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Some Implications for Mirror Research of the Coupling Between Fusion Economics and Fusion Physics

R. F. Post

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Lawrence Livermore Laboratory

SOME IMPLICATIONS FOR MIRROR RESEARCH OF THE COUPLING BETWEEN FUSION ECONOMICS AND FUSION PHYSICS*

R. F. Post

Lawrence Livermore Laboratory

University of California, Livermore CA U.S.A.

ABSTRACT

The thesis is made that physics understanding and innovation represent two of the most important ingredients of any program to develop fusion power. In this context the coupling between these and the economics of yet-to-be realized fusion power plants is explored. The coupling is two-way: Realistic evaluations of the economic (and environmental) requirements for fusion power systems can influence the physics objectives of present-day fusion research programs; physics understanding and innovative ideas can favorably impact the future economics of fusion power systems. Of equal importance is the role that physics/innovation can have on the time scale for the first practical demonstration of fusion power. Given the growing worldwide need for long-term solutions to the problem of energy it is claimed to be crucial that fusion research be carried out on a broad base and in a spirit that both facilitates the growth of physics understanding and fosters innovation. Developing this theme, some examples of mirror- based fusion system concepts are given that illustrate the coupling here described.

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I have been asked to speak on the topic: Implications for mirror research of the coupling between fusion physics and fusion economics. In considering what I might say about such a subject I very soon came to the realization that it would not be possible at this point in time to lay out a well-defined set of relationships between these two elements of the fusion problem. Perhaps only the historians will be able to accomplish that with any success - after fusion power plants have been developed and deployed. My talk will consequently not really properly answer the question that I was asked to discuss. What I have to say could therefore be better described as an attempt to raise some new questions. If we could find the answers to these questions through physics plus innovation I believe it could have a highly positive impact on the future economics of fusion power. In raising these questions I am hoping to stimulate your imagination, and thereby perhaps to encourage you to think about some different aspects of the physics/economics coupling.

To set the tone of this talk I will therefore use a quotation from a rather well-known scientist:

"IMAGINATION IS MORE IMPORTANT THAN KNOWLEDGE"

A. Einstein

I am sure that Einstein did not mean to deprecate or to undervalue knowledge in making this statement. In my opinion he was only pointing out that knowledge without imagination is sterile and by itself leads nowhere. Conversely, at least in research, imagination, if it is exercised in a physics knowledge vacuum tends to be like blowing in the wind - it represents an intellectual exercise that is detached from reality.

In discussing fusion research it is clear that we are dealing with what will someday become a whole new industrial technology, a technology created to satisfy a widely perceived need. In the case of fusion this is of course the need for a safe and inexhaustible source of high quality energy. We can also be equally sure that the actual achievement of fusion will have been preceded by cycles of knowledge growth and leaps of the imagination. We have in our lifetime seen a spectacular example of just such a cyclic sequence. This example is the evolution of the computer. As

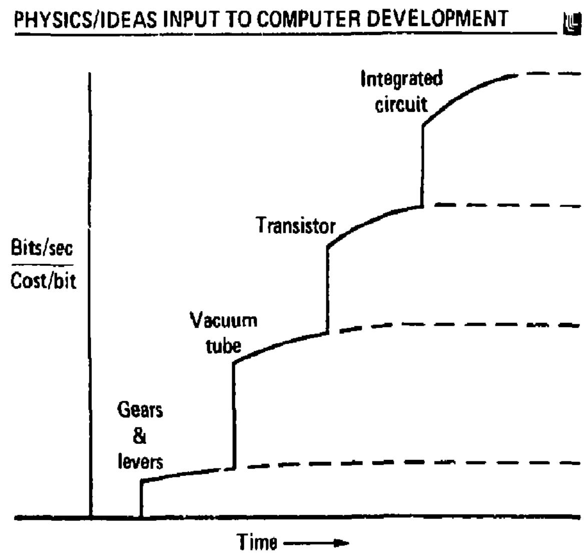

Figure 1

is shown schematically (and not to scale) in Fig. 1, successive cycles of the introduction of new ideas and new physics and technology have in the short period of 30 years transformed the computer from a clumsy assembly of gears and levers - as in the old mechanical calculators that I used to use - to today's spectrum of devices ranges from tiny microprocessors to the CRAY-1 computer. The increase of the figure of merit that I have used -

bits per second divided by cost per bit - between the mechanical calculator and the CRAY-1 has been more than 15 orders of magnitude!

We of course cannot hope for that kind of performance in fusion - or probably even for gains of one order of magnitude in output per unit cost relative to conventional methods of generating power. But the essential part of the message remains the same - physics knowledge (which was solid state physics for the computer, and is mainly plasma physics for fusion), coupled with innovative leaps can have a remarkable effect in promoting the development of a new technological system.

