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Intermediate Simulation of
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John E. Brolley

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INTERMEDIATE SIMULATION OF THE INVERSE SEISMIC PROBLEM

by

John E. Brolley

ABSTRACT

An introductory study of the inverse seismic problem is performed. The complex cepstrum of a seismogram generated by the convolution of three factors 1) the Seggern-Blandford source function of an explosion, 2) the Futterman mantle transfer function, and 3) the SRO seismometer transfer function, is used. For a given Q and yield, a synthetic seismogram is computed. Arbitrary values of Q and yield are introduced and a search is conducted to find that pair of values which minimized the cepstral difference between the original and arbitrary seismograms. The original values are accurately recovered. Spectral and amplitude characteristics of the various factors are presented. Possible application to the problem of studying a medium intervening between a source and receiver is discussed.

1. INTRODUCTION

The inverse seismic problem is defined as the process of tracing a seismic signal backwards from the seismometer to the source. The goal in this case is to learn as much as possible about the source. Information concerning the intervening media that conduct the signal is also sought where it is not known. This problem^{1,2} and its inverse have been studied for some time.

The present study presents a simplified problem wherein the earth is regarded as a one-dimensional system with the seismic source and seismometer embedded in a mantle as depicted in Fig. 1. The approach is exegetical and provides a foundation for more elaborate calculations wherein the effects of the earth's crust may be considered as well as more complicated mantle models.

A convenient model for the underground nuclear explosion is that of Seggern and Blandford.³ Futterman's⁴ descriptor of the single layer mantle is

Fig. 1. A simplified model for the inverse seismic problem.

used for the propagating medium. Transfer functions of the Albuquerque Seismic Research Observatory⁵ (ABQ/SRO) are used to model the seismometer response.

In the following, some of the characteristic features of these three terms are presented and then a synthesis is effected to produce a seismogram. The complex cepstrum of the synthetic seismogram is computed for specific values of Q and yield of the underground explosion. A second complex cepstrum is computed for some arbitrary Q and yield. The cepstral difference is taken and an optimization driver seeks to find those values of the arbitrary Q and yield that minimize the cepstral difference. This procedure is then considered in another context: the study of a transmitting medium intervening between a source and receiver.

II. SEISMOMETER TRANSFER FUNCTION

A specific short period transducer will be considered. The worldwide SRO network is nearly completed and represents a modern, high-performance, completely digital system. Average transfer functions are known for all of the installations. A measured transfer function exists for the ABQ/SRO and will be used here.

The transfer function of the seismometer is an expression in the complex domain that describes the frequency response and phase characteristics. It is defined in a frequency space. Its counterpart in time space is a real quantity called the impulse function. The latter is the seismogram that would be obtained if the seismometer were impinged by a pulse of very short duration.

It is possible to develop a transfer function from first principles^{6,7} by considering the properties of the various elements, both mechanical and electrical, that constitute the seismometer. Such a transfer function can be further refined by empirical measurements, thus taking into account manufacturing tolerances and other deficiencies. From such considerations the ABQ transfer function is obtained.

$$I(s) = s^4(s + z_1)(s + z_2)/(s + p_1)/(s + p_2)/(s + p_3)/(s + p_4)/(s + p_5)/(s + p_6)/(s + p_7)/(s + p_8)/(s + p_9)/(s + p_{10})/(s + p_{11})/N, \quad (1)$$

where s is the angular frequency and the z and p are the zeroes and poles, respectively. Particular values, as determined by measurement, for ABQ/SRO are given in Table I. The normalizing constant N is chosen so that gain of the seismometer will be unity at 1 Hz. In what follows certain other multiplicative constants associated with the seismometer and source function will be ignored, as they are not essential to the present discussion.

The gain of the seismometer may be calculated either in the frequency domain or in the time domain. Consider the frequency domain. The real and imaginary parts of Eq. (1) are plotted in Figs. 2 and 3, respectively. The

TABLE I

Poles and zeroes of the short period
ABQ/SRO as determined by measurement

$$N = 1.533 \times 10^{-10}$$

Zeroes		Poles	
Real Part	Imaginary Part	Real Part	Imaginary Part
0.0000	0.0000	4.4597	-3.3985
0.0000	0.0000	4.4597	3.3985
0.0000	0.0000	0.1514	0.0000
0.0000	0.0000	38.5700	0.0000
0.1621	0.0000	100.0000	0.0000
47.6200	0.0000	0.1613	0.0000
		264.0000	0.0000
		16.7300	-3.3970
		16.7300	3.3970
		63.2900	0.0000
		63.2900	0.0000

Fig. 2. Real part of the complex ABQ/SRO transfer function.

