CONF-8509159-2
UCRL-93248
PREPRINT

RECEIVED BY OSTI SEP 17 1985

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS
IN THE CONTINUUM

M. Blann, W. Scobel, and E. Plechaty
Lawrence Livermore National Laboratory
University of California
Livermore, CA 94550

This paper was prepared for submittal to
the 17th International Summer School
on Nuclear Physics
Mikolajki, Poland
September 2-14, 1985

August 1985

Lawrence
Livermore
National
Laboratory

This is a preprint of a paper intended for publication in a journal or proceedings. Since changes may be made before publication, this preprint is made available with the understanding that it will not be cited or reproduced without the permission of the author.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS IN THE CONTINUUM

M. Blann, W. Scobel,[†] and E. Plechaty
Lawrence Livermore National Laboratory
University of California
Livermore, CA 94550

UCRL--93248

DE85 017832

Angular distributions for nucleon induced reactions (incident energies 14 to 90 MeV) leading to precompound nucleon emission in the continuum (emitted particle energies 9-70 MeV) are calculated based on nucleon-nucleon scattering kinematics for an incident nucleon on a Fermi gas. Analytic expressions due to Kikuchi and Kawai are used for the single scattering kernel. The geometry dependent hybrid model is used to generate the differential cross sections for first, second, etc. order scattering, these weightings being used to fold the single scattering kernel. Results are found to reproduce all experimental angular distributions quite well at angles in the 20° to 90° range. Ad-hoc modifications to approximate quantal effects and Coulomb deflections are explored, but the results do not seem to offer a consistent means of reproducing back angle yields, and give generally poorer results at very forward angles.

1. INTRODUCTION

The goal of the present work is to consider the interpretation of angular distributions of nucleons emitted in the continuum for reactions induced by 14 to 90 MeV nucleons. This will be done in terms of nucleon-nucleon scattering kinematics between an

[†] Permanent address: I. Institut für Experimentalphysik, Univ. Hamburg, Hamburg, Federal Republic of Germany.

MASTER

M. BLANN

incident nucleon and target nucleons having a Fermi momentum distribution. Some of the geometric (local density) information as well as single versus multiple scattering fractions and emission cross sections will be provided by using the geometry dependent hybrid model¹ (GDH) in conjunction with nucleon-nucleon scattering kinematics for a projectile scattering with nucleons having a Fermi gas energy distribution.² We will attempt to place the results of this work in perspective with other closely related approaches, such as the intranuclear cascade model,³⁻⁵ and more recent and rigorous approaches involving quantum mechanical procedures.^{6,7} We will also mention exciton model approaches which precede this work.⁸⁻¹² The main new contribution of this work will be use of the GDH to give multiple scattering folding fractions. However we will also be critical in looking at both existing quantum mechanical and semiclassical treatments on this subject, and believe that some of the conclusions of this work may lead to perspectives on the subject which differ from those of some of the earlier works.

To our knowledge Goldberger (1948) presented the first analytic expressions for the angular distribution of a nucleon after scattering with nucleons having a Fermi gas momentum distribution, including restrictions due to the Pauli exclusion principle.¹³ This approach was repeated by Hayakawa, Kikuchi and Kawai (1955) who considered examples of (p,p') and (p,n) reactions at 18 MeV incident proton energy.¹⁴ These authors included estimates of refraction for incident and emitted particles, with trajectories based on impact parameter and the mean free path for nucleon-nucleon (N-N) scattering. Only a single N-N scattering process was considered.

Intranuclear cascade models predicted angular distributions based on N-N scattering kinematics, and included multiple

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

scattering processes (which was not done in Refs. 13,14). Some of the newer INC codes did include a semiclassical treatment of reflection and refraction.⁵

Exciton type models were first used by Mantzouranis *et al.*⁶ as a means of estimating single versus multiple scattering contributions to use in folding a kernel based on N-N scattering kinematics. A 90° scattering approximation was made in deriving an analytic expression for the kernel, resulting in a loss of part of the kinematic information. This point is illustrated in a more quantitative manner in Sec. 3 of this work.