In defining the nature of the technological system that fusion will represent, we might start by considering the general two-column matrix of factors of the kind that always appears in one form or another whenever the development of any new technological system is undertaken. I am referring to the matrix of objectives and constraints. In striving to achieve a new technological system we always have a series of objectives in mind; in trying to reach those objectives we are at every step bounded by a set of constraints. These constraints will vary in their relative and absolute importance with time, but the objectives are not likely to vary appreciably, if they were well founded to begin with.

Figure 2 shows a list of objectives and contraints that I see as operating in fusion research. Our objectives have not changed appreciably since fusion research began 30 years ago. They have in fact become even more relevant and more desirable as the energy crisis has deepened. Furthermore, the fusion goal itself has become more believable owing to our scientific progress.

Our constraints are very well understood by anyone who has been seriously involved in fusion research. The first one - the political base - becomes very obvious when the budgets are set, and this constraint

OBJECTIVES AND CONSTRAINTS OF FUSION R&D

Objectives	Constraints
• Early deployment	• Political base
• Competitive costs	• Conceptual pool
• Minimal hazards	• Physics understanding
• Minimal complexity	• Technology level

Figure 2

depends not only on how far the public (and the government) agrees with our objectives, but also on their perception of what we have actually accomplished and what we are doing about relieving the other three constraints.

In attempting to organize our thinking about fusion research there is also another way we could chart its progress. That way is to think of the research and development as a progression in time in a three dimensional space the coordinates of which define three critical elements. Figure 3 shows how we might view fusion progress according to this way of thinking about it. In this space a straight line drawn from the origin to the end point - practical fusion - would be expected to be the optimum path. Falling behind in progress in any one of the three coordinates would then signal a less-than-optimum approach to the goal. This way of looking at the problem even though it is clearly simplistic and non-quantitative, helps me to visualize both the tightly coupled nature of our research and the need to maintain breadth in our march toward the fusion goal.

Coming back to a more specific definition of fusion power as a technological system, I think it is worthwhile at this point to list the fusion options. It has long seemed to me that fusion stands alone among all sources of energy in the breadth of the options that it can offer,

COORDINATES OF FUSION RESEARCH AND DEVELOPMENT

- Physics
- Technology/engineering
- Economics/environmental constraints
- Time

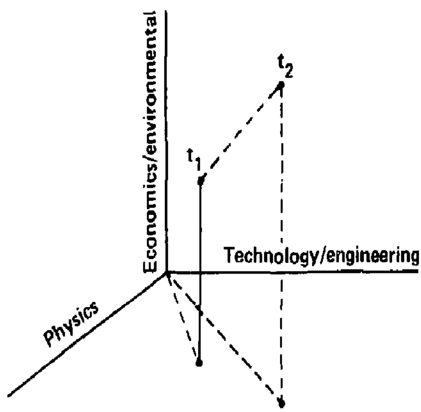

Figure 3

given the full development of its potentialities.

Figure 4 lists some of the options that I believe will be possible for fusion power plants when they are finally developed to their full potential. Within these options there will probably be particular fusion systems that are optimal for given situations as defined by economic and/or environmental considerations. For example, as has often been pointed out, the possibility of using advanced fuel cycles that minimize the radioactive inventory or that reduce the first-wall and blanket problems, or that permit direct electrical conversion, may turn out to be one of the most important attributes of fusion power in the long run.

FUSION OPTIONS

- Spectrum of unit sizes
- Modular or non-modular systems
- Fuel cycles
- Energy conversion technique
- Magnetic or inertial approach
- Electrical and/or thermal-chemical output

Figure 4

Assuming that we all agree that fusion power is a highly desirable objective, and that it possesses many options, we are still presented with the problem of making our individual and our collective choices concerning how to go about achieving the fusion goal.

It is at this point that the question of the inherent potentialities of the particular system that we propose to investigate becomes crucial. One criterion that I happen to favor very much is that of system versatility. That is to say, is the approach sufficiently versatile and adaptable to permit the optimization of its performance in terms of the set of requirements that it will have to meet arising from physics, technology and engineering, and economic and environmental considerations? It should come as no surprise to you that I believe that open-ended magnetic fusion systems that employ the magnetic mirror principle - in one or more of the many ways that this is possible - offer the highest degree of versatility of all magnetic fusion systems. All of us are at the same time aware of the shortcomings of mirror systems in their long fight to overcome the Q problem.

Since my talk is concerned with how physics and innovation can couple constructively to the goal of economic fusion power, I thought a cartoon might illustrate the situation as I sometimes view it.