Fig. 3. Imaginary part of the complex ABQ/SRO transfer function.

gain is given by the absolute value of Eq. (1) and is plotted in Fig. 4. The inverse Fourier transform of the transfer function yields the impulse function in the time domain. This is plotted in Fig. 5. This curve represents the time series output of the seismometer when it is excited by a displacement of very short duration. The gain in the time domain can be calculated by convoluting the impulse function with a sine wave. This convolution is given by Eq. (2).

$$s(t) = \int_{\tau=-\infty}^{\tau=\infty} A(\tau) B(t - \tau) d\tau, \quad (2)$$

where $s(t)$ is the seismometer output, $A(\tau)$ is a sine wave, and B is the impulse function of Fig. 5. Figure 6 displays the result of evaluating Eq. (2) with a 1-Hz sine wave of unit amplitude. Evaluation of Eq. (2) with unit amplitude sine waves of different frequencies reproduces the gain curve of Fig. 4. En passant it may be noted that the phase characteristics are also easily obtained in either the frequency or time domain.

The foregoing has been a description of the short period seismometer in terms of quantities needed for the present calculation. In terms of actual hardware, the short period vertical component of the seismic signal is sensed using a La Coste suspension. Electrical signals are generated by a capacitive transducer mounted between the frame and moving mass. The period and damping are controlled by feedback.

Fig. 4. Gain of the ABQ/SRO seismometer.

Fig. 5. Impulse function of the ABQ/SRO seismometer.

III. THE MANTLE

In this idealized problem the seismometer is embedded in an idealized mantle that is assumed to be homogeneous. In a detailed calculation of tele-seismic body wave propagation, modifications would be introduced. For example, it is known⁸ that body wave velocity is not independent of depth. This effect is illustrated in Fig. 7. The step-like discontinuities are thought to be associated with phase changes. Another example of an effect that would be incorporated in a detailed calculation is lateral asymmetry.

However, in first approximation the mantle may be regarded as being free of these various effects and an ideal transfer function may be calculated for it.

In this connection it is helpful to briefly summarize the somewhat beclouded knowledge of wave propagation in seismic media. Ricker⁹ has proposed that in the low-frequency seismic band the attenuation is proportional to the square of the frequency once the velocity is constant. Above a certain transition frequency, the velocity and attenuation increase as the square root of the frequency. He conducted some experiments in shale that were in remarkable agreement with his theory, although no direct attenuation measurements were made. Other experiments of Collins and Lee¹⁰ in sandstone indicate that the attenuation goes as the first power of the frequency. McDonal et al.¹¹ have made a detailed set of measurements in shale and observed that the attenuation varies as the first power of the frequency and could discern no dispersion.

Fig. 6. Output of the ABQ/SRO convolution integral for a 1-Hz unit-amplitude sinusoidal input.

Fig. 7. Body-wave velocity dependence on mantle depth.

The Boltzman superposition model¹⁰ has been used to construct a theory that requires the attenuation constant to vary with some power, not equal to unity, of the frequency. It may be possible in this theory to have the power approach closely to unity. Another approach by Knopoff and MacDonald¹² uses small amplitude perturbation of linear theory. This is predicated on the observation that the experimental data is evidence for the failure of Hooke's law, and therefore a nonlinear theory is required.

The discussion continues (e.g., Ref. 13), and it would appear that a clear picture is not yet in hand.

Nonlinear theories imply the failure of linear superposition. In view of the utility of this principle, most investigators have sought an ansatz that will preserve it. Futterman⁴ has approached the problem by assuming the following boundary conditions appertain: the absorption coefficient varies linearly as the frequency; linear superposition is valid; causality is valid. The last boundary condition is implemented by means of the Kramers-Kronig dispersion relations. His approach invokes no physical model and arrives at the mantle-gain expression

$$M(\omega) = \exp \left\{ \frac{\omega R}{v} \left[-\frac{1}{2Q} \left(1 - \exp \frac{-\omega}{\omega_0} \right) + j \left(1 - \frac{1}{\pi Q} \ln \frac{\varepsilon \omega}{\omega_0} \right) \right] \right\}, \quad (3)$$

$\ln \epsilon = 0.5772157$, ω_0 is a low-frequency cutoff that is required by the theory and below which no absorption occurs. $\omega_0 = 10^{-3}$ is used. ω is the angular frequency at which the gain is evaluated, R is the distance traversed, and v is the phase velocity at ω_0 . Q is a dimensionless quantity that has the same significance as in circuit theory: $2\pi/Q = \Delta W/W$, where $\Delta W/W$ is the fractional loss of energy per cycle.

