MASTER

A WORK HARDENING CORRELATION FOR
MONOTONIC LOADING BASED ON STATE VARIABLES

F. H. Huang and Che-Yu Li

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Department of Materials Science and Engineering
Cornell University, Ithaca, New York 14853

EB
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

An absolute work hardening correlation in terms of the hardness parameter and the internal stress based on the state variable approach was developed. It was found applicable to a variety of metals and alloys. This correlation predicts strain rate insensitive work hardening properties at low homologous temperatures and produces strain rate effects at higher homologous temperatures without involving thermally induced recovery processes.

INTRODUCTION

The purpose of this paper is to introduce a work hardening correlation based on the state variable approach and to show its general applicability.

In two previous papers^{1,2} we have demonstrated that the work hardening coefficient γ and the absolute work hardening coefficient Γ are a function of only current state variables where $\gamma = \left. \frac{\partial \ln \sigma}{\partial \epsilon} \right|_{\dot{\epsilon}}$ and $\Gamma = \frac{d \ln \gamma}{d \epsilon}$ with σ being the stress, ϵ and $\dot{\epsilon}$ being the nonelastic strain and strain rate respectively and γ being the hardness. Since that time, continued work on a plastic equation of state has allowed the development of a deformation model for the grain matrix processes³ which has been tested extensively by various experiments.^{4,5,6} In this model the significance of the absolute work hardening coefficient Γ is identified. In the present work we used also the constitutive relations associated with the deformation model in data analysis. In the following section we shall review briefly first the deformation model as a part of the background for this report.

We shall develop the work hardening correlation based on state variables by using the tensile data of nickel. We shall show that the correlation can be applied to the data of other metals and alloys and can be used to predict the creep properties. The significance of the present correlation will be examined and will be compared with the current concepts on work hardening.

THE DEFORMATION MODEL

The micromechanical considerations and the associated quantitative relations of the deformation model are given in References 3-6. The model is shown schematically in Figure 1 consisting of three elements and represents grain matrix deformation processes. The element 1 is called the α -element representing the stored recoverable anelastic strain α due to dislocation pile-ups. The element 2 is called the $\dot{\alpha}$ -element which determines the rate of dislocation "leakage" through the strong barriers. The strain rate component $\dot{\alpha}$ of the second element that results under the action of the internal stress σ_a yields the uncoverable plastic part α of the total observable non-elastic strain ϵ . The third element is the $\dot{\epsilon}$ -element and represents the glide friction limited dislocation flux. The effective stress operates on the $\dot{\epsilon}$ -element as σ_f such that

$$\sigma = \sigma_a + \sigma_f \quad (1)$$

The other constraint relation implied in the diagram is

$$\dot{\epsilon} = \dot{\alpha} + \dot{\epsilon} \quad (2)$$

In the absence of thermally induced structural changes, the constitutive relations of interest are

$$\sigma_a = M\alpha \quad (3)$$

$$\ln(\sigma^*/\sigma_a) = (\dot{\epsilon}^*/\dot{\alpha})^\lambda \quad (4)$$

$$\dot{\epsilon}^* = (\sigma^*/G)^m f e^{-Q/kT} \quad (5)$$

$$d \ln \sigma^* / dt = r(\sigma_a, \sigma^*) \dot{\alpha} \quad (6)$$

$$\dot{\epsilon} = \dot{\alpha}^* [(\sigma_f)/M]^M \operatorname{sgn}(\sigma_f) \quad (7)$$

In these equations M , M , m , λ and f are experimentally determined, G is the modulus of rigidity and Q is an activation energy.

The state variable hardness characterizes the current strength of the strong barriers and undergoes changes by work hardening. For the interest of the present work the hardness y is measured by the parameter σ^* . Since the stored anelastic strain does not contribute to work hardening, only the $\dot{\alpha}$ -element is involved in work hardening, we redefine absolute work hardening coefficient, r to be

$$r = d \ln \sigma^* / d \alpha \quad (8)$$

which measures the rate of change with plastic straining the significant microstructure that determines the hardness.

