SINGLE PASS COLLIDER MEMO

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TITLE: EMITTANCE GROWTH IN THE COLLIDER ARCS DUE TO TRANSVERSE WAKEFIELDS

The three main components of the dipole wakefield in the collider arcs are the resistive wall wake due to the finite conductivity of the vacuum chamber wall and the wakes due to discontinuities at the beam position monitors and vacuum chamber bellows. In the first section we estimate the dipole wakefield arising from the resistive wall wake. In the second section results of TBCI calculations of the wakes due to the BPM's and bellows are presented, and compared with the resistive wall calculation. In the final section the total wakefield is used to compute the emittance growth.

Resistive Wall Wake

The effective transverse deflecting field at a distance z behind a single electron of charge e traveling at a distance x_0 off axis in a round pipe of radius b is^{1,2},

$$E_1 = \frac{2e x_0 (c/\sigma)^{1/2}}{\pi(4\pi \epsilon_0)^{1/2} b^{3/2} z^{1/2}} \quad \text{mks units} \quad (1a)$$

$$= A \cdot \frac{x_0}{b^{3/2} z^{1/2}} \quad \frac{V}{m} \quad (1b)$$

Here σ is the dc conductivity, and $A = 2.5 \times 10^{-14} \text{ V}\cdot\text{m}^{3/2}$ for copper assuming a conductivity which is 90% of the handbook value), and $1.5 \times 10^{-13} \text{ V}\cdot\text{m}^{3/2}$ for stainless steel. We note that this relation does not hold all the way down to $z = 0$, since a point charge would then deflect itself. E_1 deviates from Eq. (1) at very small distances and reaches a maximum at a characteristic distance on the order of

$$z_m \approx (Z_0 \sigma)^{1/3} b^{2/3} \quad \text{mks units} \quad (2)$$

(here $Z_0 = 377 \Omega$), and then falls to zero for $z < z_m$. Thus $z_m = 0.04 \text{ mm}$ in a stainless steel pipe of radius $b = 5 \text{ mm}$. We can use Eq. (1) to computing the integrated wake field in a bunch of length σ_z if $\sigma_z \gg z_m$. For $\sigma_z = 1 \text{ mm}$, the use of Eq. (1) to

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compute the integrated wake field will give a slight overestimate.

The integrated wake field for a bunch of N particles with a density distribution $n(z)$ is

$$E_1(z) = \int_z^\infty E_1(z'-z)n(z')dz'.$$

For a gaussian distribution this becomes

$$\begin{aligned} E_1(z) &= \frac{NA x_0}{\sqrt{2\pi} b^3 \sigma_z} \int_z^\infty \frac{\exp(-z'^2/2\sigma_z^2)}{(z'-z)^{1/2}} dz' \\ &= \frac{N x_0}{b^3 (\sigma_{dc})^{1/2} (\sigma_z)^{1/2}} \cdot F(z/\sigma_z), \end{aligned} \quad (3)$$

where F is a universal function of z/σ_z . This expression is plotted in Fig. 1 for $x_0 = 1$ mm and $\sigma_z = 1$ mm for a copper pipe with radius $b \approx 5$ mm. Note that the peak value of the integrated wake is $\hat{E}_1 = 315$ V/m at a position $0.75 \sigma_z$ behind the bunch center. This can be scaled to $b = 4$ mm and $\sigma_z = 0.5$ mm, giving a peak value of 870 V/m. This agrees with the result obtained by Stiening² in CN-20.

In the emittance calculation to follow we will need the effective slope dE_1/dz of the wakefield over the region of the bunch where the current is significant, say between $\pm \sigma_z$. The dashed line in Fig. 1 gives the average slope of the wake over this interval,

$$\left\langle \frac{dE_1}{dz} \right\rangle = 1.0 \times 10^5 \text{ V/m}^2.$$

This corresponds to a slope $\hat{E}_1/3.2\sigma_z$ where $\hat{E}_1$ is the peak value of the wake.

