# Rare Decays and CP Asymmetries in Charged B Decays 

N.G. Deshpande<br>Institute of Theoretical Science<br>University of Oregon<br>Eugene, OR 97403


#### Abstract

The theory of loop induced rare decays and the rate asymmetry due to CP violation in charged B Decays is reviewed. After considering $b \rightarrow s \gamma$ and $b \rightarrow$ $s e^{+} e^{-}$decays, the asymmetries for pure penguin process are estimated first. A larger asymmetry can result in those modes where a tree diagram and a penguin diagram interfere, however these estimates are necessarily model dependent. Estimates of Cabbibo suppressed penguins are also considered.


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## 1 Introduction

Present and future B factories offer a unique opportunity to study rare decays and CP violation. Both these aspects of the standard model are sensitive to Physics beyond the standard model. The neutral B system, which involves $B \bar{B}$ mixing, has received much attention because it offers a clean method [1] of measuring CP violation with little hadronic uncertainty. The charged modes on the other hand are easily accessible, and as we shall see, offer an opportunity to measure direct CP violation in contrast with the CP violation in the neutral system which arises from the $B \bar{B}$ mass matrix analogus to CP violation parameter $\varepsilon$ in the $K$-system.

## 2 Rare B Decays

Theoretically the decays of $b$ quark that are forbidden at the tree level, but can proceed only at one loop in electroweak theory, are very interesting because they provide a window to contributions from heavy particles that might be present in the loop diagrams. Two such processes are $b \rightarrow s \gamma$ and $b \rightarrow s e^{+} e^{-}$. The first has weak sensitivity to top quark mass while the second has increased sensitivity although the rate is rather suppressed. The process $b \rightarrow s \gamma$ is enhanced through QCD corrections and the rate is given by $\Gamma(b \rightarrow s \gamma)=G^{2} m_{b}^{5} / 16 \pi$ where the electromagnetic vertex is $J_{\mu}=i G m_{b} \bar{s} \sigma_{\mu \nu} q^{\nu} b_{R}$ and G including QCD corrections was calculated in Ref.[2]. The process that will signal this transition is $B \rightarrow K^{*} \gamma$. Estimates for this mode vary but a reasonable estimate is $10 \pm 4 \%[3]$ of the inclusive mode. We plot the dependence of both these processes as a function of $m_{t}$ in figure 1 , where we have assumed the conservative ratio of $7 \%$ for the exclusive to inclusive mode. In $b \rightarrow s e^{+} e^{-}$decay,both the QCD corrections and the resonance background have been calculated. We present below[4] the branching ratio as a function as a function of $m_{t}$.

## 3 Overview of the Theory of CP Violation in $B^{ \pm}$ Decays

The basic theory follows from the work of Bander, Silverman and Soni[5]. The three conditions for asymmetry to arise are:

1. Two amplitudes with different Kobayashi-Maskawa phases must contribute to the same process.


Figure 1: Br for $b \rightarrow s \gamma$ and $B \rightarrow K^{*} \gamma$ as functions of $m_{t}$


Figure 2: Br of $b \rightarrow s e^{+} e^{-}$in units of $10^{-6}$ as functions of $m_{t}$


Figure 3: Diagrams for $b \rightarrow s u \bar{u}$
2. There should be a complex phase in the Kobayashi-Maskawa matrix.
3. At least one of the amplitudes must have an absorptive part. (This is some times referred to as final state interaction, though I find this terminology misleading)
The easiest way to generate an absorptive part, at least at the quark level is to consider one loop diagrams involving a gluon or a photon, usually referred to as penguin diagrams. The loop diagram generates an absorptive part when the virtual gluon four-momentum $q^{2}$ is greater than $4 m_{q}^{2}$ where $m_{q}$ is mass of the intermediate quark. For b quark decay, the intermediate quarks are $u, c, t$, so that $q^{2}>4 m_{u}^{2}$ or $4 m_{c}^{2}$ generates the absorptive part. The two amplitudes necessary for the effect can be a tree diagram and a penguin diagram, or if the tree diagram is absent, two penguin diagrams with different quark intermediate states suffice. The asymmetries are easiest to calculate at the quark level, but since the measurements will be made with a definite hadronic mode, the question of the reliability of such a calculation has to answered. We shall comment on this point in the conclusions.