Sun Ziyang *et al.*⁹ utilized the analytic expressions of Hayakawa *et al.*^{1,4} as reported by Kikuchi and Kawai² to provide a N-N scattering kernel for folding using the exciton model, thereby maintaining the kinematics due to the Fermi momentum distribution. However these authors used a kernel which was averaged over all final nucleon energies, thereby losing the information of the differences expected in angular distributions between nucleons losing small amounts of energy versus those losing large amounts of energy. Akkermans and Gruppelaar¹⁰ first followed the method of Mantzouranis *et al.*, and later¹² of Sun Ziyang *et al.*⁹ They explored the consequences of adding refraction first in the entrance channel, and then also in the exit channel.¹² More recently they have used the Hayakawa analytic expressions to give the energy dependent angular distribution of particles following the first scattering process, folding with an energy averaged result for multiple scattering processes.¹² (These authors carefully considered errors due to use of an energy averaged kernel for scattering beyond the first collision, and demonstrated that the approximations made were very reasonable.)

Several quantum mechanical formulations of the problem have been presented. Tamura *et al.*⁸ do a wave mechanical

M. BLANN

treatment of single and double N-N scattering processes with interference among intermediate two quasiparticle states. A comprehensive presentation of this approach was recently published in this journal.⁶ A somewhat less rigorous approach has been presented by Feshbach *et al.*⁷ In this approach a single scattering kernel is calculated using DWBA; a statistical argument is then invoked to fold this result incoherently, somewhat analogous to the folding of the semiclassical N-N scattering kernel by Mantzouranis *et al.*⁸ Unfortunately there appear to be some errors and/or ambiguities in the application of this model, and a correct calculation with this theory remains to be performed.^{15,16}

In Section 2 of this work we present a very brief formulation of the geometry dependent hybrid model, followed by a description of the calculation of the angular distribution for a single nucleon-nucleon scattering event in nuclear matter according to Ref. 14, and of the folding of this kernel for multiple scattering. In Section III we will compare results using the N-N scattering kernel, folded for multiple scattering processes, with experimental data for reactions induced by incident nucleons of 14.7 to 90 MeV. In Section IV we will consider ad-hoc methods of treating quantum mechanical effects in the entrance and exit channels via semiclassical approximations, and discuss possible shortcomings of the approximations in the calculational model of folding of the N-N scattering kernel. Our conclusions are given in Section V; they are intended to be critical in order to focus attention on those aspects of the subject which we believe should be considered open and awaiting improvement, and areas where we think caution is required lest we are headed up wrong paths.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

2. FORMULATIONS TO CALCULATE N-N SCATTERING DISTRIBUTIONS AND TO WEIGHT THEIR FOLDING

2.1 Hybrid and Geometry Dependent Hybrid Models

In this work we explore the applicability of N-N scattering kinematics in reproducing the observed angular distributions of (nucleon in, nucleon out) reactions. The single scattering result for N-N scattering in a Fermi gas was derived analytically by Goldberger^{1,2} and later by Hayakawa *et al.*^{1,4} We will try to extend this result to include multiple N-N scattering contributions. The vehicle to weight single versus multiple scattering results will be the geometry dependent hybrid model (GDH),¹ with some comparisons also using the hybrid model.^{1,7}

Use of exciton type models (including hybrid or GDH) to predict angular distributions is in a sense a contradiction, for these models owe their simplicity to the abandonment of angular information in their derivation. However these models follow energy partition according to the number of N-N scattering collisions, and therefore may be used to give the weighting, for a given exit channel energy, between one, two, three, etc. collision contributions. In addition to the N-N scattering events which are assumed to be represented as incoherent processes in semiclassical exciton models, we also will consider (as did Hayakawa *et al.*^{1,4} and later Costa *et al.*^{1,2}) shape-elastic (coherent) processes such as refraction of the incident projectile and emitted ejectile.

Nucleon-nucleon scattering kinematics will be dependent on the local density of the collision site; incident particle refraction will depend upon entrance channel impact parameter, and local density of the trajectory. Exit channel refraction will depend upon collision position, angle and local density of trajectory, information which is totally missing in exciton model formulations

M. BLANN

(including hybrid and GDH models). We will return to this point in Section 4.

The hybrid and GDH models have recently been reviewed in some detail.¹⁷ We very briefly present formulations here for completeness of discussion, and refer to Ref. 17 for a more complete description.