In what follows, arbitrary values $Q = 10^3$, $v = 10^4$ m/s, and $R = 10^6$ m are used except where noted. The real and imaginary parts of the mantle gain-function are plotted in Figs. 8 and 9. The gain of the mantle, as computed from the absolute value of Eq. (3), is plotted in Fig. 10. A coarse plot of the gain near zero frequency is shown in Fig. 11. Granularity of plotting restricts the plot from going to ω_0 . The theory assumes there is no absorption in the gap. The gain of the product system, seismometer and mantle, as determined from the absolute value of the product, is plotted in Fig. 12. Inverse Fourier transformation of the complex product gain produces the impulse function shown in Fig. 13. As before, this can be convoluted with a sine wave to evaluate the gain in time space. Figure 14 shows the results of convolving with a 2-Hz sine wave of unit amplitude, a result that verifies the gain curve in Fig. 12.

The system of seismometer and mantle is now ready to receive the excitation of an underground nuclear explosion.

Fig. 8. Real part of the mantle gain-function.

Fig. 9. Imaginary part of the mantle gain-function.

Fig. 10. Mantle gain curve.

Fig. 11. Mantle gain near zero frequency.

Fig. 12. Gain of the combined system of mantle and seismometer.

Fig. 13. Impulse function of the combined system of mantle and seismometer.

Fig. 14. Convolution of a 2-Hz unit-amplitude sine wave with the combined impulse function.

IV. EXPLOSION SOURCE FUNCTION

An underground nuclear explosion induces inelastic behavior in the confining medium out to some radius where it effectively becomes elastic. At this point the ground motion can be described in terms of a source function¹⁴ that is parametrized in terms of source parameters and the medium.

If the general wave equation for propagation of displacement and rotation is specialized to consider displacement, radial in this case, then the wave equation can be solved in terms of a scalar potential. The radial displacement is then given in terms of the derivative of this scalar potential. In the literature of underground nuclear explosions, this is often referred to¹⁵ as the reduced displacement potential when the scalar potential $\psi(r,t)$ is written such that $u(r,t) = \frac{\partial}{\partial r} \left[\frac{\psi(\tau)}{r} \right]$, where u is the displacement and $\tau = t - r/c$ is the retarded time, c the wave velocity, t the time, and r the distance from the source. The reduced displacement potential has been modeled by Haskell¹⁶ to be of the form

$$\frac{\psi(\tau)}{\psi(\infty)} = 1 - e^{-kt} \left[1 + kt + \frac{(kt)^2}{2} + \frac{(kt)^3}{6} - B(kt)^4 \right]. \quad (4)$$

Seggern and Blandford¹⁷ have obtained an improved correspondence with experimental data if it is written as

$$\frac{\psi(t)}{\psi(\infty)} = 1 - e^{-kt} [1 + kt - B(kt)^2] , \quad (5)$$

where k is medium dependent and assumed to scale as the inverse cube root of the yield, B is a medium dependent parameter, and t is the retarded time referred to the elastic boundary. The far-field displacement is obtained from Eq. (5) and is, apart from some multiplicative constants,

$$-k^2 t e^{-kt} (B k t - A') , \quad (6)$$

where $A' = 2B + 1$. The forward Fourier transform is

$$- \frac{2B}{(1 + j \frac{\omega}{k})^3} + \frac{2B + 1}{(1 + j \frac{\omega}{k})^2} , \quad (7)$$

where ω is the angular frequency. Other source functions¹⁷ have been proposed that have different frequency distributions.

The real and imaginary parts of Eq. (7) are displayed in Figs. 15 and 16 using the arbitrary values³ $k = 16.8$ and $B = 2.04$. A machine calculation of the inverse Fourier transform of Eq. (7) produces the far-field displacement shown in Fig. 17. A plot of the analytical expression, Eq. (6), is shown in Fig. 18 for comparison.

Fig. 15. Real part of the source function frequency distribution.

Fig. 16. Imaginary part of the source function frequency distribution.

Fig. 17. Machine inversion of the frequency representation of the source function.

Fig. 18. Far-field displacement as given by the analytical representation in time space.

With all three elements, seismometer, mantle, and source function described in frequency space, the synthetic seismogram may be constructed.

