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Self-Potential Effects Due to Hydrothermal
Convection-Velocity Crosscoupling

by

W. R. Sill

Department of Geology and Geophysics
University of Utah
Salt Lake City, Utah

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ABSTRACT

The modeling of self-potential effects by the use of electric and convection currents (Sill, 1982a) is readily adaptable to the problem of crosscoupled flows due to hydrothermal convection. In this modeling of the electrokinetic effects, all that is required is a velocity model and models of the crosscoupling parameters and the electric conductivity.

A simple model of a two-dimensional, divergence-free velocity field is used to represent the essential features of a hydrothermal convection system. With this velocity field and the physical properties models, the electric-convection current algorithm is used to calculate the electric current sources and the resultant self-potential. In the examples presented, a positive self-potential anomaly is generated over a plume of rising hot water. The magnitude of the anomaly could easily be of the order of hundreds of millivolts.

INTRODUCTION

In a previous paper (Sill, 1981) I showed how the electric-convection current approach could be used to find the self-potential (SP) effects generated by the flow of fluid (electrokinetic) and the flow of heat (thermoelectric). In the examples presented in Sill (1981) the fluid flow was caused by a pressure gradient which resulted from sources and sinks of fluid. In Sill (1982a), I noted that the crosscoupled fluid flow problem could be reformulated in terms of the velocity of flow, so that the method is applicable to problems where the flow is not derivable from the gradient of a pressure. Such is the case in geothermal areas where the fluid flow is thermally driven mass-convection. The velocity field derived from a thermal-convection problem is essentially divergence free and therefore it can be represented by the curl of a vector.

In the following sections the crosscoupled fluid flow problem is reformulated in terms of the velocity field, and some simple examples of thermal-convection SP effects are presented.

VELOCITY CROSSCOUPLING

The crosscoupled equation for the total electric current (J_T) flow driven by a velocity field (V) and an electric field (E) is

$$J_T = L_V V + \sigma E , \quad (1)$$

where

L_V is the velocity crosscoupling conductivity,

and σ is the usual electric conductivity.

The velocity crosscoupling conductivity is related to the more familiar pressure crosscoupling conductivity (L_p) or streaming potential (C_p) by

$$L_V = L_p/k , \quad (2)$$

or

$$L_V = \sigma C_p/k , \quad (3)$$

where k = fluid permeability.

For no external current sources the divergence of the total current is zero and the source (S) for the electric potential (ϕ) is given by

$$S = \nabla \cdot J_{\text{cond}} = - \nabla \cdot (L_V V) = - \nabla L_V \cdot V - L_V \nabla \cdot V \quad (4)$$

where $E = - \sigma \nabla \phi$,

and $J_{\text{cond}} = \sigma E$.

Since we will be interested in modeling divergence-free fluid flow with this development, equation (4) reduces to

$$S = - \nabla L_V \cdot V \quad ; \quad \nabla \cdot V = 0. \quad (5)$$

Thus there are current sources which generate the electric potential (SP) in those regions where there is a gradient of the crosscoupling parameter in the same direction as the velocity. Given the velocity field, equation (5) can be used to find the electric current sources which can then be used to calculate the SP effects for a given electrical structure.

NUMERICAL MODELING

The transmission surface algorithm for the electric potential problem is outlined in Sill (1981, 1982a). For the two-dimensional, lumped-element network (Figure 1) used to solve for the electric potential the source terms (equation (5)) at a node (i,j) are

$$\begin{aligned} S(i,j) \Delta x \Delta z = & Y_x(i,j) V_x(i,j) \Delta x(j) - Y_x(i,j+1) V_x(i,j+1) \Delta x(j+1) \\ & + Y_z(i,j) V_z(i,j) \Delta z(i) - Y_z(i+1,j) V_z(i+1,j) \Delta z(i+1), \end{aligned} \quad (6)$$

where the network admittance terms are $Y_x = L_v \Delta z / \Delta x,$ (7)

and $Y_z = L_v \Delta x / \Delta z.$ (8)

For the numerical problem, the velocity field $V_x(i,j)$, $V_z(i,j)$ in the branches between the nodes must be known. With these and a model for the crosscoupling conductivity L_v , the electric current sources can be calculated with equation (6). The solution for the electric potential is straightforward and is discussed in more detail in Sill (1982a).

