HIGH CURRENT BEAM TRANSPORT WITH MULTIPLE BEAM ARRAYS*

Charles H. Kim
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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ABSTRACT

Highlights of recent experimental and theoretical research progress on the high current beam transport of single and multiple beams by the Heavy Ion Fusion Accelerator Research (HIFAR) group at the Lawrence Berkeley Laboratory (LBL) are presented. In the single beam transport experiment (SBTE), stability boundaries and the emittance growth of a space charge dominated beam in a long quadrupole transport channel were measured and compared with theory and computer simulations. Also, a multiple beam ion induction linac (MBE-4) is being constructed at LBL which will permit study of multiple beam transport arrays, and acceleration and bunch length compression of individually focused beamlets. Various design considerations of MBE-4 regarding scaling laws, nonlinear effects, misalignments, and transverse and longitudinal space charge effects are summarized. Some aspects of longitudinal beam dynamics including schemes to generate the accelerating voltage waveforms and to amplify beam current are also discussed.

I. INTRODUCTION

During the past decade there has been considerable interest in high current beam transport and the stability of space charge dominated beams in accelerators, because of their potential applications to various fields such as inertial confinement fusion [1] [2], radiation testing facilities [3], neutron spallation sources [4], and other industrial applications [5]. In these accelerators, one seeks to have the space charge defocusing force almost equal to the average channel focusing force. Transverse and longitudinal beam containment and the stability of the beam is of major concern.

In most high current accelerators, transverse containment is achieved by alternating gradient (AG) focusing [6]. The beam envelope equations for AG focusing were derived first by Kapchinskij and Vladimirskij [7] using a rather specialized particle distribution -- a uniform distribution on the surface of a four dimensional sphere in transverse phase space (K-V distribution). Later, Lapostolle [8] and Sacherer [9] showed that the rms envelope equations for certain other distributions are identical to the K-V envelope equations if the rms emittances are used. The scaling laws derived from the envelope equations and some of their consequences are discussed in section II.
The Single Beam Transport Experiment (SBTE) was constructed in 1983 to test the stability of a space charge dominated beam in a long transport channel. It consists of 87 electrostatic quadrupole lenses, each of which is 10 cm long and 5 cm aperture diameter. The lattice half period is 15 cm. Typical beam parameters of the cesium +1 beam are: kinetic energy 120 keV, variable current (up to 13 mA), and variable normalized emittance (1.2 - 6.0 x 10^-7 pi rad . m). [10]*

An ion induction linac called the Multiple Beam Experiment (MBE-4) is being constructed at LBL [11] to test multiple beam transport arrays [12], and demonstrate acceleration and bunch length compression of space charge dominated ion beams [13]. Construction is scheduled to be completed late in 1986. It has 30 lattice periods each of which is 45.72 cm long and has an aperture diameter of 5.4 cm. It has 24 accelerating gaps; After each set of 4 accelerating gaps, the 5-th gap is left for diagnostic access. The 4-beam ion injector is presently operational [14]. Cesium ions, 200 keV, variable in current up to 15 mA per beamlet, and normalized emittance 1.2 x 10^-7 pi rad.m will be injected into the beam conditioning unit composed of B matching quadrupoles. A final energy of 800 keV, a current amplification of factor 3, and a spatial bunch compression of factor 1.5 are anticipated.

The transportable current limit in a channel increases as the beam is accelerated. The length of the bunch needs to be compressed in a carefully programmed fashion to exploit the increasing current-carrying capability of the channel. This topic is discussed in sections III and IV.