V. SYNTHETIC SEISMOGRAM

In the stage of approximation that considers a simple mantle and ignores crustal contributions, the seismometer time series can be written

$$s(t) = f(t) * m(t) * i(t) \quad , \quad (8)$$

where f is the impulse from the explosion, m is the mantle response to f , i represents the modification of the incident signal by the seismometer, and $*$ denotes the convolution operation.

In frequency space the corresponding relation is

$$S(\omega) = F(\omega) \cdot M(\omega) \cdot I(\omega) \quad , \quad (9)$$

where $S(\omega)$ denotes the forward Fourier transform of $s(t)$, etc. Machine inversion of Eq. (9) has been done for the following values; $v = 10^4 \text{ m/s}$, $R = 10^6 \text{ m}$, $\omega_0 = 10^{-3}$, $Q = 10^3$, $k = 16.8$, and $B = 2.04$. The result is the synthetic seismogram displayed in Fig. 19.

Fig. 19. Synthetic seismogram.

For some types of analysis it may be desirable to break down a seismogram into components, thus reversing the procedure followed so far. The technique of cepstral decomposition will be considered here.

VI. THE COMPLEX CEPSTRUM

The time series, Eq. (8), is a sequence of convolutions. A transformation is desired that changes it to a space where the factors are additive. An operator that achieves this is the complex cepstral operator (CCO). Thus, operating on Eq. (8) with the CCO produces

$$\hat{s}(T) = \hat{f}(T) + \hat{m}(T) + \hat{i}(T) \quad . \quad (10)$$

The variable is time. A description of several forms of CCOs is given in Refs. 18 and 19.

Before continuing the present problem, it is useful to consider another formal example to gain further appreciation of CCO applications. Consider the more complete formulation of the inverse seismic problem where crustal contributions are included. The seismometer time series is then

$$s(t) = f(t) * c_1(t) * m(t) * c_2(t) * i(t) \psi \quad . \quad (11)$$

The underground explosion is contained in the crustal region described by c_1 . The seismometer is contained in the crustal region described by c_2 , and ψ is a geometrical factor. Application of the CCO yields

$$\text{CCO} \left\{ \frac{s(t)}{\psi} \right\} = \hat{f}(T) + \hat{c}_1(T) + \hat{m}(T) + \hat{c}_2(T) + \hat{i}(T) . \quad (12)$$

If two different explosions are fired in approximately the same region and observed by the same distant seismometer, then

$$\text{CCO} \left\{ \frac{s_1(t)}{\psi} \right\} - \text{CCO} \left\{ \frac{s_2(t)}{\psi} \right\} = \hat{f}_1(T) - \hat{f}_2(T) , \quad (13)$$

and one can study the difference in the source functions without considering the other factors. If the same explosion is observed by two identical seismometers in different locations

$$\begin{aligned} \text{CCO} \left\{ \frac{s_1(t)}{\psi_1} \right\} - \text{CCO} \left\{ \frac{s_2(t)}{\psi_2} \right\} = \\ \hat{c}_1(T) - \hat{c}_2(T) + \hat{m}_1(T) - \hat{m}_2(T) + \hat{c}_1(T) - \hat{c}_2(T) , \end{aligned} \quad (14)$$

one can study the differences of the terrestrial components without invoking source or transducer effects.

Operating on the simpler three component seismogram, Fig. 19, with a CCO of adaptive phase type¹⁸ yields the complex cepstrum shown in Fig. 20. It is

Fig. 20. Complex cepstrum of the synthetic seismogram.

a real time series. The appellation complex is applied to differentiate it from other types of cepstra.²⁰ Most of the cepstral energy is concentrated in the low time region. If reverberatory terms had been included in Eq. (8), the complex cepstrum would display energy at higher times.²¹⁻²³ Thus in the present context the crustal terms might include reverberation. In geophysical exploration work the associated analysis may seek to separate source and reverberatory effects. This is done by using high-time or low-time filtering of the complex cepstrum.

In order to transform the results of operating in the cepstral domain back to the original time domain containing the convolutional sequence, the inverse CCO is constructed,¹⁸ CCO^{-1} . Application of CCO^{-1} to the complex cepstrum of Fig. 20 regenerates the original time series as shown in Fig. 21.

Other operations than high- or low-time filtering can be performed in the cepstral domain. In particular, the minimization of cepstral difference to recover parameters in the original series can be done.