Figure 5 contrasts two different paths to fusion - here named "Concept A" and Concept B". As you can see, Concept A is proceeding steadily up an even slope with the goal "fusion power" clearly in view. What he doesn't see is the "economics crevasse" - which might prove to be impassable. On the other side of the mountain, Concept B not only has had rough terrain to cross but also has come up against "Q cliff" - from the base of which he can't even see the fusion power gem. But suddenly there appears an idea based on new physics understanding. This new idea sends him high up in the air, and he then parachutes directly onto the goal. I sometimes wish that fusion were that easy - it isn't - but again my point is that it is entirely possible that the most obvious route to fusion is not necessarily the shortest or best one, when we consider all factors, particularly the element of innovation based on a firm physics understanding.

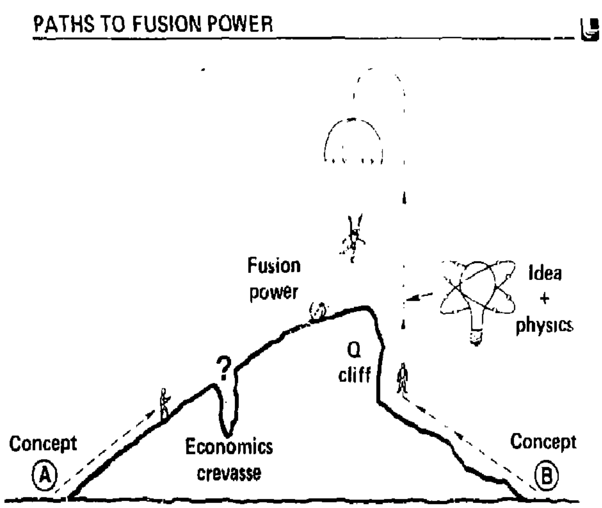

Figure 5

COMPONENT ELEMENTS OF MIRROR-BASED FUSION SYSTEMS

- Magnet coils and structure*
- Blanket*
- Injectors*
- R.F. and microwave power*
- Direct converter/plasma dump*
- Heat exchangers
- Vacuum and cryogenic systems
- Controls

} Required to be $\leq$ 50% of plant capital cost

+ Balance of plant (turbines, buildings, land, etc.)

*Cost reducible by physics/innovation

Figure 6

I would like first to consider those elements of mirror systems where physics and innovation can make a difference. Figure 6 lists the relevant elements of such a system as we now understand them. I have indicated with asterisks those elements where it appears to me that physics and innovation have a chance to make a major positive impact on the economic: - that is on capital cost or on reliability, or on equipment lifetime or on environmental compatibility. If we are considering the magnetic fusion research effort in general, and mirror research in particular, these four items are the key factors that we must take into account as we progress toward the goal of practical fusion.

In the remainder of my talk I will use two examples picked to illustrate what I am trying to convey. Please remember that I said at the beginning of my talk that I would not be so presumptuous as to try to give you answers to all the hard questions ahead, but rather to see if there are some insights that will help us to decide <u>which</u> of these questions are the most important to consider.

My first example resulted from a question that I asked an engineer from a large U.S. electric utility. My question to him concerned the potential economic benefits that an electrical utility system might gain from what I will call a "modular fusion power plant." By that I mean a power plant in which the non-fusion part of the plant - that is the steam turbines and the generators - derive their energy input from a paralleled set of fusion modules each of which is independent of the other modules. By independent I mean that if there are N modules each module supplies the fraction 1/N of the total energy input to the turbines, and each module is capable of being operated or shut down and maintained independent of the status of any of the other modules.

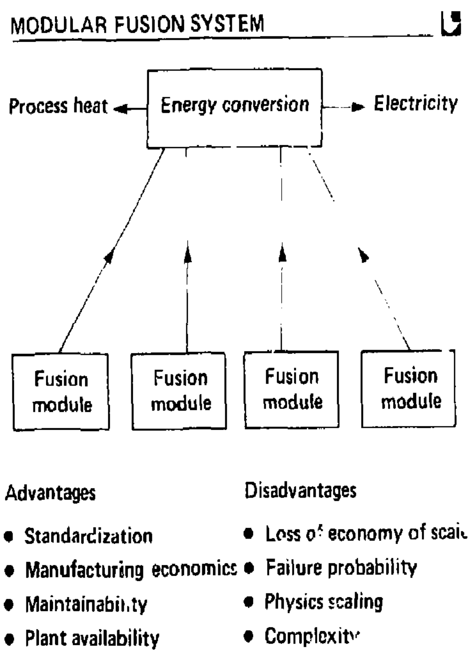

Figure 7

The fusion system that I am talking about is illustrated schematically in Fig. 7. To take a more specific example, there might be, say, 10 modules consisting of small Field-Reversed Mirror cells, each of which produces 150 megawatts of thermal energy. The total plant output would therefore be of order 500 megawatts of electricity, assuming a net plant conversion efficiency of 33 percent.

Some of the advantages and disadvantages of such a modular fusion system are listed on the figure. You will notice that in several cases a disadvantage can in principle be offset by an advantage. For example, standardization and manufacturing economies can offset the loss of economy of scale; or increased maintainability can offset the increased failure probability that comes from having several modules instead of just one.

