Image quality of figured multilayered optics

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Abstract

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The reflectivity and resolution of a multilayer structure is strongly affected by the roughness at the interfaces between two successive layers and by the amount that the constituent materials will diffuse into one another at the interfaces. Performance is also affected by the variations in individual layer thicknesses and by inhomogeneities in the materials. These deviations from the ideal multilayer will also affect the quality of the image from a figured multilayer optical element.

The theory used to model the effects of non-ideal multilayers on the image quality of figured optics will be discussed. The relationship between image quality and multilayer structure quality will be illustrated with several examples.

Introduction

We have developed two computer codes for modeling multilayer performance. The two codes are complementary in the actual computational method used, but it is straightforward to show that the theory used to predict the performance of an ideal multilayer is identical for the two methods. The first method is based on the commonly-used recursive method which originated with Parratt^{1,2,3}. The second method uses the matrix formulation described by Born and Wolf^{4,5}. The codes are complementary in the sense that the recursive method allows greater flexibility in varying the various layer thicknesses in the structure, while the matrix formulation is computationally compact for a periodic structure.

We have modified both codes to model the effects of interface roughness and the matrix formulation has been extended to model the effects of interface diffusion and layer thickness variations. These models have been used to evaluate the effect of structure imperfections on the images resulting from some simple figured optic configurations.

Theory

For the description of the theory we will choose a coordinate system such that the surface of the multilayer is in the x-y plane, and the wave vector of the incident radiation will have a component in the positive z-direction. The z-coordinate will increase as one progresses into the structure, with the back surface of the substrate lying at the maximum value of the z-coordinate. The first layer encountered by the incident radiation will be designated as layer number 1, with the last layer before the substrate being the layer number N. A subscript on a variable is used to indicate which layer the variable refers to and a superscript sign is used to indicate the direction of the z-component of the wave vector. The polarization is indicated as either TE (electric field perpendicular to the plane of incidence) or TM (magnetic field perpendicular to the plane of incidence.)

Ideal multilayer

The recursive formulation for an ideal multilayer is based on the theory presented by Parratt¹. The approximations used in the original presentation of the theory have been removed so that the exact formula

$$R_{i-1,i} = a_{i-1}^4 \times \frac{R_{i,i+1} + r_{i-1,i}}{R_{i,i+1} \times r_{i-1,i} + 1} \quad (1)$$

is used. Here we have defined

$$R_{i,i+1} = a_i^2 \times \frac{E_i^-}{E_i^+} \quad (2)$$

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$$a_i = \exp(i \times \frac{k_0 \epsilon_i d_i}{2}) \quad (3)$$

$$r_{i-1,i}^{TE} = \frac{\epsilon_{i-1} - \epsilon_i}{\epsilon_{i-1} + \epsilon_i} \quad (4)$$

$$r_{i-1,i}^{TM} = \frac{\frac{\epsilon_{i-1}}{n_{i-1}^2} - \frac{\epsilon_i}{n_i^2}}{\frac{\epsilon_{i-1}}{n_{i-1}^2} + \frac{\epsilon_i}{n_i^2}} \quad (5)$$

and

$$\epsilon_i = (n_i^2 - \cos^2 \theta_0)^{1/2}. \quad (6)$$

In these equations d_i and θ_i refer to the thickness of the i^{th} layer and the grazing angle of incidence of the radiation in the i^{th} layer respectively. The complex index of refraction for the i^{th} layer is represented by $n_i (= 1 - \delta_i + i\beta_i)$. One should also note that $r_{i-1,i}$ is the Fresnel reflection coefficient for the interface between layers $i-1$ and i .

The application of this method is based on the fact that the bottom surface of the substrate, or the bottom surface of the last layer for a very thick substrate, will have no wave traveling in the negative z -direction incident on it. This immediately yields the value of R for that interface, and one can progress up through the stack until the top surface is reached.