The stress operating on the $\dot{\alpha}$ -element is the internal stress σ_a . We are therefore interested in representing the absolute work hardening coefficient r in terms of the two state variables, σ^* and σ_a . It should be noted that the choice of other state variables is allowed according to the constitutive relations of the deformation model. In a tensile test the applied stress σ and total non-elastic strain ϵ are measured. We have to use the constitutive relations given previously to calculate the value of the relevant parameters in order to develop a correlation for r .

EXPERIMENT

The tensile test specimens with threaded ends were prepared from 1/2 inch rods of high purity nickel (Nickel 270) obtained from International Nickel Company. The gage section of the specimen was 0.75 inch (1.905 cm) long with a diameter of 0.1 inch (0.254 cm). After machining the specimens were annealed in vacuum at 650°C for three hours. The average grain size was 120 μ m.

The procedures for tensile testing and the method of data analysis were described in Reference 1. Tensile tests were performed at room temperature, 200°, 250° and 300°C with a crosshead speed of

0.02 inch/min (.0528 cm/min). The experiment yielded stress σ vs. non-elastic strain ϵ data at different temperatures.

For data analysis we required also load relaxation as well as load change data in order to evaluate some of the parameters of the constitutive relations. These latter data were given in References 7 and 8 and the method of data analysis used were described in References 4 and 9.

RESULTS AND DISCUSSION

1. Analysis of the Tensile Data of Nickel

The tensile data of nickel in a plot of stress vs. non-elastic strain ϵ are shown in Figure 2. In the figure the measured strain value was shifted by an amount of $\Delta\epsilon$ 2.3%, for the room temperature curve. This was done to avoid the overlap of curves in the figure. In the following we shall use these data to evaluate the hardness parameter σ^* , the internal stress σ_a and the plastic strain α .

For a given pair of σ and ϵ on a curve in Figure 2 we calculated the corresponding value of non-elastic strain rate $\dot{\epsilon}$ by using the method in Reference 1 and the known extension rate. For the present interest, $\dot{\epsilon}$ could be approximated by $\dot{\epsilon}$ at large values of ϵ . With this approximation and by using Equations (1), (3) and (7) and the parameters in Table 1 we obtained the value of the hardness parameter σ^* and the internal stress σ_a . The value of the corresponding plastic strain α was calculated from $\epsilon = \alpha + a$ with $a = \sigma_a/M$.

The results of the calculation were given in Figure 3 in the form of $\log\sigma^*$ vs α and $\log\sigma_a$ vs α curves. The curves show that at low temperatures σ^* and σ_a are very close as required by Equation

(4) when the value of $\dot{\epsilon}$ becomes significantly larger than that of $\dot{\epsilon}^*$. At higher temperatures σ^* and σ_a deviate from each other. According to Equation (5) this deviation will become larger at higher values of σ^* for the same plastic strain rate.

In the following we shall calculate Γ and find its dependence on σ^* and σ_a .

2. Calculation of the Absolute Work Hardening Coefficient Γ

During a constant extension rate tensile test the plastic strain rate varies as the strain is increased. We can use Equation (7) in reference 1 to calculate Γ . This equation takes the following form when applied to the $\dot{\epsilon}$ -element of the deformation model,

$$d \ln \sigma^* = 1/(1-mv) (d \ln \sigma_a - v d \ln \dot{\epsilon}) \quad (9)$$

where we took γ , ν , σ , $\dot{\epsilon}_p$ in Equation (7) of Reference 1 to be σ^* , $1/m$, σ_a and $\dot{\epsilon}$ respectively and $v = \partial \ln \sigma_a / \partial \ln \dot{\epsilon} |_{\sigma^*} = \lambda \ln(\sigma^* / \sigma_a)$.