The hybrid model has been formulated as

$$P_v(\epsilon)d\epsilon = \sum_{n=n_0}^{\bar{n}} [X_n N_n(\epsilon, U)/N_n(E)] g \cdot \{ \lambda_c(\epsilon)/(\lambda_c(\epsilon) + \lambda_+(\epsilon)) \} D_n d\epsilon \quad (1)$$

where symbols are defined in Table I. The differential cross section is given by

$$\frac{d\sigma}{d\epsilon} = \sigma_R P_v(\epsilon). \quad (2)$$

In order to reproduce local density effects to first order, the hybrid model was reformulated as a sum over each incident entrance channel partial wave ℓ .¹

$$\frac{d\sigma}{d\epsilon} = \pi \lambda^2 \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell} P_v^{\ell}(\epsilon) \quad (3)$$

where Fermi energy and particle mean free paths are evaluated in a local density approximation over an assumed (unrefracted) entrance channel trajectory. Fermi density distributions are used for the averaging process, with a sharp radius based on the Myers droplet formula.¹⁸ Details of the averaging process may be found elsewhere.¹⁹

It may be seen that the GDH formulation preserves more of the physical information necessary for treating the N-N

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

TABLE I. Definition of symbols

Symbol	Definition
$P_v(\epsilon)d\epsilon$	number of particles of the type v (neutrons or protons) emitted into the unbound continuum with channel energy between ϵ and $\epsilon + d\epsilon$ (MeV).
$\bar{n}$	equilibrium (most probable) particle plus hole (exciton) number.
n_0	initial exciton number.
nX_v	number of particles of type (proton or neutron) v in an n exciton hierarchy.
E	composite system excitation
U	residual nucleus excitation.
$N_n(\epsilon, U)$	number of ways that n excitons may be combined such that one, if emitted, would have channel energy ϵ and the remaining $n-1$ excitons would share excitation $U = E - B_v - \epsilon$, where B_v is the particle binding energy.
$N_n(E)$	number of combinations with which n excitons may share excitation energy E .
$\lambda_c(\epsilon)$	emission rate of a particle into the continuum with channel energy ϵ .
$\lambda_+(\epsilon)$	intranuclear transition rate of a particle which would have channel energy ϵ if it were emitted into the continuum.
D_n	fraction of the initial population which has survived to an n -exciton hierarchy.
σ_R	reaction cross section.
l	orbital angular momentum in units $\hbar$.
λ	reduced de Broglie wavelength.
ϵ	channel energy
g	single particle state density

M. BLANN

scattering and incident refraction questions than a hybrid/exciton model approach in which all density dependencies are averaged out. For this reason we will primarily use the GDH model for calculation of angular distributions in this work. Even in this formulation the necessary information for exit channel refraction is totally missing, and we will ultimately use a quite arbitrary means of handling this problem.

2.2 Nucleon-Nucleon Scattering Kinematics

2.2.1. Basic scattering kernel. In this subsection we present the result of Hayakawa, Kikuchi and Kawai^{2,14} for the angular distribution of nucleons following a single nucleon-nucleon scattering event in nuclear matter as a function of incident and final nucleon energies and Fermi energy. The expressions exclude scattering events for which either scattered nucleon has an energy less than the Fermi energy. The result is derived on the assumption that the total nucleon-nucleon scattering cross section is independent of energy.^{2,14} Calculations with a more reasonable $1/E$ dependence of N-N scattering cross sections yield much more complicated results¹⁴; Kikuchi and Kawai suggest that the spectral shapes are not significantly different for results using the more realistic $1/E$ energy dependence.²

The results of Hayakawa et al., as taken from the later work of Kikuchi and Kawai, are represented by

$$\frac{d^2\sigma}{dE_f d\Omega} = \frac{3\sigma_f(E_1)}{8\pi E_F} \left(\frac{E_1}{E_F}\right)^{1/2} \cdot \frac{1}{\sqrt{QE_1}} (E_1 - E_f) \quad (4a)$$

or

$$\frac{d^2\sigma}{dE_f d\Omega} = \frac{3\sigma_f(E_1)}{8\pi E_F} \left(\frac{E_1}{E_F}\right)^{1/2}$$