II. TRANSVERSE DYNAMICS OF A SINGLE BEAM

II.1 Scaling

Scaling laws for the current per channel (I), the maximum beam radius (a), and the length of the half lattice period (L) were obtained by Lambertson, Laslett, and Smith by transforming the K-V envelope equations into dimensionless forms [15]. The scaling laws for a matched beam is listed below:

\[ I = 7.86 \times 10^6 (A/q) (e^2 Q/u^2) (a/L)^2 (B_Y)^3 \]  \hspace{1cm} (1)

\[ a^2 = \frac{v^2}{\epsilon} \frac{\epsilon_a L}{B_Y} \]  \hspace{1cm} (2)

\[ L = \frac{\sqrt{(B_p)}}{B'} = \frac{\sqrt{\frac{a_0^2 v}{V q}}} \]  \hspace{1cm} (3)

where \( A \) is the atomic mass number, \( q \) charge state, \( \beta, \gamma \), the relativistic parameters, \( \epsilon \), normalized beam emittance, \( B_p \) beam momentum, \( B' \) magnetic quadrupole field gradient \( (B_0/a_0) \), \( a_0 \) aperture of the channel,

*Throughout this paper emittance is used in the sense of four times the rms normalized emittance.
VQ voltage on electrostatic quadrupole, and V is the kinetic energy of the beam in eV divided by the charge state q. The parameters ω, u and Q appear in the dimensionless envelope equations and the values have been tabulated by Laslett [16] as functions of σ and σ₀, where σ and σ₀ are the phase advance per lattice period of the betatron oscillations with and without the space charge defocusing forces. Some useful approximations to these values for electrode occupancy factor of 0.5 are shown below:

$$\frac{e^2 q}{u^2} = 6.95 \times 10^{-4} \sigma_0^{3/2} \left(1 - \frac{\sigma^2}{\sigma_0^2}\right)$$

and

$$\omega = 0.248 \sigma_0^{0.45}$$

Eq. (4) is accurate within 2% for 40° < σ₀ < 80°, and σ/σ₀ < 0.2. Eq. (5) is accurate within 1% for σ₀ < 90°. Similar approximate formulas were derived analytically by Reiser [17] and Lee, Fessenden, and Laslett [18].

Eqs. (1), (2), and (3) can be written in many convenient forms by choosing other relevant parameters as independent variables; for example, if the pole tip field, the beam emittance, and the kinetic energy are chosen as independent variables, Eq. (1) becomes functionally similar to the Maschke formula [19] but with a different numerical coefficient.

II.2 Stability

Until the late 1970's, stable beam transport was believed to be possible only if σ/σ₀ > 0.4 and σ₀ < 60°, based on the theoretical stability analysis for K-V beams [20]. This restriction had been one of the most important guide lines for designing high current accelerators during this period. During the last several years, experimental investigations and computer simulations using more realistic distributions have shown that space charge alone does not cause any emittance growth if σ₀ < 90°; only when strong space charge is coupled with misalignments and image charge effects, or with misalignments and lens nonlinearities, does the beam emittance grow [12] [28]. Rapid emittance growth followed by a saturation was predicted theoretically [21] [22], and, in deed, was observed experimentally if the particle distribution is not spatially uniform [37]. Some of the consequences of these new developments on the design of high current accelerators are discussed in this and the following section.

Effects of nonlinearities and misalignments are negligible in SBTE [23]. The initial distribution is approximately semi-Gaussian (flat in configuration space and Gaussian in velocity space) -- a typical feature of thermionic ion sources. In the space charge dominated regime (σ/σ₀ << 1), the semi-Gaussian nature is approximately preserved in the transport channel; in the emittance dominated regime (σ/σ₀ > 0.5), the spatial distribution is transformed in the channel and becomes more peaked [24].
Fig. (1) The measured stability boundaries in $\sigma - \sigma_0$ space. The region above the line marked "A" is accessible in SBTE. The envelope equation gives unstable solution in the shaded area [20]. (from Ref. 25)

The stability boundaries in $\sigma - \sigma_0$ space are discussed by M. Tiefenback and D. Keefe [25] and the results are reproduced in Fig. (1). A stable beam was inferred if the current and the emittance both remained unchanged in passing through the entire transport channel of 82 quadrupoles. These measurements largely agree with particle simulations using a semi-Gaussian distribution [26]. The simulations also showed stable beam transport for even smaller values of $\sigma$ such as $\sigma = 1.5^\circ$ and $\sigma = 60^\circ$.