The matrix method of Born and Wolf⁴ describes the effect of each layer using a 2×2 matrix of the form

$$\begin{vmatrix} U_i \\ V_i \end{vmatrix} = \begin{vmatrix} \cos(\gamma_i d_i) & -i \times \sin(\gamma_i d_i) / Y_i \\ -i \times \sin(\gamma_i d_i) \times Y_i & \cos(\gamma_i d_i) \end{vmatrix} \begin{vmatrix} U_{i+1} \\ V_{i+1} \end{vmatrix} \quad (7)$$

where U_i represents the tangential component of the electric field and V_i represents the tangential component of the magnetic field at the top of the i^{th} layer for TE polarization while U_i represents the tangential component of the electric field and V_i represents the negative of the tangential component of the magnetic field at the top of the i^{th} layer for TM polarization. Also

$$\begin{aligned} \gamma_i &= k_0 \times n_i \times \sin(\theta_i) \\ &= k_0 \times (n_i^2 - \cos^2 \theta_0)^{1/2}. \end{aligned} \quad (8)$$

We also define

$$Y_i^{TE} = n_i \times \sin(\theta_i) \quad (9)$$

$$Y_i^{TM} = \frac{n_i}{\sin(\theta_i)}, \quad (10)$$

which are referred to as the optical admittances of the i^{th} layer.

To calculate the performance of a multilayer stack, one multiplies the matrices for the various layers together, with the matrix for the first layer on the left, to determine a single 2×2 matrix M . One then calculates the reflection coefficient for the multilayer using the formula

$$r = c \times \frac{m_{1,1} + m_{1,2} \times Y_s - (m_{2,1} + m_{2,2} \times Y_s) / Y_v}{m_{1,1} + m_{1,2} \times Y_s + (m_{2,1} + m_{2,2} \times Y_s) / Y_v}, \quad (11)$$

where Y_s and Y_v are the admittances for the substrate and vacuum respectively. $c = 1$ for

TE, $c = -1$ for TM polarization, and $m_{i,j}$ represents the element from row i and column j of the matrix M .

The physical meaning of the matrix formulation can be better understood, and the effects of imperfections on the multilayer performance better modeled, if we transform to a system involving only the electric fields of the waves. This transformation yields the equation

$$\begin{pmatrix} E_i^+ \\ E_i^- \end{pmatrix} = \frac{1}{t_{i,i+1}} \begin{pmatrix} \exp(-i\gamma_i d_i) & 0 \\ 0 & \exp(i\gamma_i d_i) \end{pmatrix} \begin{pmatrix} 1 & r_{i,i+1} \\ r_{i,i+1} & 1 \end{pmatrix} \begin{pmatrix} E_{i+1}^+ \\ E_{i+1}^- \end{pmatrix} \quad (12)$$

where $t_{i,i+1}$ and $r_{i,i+1}$ are the usual Fresnel coefficients for the appropriate polarization. The first matrix propagates the wave across the layer thickness while the second matrix, along with the leading coefficient, contains the effect of the interface at the bottom of the layer. From this form of the matrix, it is a simple exercise to derive the recursion formula presented above.

Imperfect multilayer

The effects of roughness are modeled in a very straightforward way in the recursive code. If one denotes by $\Sigma_{i,i+1}^R$ the change in the reflection coefficient for the interface between layers i and $i+1$, and by $\Sigma_{i,i+1}^T$ the change the transmission coefficient, one can write the recursion relation as

$$R_{i-1,i} = a_{i-1}^4 \Sigma_{i-1,i}^R \frac{r_{i-1,i} + R_{i,i+1} \Sigma_{i-1,i}^*}{R_{i,i+1} \Sigma_{i-1,i}^R + r_{i-1,i}} \quad (13)$$

where

$$\Sigma_{i-1,i}^* = \frac{t_{i,i-1} t_{i-1,i} \Sigma_{i,i-1}^T \Sigma_{i-1,i}^T}{\Sigma_{i-1,i}^R} + r_{i-1,i}^2 \Sigma_{i,i-1}^R \quad (14)$$

We have then used the correction for the specular reflection coefficient given by Beckman and Spizzichino⁶, yielding

$$\begin{aligned} \Sigma_{i-1,i}^R &= \exp\left(-\frac{1}{2} \left(\frac{4\pi \sigma_{i-1,i} \sin(\theta_{i-1})}{\lambda_{i-1}}\right)^2\right) \\ &= \exp(-2 \times (k_{0i} \sigma_{i-1,i} n_{i-1} \sin(\theta_{i-1}))^2). \end{aligned} \quad (15)$$

This theory assumes that the roughness at successive interfaces to be completely uncorrelated, that the height of the surface imperfections can be represented by a Gaussian distribution with σ representing the rms deviation, and that the surface is composed of small-curvature irregularities. It is commonly assumed that

$\Sigma_{i-1,i}^R = \Sigma_{i-1,i}^T$. However, there is considerable evidence⁷⁻⁹ that one should use

$$\Sigma_{i-1,i}^T = \exp(-2 \times (k_0 \sigma_{i-1,i})^2 \times (n_{i-1} \sin(\theta_{i-1}) - n_i \sin(\theta_i))^2). \quad (16)$$

The experimental data presented in reference 9 indicate that this formula may only be valid for $0 < \theta < 30^\circ$. The code is currently written using the latter formula for

$\Sigma_{i-1,i}^T$.