The answer that I got to the question that I asked of the utility engineer in my attempt to quantify the benefits to a utility network of having a modular fusion power plant went somewhat as follows: First, for the increased plant availability that would result from the fact that the total plant could still be operated (at reduced capacity) with one or more of its modules shut down, a utility would probably be willing to pay a premium which he estimated to be about a 10 percent increase in the capital cost, relative to the non-modular plant. The advantage to them of the modular plant is that in the case of a non-modular plant, failure of the fusion part of the plant would result in the loss of the entire output of the plant. With a modular plant they would therefore not have to have as many additional plants in reserve somewhere to take up the large loss of power.

Second, for the economic advantage that they would gain by being able to schedule routine maintenance operations on a rotating basis, still keeping the main plant in operation, they might be willing to pay an additional 5 to 10% capital cost premium.

Third, for the standardization of maintenance procedures that would result from having a modular system, they might pay an additional premiun of 5 percent or so. Therefore a modular fusion power plant might be allowed to cost as much as 20 to 25 percent more than a non-modular plant and still be preferred by the utilities. The conclusion is that modular fusion systems present an opportunity for physics and innovation to produce an economic benefit. There would be additional advantages as well: those that would result from developing fusion modules smaller in size and lower in output than the sizes that have usually been considered. These advantages have to do with the benefits of achieving an earlier demonstration of fusion. At the Livermore Laboratory we have in fact already done a design study for the Electric Power Research Institute of a small Field-Reversed Mirror module that could become an example of such a development.

In trying further to quantify the idea of a modular system I asked myself the following question: If the design of a small fusion module necessarily resulted, because of the physics scaling laws, in reducing the Q value of the small module below that of a larger unit, how low could Q be allowed to be without raising the cost of the entire plant beyond, say, the 20 percent premium that I said the utilities might be willing to pay for modularity?

Reducing Q results directly in increasing the capital cost of two parts of the plant: First, the energy conversion system (for example the steam turbines and generators) which must supply additional recirculated power. Second, the plasma injection and heating system that uses that recirculated power to maintain the plasma density and temperature. Using results from the studies of Tandem Mirror power plants carried out at Livermore by Moir and Carlson and others,[1] I estimated the relative costs of thermal conversion and plasma heating and injection for a fusion power plant for which the Q value was 15. This was my "base case" for the comparison. It was then not difficult to derive what the increase in capital cost would be if Q was decreased, assuming comparable costs per watt for the components of each system. Figure 8 illustrates the results.* The cost comparison

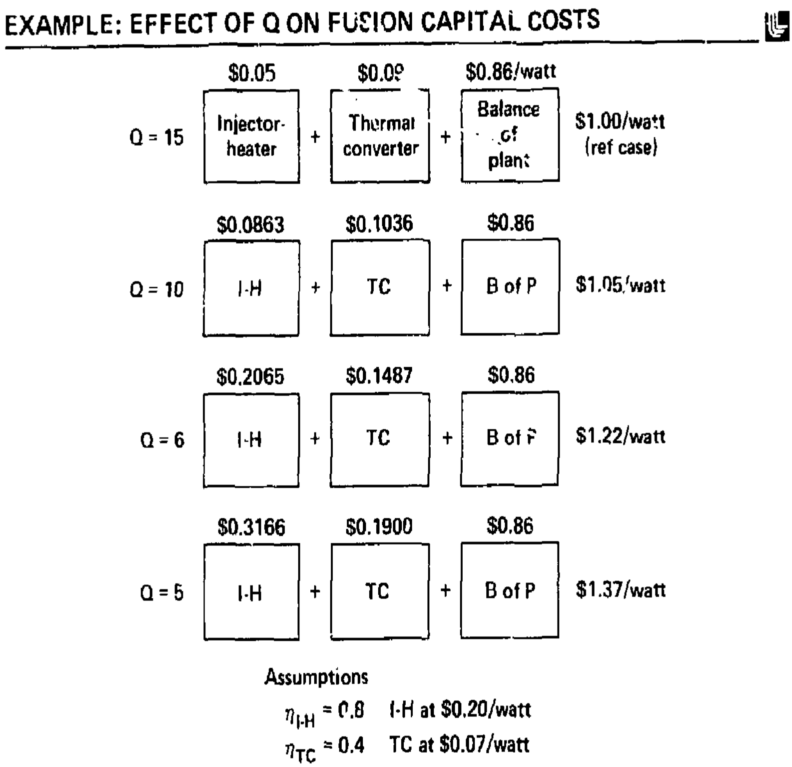

Figure 8

figures shown should be considered as being merely indicative, rather than as precise estimates. As you can see, however, based on reasonable cost estimates per electrical watt handled, the Q of the modules might be as low as, say, 6 without thereby increasing the capital cost of the plant as a

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* See Appendix I for the method of calculation.

whole by more than about 20 percent - which is the premium we said a utility might pay for modularity. Therefore, even if the confinement physics scaling laws dictate that a small fusion module could only achieve a Q value substantially lower than that of a single large fusion cell, the smaller unit might still be preferred.