Let us review the calculation with $b \rightarrow u \bar{u} s$ as a definite quark channel for illustrative purposes. The following diagrams contribute to this process:

We can write the amplitude for this process as

$$
\begin{equation*}
M(b \rightarrow u \bar{u} s)=V_{u} A_{u}+\left(\alpha_{s} / \pi\right) \sum_{i} V_{i} P_{i} \tag{1}
\end{equation*}
$$

where i runs over $u, c, t$ and $V_{i}$ is defined in terms of $\mathrm{K}-\mathrm{M}$ angles as

$$
\begin{equation*}
V_{i}=U_{b i} U_{s i}^{*} \tag{2}
\end{equation*}
$$

Unitarity requires that

$$
\begin{equation*}
\sum_{i} V_{i}=0 \tag{3}
\end{equation*}
$$

$A_{u}$ refers to the contribution from the tree diagram, while $P_{i}$ refers to the contribution from the penguin diagram. We have factored ( $\alpha_{s} / \pi$ ) so that the remaining penguin operator contributes roughly the same as the tree operator. Using unitarity we can remove $V_{t}$ dependence, and we have

$$
\begin{equation*}
M=V_{u}\left(A_{u}+\left(\alpha_{s} / \pi\right) P_{u-t}\right)+\left(\alpha_{s} / \pi\right) V_{c} P_{c-t} \tag{4}
\end{equation*}
$$

We now define $\bar{M}$ as the amplitude for $\bar{b} \rightarrow \bar{u} u \bar{s}$. This is given by

$$
\begin{equation*}
\bar{M}=V_{t b}^{*}\left(A_{u}+\left(\alpha_{s} / \pi\right) P_{u-t}\right)+\left(\alpha_{s} / \pi\right) V_{c}^{*} P_{c-t} \tag{5}
\end{equation*}
$$

The asymmetry is then given by

$$
\begin{equation*}
A=\frac{|M|^{2}-|\bar{M}|^{2}}{|M|^{2}+|\bar{M}|^{2}} \tag{6}
\end{equation*}
$$

In the denominator we may safely assume that $\left(\alpha_{s} / \pi\right) V_{c} \gg V_{u}$, and obtain

$$
\begin{equation*}
A=\frac{2 \operatorname{Im}\left(V_{u} V_{c}^{*}\right) \operatorname{Im}\left[\left(A_{u}^{*}+\left(\alpha_{s} / \pi\right) P_{u-t}^{*}\right)\left(\alpha_{s} / \pi\right) P_{c-t}\right]}{\left|V_{c}\right|^{2}\left(\alpha_{s} / \pi\right)^{2}\left|P_{c-t}\right|^{2}} \tag{7}
\end{equation*}
$$

This expression is clearly zero unless the three conditions we mentioned abure are satisfied. The maximum asymmetry occurs when tree amplitude is present and $P_{c-t}$ has absorptive part, which requires $q^{2}>4 m_{c}^{2}$. If $q^{2}<4 m_{c}^{2}$, the formula generates absorptive part due to $\left(\alpha_{s} / \pi\right)^{2}$ contribution, however, to this order there are other contributions which must be evaluated [6]. In pure penguin process, like $b \rightarrow s \bar{s} s$, we have the expression

$$
\begin{equation*}
A=\frac{2 \operatorname{Im}\left[V_{u} V_{c}^{*}\right] \operatorname{Im}\left[P_{u-t}^{*} P_{i-t}\right]}{\left|V_{c}\right|^{2} P_{c-t}^{2}} \tag{8}
\end{equation*}
$$

The asymmetry is smaller, but the estimates are less model dependent in this case.