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

$$\sigma_f(E_1) = \frac{1}{Q\sqrt{QE_1}} [E_1 E_f \sin^2 \theta - Q(E_f - E_F)] \quad (4b)$$

where $\sigma_f(E_1)$ is the total cross section of free N-N collisions at the incident laboratory energy of the particle E_1 in nuclear matter, E_F is the Fermi energy, E_f is the final nucleon energy after scattering, and where

$$Q = E_1 + E_f - 2\sqrt{E_1 E_f} \cos \theta \quad (5)$$

[Our Eq. (4a) differs from the result of Ref. 13 by a factor of 2, which we believe was a type-setting error in Ref. 13.] The energy E_1 is measured from the bottom of the Fermi sea. Whether to use (4a) or (4b) is determined by the following: if

$$E_f < E_1 - E_F, \quad \alpha_2 < \cos \theta < \beta_2, \quad \text{use (4b)}$$

$$E_f > E_1 - E_F, \quad \beta_1 < \cos \theta < \beta_2, \quad \text{use (4b)}$$

$$\alpha_1 < \cos \theta < \beta_1 \quad \text{use (4a)}$$

$$\alpha_2 < \cos \theta < \alpha_1 \quad \text{use (4a)}$$

where

$$\begin{aligned} \alpha_1 &= \{E_1 - E_F + [E_F(E_F + E_f - E_1)]^{1/2}\} \sqrt{E_1 E_f} \\ \beta_1 & \end{aligned} \quad (5a)$$

$$\begin{aligned} \alpha_2 &= \{E_f - E_F + [E_F(E_F + E_1 - E_f)]^{1/2}\} \sqrt{E_1 E_f} \\ \beta_2 & \end{aligned} \quad (5b)$$

The angular distributions for each final nucleon energy following a scattering process were calculated at 36 angles spaced 5° apart, beginning at 2.5° using Eqs. (4) and (5). For the hybrid

M. BLANN

model calculation a maximum value of $E_f = 40$ MeV was used; for the GDH calculation the angular distribution was calculated for the maximum E_f along each trajectory. For the incident channel of angular momentum l with partial cross section σ_l , the kernel was weighted as σ_l/σ_R (where σ_R is the total reaction cross section) and the results were summed over all entrance channel partial waves to get a weighted first scattering kernel. This kernel was folded with a pure hybrid model kernel (i.e. $E_f = 40$ MeV) for higher order scattering, since nucleons are no longer localized by impact parameter after an intranuclear scattering event. The folding of these histogram kernels is discussed in the next subsection.

It is instructive to look at results of Eqs. (4a-b) for the single scattering of a 90 MeV proton incident on a ^{90}Zr target nucleus, giving neutrons of final energy 5, 50 and 80 MeV. These results are shown in Fig. 1. A Fermi energy of 40 MeV was used in these calculations. Also shown in Fig. 1 is the single scattering result (kernel) due to Mantzouranis *et al.* in which the 90° approximation (target nucleons stationary) was made.^{*} We see from Fig. 1 that the 90° approximation limits scattering to 90° or less, whereas inclusion of the target nucleon Fermi momentum permits larger angle scattering for lower energy collision partners. Mantzouranis *et al.* restricted their derivation to the fast particles; however in popular usage of their model this restriction has sometimes been ignored.^{20,21} We see in Fig. 1 that the shapes of the angular distributions vary considerably with final energy; we do not therefore feel that the use of an N-N kernel averaged over energy is desirable, consistent with the conclusion of Ref. 12. Note also (Fig. 1) the large difference in shape between the single scattering kernel with the 90° approximation versus the result of the more rigorous Fermi gas calculation according to Ref. 2.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

FIG. 1. Angular distribution kernels (single scattering event) for 90 MeV protons incident on a Fermi gas with $\epsilon_f = 40$ MeV. The solid histogram is for 5 MeV neutrons (laboratory reference) from a ^{90}Zr target; dotted histograms for 50 MeV neutrons, and dashed histogram for 80 MeV neutrons. The smooth dashed curve is the result of the 90° approximation used in Ref. 8.

Additional kinematic information is summarized in Fig. 2, where the minimum and maximum scattering angles are shown versus final scattered nucleon energy for incident nucleons of 45 and 90 MeV (a 40 MeV Fermi energy was used in these calculations). It may be seen that e.g., for the 90 MeV incident nucleon case, the maximum final angles vary between 128 and 68 degrees for collision partners receiving energies between 5 and 80 MeV in the frame of the emitted nucleons. This figure re-emphasizes the point of the previous paragraph. This is of course the reason that Costa *et al*^{1,2} adopted an energy dependent kernel (as we will now do) rather than an energy averaged kernel as e.g., in Ref. 9.

