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Optimization of Modular Coils for Stellarator Fields

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ABSTRACT

Introduction of a non-sinusoidal deformation can enhance the efficacy of modular coils for generating magnetic fields with a built-in rotational transform. Techniques are developed that provide an understanding of how specific deformations affect the harmonic content of the magnetic field and thus the properties of the vacuum configuration. This provides an optimization procedure for coil design.

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I. INTRODUCTION

The stellarator concept¹ for controlled nuclear fusion was replaced by the tokamak approach in the early 1970's in the United States for three reasons. Experimental work on the Model C Stellarator² showed poor plasma confinement. Theoretical predictions of enhanced transport in the collisionless regime due to the three-dimensional nature of the configuration^{3,4} indicated that confinement would only become worse as reactor conditions were approached. Reactor studies⁵ showed that building and supporting the necessary helical stellarator windings inside the toroidal field coils pose severe problems and led to a conclusion that stellarators had no future. Recently, WENDELSTEIN VII-A⁶ and HELIOTRON E⁷ have shown that current-free plasmas can be contained in stellarators with energy confinement times as good as, or better than, those in tokamaks of comparable device parameters. Similarly, theoretical studies using Monte-Carlo techniques⁸⁻¹⁰ indicate that transport problems associated with stellarator asymmetry are not as serious as had been believed. This paper reports on the stellarator magnetic field configurations that can be generated by modular coils; this approach alleviates the engineering problem.

In a classical stellarator the forces on one set of helical windings, associated with the interaction of the current in the coil with the separately applied toroidal field, are large and point inward towards the plasma. Thus valuable volume inside the windings must be occupied by the supporting structure. In torsatrons,¹¹ the coils provide both the poloidal and the toroidal field components so that these forces are reduced and can be handled comparatively easily. Both access to the plasma and ease of maintenance are also improved because there are no longer two sets of interwoven windings.

However, the helical windings and their support structure must be toroidally connected so that disassembly for maintenance is difficult.

Another way of solving the problems associated with classical stellarator windings would be to make them modular. Rekher and Wobig¹² proposed using nonplanar modular coils to generate the stellarator field. In each coil the azimuthal position varies sinusoidally with the angle around the plasma axis. This solves both the modularity problem and the difficulties with forces on the coils but generated relatively little interest because the rotational transforms that could be achieved seemed small. The transform can be increased with additional tailoring of the coil deformation.^{13,14} Here we complete the previously reported work on this approach and extend the studies to show how to optimize the design of modular stellarators. We note that a different attack on the problem would be to specify the desired magnetic fields and calculate the poloidally-closed current loops on a prescribed toroidal surface required to set up these fields.^{15,16} This leads, in general, to fairly complicated coil configurations. Comparison of the results of this treatment with those of the present paper, where we start with coil structures of simply prescribed geometry and study the effects of changing their shapes and positions on the properties of the magnetic field, should lead to useful machine design.

Other schemes have also been studied. The helical and toroidal field coils can be combined into discrete units.¹⁷ Similarly, a system consisting of two opposing torsatron windings with different pitches can be segmented.^{18,19} Another interesting method is to use shaped toroidal field coils (elliptic for $\ell = 2$ or triangular for $\ell = 3$) with the major axis rotating from coil to coil.²⁰

In the next section we introduce our formalism and expand on our earlier

work.¹⁴ We then describe some applications and results.

II. FORMALISM

The early modular stellarator coils¹² can be represented by current filaments wound on a toroidal surface of minor radius a and major radius R which satisfy the winding law

$$\phi = \phi_j + \frac{d}{R} \sin(l\theta - \theta_j), \quad r = a; \quad j = 1, 2, \dots, N. \quad (1)$$

Here ϕ is a coordinate along the toroidal direction, θ is the poloidal angle, $\phi_j = 2\pi j/N$ measures the coil spacing, $\theta_j = 2\pi m j/N$ represents the poloidal rotation between neighboring coils, d is the deformation amplitude of the coil, N is the total number of coils, and l and m are the numbers of poloidal and toroidal field periods. The successive rotation of each coil due to θ_j provides a rotational transform and generates magnetic surfaces.