## 4 Pure Penguin Processes

These processes are much cleaner because although the rate for a given mode is dependent on hadronic wave functions, the asymmetries are not. Let us first consider the decay $b \rightarrow s \phi$ which arises at the quark level from $b \rightarrow s \bar{s} s$. We can isolate this quark process by looking at $B \rightarrow K \phi$ or $K^{*} \phi$. The quark level lagrangian is given by [7].

$$
\begin{equation*}
L=\kappa I\left(k^{2}\right) \bar{s} \lambda^{a} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{s} \lambda^{a} \gamma^{\mu} s \tag{9}
\end{equation*}
$$



Figure 4: The process $b \rightarrow s \phi$
where $\kappa=\left(G_{F} / \sqrt{2}\right)\left(\alpha_{s} / 8 \pi\right)$ and

$$
\begin{gather*}
I\left(k^{2}\right)=\sum_{i} V_{i} F_{i}\left(k^{2}\right)  \tag{10}\\
F_{i}\left(k^{2}\right)=-\int_{0}^{1} d y \int_{0}^{1-y} d x \frac{\left[2 y(x-1)+4 x(1-x)+\widehat{m}_{i}^{2}(x y+2 x(x-1))\right]}{y+\hat{k}^{2} x(x-1)+\widehat{m}_{i}^{2}(1-y)} \tag{11}
\end{gather*}
$$

where $\hat{k}^{2}=k^{2} / M_{W}^{2}, \widehat{m}_{i}^{2}=m_{i}^{2} / M_{W}^{2}$, and $\mathrm{i}=\mathrm{u}, \mathrm{c}, \mathrm{t}$. The asymmetry for the process $b \rightarrow s \phi$ can be calculated provided we know what value of $k^{2}$ to use. We use Fig. 2 to illustrate the process and argue from the fact that $\phi$ can be only formed from $s$ quark coming from b and $\bar{s}$ coming from the gluon, since $\phi$ is in a color singlet state. Simple kinematics yields $k^{2}=\left(m_{b}^{2}-m_{s}^{2}\right) / 2 \approx 12(G e v)^{2}$. We can now evaluate the dispersive and the absorptive part of the penguin diagrams. For $\widehat{m}_{i} \ll 1$, we have

$$
\begin{gather*}
\operatorname{Re} F_{1}\left(k^{2}\right)=-4 \int_{0}^{1} d x x(1-x) \ell n\left|\widehat{m}_{i}^{2}-\hat{k}^{2} x(1-x)\right|  \tag{12}\\
I m F_{1}\left(k^{2}\right)=-(\pi / 2) \theta\left(k^{2}-4 M_{i}^{2}\right)\left(b_{+}^{2}-b_{-}^{2}+b_{-}^{3} / 3-b_{+}^{3} / 3\right) \tag{13}
\end{gather*}
$$

where $b_{ \pm}=1 \pm\left(1-4 m^{2} / k^{2}\right)^{1 / 2}$. We now find:

| Quantity | Value |
| :---: | :---: |
| $\operatorname{Re} F_{1}^{u}$ | 5.3 |
| $\operatorname{Im} F_{1}^{u}$ | -2.1 |
| $\operatorname{Re} F_{1}^{c}$ | 6.4 |
| $\operatorname{Im} F_{1}^{c}$ | -1.4 |
| $F_{1}^{t}$ | .71 |

where we have chosen $m_{t} \approx M_{W}$. We find for the asymmetry

$$
\begin{equation*}
A=\frac{2 \operatorname{Im}\left[V_{u} V_{c}^{*}\right](5 / 33)}{\left|V_{c}\right|^{2}} \tag{14}
\end{equation*}
$$

If we use Wolfenstein parametrization,

$$
U=\left(\begin{array}{ccc}
1 & \lambda & A \lambda^{3}(\rho-i \eta)  \tag{15}\\
-\lambda & 1 & A \lambda^{3} \\
A \lambda^{3}(1-\rho-i \eta) & -A \lambda^{2} & 1
\end{array}\right)
$$

then

$$
\begin{equation*}
A=\lambda^{2} \eta(5 / 33)=\eta \times 10^{-2} \tag{16}
\end{equation*}
$$

the value of $\epsilon, B-\bar{B}$ mixing, $m_{t}$ and $U_{b u}$ imply $A=1.0 \pm 0.1$ and $\eta>0.1$ The asymmetry is therefore $\geq 0.1 \%$. If we had calculated the asymmetry of the Cabibbo suppressed penguin $b \rightarrow d s \bar{s}$ process, the asymmetry would be of order $\eta$ [8]. However the branching ratio of any exclusive mode like $B \rightarrow K \bar{K}$ from this process is suppressed and is expected to be of the order of $10^{-7}$. The exclusive modes $B^{-} \rightarrow K^{-} \phi$ or $B^{-} \rightarrow K^{*-} \phi$ are expected to have a branching ratio of order $10^{-5}$ [7] and the asymmetry equal to the inclusive process $b \rightarrow s \phi$.