The second example that I will use to illustrate the effect that new physics knowledge coupled with innovation could have on the economics of mirror fusion systems is based on the economic advantages of using a long axially-symmetric solenoidal magnetic field combined with the engineering, economic and environmental advantages that could result from the use of the D-D-Helium 3 fuel cycle. Again my example will leave unanswered many important physics questions. However, I hope the example will help to define areas where there exists the opportunity for substantial gains - if the physics issues can be resolved.

Figure 9 lists the two major elements of my example: The D-D-Helium 3

COUPLING BETWEEN PHYSICS/INNOVATION AND FUSION ECONOMICS

Example case:

- D-D-^{3}He Fuel cycle
- Force-free superconducting solenoid for confinement

Potential economic/environmental gains

- Fuel cycle:
  - No tritium breeding required, simplified blanket
  - Reduced first-wall problems
  - Lowered radioactive inventory
  - Suitable for direct conversion
- Magnet:
  - Higher fields
  - Lowered conductor cost
  - Lowered structure cost
  - Increased bore permits simplified construction and maintenance of interior structure

Figure 9

fuel cycle and the use of a so-called force-free superconducting solenoid to produce the main component of the confining field.

The figure also lists the potential economic, engineering and environmental gains that could result from the use of the D-D-Helium 3 cycle (that is D-D reactions with recycling of the reaction products). The advantages that could come from producing the main component of the confining field by means of a force-free solenoid are also listed. I will discuss them in more detail after I outline the fuel cycle and plasma issues.

The open-ended solenoid geometry that I am discussing will require the resolution of two key issues where new physics knowledge and, probably, innovation will be required. Figure 10 lists these two issues: end loss control and high beta MHD stability - together with some options that might

COUPLING BETWEEN PHYSICS/INNOVATION AND FUSION ECONOMICS

Example case, cont:

Physics/innovation requirements

- End loss control
  - Options: Tandem mirror concepts
    R.F. plugging
    Field-reversed mirror
    Multiple mirrors
    X_1, Y_1, Z_1, etc.
- High beta MHD stability
  - Options: Average Min-B
    Electron rings
    Field reversal
    Finite orbit effects
    Feedback stabilization
    X_2, Y_2, Z_2, etc.

Figure 10

provide workable answers. That is, to exploit the gains that could result from the fuel cycle and from the solenoid design first, sufficient control of the end losses must be achieved to attain the containment required by the fuel cycle and MHD stable high beta equilibria will be needed to produce the required fusion power densities. These two requirements are specifically issues of physics and/or innovation.

We can quantify these issues by calculating three relevant Q factors for the fuel cycle.* As shown in Fig. 11, these are: First, the conventional Q value, the one that is defined in terms of confinement time

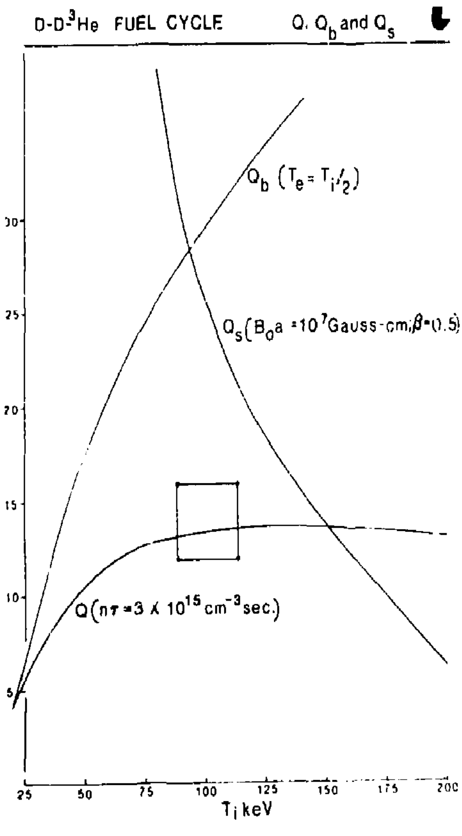

Figure 11

* See Appendix II for additional details.