An emittance growth was observed experimentally in SBTE in the normally stable region (e.g., $\sigma = 0^\circ$, $\sigma = 60^\circ$) when a deliberate misalignment was introduced. The result agrees with computer simulations with image charge forces [12] and will be discussed later in section III.1.

II.3 Adjustments of Focusing Channel as the Beam is Accelerated

Maximum transportable current increases with kinetic energy, but the exact functional dependence on $V$ is different in different limiting regimes discussed below. We assume here that the channel radius does not increase with kinetic energy for the simplicity of discussion.
(1) Low Energy Limit

For low energy beams, the quadrupole voltages are low and the transportable current limit is determined by the beam aspect ratio, a/L [15]. The value of a/L should be kept smaller than about 0.1 in order to avoid nonlinear effects. In this regime, the beam current scales approximately as \( \sigma_0^{3/2} \) as the quadrupole voltage is increased.

Thus the maximum transportable current per channel is limited by beam dynamics and not by any technological limits in this regime. For example, for a 1 MeV proton beam the maximum current is about 4 Amperes per channel. For higher total current, transverse stacking of many channels is required [15].

As the beam is accelerated in this regime, it is convenient to keep a and L constant and increase VQ proportionately with V so that \( \sigma \) is held constant. Since \( u = (1/\sigma)^{1/2} \), \( \sigma \) decreases as \( \beta \gamma^{-1} \). If \( \sigma/\sigma_0 \ll 1 \), the transportable current increases as \( V^{3/2} \). Appropriate longitudinal compression is necessary with the line density increasing as \( V \) to take advantage of the scaling.

(2) Breakdown limit

As the beam is accelerated further, the quadrupole voltage can not be increased indefinitely because of the technological limits. In this regime, the simplest configuration of the transport channel for an accelerating beam is to keep a and VQ constant and increase L as \( V^{1/2} \) to keep \( \sigma_0 \) constant. The value of \( \sigma \) does not change and the transportable current scales exactly as \( V^{3/2} \), leaving no room for longitudinal bunch length compression.

Varying the length of the quadrupoles continuously along the linac is not very convenient, but if L as well as a and VQ are kept constant, the maximum transportable current does not increase with kinetic energy.

(3) Magnetic Focusing and the Pole-Tip-Field Limit

For still higher kinetic energies, magnetic quadrupoles are more effective. Depending on the available maximum field values, the transition occurs around 0.1 - 0.5 MeV/amu. In ion induction linacs for HIF application, magnetic focusing consists most of the linac.

Again, if we let the value of \( \sigma \) vary, keep the maximum beam radius fixed, and increase L as \( V^{(1/4)} \) to keep \( \sigma_0 \) constant, then the transportable current scales as \( V \) and the line density as \( V^{(1/2)} \). The value of \( \sigma \) decreases approximately as \( V^{-1/4} \).

(4) Tune-Depression Limit

In a transport channel where misalignments and lens nonlinearities are not negligible, there may be a lower bound for the allowable values of \( \sigma \). If the aperture size and the magnetic field is held constant, L should be increased as \( V^{1/4} \) to keep \( \sigma_0 \) constant. If the current is increased as \( V^{3/4} \), \( \sigma \) decreases as \( V^{-1/4} \) and \( \sigma_0 \) remains constant.
A faster current amplification can be achieved if the aperture size is progressively decreased with the kinetic energy and the pole tip field strength is kept constant [27].

II.4 $\Delta \beta/\beta$ limit

If current amplification is required, the velocity of the tail is necessarily higher than that of the head of the bunch at a given location. If the quadrupole voltage does not vary in time at the given location, the energy and the zero-current-tune ($\sigma_1$) dependences of the beam current in Eq. 1 approximately cancel each other and thus the beam radius scales as $\sqrt{T}$. Therefore, an acceleration and bunch compression scheme which gives a constant current at a given location is convenient. This will be discussed in section IV.

Under these circumstances, the maximum beam radius remains constant for a constant current at a given location, but $\sigma_1$, $\sigma_0$, and the ellipticity of the beam shape will vary from the head to the tail of the bunch. These variations may cause mismatch oscillations and coherent betatron oscillations (if there are misalignments) which may cause particle loss if $\Delta \beta/\beta$ exceeds a certain maximum value.