The longitudinal resistive wall wake is also of interest, since this will lead to an energy loss and an increase in energy spread. The average energy loss per particle for a pipe of length L is³

$$V = (2.05 \times 10^{-11} \text{ V-m}/\Omega^{1/2}) \frac{NL}{b \sigma_z^{3/2} \sigma_{dc}^{1/2}} \text{ mks} \quad (4)$$

Using $N = 5 \times 10^{10}$, $L = 1.23$ km, $b = 5$ mm, $\sigma_z = 1$ mm, and $\sigma_{dc} = 1.3 \times 10^6 (\Omega - m)^{-1}$ for stainless steel, we compute that for one arc of the collider $V \approx 7$ MV. The energy spread will be about twice this, since the leading electrons experience no loss and the trailing electrons twice the average loss, giving $\Delta E/E_0 \approx 3 \times 10^{-4}$ for $E_0 = 50$ GeV.

Bellows and Beam Position Monitors (BPM's)

Karl Bane has computed the dipole wakefields for the bellows and BPM designs contemplated for the SLC arcs using TBCI. An example of the integrated wake for a bellows with 18 convolutions is shown in Fig. 2. Note that the form of the integrated wakefield differs from the resistive wall wake in that it reaches a peak considerably further behind the bunch center. A total of 230 bellows per arc were assumed, each having 30 convolutions, with an average spacing between bellows of 5.2 m.

Since the BPM electrodes are not cylindrically symmetric, we had to make the following approximation. Let f be the fraction of a complete circle which is filled by the electrodes. If W_h is the wake due to the housing alone without electrodes and if W_e is the wake due to the housing with a fully cylindrical electrode (no gaps), then the net BPM wake is taken to be

$$W_{BPM} = f W_e + (1 - f) W_h.$$

Again, 230 BPM's per arc were assumed, with an average spacing of 5.2 m.

In the emittance calculation to follow, the slope of the total wake along the bunch due to the arc components will be required. The slopes of the various components, and the total slope, are given in Table 1 for $\sigma_z = 1$ mm. The resistive wall wake is given for both copper (Cu) and stainless steel (SS).

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Table 1

Component		$\frac{dE_{\perp}}{dz} \cdot \frac{\sigma_z}{x_0}$
Resistive Wall (ID = 2 x 5.3 mm)	Cu	$0.9 \times 10^5 \text{ V/m}^2$
	SS	5.0×10^5
BPM's		2.5×10^5
Bellows		4.6×10^5
Total ($\sigma_z = 1 \text{ mm}$)	Cu	8.0×10^5
	SS	12.1×10^5

Effect on Wake due to Bunch Shortening in the Arcs

The longitudinal wake in the disk loaded structure of the SLAC linac, together with the fact that the bunch is phased off crest with respect to the accelerating wave, produces a correlation between particle energy and longitudinal position in the bunch. At the end of the linac, particles one σ_z behind the bunch center have an energy which is lower by about 1% compared to particles one σ_z ahead of the bunch center. As the result of momentum dispersion in the arc, lower-energy particles can overtake higher energy particles and the tail of the bunch therefore tend to catch up with the head. In CN-218 it is shown⁴ that the bunch length at the end of the arc is reduced to about 0.2 mm because of this effect. This bunch shortening in the arc may act to increase the effective amplitude of the dipole and longitudinal wakes.

K. Bane has computed the dipole wake for the arc bellows as a function of bunch length in the range $\sigma_z = 0.25$ to 1.0 mm. The slope of wake averaged between $\pm \sigma_z$ scales as

$$W \equiv \frac{\sigma_z}{x_0} \left\langle \frac{dE_{\perp}}{dz} \right\rangle \sim \sigma_z^{0.25}$$

This same quantity scales as $\sigma_z^{-0.5}$ for the case of the resistive wall wake. If we assume that the bunch length decreases linearly with distance along the arc, then the wake as defined above, averaged over the length of the arc, is computed to be

$$W(\text{Cu}) = 7.4 \times 10^5 \frac{\text{V}}{\text{m}^2}$$

$$W(\text{SS}) = 13.1 \times 10^5 \frac{\text{V}}{\text{m}^2} .$$

Taking into account bunch shortening along the arc does not lead to a significant change in the effective wake, since the components due to the bellows and the resistive wall effect scale in the opposite sense with bunch length.