The modifications to the matrix formulation to model interface roughness and layer thickness imperfections follow exactly the method of Carniglia¹⁰. This method involves the introduction of some position-dependent deviation of the multilayer structure from the ideal case. The matrices are modified to reflect this deviation and the reflection coefficient, which is a function of the position-dependent variations, is calculated. One then calculates a most probable value of the coefficient by averaging each of the variables representing deviations from the ideal over some appropriate distribution. It is usually assumed that a Gaussian distribution is appropriate.

In order to check the predictions of this method, the change in reflectivity and transmissivity of a single interface due to roughness was calculated. By assuming a Gaussian distribution, this theory yielded the values of $\Sigma_{i-1,i}^R$ and $\Sigma_{i-1,i}^T$ given above.

One can model a diffuse boundary using a method very similar to that used to model a rough interface except that the averaging process is performed before determining the reflection coefficient. This effectively produces a region at the interface where the index found by using an exponential distribution function was chosen to provide a reasonably good match to the 'ideal' complementary error function distribution. The parameter chosen to represent the thickness of the diffuse layer is equal to the value $(D \times t)^{1/2}$, where D is the diffusion coefficient for the materials in question and t is the elapsed time for the diffusion process to take place (with the implicit assumption that D is very small at room temperatures and the diffusion took place during the deposition process or during an annealing process). This method is very similar to the method used by Vidal and Vincent¹¹ to represent interface roughness, but a distribution more appropriate to material diffusion has been.

A comparison of the matrix generated using this method with one generated by dividing the region of diffusion into a large number of very thin sublayers (assuming that the optical properties are constant in each of the sublayers) indicates good agreement of the two methods. The off-diagonal elements of the matrix from the former method are identically zero while those of the latter method are on the order of 1×10^{-6} . The diagonal elements agree to within a few parts in 10^4 .

In practice the matrices resulting from these corrections are quite complex. However, it was found that considerable simplification resulted if they were transformed back to the U, V notation. After performing this transformation one finds that the matrix for a single layer can be written

$$\begin{vmatrix} \cos(\gamma_i z_i) & -i \times \sin(\gamma_i z_i) / Y_i \\ -i \times \sin(\gamma_i z_i) \times Y_i & \cos(\gamma_i z_i) \end{vmatrix} \times \underline{D}_{i,i+1} \times \underline{R}_{i,i+1} \quad (17)$$

where

$$\underline{D}_{i,i+1} = \frac{r^2}{t_{i,i+1}} \begin{vmatrix} Q_- + r_{i,i+1} Q_+ & 0 \\ 0 & (Q_- - r_{i,i+1} Q_+) \times Y_i / Y_{i+1} \end{vmatrix} \quad (18)$$

represents the effect of a diffuse interface for the TE mode and

$$\underline{R}_{i,i+1} = \frac{\sin(\theta_i)}{\sin(\theta_{i+1})} \frac{r^2}{t_{i,i+1}} \times \begin{vmatrix} Q_- - r_{i,i+1} Q_+ & 0 \\ 0 & (Q_- + r_{i,i+1} Q_+) \times Y_i / Y_{i+1} \end{vmatrix} \quad (19)$$

represents the effect for the TM mode. In these equations we define

$$Q_+ = \frac{1}{r^2 + (\gamma_i + \gamma_{i+1})^2} \quad (20)$$

$$Q_- = \frac{1}{r^2 + (\gamma_i - \gamma_{i+1})^2} \quad (21)$$

and

$$r = \frac{1.69305}{2 \times (D \times t)^{1/2}} \quad (22)$$

The constant factor in the expression for r is present to provide the best fit between the exponential curve used to model the diffusion profile and the complementary error function. The effect of interface roughness is modeled by the matrices