The winding law of Eq. (1) is shown as solid lines in Fig. 1. We introduce a set of direction angles $\alpha_1, \beta_1, \alpha_2, \beta_2, \dots, \alpha_i, \beta_i$, obtained by connecting the nearest nodal points (with currents flowing in the same direction) of neighboring coils, the next nearest nodal points, etc. The angles are given by

$$\tan \alpha_1 \equiv ma/lR, \quad (2)$$

$$\tan \beta_1 \equiv [(N-m)a/lR], \quad (3)$$

$$\tan \alpha_2 \equiv [(N+m)a/lR], \quad (4)$$

$$\tan \beta_2 \equiv [(2N - m)a/R] \quad , \quad (5)$$

and so on.

The currents in the coils can be decomposed along the direction angles. A good approximation is obtained by keeping only the smallest of the direction angles, α_1 and β_1 . These two dominant helical contributions provide a reasonable model for visualization of the magnetic field in a modular stellarator. The α_1 -angle helical line, having toroidal field period number m , corresponds to that in a classical stellarator winding, and the pitch is determined by the geometry of the coils, Eq. (2). The pitch associated with the β_1 -angle helical line, Eq. (3), which we henceforth call the "antihelical line," depends, in addition, on the number of coils in the torus. This line has a toroidal field period number given by $N-m$. The helical lines provide the basic rotational transform of the system while the antihelical lines unwind the field lines and provide an opposing transform. The fewer the number of coils, the stronger is the opposing transform. The lowest integer N/m value for which there is a non-zero rotational transform is 3. Since the pitch of the antihelical lines is large, their contribution is generally small in the plasma interior but becomes significant near the edge. This antihelical component causes the rotational transform to be smaller and the separatrix to be located at a larger radial position than in a conventional stellarator. Thus a modular stellarator differs from a classical one in having a lower edge transform and a larger usable magnetic volume.

These considerations of the competition of the fields associated with the helical and antihelical lines provide a direction for improving the configuration. It was pointed out in Ref. 14 that one can decompose the

current in the coils into poloidal and toroidal components, along θ and ϕ respectively. Small changes in the component along θ have little effect on the magnetic configuration since the toroidal field is mainly axisymmetric and relatively insensitive to where the currents are. On the other hand, small changes in j_j can have a significant effect since they modify the poloidal field. This component is large near the nodal points $\ell\theta = \theta_j + n\pi$ where $n = 0, 1, 2, \dots$ and quite small elsewhere (see Fig. 5 of Ref. 14). Thus the distribution of the current into its helical and antihelical components is largely determined by the local winding geometry at the nodal points of the coils. We therefore replace the winding law of Eq. (1) with

$$\phi = \phi_j + \frac{d}{R} \sin s(\ell\theta - \theta_j), \quad r = a; \quad j = 1, 2, \dots, N, \quad (6)$$

where s is an arbitrarily prescribed function of θ . We can pick s to deform the coil only near the nodal points that have positive slope ($\ell\theta = \theta_j + 2n\pi$, $n = 0, 1, 2, \dots$) as in Fig. 2a, near the nodal points with negative slope [$\ell\theta = \theta_j + (2n + 1)\pi$] as in Fig. 2b, or we may use a combination of the two. In the positive slope case of Fig. 2c, with s a constant greater than 1 for $[\theta_j - \pi/2s] < \ell\theta < [\theta_j + \pi/2s]$, $\phi = \phi_j + d/R$ for $[\theta_j + \pi/2s] < \ell\theta < [\theta_j + \pi/2]$, and $s = 1$ for $[\theta_j + (\pi/2)] < \ell\theta < [\theta_j + (3\pi/2)]$, etc., we can introduce the angle γ to measure the local slope of the winding with respect to the ϕ -axis at the nodal point,

$$\tan \gamma \equiv a/sd. \quad (7)$$

The current at the nodal points can then be decomposed into components along

α and β :

$$\frac{I_{\alpha(+)}}{I} = - \frac{\sin(\gamma + \beta)}{\sin(\alpha + \beta)} , \quad (3)$$

$$\frac{I_{\beta(+)}}{I} = - \frac{\sin(\alpha - \gamma)}{\sin(\alpha + \beta)} , \quad (9)$$

with the subscript (+) denoting evaluation at the positive-slope point. Similarly, at the negative-slope nodal point with the appropriate equivalent choice of s ,

$$\frac{I_{\alpha(-)}}{I} = \frac{\sin(\beta - \gamma)}{\sin(\alpha + \beta)} , \quad (10)$$

$$\frac{I_{\beta(-)}}{I} = \frac{\sin(\alpha + \gamma)}{\sin(\alpha + \beta)} . \quad (11)$$

Equation (7) shows how this winding law provides an improvement over the simple sinusoidal one. Namely, the local pitch at the place where it is most important is increased by the factor s for the same coil deformation.