III. TRANSVERSE DYNAMICS OF MULTIPLE BEAMS

In designing a multiple beam array, it is important to minimize the transverse dimensions in order to reduce the cost of the accelerator. The transverse dimension is determined by many considerations such as alignment tolerances, nonlinearities of the focusing field, image charge effects, beam-beam interaction, and others of a practical nature. In the present summary, we will consider only those effects which were important in designing the MBE-4 multiple beam array (Fig. 2).

Fig. (2) The MBE-4 multiple beam focusing array. (from Ref. 11)
III.1 Image Charge Effects

Effects on transverse beam dynamics of induced charges on the focusing electrodes was studied using a particle simulation code [12]. For a perfectly aligned beam, image charges did not cause any undesirable effects to the beam, unless the beam radius exceeded 85% of the aperture. For such large beams, particle loss occurred [26].

However, when the beam was misaligned and the tune was sufficiently depressed ($\sigma/\sigma_0 < 0.1$), an unacceptable level of emittance growth was observed (Fig. 3). The emittance growth was due to the excitation of coherent sextupole oscillations driven by the image forces [12].

III.2 Geometric Nonlinearities

Nonlinear field strengths of the MBE-4 multiple beam focusing arrays were calculated by relaxation codes for use in the simulation calculation. Three classes of nonlinearities were studied: (1) imperfections in the field of a single quadrupole, (2) nonlinearities due to the asymmetry of the geometry with respect to the beamlet of interest, and (3) end effects of the quadrupoles which are supported in an interdigital fashion. (see Fig. 2)

A dodecapole field is the most dominant nonlinearity of type (1). The dodecapole field vanishes only if the radius of the cylindrical electrode is 1.1457 times the aperture radius [29]. In MBE-4, the ratio was chosen to be 0.744 in order to increase the aperture size without increasing the overall size of the array. Another reason for this special ratio is explained below.

In the particle simulation study, if the beam was perfectly aligned, the dodecapole field did not cause any undesirable effect to the beam. However, if the beam was misaligned, and the tune was strongly depressed ($\sigma/\sigma_0 < 0.1$), and the image charge forces were neglected, an emittance growth similar to the one caused by the image forces was observed. One of the remarkable results of the simulation was that for the chosen electrode size the image forces suppressed emittance growth due to the dodecapole, or vice versa [12]. (See Fig. 3)

Effects on beam dynamics of the asymmetry nonlinearities (mainly a dipole field) and the end effects (mainly an octupole field) were found to be negligible in MBE-4 [28].

III.4 Alignment Tolerance

The effect of random misalignments of quadrupole electrodes was treated statistically by L. Smith [30]. As a result of the misalignments, a coherent betatron oscillation is excited. The probability of the amplitude of the oscillation being less than a value $\delta a$ is given by:

$$P(\delta a) = 1 - \exp \left( -\frac{\delta a^2}{A_{\text{rms}}^2} \right).$$

where $A_{\text{rms}}$ is a function of misalignments and the number of lattice periods. Thus the probability is 63% for $\delta a$ being less than $A_{\text{rms}}$ and 95% for $\delta a$ being less than $\sqrt{3}A_{\text{rms}}$. 

7
Fig. (3) Calculated emittance growth in MBE-4 for various conditions. The middle curve shows the growth of the beam emittance due to the image charge forces only. Emittance growth due to the dodecapole component only (not shown here) behaves similarly as the middle curve. Addition of a dodecapole field suppressed the emittance growth due to the image charge forces (lower curve). When the polarity of the dodecapole field was reversed (corresponds to larger size electrodes) a stronger emittance growth was observed (top curve). (from Ref. 12)