From the definition of Γ [Equation (8)] and the above equation, Γ can be expressed as

$$\Gamma = 1/(1-mv) (d \ln \sigma_a / d \epsilon - v d \ln \dot{\epsilon} / d \epsilon) \quad (10)$$

In the appendix we shall show the calculation of Γ by using the above equation. The results of the calculation were given in Figure 4 in a plot of $\log \Gamma$ vs $\log \sigma^*$ and in Figure 5 in a plot of $\log \Gamma$ vs $\log \sigma^* / \sigma_a$ for various temperatures. In the following we shall use these results to develop the correlation for the absolute work hardening coefficient Γ in terms of the hardness parameter σ^* and the internal stress σ_a .

3. Work Hardening Correlation

In Figure 5 the dependence of Γ on $\log \sigma^* / \sigma_a$ varies depending on the test temperature. We are interested in eliminating tempera-

ature as a variable in our correlation. A correlation based on σ^* and σ_a not only offers advantage both in its application and in mechanical testing^{1,2} but also demonstrates that the work hardening coefficient r is truly a state property.²

The strategy which we took was to plot by using the data in Figures 4 and 5, two of the three variables $\log r$, $\log \sigma^*$ and $\log \sigma^*/\sigma_a$ against each other while keeping the third one fixed. In Figure 6 $\log r$ was plotted against $\log \sigma^*/\sigma_a$ at different values of $\log \sigma^*$. In making this figure, for a given $\log \sigma^*$ we picked out the value of $\log r$ from the data for a particular temperature in Figure 4. From the data of the same temperature in Figure 5 we obtained the value of $\log \sigma^*/\sigma_a$ corresponding to the $\log r$ of interest. Figures 7 and 8 were constructed in the similar manner. In Figure 7, $\log r$ was plotted against $\log \sigma^*$ at fixed values of $\log \sigma^*/\sigma_a$ and in Figure 8, $\log \sigma^*$ was plotted against $\log \sigma^*/\sigma_a$ at a fixed value of $\log r$.

Figure 8 is equivalent to Figure 5.14 in Reference 2. The latter figure contained the data of 1100 aluminum based on which a power law type correlation was derived. Figure 8 contained data of a wider range. It is apparent that the power law type correlation does not apply to the present data. The data in Figure 8 shows that a plot of $\log \sigma^*$ vs $\log \sigma^*/\sigma_a$ at constant values of r produces curves of varied slopes while the power law type correlation requires a family of straight lines of the same slope.

For the present correlation we used the data in Figure 6 which showed $\log r$ to be linearly related to $\log \sigma^*/\sigma_a$ at a fixed $\log \sigma^*$ in the form of $\log r = A + B \log \sigma^*/\sigma_a$ with both A and B functions of σ^* . From the data in Figure 6 we computed the values of A and

log B and plotted them against log σ^* in Figure 9.

We obtained

$$\begin{aligned} A &= 8.28 - 1.6 \log \sigma^* \\ &= 1.6 \log \frac{1}{\sigma^*/1.49 \times 10^5} \end{aligned} \quad (12a)$$

$$\begin{aligned} \log B &= 5.18 - \log \sigma^* \\ &= \log \frac{1}{\sigma^*/1.51 \times 10^5} \end{aligned} \quad (12b)$$

The correlation for the absolute work hardening coefficient Γ can therefore be written as

$$\log \Gamma = 1.6 \log \frac{1}{\sigma^*/1.49 \times 10^5} - \frac{1}{\sigma^*/1.51 \times 10^5} \log \sigma^*/\sigma_a \quad (13a)$$

or

$$\Gamma = (\sigma^*/1.49 \times 10^5)^{-1.6} (\sigma^*/\sigma_a)^{-1/(\sigma^*/1.51 \times 10^5)} \quad (13b)$$

It is reasonable to expect that σ^* and σ_a are measured² in units of G. We note that the factors 1.49×10^5 and 1.51×10^5 in Equation (13) are close to each other within experimental value. We propose therefore a generalized correlation for the absolute work hardening coefficient Γ to be

$$\log \Gamma = \delta \log \frac{1}{\sigma^*/kG} - \frac{1}{\sigma^*/kG} \log(\sigma^*/\sigma_a) \quad (14)$$

where the parameter δ and k are to be determined experimentally. We note that for the data analysis given above the stress scale used was that of room temperature. We evaluated the parameter k by using the G value at room temperature. The G value has to be temperature corrected to calculate the value Γ at a different temperature. In the following we shall first examine the applicability of the proposed general correlation. We shall then consider its physical significance.