The above analysis applies to windings of constant pitch angle, i.e., the α and β angles are independent of their toroidal position. Other winding laws having variable pitch angle are also useful. Stellarator equilibrium studies²¹ have shown that introduction of a local vertical field can modify the shapes of the magnetic surfaces significantly. This can be accomplished by modifying the winding law of Eq. (1) so that the α_j lines of Eq. (2) are given by

$$\phi = \frac{2}{m} (\theta + \mu \sin \theta) . \quad (12)$$

The coil shapes and a typical configuration cross section for this winding law are given in Ref. 14. This broadens the harmonic spectrum and introduces a vertical magnetic field. For positive (or negative) values of u , the inner surfaces shift outward (or inward) with respect to the outer surfaces. The coil system obtained from Ref. 16 has properties similar to this. Each coil within a toroidal field period is different. This winding law can be approximated adequately by using only two types of coils. The simple analysis using the decomposition into the α and β directions is still useful; however, these lines are no longer straight in the unfolded $\theta - \psi$ plane.¹⁴

A significant contribution to the $\lambda = 2$ or $\lambda = 3$ components of the magnetic field can be obtained by using deformed coils wound on an elliptic or triangular cross section,²⁰ respectively, rather than a circular one. For example, we could introduce a winding law such as Eq. (1), (6), or (12) with the coils wound on the surface

$$r = \bar{a} [1 + u \cos 2(\theta - \theta_0/\lambda)] . \quad (13)$$

Since the fields associated with higher values of λ fall off rapidly as one moves in from the coil, generalization of this technique to more complicated cross-section shaping is not very useful.

One can also introduce a significant $\lambda = 1$ component by mounting the coils on a modulated axis,

$$R = R_0 [1 + (d_R/a) \cos n\phi] \quad (14)$$

and adopting the winding law of Eqs. (1), (6), or (12). This technique has been used by Rau²² for torsatron configurations.

It is useful to use Fourier techniques to decompose the fields associated with Eq. (6) in order to get a better understanding of why this local modification near the nodal point is so beneficial. This is quite difficult, but an analysis of a similar winding law for a straight system,

$$z \equiv z_j + \sum_n d_n \sin n(l\theta - \theta_j), \quad j = 1, 2, \dots, N \quad (15)$$

can be made using the method developed by Ohasa and Miyamoto.²³ The functional form of Eq. (6) can be approximated well by proper choice of the d_n 's. Ignoring toroidicity, we can Fourier analyze the current in the coils of Eq. (15) to obtain an equivalent surface current and then use the condition $\vec{J}^* = n \times [\vec{R}]$ to get the magnetic field. Inside the coil where $r < a$,

$$\vec{B} = \nabla \Phi \quad (16)$$

with the scalar potential

$$\Phi = \frac{NIa}{2\pi R} \sum_{h=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} \sum_{t=-\infty}^{\infty} \frac{p}{|p|} K'_{|h|} \left(\frac{|p|a}{R} \right) J_{h-2s-3t} \left(\frac{pd_1}{R} \right) \cdot J_s \left(\frac{pd_2}{R} \right) J_t \left(\frac{pd_3}{R} \right) I_{|h|} \left(\frac{|p|r}{R} \right) \sin(hl\theta - pz/R); \quad (17)$$

$$p \equiv mh + Ni. \quad (18)$$

We have arbitrarily restricted the number of harmonic terms associated with the d_n 's to three. Generalization to more terms is straightforward. The $h = i = 0$ term produces a uniform field along z . Terms with $h = 0$ but $p \neq 0$ describe bulge, or mirror, fields. The remaining terms describe helical

field components. This formula reduces to that found earlier²⁰ for the case where $d_n = 3$ when $n > 1$.

The α_i and β_i lines of Eqs. (2) through (5) can be identified with the different harmonics of $\sin(h(\theta - pz/R))$; all α_i and β_i modes correspond to $h = 1$ by definition, with α_1 having $i = 0$, α_2 having $i = 1$, β_1 having $i = -1$, etc. The most important modes tend to be those with $h = 1$ and $i = 0, \pm 1$. However, modes with $h > 1$ also contribute. Some modes with large values of $|p|$ have very large amplitudes. These short wavelength terms fall off so rapidly as one gets in away from the coil that they have little effect on the interior. They are probably responsible for the ergodic behavior observed for large radius in exact numerical calculations of the magnetic surfaces by following field lines.