- ratio of fusion power generated to heating power input to the plasma required to overcome particle losses. Second, there is what I have called Q_s, the ratio of fusion power to synchrotron radiation losses. Third, there is the Q value relative to bremsstrahlung losses - which I call Q_b. In calculating these Q values I counted in the fusion power the extra several MeV's of energy yield that would result when the neutrons from the fusion reactions are captured in sodium or aluminum. Both of these are 100 percent isotopes for which the neutron activation products have a very short half-life. In the calculation of synchrotron losses, which scale at a given beta value as the square root of the product of magnetic field and plasma radius, for my example I have chosen that product to be 10^7 gauss-cm. I have also made the worst-case assumption of zero reflection of the synchrotron radiation by the chamber wall.

As can be seen from Fig. 11, provided $n\tau$ confinement factors of order 3 X 10^{15} cm^{-3} sec can be achieved there is a near-optimum ion temperature of about 100 keV where all of the Q values should be sufficiently high to satisfy economic power balance requirements.

I will now turn to the question of the solenoid itself. Since high magnetic fields are advantageous in maximizing the Q value for synchrotron losses and to give an economically acceptable fusion power density, minimizing the cost of the high field solenoid becomes a very important issue. In fact lowering the solenoid cost sufficiently might permit the use of a solenoid with an oversize inner bore. This would in turn permit locating auxiliary coils and other equipment entirely within it without geometric interferences. This possibility could greatly simplify both construction and accessibility for maintenance.

My suggestion for a possible way to reduce the cost of a large bore superconducting solenoid is based on an old idea - the force-free

FORCE-FREE SUPERCONDUCTING SOLENOID

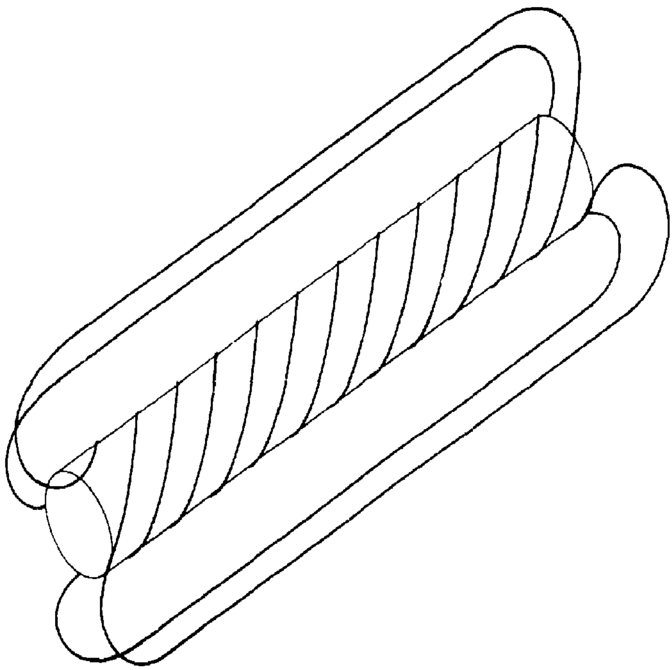

Figure 12

solenoid. This idea was discussed by Harold Furth and others over 20 years ago.[2] In a force-free solenoid the windings are arranged to lie as nearly as possible parallel to the local direction of the magnetic field lines. Figure 12 shows a simple example of such a solenoid - one with a single winding layer. Here the winding pitch angle is 45 degrees, which produces an purely axial field inside the solenoid and a purely poloidal field of the same intensity on the outer side of the winding. Return currents are carried back in the longitudinal direction, and at a larger radius so that they cancel the poloidal field, but at a much reduced value.

Figure 13 Photograph of short force-free solenoid constructed at Livermore.

Figure 13 is a photograph of a short force-free solenoid tested by Furth and Birdsall at Livermore about 20 years ago. Coils like this one, including ones with totally unsupported windings, were made and tested by them up to fields of 200 kilogauss without mechanical failure.

If a large long solenoid of force-free design is constructed using superconducting windings there should result two major cost savings relative to the cost of a conventionally designed superconducting solenoid of the same bore and length:

First, the amount of support structure needed should be greatly reduced. The solenoid windings themselves should need only enough support to keep them located mechanically. The return conductors, both because they are in a weaker field, and because they are longitudinally directed, should require a minimum of support. Their support structure also need not be as rigid as when circular windings must be supported. When circular windings are used the structure must be rigid enough to prevent the windings from stretching beyond their critical strain values of about 3/10 percent. However, when the conductors are aligned longitudinally this requirement should be much easier to satisfy.