4. Applicability of the Work Hardening Correlation

Before examining the applicability of the proposed correlation it is worth noting that work hardening (or tensile) data are usually of more limited range (especially the strain rate range) compared to load relaxation data.^{1,2,4} The accuracy of the value of the parameters, k and δ in Equation (14) are much less than that of the load relaxation parameters given in Table 1. Since it is difficult to obtain the low strain rate work hardening data experimentally, a valid generalized work hardening correlation will allow the characterization of this important property by using high strain rate data only.

In a related work on the state variable approach we used the constitutive relations given in the section on the deformation model and Equation (14) to predict the creep properties (constant load creep data) of a material.^{10,11} The parameters used in these relations were determined by experiments other than the creep test including load relaxation, load change and tensile or work hardening tests. In this work it was found that the ability of the deformation model used to predict the creep properties of a material depends strongly on the form of r as well as on the value of k and δ in Equation (14). We found that in order to be able to predict a wide range of creep and tensile data of nickel the parameters kG and δ in Equation (14) took a value of 1.04×10^5 and 1.64 respectively. These numbers compared well to the experimentally determined value of 1.50×10^5 and 1.6 respectively. The deviation was within experimental error.

At low homologous temperatures and/or high strain rates, the value of $\dot{\epsilon}$ becomes significantly larger than that of $\dot{\epsilon}^*$. According

to Equation (4), the value of σ^*/σ_a approaches one. For the tensile data of Type 316 and 304 stainless steel and niobium^{1,14} obtained at room temperature and moderate extension rates the above condition is satisfied.^{2,4,11,13} Thus in this data range, the second term on the righthand side of Equation (14) will become not important. We will expect that under these experimental conditions the work hardening properties of these materials to exhibit the following features according to the proposed correlation:

- a. The work hardening coefficient will depend only on the stress or hardness.
- b. The work hardening coefficient will be insensitive to strain rate.
- c. Constant hardness curves and constant $\dot{\epsilon}$ curves in a $\log \sigma - \log \dot{\epsilon}$ plot will have the same shape.

The data of Type 316 stainless steel and niobium given in Reference 1 are consistent with the above predictions. We have estimated the value of δ to be 1.7 and 2.5 for Type 316 stainless steel and niobium respectively based on the data given in Figures 11 and 12 of Reference 1.

In another related work we used tensile data of Type 304 stainless steel obtained under conditions where $\sigma^*/\sigma_a \sim 1$ to obtain a value of 1.33 and 0.0191 for δ and k in Equation (14) respectively.^{11,14} Experimentally we were not able to test the validity of the second term in the righthand side of Equation (14) by using the available tensile data. We were able however to use Equation (14) and the value of δ and k given above to predict the creep properties of Type 304 stainless steel under conditions where the second term was important. This exercise supported strongly the

applicability of the proposed work hardening correlation.

In the following we shall discuss the physical significance of the correlation.

5. Physical Significance of the Work Hardening Correlation

At low homologous temperatures and/or high strain rate the proposed correlation is consistent with the strain rate insensitive power law type of work hardening theories.^{15,16} Under these experimental conditions, σ^*/σ_a approaches one such that only the first term on the righthand of Equation (14) is important. For an essentially constant strain rate tensile test according to Equation (7) and $\sigma_f \sim \sigma - \sigma^*$, the hardness of σ^* will be proportional to the applied stress σ . Thus the first term on the righthand side of Equation (14) will yield a power law type relation between stress and strain. With the value of δ range between 1.33 and 2.5 given previously, the resulting power law relation will be similar to the well known parabolic relation for work hardening.