Given these magnetic fields, it is easy to calculate the magnetic surfaces and determine the rotational transform and other properties of the magnetic system by means of the stellarator expansion.²¹ It was shown in Ref. 14 that excellent agreement is obtained between the results of this analysis and the exact numerical field line integration for systems with large aspect ratio.

The properties of modular stellarators that seem most important are that (i) adjusting the harmonic content of the magnetic fields improves the magnitude and shape of the rotational transform, and (ii) changing the magnitudes of the higher harmonics affects the radius at which ergodicity sets in. This interplay between modification of the rotational transform and useful volume can be quantified by introducing the concept of an equivalent current, which can be defined as the ohmic current that would be required to produce the transform χ at the edge ρ of a straight circular plasma column of length $2\pi R$ and axial field B ;

$$I_{eq} = \pi B_0^2 x/R . \quad (19)$$

III. APPLICATION

The concepts and formalism developed in the last section provide insight into techniques for optimizing configurations.

The usefulness of the winding law of Eq. (6) can be investigated by studying the dependence of the rotational transform on the parameter s . Equations (8) and (9) are shown in Fig. 3 and Eqs. (10) and (11) are given in Fig. 4 as functions of the harmonic multiplication factor s for a configuration with $\lambda = 2$, $m = 4$, $N = 36$, $R/a = 10/1.59$ and $d/a = 0.2$. The case $s = 1$ corresponds to a basic Rehker-Wobig coil, and $s \rightarrow \infty$ corresponds to a coil having windings along the ϕ -direction at the nodal point.

Figures 3 and 4 show that higher harmonics with this functional form of coil distortion increase the component of current along the helical line and reduce the component of current along the antihelical line. It can be anticipated that increasing the harmonic content in this sense should increase the transform. [When $\gamma = \alpha$, $I_{\alpha(+)} / I = 1$ and $I_{\beta(+)} / I = 0$. Further increase of s reduces $I_{\alpha(+)}$, as shown in Fig. 3.]

The numerically computed rotational transforms at $\bar{r} = 0$ and at the last closed magnetic surface $\bar{r} = \rho$ for this configuration are shown in Figs. 5 and 6 as functions of $I_{\alpha(+)} / I$ for the positive slope case and $I_{\alpha(-)} / I$ for the negative slope case, respectively. Here $\bar{r}$ denotes the average radial position of a magnetic surface with an area $\propto \pi \bar{r}^2$, and ρ is that for the outermost closed magnetic surface. The lowest rotational transform in each

figure corresponds to $s = 1$. In general, the transform increases with I_α . This improvement is particularly significant for small deviations of s from unity. This can be understood by noting that I_α , in particular $I_{\alpha(-)}$ in Fig. 4, increases rapidly with s in this region. This is because the coil slope at the negative slope nodal points for $s = 1$ is nearly perpendicular to the α -angle (Fig. 2d), and a slight increase of s results in a significant increase of current along the α -angle line. We note that as $I_{\alpha(-)}/I$ increases, the winding geometry at the nodal points becomes increasingly like the helical windings of a classical stellarator, the separatrix moves inward, and the rotational transform increases radially outward. Eventually, the increase in rotational transform at the new outermost surface resulting from a further increase of $I_{\alpha(-)}/I$ cannot compensate the reduction in transform at this surface due to the decrease in average radius. This accounts for the transform decrease when $I_{\alpha(-)}/I > 0.8$ in Fig. 6. The equivalent current of Eq. (19) is shown in Fig. 7 as a function of I_α/I for the configurations of Figs. 5 and 6. An optimum equivalent current is reached for $I_{\alpha(+)} / I \approx 0.8$ for harmonic enhancement in the positive-slope case and for $I_{\alpha(-)} / I \approx 0.4$ for the negative-slope case. In both cases this optimum is achieved with a harmonic multiplication factor $s \approx 2$. It is useful to compare these results with what would be obtained from an idealized winding law with ψ a linear function of θ everywhere such that, at the nodal points, $\psi_{(+)} = \alpha_{(+)}$, $\psi_{(-)} = \alpha_{(-)}$, and on the connecting lines $\psi = \pi/2$. The equivalent current for this model is also shown in Fig. 7 and is far from optimum.