EMITTANCE GROWTH

The effect of a transverse electric wake field is to induce a coherent betatron oscillation. Consider a relativistic electron at point z within the bunch that feels a force $f(z) = eE_{\perp}(z)$ in the x -direction over the interval δs at position s in the arc. The differential change in slope $\delta x'(z, s)$ is

$$\delta x'(z, s) = \frac{\delta p_x(z)}{p} = \frac{f(z) \, c \, \delta t}{p \, c} \approx \frac{eE_{\perp}(z) \delta s}{W} \quad (5)$$

where p is the momentum and W is the energy of the electron. At a downstream point s_2 the differential amplitude of the oscillation thus produced is

$$\delta x(z, s_2) = \delta x'(z, s) \sqrt{\beta \beta_2} \sin(\phi_2 - \phi) \quad (6)$$

where β and β_2 are the values of β_x at s and s_2 , and $\phi_2 - \phi$ is the betatron phase difference between s and s_2 .

It is convenient to transform to normalized betatron coordinates η and η' , where $\eta \equiv x/\sqrt{\beta}$ and $\eta' \equiv dn/d\phi = x' \sqrt{\beta} + \alpha x/\beta$. Let us also express the collective betatron amplitude as a vector $\vec{\eta}(s)$ in η, η' phase space:

$$\vec{\eta}(s) \equiv \eta(s) + i \eta'(s)$$

In η, η' phase space a betatron oscillation is represented by a constant amplitude radial vector whose angular position is the betatron phase.

In the new coordinates the differential amplitude $\delta \vec{\eta}$ at s_2 due to wake field at s is

$$\delta \vec{\eta}(s_2) = \frac{i \sqrt{\beta} e E_{\perp}(z)}{W} e^{-i(\phi_2 - \phi)} ds \quad (7)$$

and the sum $\Delta \vec{\eta}$ of all contributions throughout the arc up to s_2 is

$$\Delta \vec{\eta}(s_2) = \frac{i e}{W} e^{-i\phi_2} \int_{s_0}^{s_2} E_{\perp}(z) \sqrt{\beta} e^{i\phi} ds \quad (8)$$

Using Eq. (3) and $ds = \beta d\phi$, this can be written as

$$\Delta \vec{\eta}(s_2) = \frac{i e N F e^{-i\phi_2}}{W b^3 (\sigma_{dc} \sigma_z)^{1/2}} \int_{\phi_0}^{\phi_2} x_0 \beta^{3/2} e^{i\phi} d\phi \quad (9)$$

We will assume that the perturbation $\Delta x (= \beta \Delta \eta)$ remains small relative to the driving term, x_0 .

We recall that x_0 is the displacement of the beam pulse relative to the center of the pipe. It has contributions from collective betatron

motion relative to a central orbit, from distortions of the central orbit due to magnetic errors and magnet misalignments, and from misalignment and distortions of the pipe. Because of the $e^{i\phi}$ factor in the integral, it is apparent that the greatest contribution to $\Delta \vec{n}$ comes from the component of x_0 that is at or near the betatron frequency.

Consider the case where x_0 is a pure betatron oscillation:

$$x_0 = a \sqrt{\frac{\beta}{\beta_0}} \cos(\phi + \phi_a)$$

where a is the amplitude where the betatron function is maximum (β_0), and ϕ_a is a constant. Then $\Delta \vec{n}$ can be written as

$$\Delta n(s_2) = \frac{ieaNF e^{-i\phi_2}}{2W b^3 (\sigma_{dc} \sigma_z \beta_0)^{1/2}} \int_0^{\phi_2} e^{i\phi_a} \beta^2 e^{2i\phi} d\phi + e^{-i\phi_a} \int_{\phi_2}^{\phi_2 + 2\pi} \beta^2 e^{2i\phi} d\phi \quad (10)$$