M. BLANN

FIG. 2. Kinematic limits for single nucleon-nucleon scattering events in a Fermi gas. The dashed and solid curves represent the minimum and maximum angles versus final nucleon energy for 45 MeV incident (dashed lines) and 90 MeV incident (solid lines) nucleons and a Fermi energy of 40 MeV.

2.2.2. Folded kernel for multiple scattering. There are two points which require discussion in going from the single scattering kernel described by Eqs. (4) and (5), to a result folded for second and higher order scattering. The first is the method of evaluating the kernel for a second scattering when the energy E_1 is not so simply defined as for the incident projectile, which is used for the primary scattering kernel. The second point is the method of folding on a proper unit sphere, i.e., in three dimensional space.

We have made an approximation in evaluating the kernel for second and higher order scattering; we have assumed that the kernel appropriate to E_1 corresponding to the projectile energy may be used, rather than a weighted sum of kernels corresponding to a weighting of e.g. parent exciton populations which could

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

contribute to a given final exciton energy. There are reasons why we expect this to be a quite satisfactory approximation. The first is that for most cases we investigate, the first ($n=3$) exciton term dominates the emission process, and for this term E_1 is unambiguously the projectile energy. Since higher order ($n>3$) terms are in a sense a perturbation on the dominant first term, the error in using the primary kernel rather than a secondary-weighted kernel becomes a perturbation on a perturbation, i.e., a second order error.

Reference to Fig. 2 shows that whether E_1 is 45 MeV or 90 MeV, the kinematically determined maximum scattering angles do not differ by any large amount for given final exciton energies. From this it may be inferred that use of the primary kernel should not, as argued above, cause a significant error.

Tests of a related nature for the point under discussion were made by Costa et al.^{1,2} who performed one set of calculations using the primary kernel as we have for all orders of folding. They then performed calculations using the primary kernel only for $n = 3$, and a single kernel averaged over all emission energies for $n > 3$. They found that the differences in final results between these extremes were small enough that they could comfortably use the energy averaged kernel for $n > 3$. Quantitative estimates of differences between these two procedures may be found in Table II of Ref. 12.

We also note that use of a more rigorously consistent parent weighted scattering kernel would in all cases decrease the final maximum scattering angle for a given exit channel energy. Consequently the deficiencies we will observe in calculating back-angle yields cannot be attributed to this approximation which is in the opposite direction.

The second point involves the method of folding the kernel defined by Eqs. (4) and (5) for multiple scattering, or for attempts

M. BLANN

to estimate quantum mechanical effects which may be another means of scattering the incoming particles. To do so we have used a subroutine which follows the direction cosines in spherical coordinates using a deterministic method as follows:

- (1) The initial scattering angle Θ_1 about the beam direction is selected as 2.5°
- (2) The initial azimuthal angle φ_1 is selected as 2.5°
- (3) A scattering angle Θ_2 with respect to the direction Θ_1 is selected at 2.5°
- (4) An azimuthal angle of $\varphi_2 = 2.5^\circ$ is selected.

The direction cosines are now computed with respect to the original beam direction. Next φ_2 is rotated in 5° increments from 2.5° to 357.5° , recomputing direction cosines for each case. This process is repeated as Θ_2 is advanced from 2.5° to 177.5° in 5° increments. The above processes are repeated as φ_1 is advanced from 2.5° to 87.5° (due to considerations of azimuthal symmetry), and as Θ_1 is advanced from 2.5° to 177.5° . Weighting factors of

$$\frac{d^2\sigma_1}{d\epsilon_1 d\Omega} \sin(\Theta_1)$$

and

$$\frac{d^2\sigma_2}{d\epsilon_2 d\Omega} \sin(\Theta_2)$$

are multiplied to get a relative weighting of each element in this folding process. We are only interested in those elements which would be detected by e.g. a counter in a plane about the target. We therefore only process and store those events which fall into a range of $\pm 5^\circ$ about such a plane. As a check we expanded the $\pm 5^\circ$ acceptance angle to $\pm 10^\circ$. No significant change in

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

angular distribution resulted. For folding of the scattering kernel to approximate quantal effects, θ_1 (or θ_2) is restricted to a range of angles which is based on quantal spreading limits we try to fold in, with a $\sin \theta_1$ (or $\sin \theta_2$) weighting factor. The scattering kernel retains the

$$\frac{d^2\sigma_2}{d\epsilon_2 d\Omega} \sin(\theta_2)$$

weighting, or the folded weighting factor for the appropriate angle (whether a primary kernel or folded result depends on the stage of the calculation, i.e., $n = 3$ or $n > 3$).

