutron background count was made. The difference between the average neutron counts of the prompt and background gates defined the quantity denoted by $\bar{R}$. Gamma rays of different energies, shown in Table I, were used to initiate the ${}^2\text{H}(\gamma, n){}^1\text{H}$ reaction and, hence, measure $\bar{R}$ as a function of recoil neutron energy. The values of $\bar{R}$ were corrected for five effects:

- 1) Count-rate effect
- 2) Pulse-overlap correction (i.e., neutron-background overlaps in the prompt gate)
- 3) Effect of the ion chamber on efficiency
- 4) Correction for neutrons captured after the prompt gate has closed
- 5) Variation of efficiency with position of the ion chamber in the central channel.

Table I

Results of Efficiency Measurements

<u>Gamma-Ray</u>	<u>Gamma-Ray Energy (MeV)</u>	<u>Neutron Energy (MeV)</u>	<u>$\bar{R}'$</u>
ThC	2.618	0.190	0.6440 ± 0.0039
^{24}Na	2.757	0.265	0.6457 ± 0.0013
$^{19}\text{F}(p, \alpha\gamma)$	6.14, 6.92, 7.12	2.0	0.6433 ± 0.0016
$^{27}\text{Al}(p, \gamma)$	11.135, 12.915	4.9	0.6500 ± 0.0081

Each corrected value of $\bar{R}$, denoted by $\bar{R}'$ in Table I and in Ref. (1), is the resulting measured efficiency of neutron detection for neutrons of the corresponding energy in Table I. These values of $\bar{R}'$ were used to determine the effective efficiency $\bar{\epsilon}$ in Ref. (1) using the following procedure. In Ref. (1) the energy dependent efficiency $\epsilon(E)$ was assumed to be given by a straight line fit to the $\bar{R}'$ measurements up to 2.0 MeV. Above 2 MeV the energy dependence was taken to be that yielded by S_n calculations of a homogenized spherical equivalent of the boron pile normalized to the measurement at 2.0 MeV. Subsequent averaging over the ^{252}Cf fission neutron spectrum yielded $\bar{\epsilon}$ (Eq. 3a).

The other quantity necessary to determine $\bar{v}_p(^{252}\text{Cf})$ is the average measured number of ^{252}Cf fission neutrons detected per fission event, $\bar{v}_q(^{252}\text{Cf})$. In this measurement a fission chamber containing a ^{252}Cf source was placed in the "key" shaped hole at the center of the pile. Fission pulses triggered the same gating system outlined above, and an averaged result (over many single fission events) including corrections similar to those above applied to the values of $\bar{R}$ yielded $\bar{v}_q(^{252}\text{Cf})$. The final measured value of $\bar{v}_p(^{252}\text{Cf})$ was given by

$$\bar{v}_p(^{252}\text{Cf}) = \frac{\bar{v}_q(^{252}\text{Cf})}{\bar{\epsilon}} . \quad (4)$$

III. CALCULATIONS AND RESULTS

As outlined above, the determination of the energy dependence of the neutron detection efficiency is crucial in the measurement of $\bar{v}_p(^{252}\text{Cf})$. In this section the detailed Monte Carlo calculations carried out in this work of the relative energy dependence of the neutron detection efficiency of the boron pile are described and results presented. These calculations

yielded new values for the effective efficiency $\bar{\epsilon}$ (depending on assumption). These were combined with the original experimental value of $\bar{\nu}_Q(^{252}\text{Cf})$ to yield revised values for $\bar{\nu}_P(^{252}\text{Cf})$ (Table II). A "best" value for $\bar{\nu}_P(^{252}\text{Cf})$ was then adopted.