Although the coil aspect ratio for this case is not very large, $R/a \approx 6.285$, it is useful to treat the system as straight and use Fourier analysis to decompose the magnetic field as in Eq. (17). We see from Fig. 7 that using

$s = 2$ in the increasing slope region $\theta_j + (2n - 1/2)\pi < \varphi < \theta_j + (2n + 1/2)\pi$,
 $s = \theta_j + d/R$ for $\theta_j + (2n + 1/2)\pi/s < \varphi < \theta_j + (2n + 1/2)\pi$ and
 $\theta_j + (2n + 3/2)\pi < \varphi < \theta_j + (2n + 7/2)\pi/s$, and $s = 1$ elsewhere or the analogous
negative-slope construction are optimum. We therefore consider these two
cases. The first of these can be modeled by setting $d_1/a = 0.2128$, $d_2/a =$
 0.0181 , $d_3/a = 0.0128$; for the second, $d_1/a = 0.2128$, $d_2/a = -0.0181$, $d_3/a =$
 0.0128 . A model with both nodal-point shapings introduced corresponds to d_1/a
 $= 0.2$, $d_2/a = -0.0362$, $d_3/a = 0$ and provides less improvement over the simple
winding law than the partial ones. The analogous Rehker-Wohig coil with $s = 1$
everywhere would have $d_1/a = 0.2$, $d_2/a = 0$, $d_3/a = 0$. Plots of the rotational
transform obtained using the stellarator expansion formalism are shown in Fig.
9. A somewhat different symmetric configuration with $d_1/a = 0.24$, $d_2/a = 0.0$,
 $d_3/a = 0.04$ leads to a larger transform. The absolute value of the rotational
transform found for this configuration is a little smaller than that discussed
in Reference 14. This is due to using smaller coil distortions. The
usefulness of considering decomposition into the α_1 and β_1 angles of Eqs.
(2) and (3) can be seen in Fig. 9 where we show the contribution to the
rotational transform from the major components. Note that α_2 also
contributes to the transform. In most cases very few terms need be considered
as long as one stays well inside the coil. Some of the very short wave length
[large p in Eq. (17)] components have large amplitude, and these make the
magnetic field lines quite ergodic in the outer region.

The winding law of Eq. (12) provides the possibility of adding a vertical
magnetic field which introduces flexibility for positioning and shaping the
plasma cross section. To investigate this we consider a system with $l = 2$,
 $m = 6$, $N = 36$, $R/a = 4.44$, $d/a = 0.3$ and $\mu = 0$. The magnetic surfaces are
shown in Fig. 10 for this case. Changing to $\mu = 0.6$ distorts the cross

sections to those of Fig. 11. The dotted and dashed curves are at two different toroidal angles and show how the shapes change in going around the system. Modulation of the coil axis to introduce an $\ell = 1$ component of the magnetic field provides an additional degree of control. Keeping the same parameters as in the $\mu = 0.6$ case but using Eq. (14) with $d_R/a = 0.025$ and $n = 1$ leads to the cross sections of Fig. 12.

Comparison of these figures shows that the main effect of the winding-law modification of Eq. (12) is to push the magnetic axis outward with respect to the plasma surface. The magnetic field lines therefore spend less time in the outer regions of unfavorable curvature than on the inside, so the configuration has a magnetic well. This well is quite deep, as can be seen in Fig. 13 where $V'(\psi) = \langle \int dl/B \rangle$ (the average weighted line length over a toroidal periodicity length on a magnetic surface) is shown. Here V is the volume inside a surface containing the toroidal flux ψ . The introduction of an $\ell = 1$ component has decreased the relative shift and thus the depth of the magnetic well, but, as can be seen from Fig. 13, it still is quite good. The rotational transform for all three of these cases is quite small, $\chi \lesssim 0.3$. Coil shaping by introducing the modifications of Eq. (6) could improve this.

A major reason for introducing a modulation of the curve defining the centers of the coils, as in Fig. 12, is to minimize the variation of $\int dl/B$ on a surface. The most important consequence of this is the reduction of secondary currents due to charge separation. The integrand of this function over a field period is shown in Fig. 14 for the three cases. The solid curve corresponds to the case with $\mu = 0.0$, $d_R/a = 0.0$; the dashed curve to $\mu = 0.6$, $d_R/a = 0.0$; and the dotted curve to $\mu = 0.6$, $d_R/a = 0.025$. Significant reduction in the variation of $\int dl/B$ can be obtained. Clearly,

this work can augment the optimization studies being done elsewhere¹⁶ by providing an understanding of how specific coil windings affect the magnetic field properties.