The second economic gain from the use of force-free windings can be expected to come from the properties of superconductors themselves. It turns out that a magnetic field component parallel to the current flow direction does not degrade the critical current density as is the case for magnetic field perpendicular to the current direction. In fact it may actually increase the critical current. This effect was shown, for example, in 1971 at Livermore in some unpublished tests on niobium-titanium conductors carried out by E. M. Jones in connection with design of the superconducting Levitron. In these tests it was found that a reduction in the perpendicular component of magnetic field always increased the critical current, even when the total intensity of the magnetic field remained constant. For example, at 50 kilogauss applied field, changing the orientation of the conductor from perpendicular to the field to an angle of 26 degrees, where B parallel was twice B perpendicular, increased the critical current by a factor 3. Furthermore under these conditions at 50 kilogauss total field the critical current was still twice as high as it would have been in a perpendicular field of 25 kilogauss. I am not aware of a similar direct demonstration of this effect for niobium-tin, but the marked effect of reducing the perpendicular component of B in a Niobium-Tin superconductor of the type we have developed for our high field magnet studies at Livermore is shown in Fig. 14.[3] Using these data as a guide I estimate that for a 120 kilogauss solenoid, for example, the conductor in a force-free winding should be able to carry at least 10 times more current without quenching than it would carry if it were used in a conventional superconducting solenoid where B is perpendicular to the winding direction.

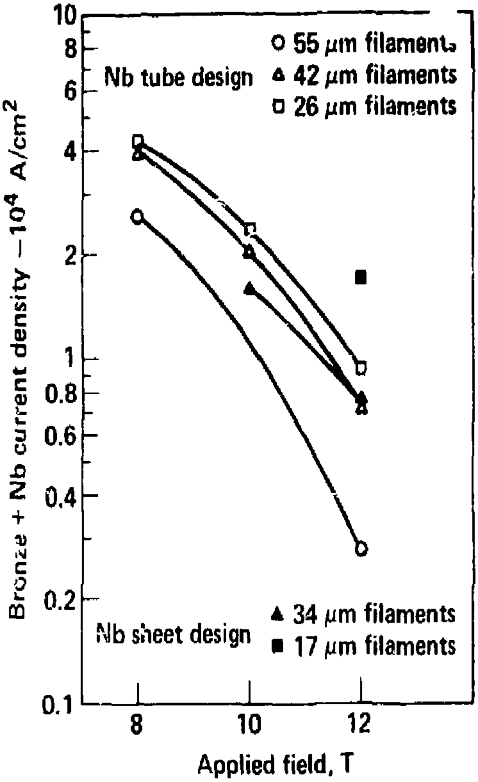

Figure 14 Critical current in multi-filament Niobium-Tin superconductor as a function of applied transverse magnetic field.

I have made some approximate cost comparisons between force-free and conventional coil design based on the estimated cost savings in structure and savings from increased critical current for a 120 kilogauss solenoid. The results are shown on Fig. 15. If my estimates are valid it appears that there could be as much as one order of magnitude decrease in cost for the force-free design as compared to a conventional solenoid of the same bore. Alternatively, a much larger bore force-free solenoid could be constructed at a comparable or even a lower cost than that of a conventional solenoid of much smaller bore.

EXAMPLE: ESTIMATED COST REDUCTION FROM FORCE-FREE SOLENOID

	Conventional solenoid (ref case; 120 kilogauss)	Force-free solenoid
Conductor at \$0.0025/AM	2×10^6 \$/meter	4×10^5 \$/meter
Support structure at \$22/kG	6×10^6 \$/meter	5×10^5 \$/meter
	8×10^6 \$/meter	0.9×10^6 \$/meter

$$\frac{\text{Cost F-F}}{\text{Cost conven.}} = \frac{0.9 \times 10^6}{8 \times 10^6} \approx 0.113$$

Figure 15

To summarize, I have attempted to illustrate the two-way coupling that exists between, on the one hand, the economic prospects for mirror fusion systems and physics and innovation on the other hand. Because of the versatility of open-ended systems and because the physics understanding of mirrors is increasing rapidly it seems to me that there are now major opportunities to move ahead rapidly toward the realization of mirror fusion power systems, given a vigorous research effort. Under these circumstances I think we know what we should do: "Tetsu Wa Atsui Uchi Ni Ute!" (Hit while the iron is hot!).

Appendix I

Estimated Effect of Q on Fusion Plant Capital Costs

Consider a fusion system where recirculated power is required to heat and maintain the plasma. If this power is converted thermally from fusion energy to electricity with efficiency η_{TC} and is utilized for injection/heating at efficiency η_{I-H} then for Q units of fusion power:

$$P_{net} = \eta_{TC} Q - \frac{1}{\eta_{I-H}} \quad \text{net electrical power}$$

$$P_{recirculated} = \frac{1}{\eta_{I-H}}$$

$$\frac{P_r}{P_{net}} = \left[\eta_{I-H} \eta_{TC} \, Q - 1 \right]^{-1}$$

For ref. case (Q = 15; η_{I-H} = 0.8 at \$0.20/watt ele.; η_{TC} = 0.4 at \$0.07/watt elec.),

we have

P_r = 0.26 P_{net} = \$0.05/watt for injection/heating

= \$0.09/watt for thermal converter

Total, including balance of plant at \$0.86/watt = \$1.00/watt

At Q = 6 these figures become:

P_r = 1.087 P_{net} = \$0.21/watt for injection-heating

= \$0.015/watt for thermal converter

Total, including B. of P. at \$0.86/watt = \$1.22/watt.