VII. CEPSTRAL MINIMIZATION BY AN OPTIMIZATION DRIVER

As an example the time series of Fig. 19 may be regarded as containing two unknowns, the yield, Y , of the explosion and the Q of the mantle. It is actually constructed from a $Q = 10^3$ and a yield of $Y = 1$ arbitrary unit. If a complex cepstrum for $Q = 1.2 \times 10^3$ and $Y = 1$ is constructed, the difference in the two complex cepstra ($Q = 10^3$, $Q = 1.2 \times 10^3$) can be computed and displayed in Fig. 22. There is substantial energy in the difference. Moreover, according to the ideas embodied in Eqs. (13,14), it may be inferred that only the mantle differences are involved. The source and seismometer differences are zero. In addition to Q , Y may be added as a search variable and cepstral differences constructed. It is possible to apply an optimization driver to this problem to minimize the sum of squares of the cepstral differences at each point in time. For this problem OPAC²⁴ was used to attempt to find values of Q and yield that satisfied the original seismogram, Fig. 19. OPAC contains, among other things, a version of the Levenburg-Marquardt algorithm. Starting values of $Q = 1.2 \times 10^3$, $Y = 0.1$ were used. OPAC made a simultaneous search over both variables and returned the values $Q = 9.9999999999760 \times 10^2$ and $Y = 1.00000000014785$, which agree with the original values.

Fig. 21. Regeneration of the original seismogram from the complex cepstrum.

Fig. 22. Difference of complex cepstra for $Q = 10^3$ and $Q = 1.2 \times 10^3$.

The cepstral optimization calculation can be extended to more complicated problems involving more variables. Other types of optimization drivers are available also.

VIII. OTHER PROBLEMS

Other convolutional problems in astrophysics, e.g., pulsar pulse shapes, physiology (speech structure), and geophysics may be contemplated. A conjectural application to a specific geophysical problem will be delineated.

A classic problem is to determine the nature of a medium between two bore holes by noninvasive techniques. The problem is illustrated schematically in Fig. 23. The radiation source may be either acoustic²⁵ or electromagnetic,^{14,26} although an acoustic source is sketched. Equation (8) applies except that f now represents the cyclable acoustic or electromagnetic source, m contains information about the propagating medium, and i represents the characteristics of the receiver.

The actual impulse fed to the medium by the source is sometimes not known as well as desired because of variable and uncertain coupling.^{11,25} A similar situation may prevail at times for the receiver. Suppose the problem to be specialized to the case where the receiver is a satisfactorily implanted seismometer of known transfer function. The problem then becomes that of modeling the transfer function of the propagating medium and then estimating its and the

Fig. 23. Schematic diagram of a configuration to noninvasively study a geologic structure.

source's characteristics from the seismometer data. In the cepstral space, Eq. (8) becomes Eq. (10). The goal would be to obtain an estimate of $\hat{m}$, which could then be subjected to geophysical modeling to estimate medium parameters. However $\hat{f}$ is often not well known.

It may be possible to exploit the nonlinear properties of CCO to obtain an estimate of $\hat{f}$. Suppose the source has variable power output and constant frequency distribution. Then for two different measurements when the power levels are varied

$$\hat{s}_i(T) - \hat{s}_j(T) = \hat{f}_i(T) - \hat{f}_j(T) \quad . \quad (15)$$

The effect of the nonlinear operator CCO on f can be observed by comparing the complex cepstra of the Seggern-Blandford source function shown in Figs. 24 and 25. They correspond to an amplitude change of a factor of 2. It may then be possible to obtain an expansion of $f(t)$ in terms of a set of orthonormal functions that satisfies a sequence $\Delta_{ij} = \hat{s}_i(T) - \hat{s}_j(T)$. It may be useful to supplement the acoustic studies with electromagnetic measurements. In the case of a fractured propagation medium, this measurement might be enhanced by filling the fractures with brine. The validity of this and other schemes, of course, must be validated by actual computations.

IX. CONCLUSIONS

The inverse seismic problem has been formulated in terms of three components. Others can be added. The components were a Seggern-Blandford model

Fig. 24. Complex cepstrum of the Seggern-Blandford source function for a unit amplitude factor.

Fig. 25. Complex cepstrum of the Seggern-Blandford source function with the amplitude factor doubled.

of the underground explosion, Futterman's crustal model, and a representation of the short period ABQ/SRO seismometer. A synthetic seismogram for a given mantle Q and yield was constructed as a datum. Minimization of the cepstral difference between the datum and seismogram for arbitrary Q and Y permitted the original values of Q and Y to be recovered. The next stage is to study the introduction of noise and then crustal factors. More complicated mantle functions may also be developed. The introduction of real data could then commence.

It may be that variations of the three-component problem can be applied to the study of propagating media between bore holes.

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