The change in the magnitude of the magnetic field in following a magnetic field line around the system is another important property. This affects the freedom of particles to flow and determines how many are trapped in local magnetic wells. This function is shown in Fig. 15 for one surface in the configuration of Fig. 12. The most significant feature of this is that the minimum value of B as the lines go around the inside of the torus shows essentially no toroidal variation. An investigation of particle confinement in this configuration¹⁰ has shown that it possesses good transport properties.

Elliptical shaping of the coils, combined with nonplanar distortion, can provide significant improvement of the magnetic field properties. A typical application is shown in Fig. 16. Here the ellipticity of Eq. (13) with $v = 0.2$ ($e = 3/2$) and $v = 0.33$ ($e = 2$) is introduced into a system defined by Eq. (1) with $R/a = 6.32$, $d/a = 0.14$, $l = 2$, $m = 4$, $N = 36$. The rotational transform is increased significantly. There is room for incorporation of more than one of these ideas at a time. However, the understanding gained by examining these ideas separately should be useful in obtaining an optimized system with achievable coils.

IV. DISCUSSION

We have shown how to develop coil winding laws that optimize a modular stellarator configuration in the sense of having as large a rotational transform as possible over a useful volume with good magnetic surfaces. Further, we have seen how changes in the winding law affect the shape and positions of the magnetic surfaces and thus the plasma equilibrium

properties. This ability to obtain a reasonable understanding of the nature of the magnetic fields that can be achieved in modular stellarators from the consideration of only a few of the properties of the coils should lead to improved device design.

Most of the work that has been done on modular stellarators has treated only vacuum magnetic fields. It is clear that in a reactor the beta values must be so high that the effects of plasma will be important. A difficulty in studying stellarator optimization lies in the calculation of three-dimensional configurations containing plasma. Codes that can do this exist^{24,25} but are restricted in that the outer boundary of the plasma, rather than the vacuum magnetic field configuration, must be specified. The Fourier decomposition employed here, coupled with stellarator expansion techniques provides a method for carrying through a self-consistent investigation. This is being done.²⁶

The first phase of a study to determine the efficacy of a modular stellarator reactor has been completed²⁷. It showed that, even with conservative physics assumptions, such a device could be competitive with other fusion approaches. The coil configurations used in that study were not optimized. Such optimization can be performed using the techniques described in this paper.

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References

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- [1] SPITZER, L. JR., Phys. Fluids 1 (1958) 253.
- [2] YOUNG, K. M., Plasma Phys. 16 (1974) 119.
- [3] STRINGER, T. E., Nucl. Fusion 12 (1972) 689.
- [4] CONNOR, J. W., and HASTIE, R. J., Nucl. Fusion 13 (1973) 221.
- [5] GIBSON, A. Nuclear Fusion Reactor Conference, Culham, Sept. 17-19, 1969, British Nuclear Energy Society (1970), p. 233.
- [6] BARTLETT, D. V., CANNICI, G., CATTANEI, G., DORST, D., GRIEGER, G., HACKER, H. H., HOW, J., JÄCKEL, H., JAENICKE, R., JAVEL, P., JUNKER, J., KICK, M., LATHE, R., MEYER, J., MAHN, C., MARLIER, S., MÜLLER, G., OHLENDORF, W., RAU, F., RENNER, H., RINGLER, H., SAPPER, J., SMEULDERS, P., TUTTER, M., ULRICH, B., WELLER, A., WÜRSCHING, E., WOBIG, H., ZIPPE, M., COOPER, D., FREUDENBERGER, K., LISTER, G., OTT, W., and SPETH, E., in Plasma Physics and Controlled Nuclear Fusion Research, 1980 (International Atomic Energy Agency, Vienna 1981), Vol. 1, p. 185.
- [7] IIYOSHI, A., SATO, M., MOTOJIMA, O., MUTAH, T., SUDO, S., IIMA, M., KINOSHITA, S., KANEKO, H., ZUSHI, H., BESSHOV, S., KONDO, K., and UO, K., IAEA Conference on Alternate Fusion Concepts, Erice, Italy, March 23-27, 1981.
- [8] POTOK, R., POLITZER, P. A., and LIDSKY, L. M., Phys. Rev. Lett. 45 (1980) 1328.
- [9] BOOZER, A. H. and KUO-PETRAVIC, G., Phys. Fluids 24 (1981) 851.
- [10] MYNICK, H. E., CHU, T. K. and BOOZER, A. H., Princeton University Plasma Physics Laboratory Report PPPL-1822 (August, 1981).