The Monte Carlo calculations, which treated the energy range from 0.0 - 10.0 MeV, were made using the program written by N. R. Candelore and R. C. Gast⁽⁷⁾. From 5.53 keV to 10.0 MeV, multigroup smooth cross sections over 25 MUFT⁽⁸⁾ groups could be provided. A 25 x 25 multigroup inelastic matrix for isotropic inelastic scattering for each nuclide was also permitted. For each nuclide the elastic scattering process was handled using the first four Legendre components (specified for each of the 25 multigroups) of the differential scattering cross section in the center of mass system. The above energy range was also spanned by 3000 energy intervals which could be used, if desired, to represent pointwise cross section detail. Between 5.53 keV and 0.625 eV, multigroup smooth reaction cross sections over 29 MUFT groups were provided. For resonance nuclides, 26,000 energy points could be used to represent resolved and unresolved resonance detail. Isotropic elastic scattering in the center of mass system was permitted, and no inelastic scattering was allowed. The thermal range extended from 0.625 eV to 0.0 eV. Smooth reaction cross sections were provided for 25 energy groups in this interval, and a P-3 thermal scattering kernel could be treated through a 25 x 25 transfer matrix. A P-1 laboratory system scattering pattern without energy exchange was used for all nuclides not treated by kernels.

The cross sections for carbon were taken from ENDF/B-III. A pointwise description of the carbon scattering cross section was used from 5.53 keV to 10.0 MeV. From 5.53 keV to 0.625 eV, multigroup averages of the scattering

cross section for carbon were used. A pointwise description of the carbon absorption cross section from 5.0 MeV to 10.0 MeV was also used. The boron, copper, and aluminum cross sections were taken from ENDF/B-III.

In the Monte Carlo calculations reported here, the pile core and reflector, including the cadmium shield, were explicitly represented. The $^{10}\text{BF}_3$ tubes were specified as laid end to end in each of the circular holes. The $^{10}\text{BF}_3$ counter boxes, which were mostly an aluminum-copper alloy, could not be exactly represented, but an equivalent amount of aluminum and copper was placed homogeneously in the gaps, which actually contained the counter boxes, between the core and reflector on the two sides of the boron pile perpendicular to the lattice direction. (They are not shown in Figure 1.)

To determine $\bar{\nu}_p(^{252}\text{Cf})$, it is necessary to compute the effective efficiency for detection of ^{252}Cf fission neutrons. Thus, it was assumed that calculations of the relative absorption in ^{10}B for neutron energies between 0 and 10 MeV was an accurate description of the neutron energy dependence of the pile efficiency. Monte Carlo calculations, in which the nearly exact geometry of the boron pile was specified, were used to compute the relative absorption rates in ^{10}B . Source neutrons were started isotropically at the center of the pile and were tracked through subsequent collisions until they were absorbed or leaked from the system. In Figure 2 are shown the experimental data, the calculated points (normalized to experiment at 265 keV), and a linear fit to the experimental data. The following observations can be made:

- 1) The calculated points do not fit the data very well. For energies > 2.0 MeV, the calculated energy dependence of the efficiency disagrees with the measurement at 4.9 MeV and

FIGURE 2. VARIATION OF BORON PILE EFFICIENCY WITH ENERGY

decreases much faster than the calculated results in Ref. (1) (not shown). This is believed to be due to the inclusion of carbon and copper absorption, a better treatment of leakage (streaming of neutrons along the lattice), and proper allowance for end effects in the counters.

- 2) The structure in the calculations at high energies is due to the detailed structure in the carbon absorption cross sections employed in the calculations.
- 3) The calculated rate of decrease in the efficiency between 265 keV and 2.0 MeV is greater than measured. Assuming that the Monte Carlo calculations give an adequate description of the energy dependence of the relative efficiency, this observation implies that the systematic renormalization, which yield the corrected count rates $\bar{R}'$ in Table I, are in error for either one or both of the accurate measured experimental points at 265 keV and 2.0 MeV. It is possible that an undetected energy dependence of some of the corrections may exist.

Recall that one of the corrections applied to the experimental count rates was to compensate for the reduction in efficiency of the pile due to the deuterium ion chamber in the efficiency measurements and the fission chamber in the fission spectrum measurements. The effects were measured by observing the reduction in the counts for a Radio Thorium D₂O source and a spontaneous ²⁴⁰Pu fission source when these two sources were placed at the center of the pile. It was impossible to verify the resulting corrections since a detailed description of the deuterium ion chamber could not be obtained.