In MBE-4, a unit consisting of a doublet for each of the four beamlets (see Fig. 2) will be assembled on a bench and brought to the beamline subsequently. In this case, the following three types of misalignment are most serious: transverse misalignments of each lens (A); transverse misalignments of doublets (4); and angular tilt of doublets (α). The functional dependence of Arms on these misalignments is:

\[
A_{rms} = \left\{(11 \delta_{rms})^2 + (15 L e_{rms})^2 + (22 \Delta_{rms})^2\right\}^{1/2} \sqrt{N/30}
\]

where N is the number of lattice periods [30]. The coefficients are relatively insensitive functions of lattice parameters. As an example, if the alignment tolerances are such that \(\delta_{rms} = 2\Delta_{rms} = L e_{rms} = 0.14 \text{ mm}\), then \(A_{rms} = 3 \text{ mm}\) and the probability of the amplitude being smaller than 10 mm is 99.99%.

For a long wavelength sinusoidal misalignment, a coherent betatron oscillation will be excited resonantly if the wavelength of the misalignment is
comparable to that of the coherent betatron oscillation. The coherent oscillation will grow linearly with the distance. The calculated growth rate is 1 mm per wavelength if the misalignment is 0.25mm [31].

III.5 Optimization of the Transverse Dimension of a Multiple Beam Array

In order to minimize the transverse dimensions of a multiple beam array, it is sometimes convenient to maximize the effective current density, \( \frac{I}{aQ} \), where \( I \) is the current per channel and \( aQ \) is the aperture radius. This will insure that the overall radius of the array is minimized for a given total current.

We now assume that misalignments and lens nonlinearities are not negligible and that the maximum beam radius and the aperture radius are related as:

\[
\alpha + \delta a = \alpha a_Q \; ; \quad a_Q > \left( \frac{\delta a}{\alpha} \right),
\]

where \( \delta a \) provides the required beam clearance due to the misalignments of the channel and \( \alpha \) provides the required beam clearance due to the image charge forces, lens nonlinearities, mismatch oscillations and coherent betatron oscillations. (\( \alpha = 0.8 \))

For each of the transverse focusing regimes discussed in section II.3, one can now find an expression for the effective current density as a function of \( aQ \) only. As an example, consider an electrostatic quadrupole channel operated at the breakdown limit. The breakdown voltage is believed to be related to the aperture size as:

\[
V_Q = \text{const} \; a_Q^s
\]

where \( s \) is a constant approximately between 0.5 and 0.7 [32]. Eq.(9) can be used to eliminate \( V_Q \) in eq.(3), which in turn can be used to eliminate \( L \) in eq.(1). By using eq.(6) to eliminate \( a \) in eq.(1), one has an expression for \( I/aQ \):

\[
\frac{I}{aQ} = \text{const} \frac{Q}{\sqrt{V}} \left( \frac{\alpha a_Q - \delta a}{\alpha} \right)^{1/2} \frac{(4 - s)}{\alpha (4 - s)}
\]

which becomes maximum when:

\[
a_Q = \frac{\delta a (4 - s)}{\alpha (2 - s)}
\]

independent of the values of \( \alpha, a_Q \) and \( V \).

Similar optimizations give slightly different optimum values for different focusing regimes discussed in section II.3. For the aspect ratio limited regime the optimum aperture is \( 2\delta a/\alpha \); for the magnetic pole tip field limited regime the optimum aperture is \( 3\delta a/\alpha \).

The value of \( \delta a \) depends not only on the alignment tolerances but also on the frequency of beam steering and on the choice of the acceptable probability of losing the particles. (Eq. 6)
IV. LONGITUDINAL DYNAMICS

As discussed in section II.3, the current carrying capability of the transport channel increases with beam kinetic energy, making room for current amplification. Current amplification is a necessity during acceleration if the capability of the induction linac to accelerate high current is to be exploited. In order to realize current amplification, the tail has to be accelerated more than the head of the bunch in a carefully programmed fashion. It was also pointed out in section II.4 that a constant current at a fixed location is desirable because the beam radius scales as $1/T$ in a given quadrupole. Current fluctuations have to be minimized in order to prevent consequent particle losses and longitudinal emittance growth. A procedure to generate accelerating voltage wave forms which satisfy these conditions was devised by Kim and Smith. [33]

IV.1 Current Self-replicating Scheme

High current ion induction linacs have narrow accelerating gaps which are separated by bulky transverse focusing lenses. Both the particle velocity and the line charge density undergo discontinuous jumps as the bunch passes across accelerating gaps. However, the beam current is continuous across each gap.