EXAMPLES OF VELOCITY-CROSSCOUPLED MODELS

Velocity and Temperature Models

In order to solve the electrical problems outlined previously, the velocity field for a divergence-free flow must be specified. In the study of any specific geothermal system this velocity field could be the result of the numerical modeling of the thermally driven convective system in the reservoir. In the examples that follow, an analytic expression for divergence-free velocity field will be used to illustrate the use of the algorithm and the type of response to be expected. The two-dimensional velocity field is given by

$$V_x = V_0 \frac{a^2}{(a^2 + x^2)} \beta x(1 - \beta z) \exp(1 - \beta z), \quad (9)$$

$$V_z = -V_0 a^2 \frac{(a^2 - x^2)}{(a^2 + x^2)^2} \beta z \exp(1 - \beta z), \quad (10)$$

where a determines the horizontal scale of the convection system and β determines the vertical scale. The maximum velocity V_0 occurs in the vertical direction at a depth where $\beta z = 1$. Contours of the velocity components in the vertical plane (x, z) are shown in Figures 2 and 3, for $a = .5$, $\beta = .5$ and $V_0 = 1$. V_x is antisymmetric in x and V_z is symmetric. In these figures negative values of V_x indicate flow to the left (negative x direction) and negative V_z is upward (negative z direction). The flow pattern that equations (9) and (10) describe is shown schematically in the inset in Figure 3. This is a relatively simple convection pattern but it contains the essential features of the thermal-convection patterns we might wish to model. The upward flow is concentrated in a thin plume between $x = \pm a$ and the outward flow is concentrated just below the surface.

In thermal convection problems of this sort, the temperature is proportioned to the vertical velocity (Lapwood, 1948) so that the isotherms show the same pattern as the contours in Figure 3. The positive temperature region (the "reservoir") is between the planes at $x = \pm a$ and the hottest point is at $x = x, z = 1/\beta$, where $V_z = -V_o$.

Convection in a porous medium will take place if the Rayleigh number (Ra) is greater than $4\pi^2$ (Combarrous and Bories, 1975), where the Rayleigh number is

$$Ra = g \alpha k (\rho c / \sigma_T) T_m \Delta z > 4\pi^2, \quad (11)$$

and g = acceleration of gravity,

α = rate of change of water density with temperature,

$(\rho c / \sigma_T)$ = thermal diffusivity,

T_m = maximum temperature change over the vertical distance Δz .

The results of Lapwood's (1948) study of convection in porous media can be used to show that

$$T - T_o \cong - 2V_z / \alpha g k, \quad (12)$$

Where T_o is the reference temperature where $V_z = 0$. Using this relation, the condition for convection can be written as

$$Ra = 2V_o(\rho c / \sigma_T)\Delta z > 4\pi^2, \quad (13)$$

Where V_o is the upward velocity at $T - T_o = T_m$. Using our model (equation (10)) for the vertical convection and $\Delta z = \beta^{-1} = 10^3$ meters, we find that the velocity V_o is at least 0.5 meter/year and the corresponding turnover time $(1/\beta V_o)$ is around a thousand years. With a maximum temperature change of 200°C , the permeability must be at least 60 millidarcy (equation (12)), which

does not seem like an unreasonable number. If the permeability is larger, the velocity will increase proportionally.

Equations (10) and (12) can be used to give an approximate relation for the temperature distribution in our model,

$$T(x,z) = T_o - T_m V_z(x,z)/V_o. \quad (14)$$

The isotherm pattern is similar to that in Figure 3. Since the temperature changes in a geothermal system can be appreciable, the effects of temperature on the electric conductivity and the crosscoupling parameter should be included in any modeling.

Crosscoupling and Conductivity Models

In order to calculate the electrical source terms (equations (6) to (8)), a model for the crosscoupling parameter (L_v) must be specified and finally the solution of the electrical problem requires a conductivity (σ) model. The specification of these models is guided by the geologic constraints and, as noted above, the effects of temperature should be included. A method for estimating the primary and crosscoupling parameters of rocks is discussed in Sill (1982b). According to this development and equation (2), the velocity crosscoupling parameter is

$$L_v = -8 \zeta \epsilon \exp(-2m_x) \quad (15)$$

where ζ = zeta potential (usually negative)

ϵ = dielectric constant of water

and m_x = mean of the lognormal distribution of pore sizes.