Table II

Neutron Detection Efficiency for ^{252}Cf Fission Neutrons

<u>Assumed Variation of Efficiency with Neutron Energy</u>	<u>Efficiency</u>	<u>$\bar{\nu}_p(^{252}\text{Cf})$</u>
I. Linear least-squares fit to the experimental data.	0.6438	3.699
II. Straight line through 265 keV and 2.0 MeV data for $E < 2.0$ MeV. Above 2.0 MeV, the calculated curve normalized at 2.0 MeV.	0.6398	3.722
III. Calculated curve normalized at 265 keV.	0.6359	3.744 ± 0.015
IV. Calculated curve normalized at 2.0 MeV.	0.6413	3.713 ± 0.019

This situation makes it very difficult to obtain a "best value" of the pile efficiency for ^{252}Cf fission neutrons. Table II lists values of the effective efficiency obtained for various assumptions. The effective efficiency was computed from

$$\bar{\epsilon} = \int_0^{\infty} \epsilon(E) M(E,T) dE, \quad (5)$$

where $M(E,T)$, the fission neutron spectrum, was taken to be Maxwellian with $T = 1.406$ MeV. This temperature was yielded by a Maxwellian fit to the spectrum measured in Ref. (9). Calculations using the fitted complex spectrum of Ref. (9) yielded similar results.

Equation (5) was computed numerically taking the pointwise calculated values for $\epsilon(E)$ and analytically using a polynomial representation of $\epsilon(E)$. In the latter case, $\epsilon(E)$ was expressed as

$$\epsilon(E) = \sum_{\ell=0}^n a_{\ell} E^{\ell}, \quad (6)$$

where the a_{ℓ} were obtained by least-squares fitting. The effective efficiency was then given by

$$\bar{\epsilon} = \sum_{\ell=0}^n a_{\ell} \bar{E}^{(\ell)}, \quad (7)$$

where

$$\bar{E}^{(\ell)} = \int_0^{\infty} E^{\ell} M(E,T) dE. \quad (8)$$

Values of $\bar{E}^{(\ell)}$ for $T=1.406$ MeV are listed in Table III.

In Table II, the corresponding values of $\bar{v}_p(^{252}\text{Cf})$ for cases III and IV were calculated using Eq. (4)

Table III

Energy Moments of a Maxwellian Fission Neutron Spectrum (T = 1.406 Mev)

$\bar{E}^{(0)}$	1.00000
$\bar{E}^{(1)}$	2.10900
$\bar{E}^{(2)}$	7.41314
$\bar{E}^{(3)}$	36.48004
$\bar{E}^{(4)}$	230.80920

$$\bar{\nu}_p = \frac{\bar{\nu}_Q (^{252}\text{Cf fission neutrons})}{\bar{\epsilon}},$$

where $\bar{\nu}_Q (^{252}\text{Cf fission neutrons}) = 2.3811 \pm 0.0025$ in Ref. (1). A negligible difference was seen between the results obtained from evaluating Eq. (5) numerically or analytically as described above. The uncertainties assigned to these values will be discussed in Sec. IV.

The effective efficiency yielded by assumption II in Table 2 was used by Colvin and Sowerby as their best estimate. Their value was 0.6428 versus 0.6398 found in the present work. The difference is mostly due to the more accurate treatment of cross sections in this analysis. However, in this work it was assumed that the calculated energy dependence of the efficiency was adequate for the whole energy range. This leads to the question as to which of the two more accurately known experimental points at 265 keV and 2.0 MeV is more suitable for normalization. This has no immediate answer since one of the more important corrections used in reducing $\bar{R}$ to $\bar{R}'$, namely that for materials in the central hole, could not be adequately studied at either experimental point. There are some reasons for believing that the calibration point at 265 keV is more reliable. These are:

- 1) The absolute calibration point at 265 keV has a lower experimental uncertainty.
- 2) The effect of the pile anisotropy was studied experimentally solely at this energy.
- 3) It is not clear whether the approximately 2.0 MeV neutrons from the $^2\text{H}(\gamma, n)^1\text{H}$ reaction had a small energy spread, since the gammas, which originated from the reaction $^{19}\text{F}(P, \alpha\gamma)$, had energies of 6.14 MeV, 6.92 MeV, and 7.12 MeV.