Since Equation (14) suggests a strain rate insensitive work hardening coefficient in the region where σ^*/σ_a approaches one and since in the state variable approach, plastic strain is measured² by σ^* , the proposed correlation is therefore consistent with some of the current work hardening theories which use plastic strain as the important variable.^{15,16}

At higher homologous temperatures and/or lower strain rate, the work hardening coefficient is known to depend on strain rate.

We are interested in a particular temperature region where the strain rate effects exist but the temperature is not sufficiently high to cause recrystallization during the tensile experiment. This is the same temperature region where the absence of thermally induced

recovery of hardness can be tested by load relaxation test.² In this temperature region the proposed correlation differs from some of the current theories in that it will produce strain rate dependence without relying on thermally induced recovery processes.

The factor

$$\frac{1}{(\sigma^*/\sigma_a)^{1/(\sigma^*/kG)}}$$

in Equation 13b gives the strain rate dependence which has a maximum value of one when σ_a approaches σ^* . The data shown in Figure 7 are in the range such that the factor is close to one. As the applied stress is decreased which results in a lower strain rate, the ratio of σ^*/σ_a will increase and will lead to a reduced work hardening coefficient. Thus in the temperature region described in the previous paragraph the proposed correlation is able to account for the strain rate dependence of work hardening without utilizing any thermally induced recovery processes.

The ratio of σ^*/σ_a measures the size of dislocation pile-ups at a given hardness level.^{2,6} At high strain rates and/or low temperatures σ^*/σ_a approaches one which represents the maximum extent of dislocation pile-ups and produces strain rate insensitive work hardening. As the homologous temperature increases and/or strain rate decreases, the ratio of σ^*/σ_a will increase corresponding to reduced dislocation pile-ups and a reduced work hardening coefficient. It is possible that the size of dislocation pile-ups will influence the dislocation interactions during straining such that the rate of work hardening will be influenced.

It should be emphasized that the proposed work hardening correlation was derived based on a careful analysis of the experimental

data by using the state variable approach. Its general applicability has to be tested by experiments on other materials and by applying it to predict, for example, the creep properties of these materials. If its general applicability is shown, it can be used to characterize the work hardening properties of a material with considerable savings in mechanical testing and can be used as a basis for further theoretical work in this area.

SUMMARY

1. An absolute work hardening correlation was developed based on the tensile data of nickel and the state variable approach.
2. The two state variables involved are the hardness parameter and the internal stress.
3. This correlation was found applicable to other metals and alloys and was used to predict the creep properties of the materials.
4. This correlation produces a strain rate dependence of the work hardening coefficient without involving thermally induced recovery processes.

ACKNOWLEDGEMENT

This work is supported by Energy Research and Development Administration. The authors wish to acknowledge also discussion with E. W. Hart.

APPENDIX

Calculation of the Work Hardening Parameter r

We can rewrite Equation (10) in terms of the measured non-elastic strain ϵ ,

$$r = 1/(1-mv) (\dot{\sigma}_a / \dot{\epsilon} - v \dot{\sigma} / \dot{\epsilon}) \dot{\epsilon} / \dot{\sigma} \quad (A1)$$

In the above equation $\dot{\sigma}$ was calculated from

$$\dot{\sigma} = \dot{\epsilon} - \dot{\sigma} / M \quad (A2)$$

The parameter v was calculated from $v = \lambda \ln(\sigma^* / \sigma_a)$. The value of $\dot{\sigma}_a / \dot{\epsilon}$ was calculated from the data in Figure 2 and $\sigma_a = \sigma - \sigma_f$. The value of $\dot{\sigma} / \dot{\epsilon}$ was calculated from Equation (A2) and the data in Figure 2.