From the data in Ishido and Mizutani (1981) on the change in the streaming potential with temperature, we find that

$$\frac{1}{\zeta\epsilon} \frac{\partial \zeta\epsilon}{\partial T} \cong 10^{-2}/^{\circ}\text{C} . \quad (16)$$

Thus the change in crosscoupling as a function of temperature is

$$L_v(\Delta T) = L_v^0 (1 + 10^{-2} \Delta T) , \quad (17)$$

Where L_v^0 is the value at the reference temperature T_0 , and $\Delta T = T - T_0$ is the temperature change.

The resistivity of electrolytes depends on temperature in a manner similar to that of the viscosity, at least for temperatures below 300°C . The functional dependence of conductivity on temperature can then be given as

$$\sigma(\Delta T) = \sigma_0 \eta_0 / \eta(\Delta T) \quad (18)$$

where σ_0 , η_0 are the conductivity and viscosity at the reference temperature and $\eta(\Delta T)$ is the viscosity at the temperature T .

In the modeling that follows, the crosscoupling and conductivity models are specified at the reference temperature and equations (17) and (18) are used to adjust the values according to the temperature change calculated from equation (14).

Model Results

Figure 4 shows the surface potential profile for the velocity field of equation (9) and (10) and the two physical properties models shown in the inset. For model 1, curve 1 shows the results for a model which is homogeneous in L_v and σ , at the reference temperature (20°C). Since the temperature change at depth is proportional to the upward velocity (Figure 3), the velocity crosscoupling parameter and the conductivity are both elevated between $x = \pm a$, and they are slightly depressed for the region $x > |a|$. At z

$= \beta^{-1}$ and $T_m = 200^\circ\text{C}$, the maximum value for the crosscoupling is about 3 times its value at the reference temperature and the conductivity is increased by almost a factor of 10.

If in addition to the thermal effects, we reduce the crosscoupling exterior to the "reservoir" as in model 2; curve 2 (Figure 4) shows that the positive anomaly is increased. This increase is due to the suppression of negative current sources for $x > |a|$. Figures 5 and 6 show the contoured potentials in a vertical plane for models 1 and 2. The patterns are similar, with a strong positive region just below the surface and a negative region at depth.

If an overburden is included in the model so that the velocity goes to zero at some depth below the surface, we get the results shown in Figure 7. In this case the velocity field was translated downward by one-half a model unit giving an "overburden" of the same thickness. Comparing Figures 4 and 7 shows that the surface potentials have been reduced by 20 to 40 percent. Due to temperature corrections (equations (17) and (18)), the potentials are not linear with respect to the maximum temperature, T_m (equation (14)). Figure 8 shows the self-potential profiles for a maximum temperature of 100°C . The results for model 5 are about 40 percent smaller than those of model 1 in Figure 3, while the peak value for model 6 is only about 11 percent smaller than that for model 2 of Figure 4. The smaller reduction in the case of model 6 is due to the fact that there are strong sources at the boundaries at $x = \pm a$, due to the change in L_v , that are relatively unchanged by the temperature differences. In models 2 and 6, the physical properties changes are relatively more important than the temperature changes.

DISCUSSION

The potentials (ϕ_n) shown in the figures are in effect normalized in terms of the velocity, crosscoupling parameter, conductivity and the model dimensions but not in terms of the temperature. The real potential ϕ can be calculated from

$$\phi = \phi_n L'_v V_o' \Delta x' / \sigma' \quad , \quad (19)$$

where the prime indicates the real value. In the section on Velocity and Temperature Models, we discussed the case where

$$\beta^{-1} = 10^3 \text{ m} \quad ,$$

$$T_m = 200^\circ\text{C} \quad ,$$

and we found that

$$V_o = 2 \times 10^{-8} \text{ m/s},$$

and

$$k = 2 \times 10^{-11} \text{ (Mks)}.$$

For the given value of β

$$\Delta x' = 500 \text{ m},$$

and if we take

$$\sigma' = .01 \text{ s/m},$$

and

$$L'_v = 50. \text{ coul/m}^3,$$

we find that the true maximum potentials in Figure 4 are 13 mv for model 1 and 23 mv for model 2. The given value of L'_v corresponds to a streaming potential of about 10 mv/atm which is a common value for rocks. The self-potential anomalies generated in this case are rather modest; whether they are very realistic depends mostly on our estimate of V_o' . With the same maximum temperature, the velocity would increase as the permeability increases. Since the permeability used was also modest, it seems likely that larger velocities are reasonable. An increase in permeability and V_o' by ten would result in SP

anomalies of the order of hundreds of millivolts. An anomaly of this magnitude would be much more likely to be detected considering the usual noise level of tens of millivolts.