$$\underline{R}_{i,i+1} = \frac{1}{t_{i,i+1}} \begin{vmatrix} C_- + r_{i,i+1} C_+ & \\ -i \times (S_- + r_{i,i+1} S_+) \times Y_i & \\ & -i \times (S_- - r_{i,i+1} S_+) / Y_j \\ & (C_- - r_{i,i+1} C_+) \times Y_i / Y_j \end{vmatrix} \quad (23)$$

for the TE mode and

$$R_{i,i+1} = \frac{\sin(\theta_i)}{\sin(\theta_{i+1})} \frac{1}{t_{i,i+1}} \left| \begin{array}{c} C_- - r_{i,i+1} C_+ \\ -i \times (S_- - r_{i,i+1} S_+) \times Y_i \\ -i \times (S_- + r_{i,i+1} S_+) \times Y_j \\ (C_- + r_{i,i+1} C_+) \times Y_i / Y_j \end{array} \right| \quad (24)$$

for the TM mode. Here

$$C_+ = \cos[(\gamma_i + \gamma_{i+1}) \times f_{i,i+1}] \quad (25)$$

$$C_- = \cos[(\gamma_i - \gamma_{i+1}) \times f_{i,i+1}] \quad (26)$$

$$S_+ = \sin[(\gamma_i + \gamma_{i+1}) \times f_{i,i+1}] \quad (27)$$

$$S_- = \sin[(\gamma_i - \gamma_{i+1}) \times f_{i,i+1}]. \quad (28)$$

In the calculation of the reflection coefficient, one does not use the matrix for roughness as written since the averaging process is done through the use of a Taylor expansion and only the first and second derivatives of this matrix (with $f_{i,i+1} = 0$) are necessary.

One point to note is that the matrices for roughness and interface diffusion do not commute--the order of the matrices is significant. We have assumed that the diffusion matrix should be to the left of the roughness matrix, but a careful determination of the proper order has not been done.

Application to figured optics

For simplicity we have used the configuration of the Von Hamos spectrograph to illustrate the effects of structural imperfections on the image quality. This configuration consists of a multilayer bent in a perfectly cylindrical shape. A point source is placed on the axis of the cylinder near one end of the multilayer with a line detector on the axis near the other end (see Fig. 1). One then evaluates the quality of the image of the point source at the detector.

Figure 1. The configuration of the Von Hamos spectrograph used in evaluating the effect of multilayer imperfections on image quality. The point source and the detector are located on the axis of a cylindrically curved multilayer.

We have used two typical configurations as examples. The first multilayer consists of 21 layer pairs with 43.0 Å of vanadium and 100.33 Å of carbon, operating at 2.34 Å. The second multilayer consists of 16 layer pairs with 32.0 Å of nickel and 44.5 Å of carbon, operating 71.3 Å. The characteristics of the materials at the specified wavelengths are summarized in Table I. These two configurations were chosen because of the availability of experimental data^{4,5,12} for comparison of the results.

TABLE I. Characteristics of the Multilayers Used in Sample Calculations

layer	material	thickness (Å)	δ	β
<u>V/C multilayer with 21 layer pairs</u>				
ODD	V	43.00	3.413×10^{-5}	8.740×10^{-7}
EVEN	C	100.33	1.488×10^{-5}	5.867×10^{-8}
SUBSTRATE	Si	—	1.768×10^{-5}	8.773×10^{-7}
<u>Ni/C multilayer with 16 layer pairs</u>				
ODD	Ni	32.00	3.014×10^{-2}	1.943×10^{-2}
EVEN	C	44.50	8.857×10^{-3}	8.133×10^{-4}
SUBSTRATE	Si	—	9.334×10^{-3}	1.209×10^{-2}

When placed in a Von Hamos configuration it is assumed that the V/C multilayer is bent with a 1 cm radius of curvature and that the center of the detector region is placed 200 cm from the source. This was necessary to accommodate the peak reflectivity angle for the first order at 0.6° . The Ni/C multilayer is bent with a 5 cm radius of curvature with the detector centered at 17.3 cm from the source point.

Diffuse interfaces

The effect of a 10 Å diffuse interface on the reflectivity curve of the two multilayers being considered is shown in Figures 2 and 3. Only the curve for TE-polarization is shown for the V/C multilayer since there is no difference between TE- and TM-polarization for this sample.