Because of the factor $e^{2i\phi}$ the first integral is negligible relative to the second integral. Thus

$$\Delta n(s_2) = \frac{ieaNF \beta^2 \phi_2 e^{-i(\phi_2 + \phi_a)}}{2W b^3 (\sigma_{dc} \sigma_z \beta_0)^{1/2}} \quad (11)$$

or, using $\beta^2 \phi_2 \approx \beta L$ (L being the length of the system), we see that the magnitude of the perturbation Δn ($\approx |\Delta n|$) at the point z (relative to the center of the bunch) is approximately

$$\Delta \eta_L(z) \cong \frac{ea N \bar{\beta}_L F(z/\sigma_z)}{2W b^3 (\sigma_{dc} \sigma_z \beta_0)^{1/2}} \quad (12)$$

The original distribution of electrons in the η coordinate $f_0(\eta)$ was Gaussian and independent of z .

$$f_0(\eta) = C_0 e^{-\eta^2/2\sigma_\eta^2}$$

where σ_η^2 is the original beam emittance ϵ . To get the final distribution $f_L(\eta)$ we must add in the systematic perturbation $\Delta \eta_L$ at each point in the bunch and integrate over the Gaussian distribution in z :

$$f_L(\eta) = C_L \int_{-\infty}^{+\infty} dz e^{-z^2/2\sigma_z^2} e^{-(\eta - \Delta \eta_L(z))^2/2\sigma_\eta^2} \quad (13)$$

This integral can be readily evaluated if we represent the wake function $E_L(z)$ as a linear function in z . Equivalently we can write the amplitude function $\Delta \eta_L(z)$ as

$$\Delta \eta_L(z) \cong \Delta \eta_0 + \Delta \eta' \cdot z \quad (14)$$

where $\Delta \eta_0$ is the value at $z = 0$ and $\Delta \eta'$ is the average slope. In this case $f_L(\eta)$ is found to be of the form

$$e^{-\left(\eta - \Delta \eta_0\right)^2/2(\sigma_\eta^*)^2} \quad (15)$$

which represents a Gaussian distribution in η whose center has been shifted by $\Delta \eta_0$ and whose rms width has been increased to

$$\sigma_n^* = \left[\sigma_n^2 + (\Delta\eta' \cdot \sigma_z)^2 \right]^{1/2} \quad (16)$$

Although the original beam emittance ϵ was equal to σ_n^2 , the final, somewhat enlarged emittance ϵ^* is not equal to $(\sigma_n^*)^2$. The distribution of the incident beam at s_0 was round in η, η' phase space (if the beam was optically matched). However, at the end of the system the distribution is no longer round because, as indicated in Eq. (11), the perturbation $\Delta\vec{\eta}_L(s_L)$ is perpendicular to the vector $\vec{\eta}_0$ ($\equiv \Delta\vec{x}_0/\sqrt{\beta}$) describing the original collective oscillation of the beam.

Thus the rms width of the final distribution is σ_n^* in the direction perpendicular to $\vec{\eta}_0$ but remains equal to σ_n in the direction parallel to $\vec{\eta}_0$.

Therefore, the final emittance ϵ^* is

$$\epsilon^* = \sigma_n \sigma_n^* = \epsilon \left[1 + \left(\frac{\Delta\eta' \sigma_z}{\sigma_n} \right)^2 \right]^{1/2} \quad (17)$$

Since the fractional increase in emittance $\Delta\epsilon/\epsilon$ ($= \epsilon^*/\epsilon - 1$) has been assumed to be small, it can be approximated as

$$\begin{aligned} \frac{\Delta\epsilon}{\epsilon} &\approx \frac{1}{2} \left(\frac{\Delta\eta' \sigma_z}{\sigma_n} \right)^2 \\ &= \frac{1}{2} \left(\frac{eaN\bar{b}LF'}{2W b^3 (\sigma_{dc} \sigma_z \beta_0 \epsilon)^{1/2}} \right)^2 \end{aligned} \quad (18a)$$

or

$$= \frac{1}{2} \left(\frac{ea\bar{b}L \left\langle \frac{dE_{\perp}}{dz} \right\rangle \frac{\sigma_z}{x_0}}{2W \sqrt{\beta_0 \epsilon}} \right)^2 \quad (18b)$$

where $\langle dE_{\perp}/dz \rangle$ is evaluated for given values of b , σ_z , σ_{dc} , and x_0 .