A current self-replicating scheme is realized if all the particles emerging from an accelerating gap are headed toward a common focal spot in z-t space. The common focal spot is different for different gaps as illustrated in Fig. 4. In this scheme, the shape of the beam current waveform as a function of time at a fixed location is preserved at each position along the linac, as the beam is accelerated and compressed in time. If the current is constant in time at the injection point, it continues to be so at each point along the linac, a desirable feature for transverse focusing as discussed in section II.4. The longitudinal focusing feature is also convenient for certain applications where the bunch is required to be focused to a final focal spot. The space and time dependence of the current, line density, and the accelerating voltage waveforms were calculated analytically for this scheme. [33]

IV.2 Physics Requirements and Technological Constraints

In generating the accelerating voltage waveforms, there are the following physics requirements and constraints of a practical nature.

The requirements dictated by the beam dynamics are: (1) If the beam is not to increase in length on entering the accelerator, a situation which is difficult to overcome by subsequent manipulation of voltage wave forms, the head must not be accelerated before the tail has entered. (2) As explained in section II.4, the value of $\Delta B/B$ should not exceed a certain upper limit. This relative limit is encountered early in the accelerator, where $B$ is small. (3) Once free of the $\Delta B/B$ limit, with $B$ continuing to increase, one can use up the available margin in $\Delta B/B$ to initiate spatial compression of the bunch, by making the acceleration rate of the tail greater than that of the head at a fixed time. The resulting current amplification should follow the required energy dependence described in section III.3.
Fig. (4) An illustration of the current self-replicating scheme. Particles emerging from an accelerating gap are headed to a common focal spot. (From Ref. 33)

There are also constraints of a practical nature: (1) The flux swing of the magnetic material of an accelerating module should not exceed a certain upper limit determined by cost. (2) The acceleration rate should not exceed the high voltage breakdown limit. [33][34]

These requirements and constraints provide the necessary and sufficient conditions for determining the head and tail accelerating voltages at each gap; thus the common focal spot for each gap.

IV.3 Synthesis of the Accelerating Voltage Waveforms

MBE-4 can be operated in a wide range of beam parameters. An illustrative set of theoretically desired accelerating voltage waveforms for MBE-4 is shown in Fig. 5. These waveforms were approximately synthesized in a numerical study [13] by adding elementary waveforms which were similar to those obtained in laboratory tests [35][36]. They are curves of positive and negative initial curvatures and an almost flat-top curve with finite rise and fall times.
Fig. (5) An example of ideal accelerating voltage waveforms of MBE-4. (from Ref. 13)

A computer code was used to study the effects of the imperfect waveforms on the longitudinal beam dynamics in the presence of strong space-charge forces [13]. Current and kinetic energy errors which are reproducible from pulse-to-pulse can be corrected in principle to an arbitrary accuracy; however, fluctuations due to timing and voltage jitter cannot be corrected easily. The effects of jitter depend not only on the magnitude of the jitter but also on the way the waveforms are synthesized.

V. SUMMARY

The single beam transport experiment has shown that stable beam transport is possible in the highly space charge dominated regime where the space charge defocusing force is almost equal to the average channel focusing force. However, particle simulations showed that misalignments and the relative size of the beam to the aperture radius played important roles in the stability of space charge dominated beams.
A multiple beam ion induction linac (MBE-4) is under construction at LBL, which will permit study of three dimensional beam dynamics of space charge dominated beams as they are accelerated and compressed longitudinally. It will also permit study of effects of misalignments, image charge forces, and geometric nonlinearities on the stability and emittance growth of the beams.

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REFERENCES


(19) A. Maschke, Quoted by E.D. Courant, *Final report of ERDA Summer Study of Heavy Ions for Inertial Fusion*, 72 (1976)


(34) E.P. Lee also considered these characteristic regions independently. Private communications (1984).