- 4) The value of $\bar{\nu}_p(^{252}\text{Cf})$ deduced from the normalization to the 265 keV measurement is in close agreement with the value from a similar reanalysis⁽⁵⁾ of Boldeman's measurement of $\bar{\nu}_p(^{252}\text{Cf})$.

However, despite these arguments it was judged that the most reliable value to take for $\bar{\nu}_p(^{252}\text{Cf})$ from this analysis was a weighted average of values for cases III and IV in Table II; i.e., $\bar{\nu}_p(^{252}\text{Cf}) = 3.733 \pm 0.012$.

It was further judged appropriate to add an additional 0.5% uncertainty in quadrature to account for the uncertainty due to not being able to calculate the effects of materials in the central hole for the 265 keV and 2.0 MeV calibration measurements. The final value of $\bar{\nu}_p$ from this analysis is $\bar{\nu}_p = 3.733 \pm 0.022$.

IV. DISCUSSION OF UNCERTAINTIES

For the values of $\bar{\nu}_p(^{252}\text{Cf})$ in Table II for cases III and IV the associated uncertainties arise from several sources. These are:

- 1) Experimental statistical counting error for the measured efficiencies ($\bar{R}'$) and $\bar{\nu}_Q$ and the Monte Carlo statistical uncertainty.
- 2) Uncertainty in the characterization of the ^{252}Cf fission neutron spectrum used to deduce values of $\bar{\epsilon}$.
- 3) Existence of anisotropy in the pile efficiency.
- 4) Correction for prompt neutrons captured after the prompt counting gate has closed.
- 5) Miscellaneous experimental equipment errors.

Each of the first four of these effects is discussed in turn in this section, and results are summarized in Table IV. For item 5 the uncertainty reported in Ref. (1) was assumed.

A. Statistical Experimental Neutron Counting and Monte Carlo Errors

The error assigned to each efficiency includes the reported experimental

Table IV

Errors in $\bar{\nu}_p(^{252}\text{Cf})$

<u>Error</u>	Contribution to error in $\bar{\nu}_p(\%)$	
	<u>(Case III)</u>	<u>(Case IV)</u>
Efficiency	0.290	0.321
$\bar{R}'$ for fission	0.110	0.110
Fission spectrum	0.045	0.045
Anisotropy in pile efficiency	0.220	0.370
Neutrons captured after the prompt gate	0.100	0.100
<u>Others*</u>	<u>0.066</u>	<u>0.066</u>
Total Error	0.401	0.518

* Includes loss of fissions below bias and electrical pick-up.

uncertainty in the efficiency measurement and the Monte Carlo statistical uncertainty (0.2%) of the present calculations. Both contributions were added in quadrature. In Ref. (10) it was stated that the uncertainty in the experimental efficiencies included contributions due to counting statistics, pile stability, and the error in the correction for materials in the center hole. Likewise, the error in $\bar{v}_0$ for ^{252}Cf fission neutrons included counting statistics, pile stability, and the error in the correction for the reduction in counting due to the presence of the fission chamber.

B. Fission Spectrum

The efficiency of the boron pile was computed assuming a certain mean energy of the fission spectrum given by $\bar{E} = 3/2 T = 2.109 \text{ MeV}$. This mean energy was not known precisely and was assigned an uncertainty of $\pm 0.05 \text{ MeV}$. The corresponding error in $\bar{v}_p$ was 0.045%.

C. Anisotropy in Pile Efficiency

The anisotropy of the pile efficiency was caused by two effects:

- 1) The effect of the counter holes
- 2) Capture of neutrons in the dead spaces at the end of the counters.

It is possible to imagine that the first effect would cause neutrons streaming down the lattice to be detected less efficiently than those starting out perpendicular to the lattice. On the other hand, the second effect would cause the efficiency of detection of neutrons emitted perpendicular to the lattice to be lower.

Due to the directional dependence of the efficiency, an allowance had to be made for the difference in neutron angular distribution between an isotropic fission source and a photodisintegration source, the latter having a differential cross section proportional to $\sin^2 \theta$. Here θ is the angle between the outgoing neutron and the incoming gamma ray which was along the "key" shaped hole and, hence, parallel to the lattice direction. Thus neutrons from the $^2\text{H}(\gamma, n)^1\text{H}$ efficiency

measurements were emitted predominantly perpendicular to the pile lattice. Colvin and Sowerby, using ^{24}Na γ -rays and the anisotropy of the photodisintegration reaction, concluded that the inference of the pile efficiency for an isotropic source was affected by the anisotropy of the pile efficiency by no more than 0.2%.