Let us use this expression to find a_m , the maximum amplitude of collective oscillation that is permissible for a given emittance growth $\Delta\epsilon/\epsilon$:

$$a_m = 2\sqrt{2} \left(\frac{\Delta\epsilon}{\epsilon} \right)^{1/2} \frac{W \sqrt{\beta_0 \epsilon}}{\bar{b}L \left\langle \frac{dE_{\perp}}{dz} \right\rangle \frac{\sigma_z}{x_0}} \quad (19)$$

We have evaluated the amplitude a_m that is permissible in the SLC arc for four cases of interest: for two vacuum-pipe materials (copper and stainless steel) and for two pulse-length scenarios (with constant $\sigma_z = 1$ mm throughout the arc and with bunch shortening such that σ_z decreases linearly from 1 to 0.2 mm in traversing the arc). Using $W =$

50 GeV, $\beta_0 = 8$ m, $\bar{\beta} = 5$ m, $\epsilon = 3 \times 10^{-10}$ rad-m, $L = 1.23$ km, $\Delta\epsilon/\epsilon = 0.1$, and the listed values of $\langle dE_L/dz \rangle \sigma_z/x_0$ (which represent the total wake fields from wall resistivity, beam position monitors, and bellows, as discussed earlier), we obtain:

Table II

<u>Pipe Material</u>	<u>σ_z</u>	<u>$\langle dE_L/dz \rangle \frac{\sigma_z}{x_0}$</u>	<u>a_m</u>
copper	1 mm	8×10^5 V/m ²	0.45 mm
stainless	1	12.1×10^5	0.29
copper	1 $\Rightarrow$ 0.2	7.4×10^5	0.48
stainless	1 $\Rightarrow$ 0.2	13.1×10^5	0.27

It should be noted that the amplitude a_m is for the component of the beam orbit relative to the center of the vacuum pipe that is at the betatron frequency. Other frequencies that are likely to occur are the magnet, the cell, and the girder frequencies, which are, respectively, 6.7, 3.3, and 1.7 times the betatron frequency. It is pertinent also that the arc length (1.23 km) is about 70 betatron wavelengths. Repeating the analysis for these other orbit-frequency components shows that the tolerable amplitudes are all about two orders of magnitude larger than for the betatron-frequency component.

The beam orbit $X_0(s)$ relative to the pipe center can be considered as the sum

$$X_0(s) = X_B(s) + X_C(s) - X_P(s) \quad (20)$$

where $x_b^{(s)}$ describes the collective betatron oscillation relative to the central ("closed") orbit $x_c(s)$, and $x_p(s)$ describes the position of the pipe relative to the ideal orbit. As was noted in the previous paragraph, we need be concerned with only the betatron-frequency amplitude of the x_c and x_p .

Kheifets⁵ has shown that the betatron-frequency amplitude of the corrected central orbit is only about one-fifth the rms amplitude, if the orbit-correction scheme is the one now envisaged. Sample calculations by Kheifets show a total rms amplitude for the corrected central orbit of about 150 microns, so the expected betatron-frequency amplitude of x_c should not be greater than about 30 microns.

The present analysis of the corrected central orbit has been made relative to the ideal orbit. Kheifets is preparing to calculate it also relative to the center of a vacuum pipe that is positioned relative to the magnets, whose positions are subject to misalignments and orbit-correction procedures. The final tolerance on the allowed collective betatron amplitude must await these latter central-orbit results.

References

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Fig.1 -- Resistive Wall Wakefield for the Collider Arc.

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Fig. 2 -- Integrated dipole wakefield for an arc bellows with 18 convolutions

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