Figure 2. Comparison of the intensity of the image of a 2.34 Å point source for an ideal V/C multilayer (solid line) and one with a 10 Å diffusion layer at each interface (dashed line).

As would be expected, a diffuse interface tends to decrease the peak reflectivity of the multilayer. For most applications this will lead to a decrease in contrast in the image, but will not cause any distortion. It will also be noted that diffusion tends to shift the positions of the reflectivity peaks very slightly. This effect will only be important if the optical system requires angular accuracies better than approximately 0.1° . In the Von Hamos configuration such an effect is hardly noticeable.

A more important effect is clearly illustrated by the curve of Figure 2 for the V/C multilayer. The addition of a diffuse interface leads to a significant increase in the resolution of the device. This increase in resolution can be easily understood when one considers that interfacial diffusion tends to increase the transmission of an interface. This allows the incident radiation to penetrate deeper into the multilayer stack, effectively increasing the number of layers involved in the interference effects. This effect is not as evident in the Ni/C multilayer because of the much greater absorption of the x-rays by the constituent materials at the longer wavelength. A careful examination of the reflectivity curves of Figures 3a and 3b for the Ni/C multilayer show an increase in resolution, but the decrease in reflectivity is so great that the change in resolution is not immediately apparent.

Figure 3. Image intensity of a 71.3 Å point source using an ideal Ni/C multilayer (solid line) and one with a 10 Å diffusion layer at each interface (dashed line). Figure (3a) is for TE polarization and (3b) is TM polarization.

Figure 4 illustrates one further effect of interface diffusion on the phase of the reflected radiation. For the Ni/C multilayer (only TE-polarization is shown, TM-polarization is effected in the same way) the addition of diffusion tends to reduce the phase difference in waves that are reflected at different angles. For a relatively low resolution device of this type, the phase difference could be an important consideration in designs where coherence effects are important such as laser cavities or x-ray holography systems. This may also be of importance if the multilayer is to be used for very short-pulse applications because of temporal broadening of the pulse¹³.

Figure 4. Comparison of the phase of the image of a TE-polarized 71.3 Å point source for an ideal Ni/C multilayer (solid line) and one with a 10 Å diffusion layer at each interface (dashed line).

Figure 5. Intensity of the image of a 2.34 Å point source using an ideal V/C multilayer (solid line) and one with 5 Å of rms roughness on each interface (dashed line). Figure (5a) was calculated using the matrix formulation and (5b) using the recursion formulation.

Rough interfaces

Figures 5 and 6 illustrate the effect of 5 Å of interfacial roughness on the Von Hamos image for the specified multilayers. The results calculated from both the recursive method and the matrix method are shown since there are some differences in the results for the V/C multilayer, but no significant differences for the Ni/C system. The reason for

the difference is not clear, but is apparently only of significance for either short wavelengths or for very small angles of incidence.

Figure 6. Intensity of the image of a 71.3 Å point source using an ideal Ni/C multilayer (solid line) and one with 5 Å of rms roughness on each interface (dashed line). Figures (a) and (c) are TE polarization, (b) and (d) are TM polarization. Figures (a) and (b) were calculated using the matrix formulation, (c) and (d) using the recursion formulation.

The most noticeable effect of roughness is a universal decrease in the intensity of the reflected radiation. There is no significant change in the position of the reflection peaks or in their resolution. This should result in a decrease of the contrast of the image.

The equations used to generate the plots of reflectivity as a function of position deal only with the specularly reflected radiation. A further effect to be examined is that of diffuse reflections. In order to examine this effect we will make use of the equations developed by Beckmann⁶. There is experimental evidence that this theory is not entirely correct¹⁴, but it will allow a straightforward examination of the scattered radiation. The total reflectivity is given for one-dimensional roughness by

$$\langle \rho \rho^* \rangle = \exp(-g) \times (\rho_o \rho_o^* + \frac{\pi^{1/2} T^2}{2L} \times \sum_{m=1}^{\infty} \frac{g^m}{m^{1/2} m!} \times \exp(-\frac{v^2 x^2}{4m})) \quad (20)$$

where

$$g = (k \times \sigma \times (\sin(\theta_1) + \sin(\theta_2)))^2 \quad (30)$$

$$v_x = k \times (\cos(\theta_1) - \cos(\theta_2)) \quad (31)$$

$$F = \frac{1 - \cos(\theta_1 + \theta_2)}{\sin(\theta_1) \times (\sin(\theta_1) + \sin(\theta_2))} \quad (32)$$