As part of the present study, calculations were made at each calibration energy in which neutrons were started at the center of the pile at discrete angles to the lattice direction (0 degrees was parallel to the lattice.) For example, the relative pile efficiencies for 265 keV neutrons are plotted in Fig. 3 as a function of angle. They were respectively averaged over an isotropic distribution,

$$\bar{\epsilon}_1 = \frac{\int \epsilon(\theta) d\Omega}{\int d\Omega}, \quad (9)$$

and a photoneutron angular distribution $\sigma(\theta)$,

$$\bar{\epsilon}_2 = \frac{\int \epsilon(\theta) [\sigma(\theta)] d\Omega}{\int [\sigma(\theta)] d\Omega}. \quad (10)$$

In the case of the photo-disintegration of deuterium by photons of energies below 20 MeV the angular distributions in the center of mass system of the emitted protons or neutrons has been found to be adequately represented by

FIGURE 3. RELATIVE EFFICIENCY VS ANGLE OF EMISSION FOR 265 keV NEUTRONS.

$$\sigma(\theta_{cm}) \sim a + \sin^2 \theta_{cm} (1 + \alpha \cos \theta_{cm}). \quad (11)$$

The constants a and α depend on the energy of the incident photon.

In the laboratory system Eq. (11) is given approximately by

$$\sigma(\theta_\ell) \sim a + \sin^2 \theta_\ell \left(1 + \left[\alpha + \frac{2\beta \hbar\omega}{\hbar\omega - W} \right] \cos \theta_\ell \right), \quad (12)$$

where $\beta = \left[(\hbar\omega - W)/Mc^2 \right]^{1/2}$, $\hbar\omega$ is the photon energy, Mc^2 is the neutron or proton rest mass, and W is the threshold for photodisintegration of deuterium. Marshall and Guth⁽¹¹⁾ have shown that under a special assumption about the neutron and proton forces α is equal to 2β . In Eq. (11) the isotropic term (a) arises from magnetic dipole transitions and the $\sin^2 \theta_{cm}$ term arises from electric dipole transitions. The term proportional to $\cos \theta_{cm}$ originates from the interference between the electric dipole and electric quadrupole terms and leads to a forward asymmetry in the angular distribution.

For photon energies below 3 MeV the electric quadrupole term is less important, and Eq. (11) reduces to

$$\sigma(\theta_{cm}) \sim a + \sin^2 \theta_{cm}, \quad (13)$$

and Eq. (12) becomes

$$\sigma(\theta_\ell) \sim a + \sin^2 \theta_\ell \left(1 + \frac{2\beta \hbar\omega}{\hbar\omega - W} \cos \theta_\ell \right). \quad (14)$$

Experimental values⁽¹²⁾ of the constant a for photon energies used in calibrating the boron pile (Table I) are shown in Table V. The ratios of $\bar{\epsilon}_2/\bar{\epsilon}_1$ for 2.615 and 2.757 MeV photons were computed using the calculated values of $\epsilon(\theta_\ell)$ for 0.190 and 0.265 MeV neutrons, respectively, and Eq. (14). The results are listed in Table V.

For the 6-7 MeV photons associated with the $^{19}\text{F}(p,\alpha\gamma)$ reaction the ratio of $\bar{\epsilon}_2/\bar{\epsilon}_1$ was computed using the experimental results of Fuller⁽¹³⁾ who assumed the form of Eq. (14). In the case of 7 MeV photons

$$\sigma(\theta_\ell) \sim 0.05 + \sin^2\theta_\ell(1 + 0.32 \cos \theta_\ell). \quad (15)$$

Finally, as $h\nu$ increases to about 11 MeV, the last case in Table V, the isotropic term a seems to go to zero, and the study of Fuller yields

$$\sigma(\theta_\ell) \sim \sin^2\theta_\ell(1 + 0.43 \cos \theta_\ell), \quad (16)$$

which was used to compute $\bar{\epsilon}_2/\bar{\epsilon}_1$.