In the following section we will present results of the semiclassical N-N scattering kernel, for multiple scattering using the GDH model of Eq. (3), with a few comparisons also with the hybrid model of Eq. (2). We mention that Mantzouranis performed a closed form hybrid model calculation²⁰ using the kernel described in Ref. 8.

In Section 4 we will discuss some of the quantal effects which should cause spreading of the angular distributions beyond the range resulting from the nucleon-nucleon scattering kernel, and show results of some very ad-hoc semiclassical estimates of such effects. We will want to emphasize the very arbitrary nature of these add-ons to the N-N scattering; nonetheless it is worth an attempt to improve deficiencies in the pure N-N scattering approach for use in nuclear technology, and perhaps as a probe of what success is possible in a semiclassical framework.

3. RESULTS USING A FOLDED N-N SCATTERING KERNEL

In Figs. 3-9 experimental (n,n') and (p,n) angular distributions are shown and compared with calculated folded N-N scattering results using equations (4a-b) due to Kikuchi and Kawai with Eqs. (2) or (3). Data are from Refs. 23-27.

M. BLANN

FIG. 3. Experimental and calculated angular distributions for the $^{93}\text{Nb}(n, xn)$ reaction with 14.7 MeV incident neutrons. Results are shown for 4 MeV and 9 MeV emitted neutrons. Data (circles, squares and triangles) are from Ref. 23. The dotted lines are the calculated equilibrium components which have been added to the histogram results. The dot-dash histograms are the results of the semiclassical calculation using the folded N-N scattering result described in the text. The solid histograms are the results of the same calculation but with an additional folding for 'refraction' in the entrance channel as described in the text.

For the case of 14.7 MeV incident neutrons (Fig. 3) the result of this calculation plus the assumed isotropic evaporation components (as given by the Alice/Livermore 82 code) are in quite satisfactory agreement with the experimental results. However a large part of the back angle yields is attributable to equilibrium evaporation, decreasing the sensitivity of the comparison to the shape of the the N-N scattering kernel.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

FIG. 4. Results as in Fig. 3 for an incident neutron energy of 25.7 MeV, for 14 and 19 MeV exit channel energies. Data (circles) are from Ref. 24. The dot-dash histogram is the pure folded N-N scattering result; the solid histogram includes the folding for entrance channel refraction described in the text.

M. BLANN

FIG. 5. Results as in Fig. 4 for the $^{90}\text{Zr}(p,n)$ reaction at 25 MeV incident proton energy. Experimental results are from Ref. 22 and 25 (horizontal bars) and from Ref. 26 (circles). Calculated results are as in Fig. 4. The data of Refs. 20, 23 are more reliable than those of Ref. 24 at angles forward of 20° . The isotropic (dotted and dashed) components are the calculated evaporation contributions which have been added to the precompound results shown.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

FIG. 6. As in Fig. 5 for an incident proton energy of 45 MeV. Data (horizontal bars) are from Refs. 22, 25. Calculated results are as in Fig. 4 for exit channel energies of 20 and 30 MeV.

M. BLANN

FIG. 7. As in Fig. 5 for an incident proton energy of 90 MeV. Data are from Ref. 27 for exit channel energies as shown. Solid curves have been drawn through the data for 50 and 70 MeV exit channel energies to guide the eye. Histograms are folded N-N scattering results without refraction. The 20 MeV exit channel result includes a significant equilibrium contribution.

FIG. 8. As in Fig. 4 for the $^{209}\text{Bi}(n, xn')$ reaction with 25.7 MeV incident neutrons. Data are from Ref. 24; calculated histograms are as in Fig. 4.

In Fig. 4 similar comparisons are shown for 25.7 MeV incident neutron energy, and in Figs. 5-7 for the $^{90}\text{Zr}(p, n)$ reaction at incident energies of 25, 45 and 90 MeV. In Figs. 8 and 9 results are shown for $^{209}\text{Bi}(n, xn)$ at an incident neutron energy of 25.7 MeV. Hybrid model results (Eq. 2) are shown in Figs. 6, 7 and

M. BLANN

FIG. 9. As in Fig. 8 for 19 MeV exit channel energy.