## 5 Tree-Penguin Interference

The largest observable asymmetry is expected to arise in processes where tree diagram and penguin contribute approximately the same amount. At the quark level the process is $b \rightarrow s \bar{u} u$, and the asymmetry is given by (7), which can be written as

$$
\begin{equation*}
A=\eta\left(A_{u} /\left|P_{c-t}\right|\right)\left(P_{c-t}^{A} /\left|P_{c-t}\right|\right) \tag{17}
\end{equation*}
$$

We need the ratio of tree and penguin contributions, as well as the absorptive to dispersive ratio. The former can be calculated in from the lagrangian if factorization approximation is used. A large number of processes have been evaluated in [9] using this approximation. We present below the best modes with the corresponding ratio $R=A_{u} /\left|P_{c-t}\right|$. To calculate the absorptive part we need to know the value of $q^{2}$ to be used. If the argumert used in the previous section can be applied, $12(\mathrm{GeV})^{2}$ might be suitable. The value of $P_{c-t}^{A} /\left|P_{c-t}\right| \approx 0.25$ in this case.

| Mode | $R=A_{u} /\left\|P_{c-t}\right\|$ |
| :--- | :---: |
| $B^{-} \rightarrow K^{-} \pi^{\circ}$ | 1.1 |
| $B^{-} \rightarrow K^{*-} \pi^{\circ}$ | 1.4 |
| $B^{\circ} \rightarrow K^{+} \pi^{-}$ | 0.8 |
| $B^{\circ} \rightarrow K^{*+} \pi^{-}$ | 1.4 |

The asymmetry then is given by

$$
\begin{equation*}
A \cong \eta R / 4 \tag{18}
\end{equation*}
$$

This could be as high as $3.5 \%$ for $\eta=0.1$, and clearly in an observable range.

## 6 Conclusions

The above estimate are based on the applicability of the one loop quark diagram to a hadronic decay, a procedure that has been criticized recently by Wolfenstein [10]. In my opinion the use of one loop diagram is clearly possible only if the quark diagram is a short distance operator and long distance effects are negligible. This might be true for a heavy quark decay where large momenta are involved in the decay products. A more troubling question of the appropriate $k^{2}$ to be used will have to deferred until a better understanding emerges regarding hadronic decays. Importance of detecting direct CP violation outweighs the fact that the theoretical calculations are not at present very reliable. We expect the situation to improve as the penguin processes are measured.

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## References

1. I. Bigi and A. Sanda, Nucl. Phys. B193, 85 (1981); Phys. Rev. D29, 1393 (1984), I. Bigi, V.A. Khoze, N.G. Utaltsev and A. Sanda, in CP Violation, edited by C. Jarlskog (World Scientific, Singapore 1989), p175.
2. B. Grinstein, R. Springer and M. Wise, Phys. Lett.B, 138 (1988).See also R.Grigjanis et al,Phys. Lett. B237,252(1990).
3. N. G. Deshpande, P. Lo and J. Trampetic, Z. Phys. C 40,369 (1988);See also Brief Review by N. G. Deshpande in Mod. Phys. Lett. A 4,2095 (1989)
4. Our figure is from N. G. Deshpande and K. Panose, to be published; see also R. Grigjanis et al,Phys. Rev. 42,245 (1990); and W. Jaus and D.Wyler,Phys. Rev. D41,3405 (1990).
5. M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 43, 142 (1979).
6. J.M. Gerard and W.S. Hou, Phys. Rev. Lett., 62, 855 (1989).
7. N.G. Deshpande and J. Trampetic, Phys. Rev. D41, 2926 (1990).
8. J.M. Gerard and W.S. Hou, Preprint UCL-1P'T-90-10.
9. N.G. Deshpande and J. Trampetic, Phys. Rev. D41, 895 (1990).
10. L. Wolfenstein, preprint, NSF-ITP-90-29