SUMMARY

The formulation of the crosscoupled fluid flow-electric potential problem in terms of the velocity of the fluid flow is straightforward. This formulation permits the calculation of SP effects due to thermal convection with a slight alteration of the algorithm outlined in Sill (1982a). In the modeling of SP effects, this algorithm can be combined with numerical solutions which provide the velocity field on a grid. In the examples presented, a simple two-dimensional velocity field is used to illustrate the technique. For these examples, positive SP anomaly is generated above the upwelling plume of hot water.

REFERENCES

- Combarnous, M. A., and Bories, S. A., 1975, Hydrothermal convection in saturated porous media, in Advances in Hydroscience, v. 10, ed. V. T. Chow, Academic Press, New York.
- Ishido, T., and Mizutani, H., 1981, Experimental and theoretical basis of electrokinetic phenomena in rock-water systems and its application to geophysics: J. Geophys. Res., v. 86, p. 1763.
- Lapwood, E. R., 1948, Convection of a fluid in a porous medium: Proc. Cambridge Philos. Soc., 44, p. 508.
- Sill, W. R., 1981, Self-potential modeling from primary flows: DOE/DGE Report No. DOE/ID/12079-42, Dept. of Geology and Geophysics, University of Utah.
- Sill, W. R., 1982a, Self-potential modeling from primary flows: Geophysics, to be published in volume 47.
- Sill, W. R., 1982b, A model for the crosscoupling parameters of rocks: DOE/DGE Report in preparation.

Figure Captions

- Figure 1. The two-dimensional, lumped element network which is used to calculate the current source at a node.
- Figure 2. Contours of the horizontal velocity (V_x) in the vertical plane. Negative values denote a velocity in the negative direction. V_x is antisymmetric about $x = 0$.
- Figure 3. Contours of the vertical velocity (V_z) in the vertical plane. Negative values denote a velocity in the negative direction. V_z is symmetric about $x = 0$.
- Figure 4. Self-potential profiles ($z = 0$) for models 1 and 2, where the velocity field extends to the surface and $T_m = 200^\circ\text{C}$.
- Figure 5. Contours of the self-potential in the x,z plane for model 1.
- Figure 6. Contours of the self-potential in the x,z plane for model 2.
- Figure 7. Self-potential profiles ($z = 0$) for models 3 and 4, where the velocity field is overlain by an overburden and $T_m = 200^\circ\text{C}$.
- Figure 8. Self-potential profiles ($z = 0$) for models 5 and 6, where the velocity field extends to the surface and $T_m = 100^\circ\text{C}$.

FIGURE 1

FIGURE 2

HORIZONTAL VELOCITY

$\alpha = .5$
 $\beta = .5$

$V_0 = 1.0$

FIGURE 3

SELF POTENTIAL PROFILE

$V_0 = 1.0$ MODEL 1 $L_1 = L_2 = 1$; $\sigma = 1$

$a = 0.5$

$\beta = 0.5$ MODEL 2 $L_1 = 1, L_2 = 0$; $\sigma = 1$

$T_{\max} = 200$

FIGURE 4

FIGURE 5

SELF POTENTIAL (ϕ_n) VERTICAL PLANE

MODEL 1 $L_1 = L_2 = 1; \sigma = 1$

$V_0 = 1.0$

$\beta = 0.5$

$\alpha = 0.5$

$T_{\max} = 200$

FIGURE 6

SELF POTENTIAL (ϕ_n) VERTICAL PLANE

MODEL 2 $L_1=1, L_2=0; \sigma=1$

$V_0=1.0$
 $\alpha=0.5$

$\beta=0.5$
 $T_{\max}=200$

SELF POTENTIAL PROFILE

$V_0 = 1.0$ MODEL 3 $L_1 = L_2 = 1$; $\sigma = 1$
 $\alpha = 0.5$
 $\beta = 0.5$ MODEL 4 $L_1 = 1, L_2 = 0$; $\sigma = 1$
 $T_{\max} = 200$

FIGURE 7

SELF POTENTIAL PROFILE

$V_0 = 1$ MODEL 5 $L_1 = L_2 = 1 ; \sigma = 1$
 $a = .5$
 $\beta = 0.5$ MODEL 6 $L_1 = 1, L_2 = 0 ; \sigma = 1$
 $T_{\max} = 100$

FIGURE 8