Table V

Experimental Values of a and Calculated Values of $\bar{\epsilon}_2/\bar{\epsilon}_1$

<u>Photon Source</u>	<u>Photon Energy $\hbar\omega$ (MeV)</u>	<u>a</u>	<u>$\bar{\epsilon}_2/\bar{\epsilon}_1$</u>
ThC''	2.615	0.28	1.0020
Na ²⁴	2.757	0.20	1.0022
¹⁹ F(p, $\alpha\gamma$)	6.1, 6.9, 7.1	0.05	1.0037
²⁷ Al(p, γ)	11.1, 12.9	~0.0	1.0042

The calculations for neutrons produced by Na^{24} γ -rays confirm the investigation made by Colvin and Sowerby into the effects of anisotropy. They made no measurements at other neutron energies, however. The present calculations indicate that for the neutron energies at which the boron pile was calibrated the ratio of the efficiency from $^2\text{H}(\gamma, n)^1\text{H}$ measurements to that for isotropic emission at the same energy is only slightly dependent on the neutron energy and the corresponding angular distribution. In Table IV the sensitivities to the pile anisotropy were taken to be 0.22% and 0.37% for cases III and IV, respectively.

D. Correction for Neutrons Captured After the Prompt Gate

A small fraction of the neutrons are captured after the prompt gate has closed. If the counting interval is long enough so that it includes the interval in which the neutrons have reached an asymptotic energy distribution within the moderator, then the neutron density decays exponentially. The time constant for this decay depends on the absorption rate averaged over the neutron-energy spectrum. Under these conditions the number of neutrons per event counted up to a time t is given by

$$R(t) = R(\infty) \left[1 - e^{-t/\langle t \rangle} \right], \quad (17)$$

where $R(\infty)$ is the counting rate for an infinite gate time and $\langle t \rangle$ is the mean neutron lifetime. Experimentally, for times greater than 2 ms after the opening of the prompt gate the time rate of capture was exponential. In Table V of Ref. (1), Colvin

Table VI

Neutrons Detected per Event from Prompt
Gates of Different Length

<u>Experiment</u>	<u>No. of Neutrons Detected per Event</u>		
	<u>2-ms Gate</u>	<u>4-ms Gate</u>	<u>8-ms Gate</u>
^{240}Pu Spontaneous Fission	1.3608 $\pm$ 0.0040 $\pm$	1.3567 $\pm$ 0.0050 $\pm$	1.3578 $\pm$ 0.0057 $\pm$
$^{24}\text{Na } ^2\text{H}(\gamma, n)^1\text{H}$	0.6453 $\pm$ 0.0008 $\pm$	0.6491 $\pm$ 0.0009 $\pm$	0.6476 $\pm$ 0.0015 $\pm$
Typical Correction (%)	20.7	4.3	0.1

and Sowerby gave the results of several experiments for the number of neutrons detected per event as a function of prompt gates of different length. The experiment using $^{24}\text{Na } ^2\text{H}(\gamma, n)^1\text{H}$ has been questioned⁽¹⁴⁾. At first glance the results for the important $^{24}\text{Na } ^2\text{H}(\gamma, n)^1\text{H}$ measurements (Table VI) appear to have a precision of 0.5%. The correction for the 2 ms gate appears to be systematically low. (The decay constant $1/\langle t \rangle$ inferred from the 2 ms gate measurements is too high). This may have a physical explanation since for times shorter than 2 ms the decay in a graphite system is not a simple exponential. The neutron density rather has the form

$$N(E, t) = A_0 N_0(E) e^{-\lambda_0 t} + \int_{(v\Sigma_s)_{\min}}^{\infty} A(\lambda) N_{\lambda}(E) e^{-\lambda t} d\lambda, \quad (18)$$

where

$$\lambda_0 = v\Sigma_a = \text{average thermal absorption probability per unit time}$$

$$(v\Sigma_s)_{\min} = 2900 \text{ sec}^{-1} \quad (15) \quad \text{for graphite}$$

$N_0(E)$ = fundamental eigenfunction which is closely approximated by a Maxwellian

$N_{\lambda}(E)$ = eigenfunctions belonging to the continuous range of eigenvalues.