9. All of these results may be summarized as follows: the N-N scattering kernel gives a very good representation of the slope of the angular distribution for all incident and exit channel energies at angles between 20° and 80 to 100° where single scattering dominates. Beyond this angle, as may be anticipated from Fig. 10 (see Section IV), there is insufficient yield predicted at far back angles. This is similar to the case of the intranuclear cascade calculation; results of the INC for the $^{90}\text{Zr}(p,n)$ reaction are shown in Fig. 11. Here the deviation between experimental and calculated results is large at 60° . An important point to bear in mind is that the shapes of the angular distributions calculated solely by N-N scattering kinematics gives an excellent reproduction of the major part of the emission cross section for all exit

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

energies. While the discrepancies at back angles in Figs. 3-9 may appear large on a semi-log plot, the cross section involved is very small. The important point is that if one is seeking angular distribution information for technology, the result using the folded kernel of N-N scattering is quite good. The comparisons in Figs. 3-9 are un-normalized, and obviously give good absolute agreement with data (in the range 20° to $80-100^\circ$) when the GDH model gives a good result for the angle integrated cross section. We observe in Figs. 6, 7 and 9 that the shape of the angular distribution resulting from the GDH model is better than that resulting from the hybrid model.

FIG. 10. Angular distributions for $^{208}\text{Pb}(p,n)$ at 45 MeV incident energy. The data (circles) are from Ref. 22; the dotted curve is the single scattering result of Ref. 6, and the solid curve the result of single plus double scattering according to the theoretical result of Ref. 6. The solid histogram is the semiclassical single scattering result due to Eqs. (4-5) based on the work of Refs. 2, 13, and 14.

FIG. 11. Experimental and calculated $^{90}\text{Zr}(p,n)$ spectra versus angle of measurement for 45 MeV incident protons. The calculated results are based on the intranuclear cascade model and are from Ref. 29. The histogram is the cascade model result; solid irregular lines represent experimental results.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

We feel that the conclusions above give the main essence of physics of the N-N scattering approach to the question of continuum angular distributions in the precompound region. The comparison with the results of Tamura (Fig. 10), and the information of Fig. 12 (to be discussed in Section IV), suggests that if one wishes to reproduce the small cross section at back angles as well, a more rigorous quantum mechanical treatment is required. Alternatively one may explain scattering into the backward hemisphere in a semiclassical model and in agreement with both energy and momentum conservation, if part of the linear momentum is transferred to a heavier system. In this case the energy transfer is negligibly small. On these grounds, there has been a tendency to attempt to modify the semiclassical treatment described thus far by ad-hoc addition of semiclassical effects intended to approximate quantal phenomena. Such procedures become very tenuous; they are valuable and legitimate to gain physical insights. However, our personal viewpoint is that attempts to interpret them beyond the 'insight' stage must be made with extreme caution. Having stated this caveat we will explore such 'add-ons' to the nucleon-nucleon scattering approach.

4. QUANTAL EFFECTS AND SEMI-CLASSICAL APPROXIMATIONS

In this section we will discuss constraints on nucleon angular distributions due to diffraction and refraction. Diffraction is a purely quantum mechanical phenomenon which has no semiclassical analogue. It is related to nuclear size and nucleon wave number. Refraction has a semiclassical limit, but it is questionable that this limit is reached in nuclear reactions in the energy range of interest in this work. Refraction will depend on the change in potential between the medium and the vacuum, and on the geometry of the nucleon trajectory. However, exciton

FIG. 12. Half angle versus nucleon energy due to the relationship of Eq. (6) for targets of $A = 27, 90$ and 200 . The radius used is the half density radius of the Myers' droplet model. Half angle refers to one half the angular opening symmetric about 0° .

models (including the GDH model) do not follow nucleon geometries. Therefore treatment of refraction in these models will require arbitrary procedures. It will be shown that diffraction limits will be comparable to or greater than expected refractive effects for some cases of interest and less in others.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

We therefore approach these modifications to the N-N scattering result with pessimism as to their physical content. Nonetheless, if it is possible to obtain an overall improvement in calculated angular distributions as reported in Ref. 12, the result is valuable for technology.