Here ρ_o represents the reflection coefficient of the ideal surface, k represents the wave vector of the incident radiation, σ represents the rms deviation of the surface from perfect, θ_1 represents the incident angle, θ_2 the reflected angle, T represents the correlation length of the surface, and L is the dimension of the reflector. One should note that the specular direction is the case when $\theta_1 = \theta_2$. It is assumed that the deviations of the surface from the ideal are completely random, that the deviations follow a Gaussian distribution, and that the surface is locally flat.

The term of interest is the second one which represents the diffuse portion of the reflected radiation. The first point to note is that the correlation length T will govern how broad the scattering pattern is through the exponential term in the sum. A large correlation length will result in very localized scattering near the specular direction, while a small correlation length will result in a very broad pattern. The amplitude of the scattering is governed by both the value of g and the ratio of the correlation length to the reflector dimension L . It would be expected that T would be much smaller than L for a random surface with more than a few irregularities in the reflector, so that the leading coefficient in the diffuse term will be on the order of (or less than) 0.1. If the value of g is much less than unity, it is only necessary to keep the first term of the series and the peak value of the diffusely reflected radiation is less than 1% of that in the specular beam. As g becomes larger it is necessary to keep approximately $2.5 \times g$ terms of the series. The maximum value of the sum (without the leading coefficient and with the exponential equal to 1.0) for a few values of g are shown in Table II. One should note that at $g=3$ the diffuse scattering will be approximately equal to the specular reflection.

TABLE II. Value of the sum $\sum_{m=1}^{\infty} \frac{g^m}{m^{1/2} m!}$ for several values of g

g	number of terms kept	$\sum_{m=1}^{\infty} \frac{g^m}{m^{1/2} m!}$	value of last term
0.01	3	0.010035	9.62×10^{-8}
0.1	4	0.103634	2.09×10^{-6}
0.5	6	0.601844	8.86×10^{-6}
1.0	7	1.474981	7.50×10^{-5}
1.5	8	2.761988	2.25×10^{-4}
2.0	10	4.685297	8.92×10^{-5}
3.0	12	12.03294	3.20×10^{-4}
4.0	14	29.46843	8.23×10^{-4}
5.0	15	71.91781	6.03×10^{-3}
10.0	24	7273.687	0.329

In general, if g is sufficiently small (<0.1) the diffuse scattering should cause no significant change in the quality of the image, but for larger values of g the reduction

in resolution and in image contrast could be significant. Using the Bragg condition for the n^{th} order reflection peak, we can rewrite this condition as

$$g = 2\pi \times n \frac{\sigma}{d} < 0.1 \quad (33)$$

or

$$\sigma < 0.016 \frac{d}{n} \quad (34)$$

where d is the thickness of a single layer pair. Noll¹⁵ has also discussed the effect of scattering on image contrast of x-ray optical systems using a method based on this theory.

Interfacial roughness has no detectable effect on the phase of the specularly reflected radiation.

Summary and Discussion

The effects of diffusion on image quality can be beneficial for some combinations of wavelength and materials. The increase in resolution can be quite significant, but a similar increase could be obtained by using materials with low absorption and with a very small difference in the real part of the optical constants to maximize the number of layers involved in the reflection process.

The phase modification caused by a diffuse boundary could be used to improve the characteristics of a multilayer used in such devices as Fabry-Perot etalons or gratings if a method of controlling the diffusion profiles of the interfaces during the fabrication process is found. The benefits of interface diffusion must, of course, be balanced with the loss of reflectivity.

There appear to be no beneficial effects of interface roughness. This roughness must be small to avoid a significant loss of resolution and image contrast. It is apparent, however, that roughness not exceeding approximately 2% of the layer pair thickness will be adequate for most applications. This level of roughness will lead to about a 10% loss of intensity in the specular beam and the diffusely scattered radiation will be on the order of 1% of the specular intensity.

The extension of these results to more complex optical systems is conceptually straight forward. All defects will lead to a loss of intensity in the image. Some designs may be benefited by the presence of interface diffusion, especially where phase effects can be used to enhance the image or where the phase is the most important aspect of the image.

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