In Eq. (18) A_0 and $A(\lambda)$ are expansion coefficients, and the second term corresponds to the contribution to $N(E, t)$ from the continuous range of eigenvalues λ . This term represents at short times

neutrons that are not even in the thermalization region and are slowing down according to classical slowing-down-theory. (Note for graphite only one discrete eigenvalue λ_0 , the fundamental, is thought to exist⁽¹⁵⁾.) What emerges from Eq. (18) is that any decay constant determined for times t which are not

$\gg [(\Sigma_s)_{\min}]^{-1}$ will undoubtedly be some average of the discrete and continuum values. Hence, it will be larger than λ_0 , the decay constant of the asymptotic distribution. This may explain why the corrected count rate per event for the 2 ms gate is lower. For 4 ms and 8 ms gate times the neutron distribution is dominated by the first term in Eq. (18), and the correction for neutrons captured after the prompt gate can be made using Eq. (17) with $\langle t \rangle = \lambda_0^{-1}$.

From these observations, it is felt that no serious problem exists with the correction for neutrons captured after the prompt gate has closed for the 4 ms case, which was used for most of the measurements. Since any errors in the fission and ${}^2\text{H}(\gamma, n){}^1\text{H}$ measurements are likely to cancel, the error of 0.1% in $\bar{v}_p$ given in Ref. (1) was judged adequate in this work.

V. CONCLUSIONS

The boron pile measurement of $\bar{v}_p({}^{252}\text{Cf})$ has been subjected to a detailed Monte Carlo analysis. Specifically, the energy dependence of the neutron detection efficiency was calculated, and several sources of error in $\bar{v}_p$ were examined. The following conclusions can be made:

- 1) The calculated efficiency versus neutron energy (Fig. 2) does not fit the measured data very well. If it is assumed that the calculated curve adequately represents the energy dependence of the neutron detection efficiency of the pile, it must be concluded that one or more of the systematic corrections used in reducing the values of $\bar{R}$ to $\bar{R}'$ are possibly incorrect. Regrettably, one of the larger corrections, that for materials in the central channel, could not be calculated because of a lack of information.
- 2) The calculations normalized to experiment at 265 keV yield $\bar{\nu}_p = 3.744 \pm 0.015$ ($\bar{\nu} = 3.753 \pm 0.015$). If normalization is made to experiment at 2.0 MeV, $\bar{\nu}_p = 3.713 \pm 0.019$ ($\bar{\nu} = 3.722 \pm 0.019$). A weighted average of these two results yields $\bar{\nu}_p = 3.733 \pm 0.012$ ($\bar{\nu} = 3.742 \pm 0.012$). If an additional 0.5% uncertainty is added in quadrature to account for the inability to check calculationally the effects of materials in the central channel, then $\bar{\nu} = 3.742 \pm 0.022$.
- 3) It is tempting to choose the measured efficiency at 265 keV as best for normalization since it has the lowest experimental uncertainty and yields a value of $\bar{\nu} = 3.753 \pm 0.015$ which is in good agreement with the value $\bar{\nu} = 3.756 \pm 0.018$ reported from the analysis⁽⁵⁾ of the liquid scintillator measurement due to Boldeman. Since both experiments were reanalyzed using the same Monte Carlo code and cross section data base, this would eliminate the long-standing disparity between the boron pile and liquid scintillator measurements. However, even the weighted

average in item 2 above is not inconsistent with the revised value of Boldeman.

- 4) Several sources of error in $\bar{\nu}_p(^{252}\text{Cf})$ were examined. The results are supplied in Table IV. Notably, the effects of anisotropy in the pile efficiency for the $^{24}\text{Na } ^2\text{H}(\gamma, n)^1\text{H}$ measurement were found to be consistent with the measurements made by Colvin and Sowerby. However, the effects of anisotropy for the $^{19}\text{F}(p, \alpha\gamma) ^2\text{H}(\gamma, n)^1\text{H}$ and $^{27}\text{Al}(p, \gamma) ^2\text{H}(\gamma, n)^1\text{H}$ measurements were slightly greater.

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