Appendix II

Calculation of Q and Q_s for D-D-^3H_e Reactions

A. D-D-^3H_e Reaction Rate Parameters

We have in steady-state,[4]

$$p_f = \frac{1}{4} n_1^2 \langle\sigma v\rangle_{11} W_{11p} + \frac{1}{4} n_1^2 \langle\sigma v\rangle_{11} (W_{11n} + W_c)$$

$$+ n_1 n_2 \langle\sigma v\rangle_{12} (W_{12} + W_c) + n_1 n_3 \langle\sigma v\rangle_{13} W_{13}$$

1 = deuteron

2 = triton

3 = Helium-3

W_{11p} = 3.25 MeV; W_{11n} = 4.0 MeV; W_{12} = 17.6 MeV; W_{13} = 18.3 MeV

W_c ≈ 8-9 MeV for average neutron capture energy deposited in Na or Al

$$n_3 = \frac{1}{4} n_1 \frac{\langle\sigma v\rangle_{11}}{\langle\sigma v\rangle_{13}} \qquad n_2 = \frac{1}{4} n_1 \frac{\langle\sigma v\rangle_{11}}{\langle\sigma v\rangle_{12}}, \quad n = n_1 + n_2 + n_3$$

in equilibrium (reinjection of T and ^{3}He reaction products) substituting,

$$p_f = \frac{1}{4} n^2 \overline{\langle\sigma v\rangle}_{11} \left[W_{11n} + W_{11n} + W_{12} + W_{13} + 2W_c\right]$$

$$= \frac{1}{4} n^2 \overline{\langle\sigma v\rangle}_{11} \left[60 \text{ MeV}\right]$$

$$\overline{\langle\sigma v\rangle}_{11} = \overline{\langle\sigma v\rangle}_{11} \left\{1 + \frac{1}{4}\left[\frac{\langle\sigma v\rangle_{11}}{\langle\sigma v\rangle_{12}} + \frac{\langle\sigma v\rangle_{11}}{\langle\sigma v\rangle_{13}}\right]\right\}^{-2}$$

or

$$P_f = 77.7 \quad \beta_i^2 \; \frac{B_o^4 \, W_n \, a^2 \langle\overline{\sigma v}\rangle_{11}}{T_i^2} \text{ watts/cm}$$

T_i	$\langle\overline{\sigma v}\rangle_{11}$
20 keV	2.7×10^{-18} cm^3 sec^{-1}
40 keV	1.2×10^{-17}
60 keV	2.4×10^{-17}
80 keV	3.4×10^{-17}
100 keV	4.4×10^{-17}
200 keV	8.8×10^{-17}

B. Calculation of Q

$$Q = \frac{\frac{1}{4} n^2 \langle\overline{\sigma v}\rangle_{11} W_n}{I \, W_i}$$

$$I = \frac{n}{\tau_i} \qquad \overline{W}_i = \frac{3}{2} T_i$$

$$Q = 1.67 \times 10^2 \, (n \, \tau_i) \, \langle\overline{\sigma v}\rangle_{11} \, \frac{W_n}{T_i}$$

T_i	Q ($n\tau = 3 \times 10^{15}$ cm^{-3} sec)
20	4.1
40	9.3
60	12.0
80	12.9
100	13.2
200	13.2

C. Calculation of synchrotron radiation and Q_s

Based on work of Trubnikov and Drummond and Rosenbluth as used in calculations of Post[4],

$$p_s = \frac{1}{6\pi} \left(\frac{e}{mc}\right)^3 \frac{1}{c^2} (m^*)^3 \, kT_e \, B_o^3 \, a \quad \text{ergs sec}^{-1} \text{ cm}^{-1}$$

$$(m^*)^3 \approx 0.5 \, (\overline{\beta}_e \, B_o \, a)^{1/2}$$

$$p_s = 2.6 \times 10^{-17} \, \overline{\beta}_e^{\,1/2} \, B_o^{7/2} \, a^{3/2} \, T_e \quad \text{watts/cm}$$

Define

$$Q_s = \frac{p_f}{p_s}$$

From relations above

$$Q_s = 3.0 \times 10^{18} \, \frac{\beta_i^{3/2} \, (B_o a)^{1/2} \, W_n \, \overline{\langle\sigma v\rangle}_{11}}{T_e^{3/2} \, T_i^{3/2}}$$

Take $T_e \approx \frac{1}{2} T_i$; $\beta_i = 0.5$

$$Q_s = 1.8 \times 10^{20} \, (B_o a)^{1/2} \, \frac{\overline{\langle\sigma v\rangle}_{11}}{T_i^3} \; .$$

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