- [11] GOURDON, C., MARTY, D., MASCHKE, E. K., and DUMONT, J. P., in Plasma Physics and Controlled Nuclear Fusion Research, 1968 (International Atomic Energy Agency, Vienna, 1969) Vol. 1, p. 845.
- [12] REHKER, S. and WOBIG, H., Proceedings of the Seventh Symposium on Fusion Technology (Grenoble, 1972), p. 345; Max-Planck Institut für Plasmaphysik Report IPP 2/215 (1973).
- [13] CHU, T. K., FURTH, H. P., and LUDESCHER, C., Bull. Am. Phys. Soc. II, 24 (1979) 869.
- [14] CHU, T. K., FURTH, H. P., JOHNSON, J. L., LUDESCHER, C., and WEIMER, K. E., IEEE Transactions on Plasma Science (December, 1981).
- [15] CHODURA, R., DOMMASCHK, W., LOTZ, W., NÜHRENBURG, J., SCHLÜTER, A., GRUBER, R., HERRNEGGER, F., KERNER, W., SCHNEIDER, W., and TROYON, F., Plasma Physics and Controlled Nuclear Fusion Research, 1980 (International Atomic Energy Agency, Vienna, 1981) Vol. 1, p. 807.
- [16] CHODURA, R., DOMMASCHK, W., HERRNEGGER, F., LOTZ, W., NÜHRENBURG, J., and SCHLÜTER, A., IEEE Transactions on Plasma Science (December, 1981).
- [17] HAMBERGER, S., SHARP, L. E., and PETERSON, L. E., Ninth European Conference on Controlled Fusion and Plasma Physics (Oxford, 1979) Paper BP-20.
- [18] ANDERSON, D. T., CALLEN, J. D., DERR, J. A., HARRIS, J. H., HOFFMAN, D. J., KRUCKEWITT, T. W., MANTEI, T. D., PRAGER, S. C., SCHAEFFER, R. C., SHOHEI, J. L., TALMADGE, J. N., TREFFERT, J. D., BAUER, F., BETANCOURT, O. L., GARABEDIAN P., TATARONIS, J. A., ASPINALL, J. G., LIDSKY, L., POLITZER, P. A., POTOK, R. E., CHU, T. K., FURTH, H. P., JOHNSON, J. L., LUDESCHER, C., and WEIMER, K. E., in Plasma Physics and Controlled Nuclear Fusion Research, 1980 (International Atomic Energy Agency,

- Vienna, 1981) Vol. 1, p. 793.
- [19] ANDERSON, D. T., DERR, J. A., and SHOHET, J. L., IEEE Transactions on Plasma Science (December, 1981).
- [20] POPOV, S. N. and POPRYADUKHIN, A. P., Soviet Physics-Technical Physics 11 (1966) 284.
- [21] GREENE, J. M. and JOHNSON, J. L., Phys. Fluids 4 (1961) 875.
- [22] RAU, F., and W VII-A Team, Ninth European Conference on Controlled Fusion and Plasma Physics (Oxford, 1979), Paper B-22; also, private communication.
- [23] OHASA, K. and MIYAMOTO, K., Japan J. of Appl. Phys. 16 (1977) 813.
- [24] BAUER, F., BETANCOURT, F., and GARABEDIAN, P., A Computational Method in Plasma Physics, Springer Series in Computational Physics (Springer-Verlag, New York, 1978).
- [25] CHODURA, R., SCHLÜTER, A., KAUFMANN, M., and LOTZ, W., in Plasma Physics and Controlled Nuclear Fusion Research 1978 (International Atomic Energy Agency, Vienna, 1979) Vol. 2, p. 335.
- [26] ANANIA, G., JOHNSON, J. L., and WEIMER, K. E., Bull. Am. Phys. Soc. II 26 (1981) 956.
- [27] MILLER, R. L. and KRAKOWSKI, R. A., Los Alamos National Scientific Laboratory Report LA-8978-MS (August, 1981).