TABLE 1

The Values of Parameters Used to Formulate Equation (14)

$$\lambda = 0.15, \quad M = 7.6$$

$$M = 0.2, \quad Q = 60.5 \text{ kcal/mole}$$

Temperature (°C)	M (PSI)	$M\alpha^{-1/M}$ (PSI)	lnD (PSI) *
25	1.55×10^7	3491	26.56
200	1.23×10^7	9970	19.05
250	1.30×10^7	12800	17.82
300	1.33×10^7	15800	16.80

* $\ln D = \ln G - M \ln f + MG/kT$ as derived from Equation (5).

REFERENCES

1. Wire, G. L., Ellis, F. V. and Che-Yu Li, Acta Met. 24, 677 (1976).
2. Hart, E. W., Che-Yu Li, Yamada H., and Wire, G. L., in Constitutive Equations in Plasticity, A. Argon, Ed., p. 149, MIT Press (1975).
3. Hart, E. W., J. Eng. Mater. Technol. 98, 193 (1976).
4. Huang, F. H., Ellis, F. V. and Che-Yu Li, Met. Trans. 8A, 699 (1977).
5. Nir, N., Huang, F. H., Hart, E. W., and Che-Yu Li, Met. Trans. 8A, 583 (1977).
6. Nir, N., Hart, E. W., and Che-Yu Li, "Relationship Between Abrupt Load Change Tests at Creep Stress Levels and Low Stress Anelastic Behavior for Monocrystalline Aluminum," Acta Met., in press.
7. Chao, S. J., Thesis (M.S.), Cornell University (1976).
8. Nir, N., unpublished work.
9. Nir, N., Hart, E. W. and Che-Yu Li, Sci. Met. 10, 189 (1976).
10. Kumar, K., Huang, F. H., Che-Yu Li, unpublished work on Ni creep.
11. Kumar, K., Huang, F. H. and Che-Yu Li, "Creep Simulation by Using the State Variable Approach," Submitted to Scripta Met.
12. Yamada, H. and Che-Yu Li, Met. Trans., Vol. 4, 2133 (1973).
13. Yamada, H. and Che-Yu Li, Acta Met., Vol. 22, 249 (1974).
14. Huang, F. H., Yamada H., and Che-Yu Li, "Constitutive Relations for Non-Elastic Deformation," to be presented in ASME Symposium on "Characterization of Materials Intended for Services at Elevated Temperatures," Montreal, June 1978, to be published in the Proceedings of the Symposium.
15. Work Hardening Met. Soc. Conf., Chicago, 1966, Ed. by J. P. Hirth and J. Weertman, Gordon & Breach, N.Y. (1968).
16. Ashby, M.F., The Deformation of Plastically Non-Homogenous Alloys, in "Strengthening Methods in Crystals," Ed. by A. Kelly and R. B. Nicholson, Elsevier, N.Y. (1971).

Figure Captions

- Fig. 1 A schematic rheological diagram representing Hart's model of the constitutive relations for metal grain matrix nonelastic flow.
- Fig. 2 Logarithm of stress versus nonelastic strain as a function of temperature.
- Fig. 3 $\ln \sigma^*$ and $\ln \sigma_a$ versus α plots for different temperatures.
- Fig. 4 $\log \Gamma$ versus $\log \sigma^*$ plots for different temperatures.
- Fig. 5 $\log \log \Gamma$ versus $\log (\sigma^*/\sigma_a)$ plots for different temperatures.
- Fig. 6 $\log \Gamma$ versus $\log (\sigma^*/\sigma_a)$ plots for different values of σ^* .
- Fig. 7 $\log \Gamma$ versus $\log \sigma^*$ plots for different values of σ^*/σ_a .
- Fig. 8 $\log \sigma^*$ versus $\log (\sigma^*/\sigma_a)$ plots for different values of Γ .
- Fig. 9 The y-intercepts and the logarithm of the slopes of lines in Fig. 6 plotted as a function of $\log \sigma^*$.

Fig. 1

Fig. 2

Fig. 5

$$\text{LOG } \Gamma = A + B \text{ LOG } (\sigma^*/\sigma_0)$$

Fig. 6

Fig. 7

Fig. 8