Figure Captions

- Fig. 1. Winding law, Eq. (1); for a simple modular coil showing (a) the α_i angles of Eqs. (2) and (4) and (b) the β_i angles of Eqs. (3) and (5).
- Fig. 2. Winding law, Eq. (6), for (a) and (c) the positive slope case, and (b) and (d) the negative slope case.
- Fig. 3. Current decompositions for the positive-slope case, Eqs. (8) and (9), for a configuration with $\lambda = 2$, $m = 4$, $N = 36$, $R/a = 10/1.59$, and $d/a = 0.2$. For simplicity, we plot the absolute value of $I_{\alpha(+)} / I$.
- Fig. 4. Current decompositions for the negative-slope case, Eqs. (10) and (11), for the parameters of Fig. 3.
- Fig. 5. Rotational transform at the magnetic axis (open circles) and at the outermost closed surface (solid circles) for the positive-slope configuration of Fig. 3.
- Fig. 6. Rotational transform at the magnetic axis (open circles) and at the outermost closed surface (solid circles) for the negative-slope configuration of Fig. 4.
- Fig. 7. Equivalent current, Eq. (19), for the positive-slope configuration of Fig. 3 (solid circles) and the negative-slope configuration of Fig. 4 (open circles). The "classical stellarator" point is indicated in the lower right-hand corner.
- Fig. 8. Stellarator expansion calculations of the vacuum-field rotational transform for a configuration with $\lambda = 2$, $m = 4$, $N = 36$, $R/a = 6.285$ for the Rehker-Wobig coil, $d_1/a = 0.2$, $d_2/a = d_3/a = 0.0$ (solid curve), the positive-slope configuration, d_1/a

$= 0.2128$, $d_2/a = 0.0181$, $d_3/a = 0.0128$ (broken curve), the negative-slope case, $d_1/a = 0.2128$, $d_2/a = -0.0181$, $d_3/a = 0.0128$ (dashed curve), and the more symmetric case $d_1/a = 0.24$, $d_2/a = 0.0$, $d_3/a = 0.04$ (dotted curve).

Fig. 9. Contribution to the rotational transform from the major α_i and β_i components for the symmetric case of Fig. 8.

Fig. 10. Magnetic surfaces with winding law of Eq. (12); $l = 2$, $m = 6$, $N = 36$, $R/a = 4.44$, $d/a = 0.3$, $\mu = 0.0$.

Fig. 11. Magnetic surfaces for the configuration of Fig. 10, but with $\mu = 0.6$. The dotted and dashed surfaces are at different toroidal angles.

Fig. 12. Magnetic surfaces for the configuration of Fig. 11, but with the coil locations given by Eq. (14) with $d_R/a = 0.025$ and $n = 1$. The dotted and dashed surfaces are at different toroidal angles.

Fig. 13. $\Delta\langle dL/B \rangle \equiv \left\{ \left[\frac{dL}{B} \right]_{r=\bar{r}} - \left[\frac{dL}{B} \right]_{r=0} \right\} / \left[\frac{dL}{B} \right]_{r=0}$, the variation of the well, for the configurations of (a) Fig. 10, (b) Fig. 11, and (c) Fig. 12.

Fig. 14. Variation of $Q \equiv \int dL/B$, evaluated over one toroidal field period of the system, for the configurations of (a) Fig. 10, (b) Fig. 11, and (c) Fig. 12. The surface when $\mu = 0.0$, $d_R/a = 0.0$ (case a) has an average radius $\bar{r}/a = 0.43$ and $\chi = 0.27$; when $\mu = 0.6$, $d_R/a = 0.0$ (case b), $\bar{r}/a = 0.30$ and $\chi = 0.27$; when $\mu = 0.6$, $d_R/a = 0.025$ (case c), $\bar{r}/a = 0.30$ and $\chi = 0.16$.

Fig. 15. Variation of the magnitude of the magnetic field along a field line for the magnetic surface at $\rho/a = 0.3$ in Fig. 12.

Fig. 16. Improvement of the rotational transform with elliptic distortion of the coils; $l = 2$, $m = 4$, $N = 36$, $R/a = 6.32$, $d/a = 0.14$ in Eq. (1); $u = 0.2$ when $e = 3/2$ and $u = 0.33$ when $e = 2$.

81 X 1363

Fig. 1

#81X0744

(a)

(b)

Fig. 2

#81X0742

Fig. 3

Fig. 4

#81X1360

Fig. 5

Fig. 6

#81X1361

Fig. 7

Fig. 8

#81T0363

Fig. 9

81T0375

Fig. 10

#81X0791

Fig. 11

#81X0790

Fig. 12

81X1367

Fig. 13

81X1368

Fig. 14.

Fig. 15

81X1365

Fig. 16