4.1 Localization Considerations

An important point which was emphasized by Mantzouranis *et al.*⁸ is that the localization of the angular distribution resulting from the N-N scattering kinematics is comparable to (sometimes less than, and sometimes greater than) the minimum value required by limits due to wave mechanical considerations. In Fig. 12 we show the minimum half angle for localization due to the relation given by Mantzouranis *et al.*,⁸

$$R\Delta\theta > 2\pi/k, \quad (6)$$

where k is the nucleon wave number and where the sharp radius from the droplet model of Myers¹⁸ was used,

$$R = 1.18A^{1/3} [1 - 1/(1.18A^{1/3})^2] . \quad (7)$$

From this result it may be seen, in particular for nucleons of the lower kinetic energies of interest (e.g. ≤ 20 MeV), that a semiclassical treatment of angular distributions as due to N-N scattering kinematics is questionable. From this discussion and Fig. 12, we may anticipate a failure of the N-N kernel approach due to localization constraints.

One semiclassical approach which has been investigated has been the addition of the semiclassical concept of refraction, as defined by Snell's law. If the failure is due to localization constraints, (diffraction) it is doubtful that such a treatment (refraction) will provide a remedy which will scale over a wide

M. BLANN

range of target mass and projectile/ejectile energies (see Fig. 12).

In the context of the above statement it is worthwhile to compare the single scattering result of Tamura *et al.*⁶ with the semiclassical single scattering kernel calculated with Eq. (4). This is done in Fig. 10 for the reaction $^{208}\text{Pb}(p,n)$ for 45 MeV incident proton energy and for exit channel energies of 26 and 36 MeV. The experimental results of Galonsky *et al.* are also shown.²² The semiclassical result is for a Fermi gas with $E_f = 40$ MeV. The point of Fig. 10 is that the semiclassical result is definitely unable to reproduce the back angle yields for a single scattering event. From this comparison we may understand the deficiency as due to coherent processes not treated in the semiclassical approach. We may therefore immediately anticipate the inability of the N-N scattering approach to reproduce data much beyond 90° at those particle emission energies which result primarily from single scattering processes (the higher energies), as observed in the comparisons in the preceding section, and by all the work with this approach preceding this paper. We may see that quantum mechanical calculations which properly include refraction and diffraction are able to reproduce the experimental result, and we therefore understand the deficiencies and reasons for failure of the semiclassical result.

We next explore semiclassical modifications, purely ad-hoc in nature, to see if quantal effects may be incorporated in a simple manner into the N-N scattering approach, as adjudged solely by success in reproducing experimental results. As will be discussed further on, quantal theories on this subject are not yet at as useful a stage of development as the simpler semiclassical models.

4.2 Entrance channel refraction

Here we discuss the possibility of increasing the intensity of nucleons scattered at large angles by considering refraction of the

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

incident and emitted particles. This was done by Hayakawa *et al.* for (p,n) and (p,p') reactions induced by 18 MeV protons.^{1,4} Refraction and reflection were included in some intranuclear cascade codes,⁵ and refraction has been treated in exciton model N-N folding calculations by several authors (e.g., Ref. 12). We will first consider refraction in the entrance channel; this has some minimal justification since in the GDH model we can at least localize each entrance channel partial wave to a disk of cross sectional area $\pi k^2 T_q (2l + 1)$ with impact parameter between l and $(l + 1)$. We will apply Snell's law as was done by Hayakawa *et al.*^{1,4} and by Costa *et al.*^{1,2} However, the justification for using this approach is not obvious when the incident particle wavelength is comparable to the nuclear skin thickness, sounding yet another word of caution.

A refractive index 'n' of the nucleus may be defined as

$$n = k_{in} / k_{out} \quad (8a)$$

where k_{in} (k_{out}) is the wave number of the projectile inside (outside) the nucleus and the refraction of a nucleon entering nuclear matter is given by Snell's law as

$$k_{out} \sin \alpha = k_{in} \sin \beta \quad (8b)$$

where Fig. 13 shows the angular relationships between incident and refracted waves.

In using Snell's law with the GDH model and N-N scattering we proceed in an iterative manner as follows: The average density along the particle (projectile) trajectory prior to any refraction is calculated for impact parameter l . The corresponding Fermi energy E_F^l for this density is calculated; the projectile binding energy B is reduced by the ratio E_F^l / E_F to an effective value B_l where E_F is the average Fermi

FIG. 13. Relationship of angle of incidence (α) and angle of refraction (β) used in Snell's law as discussed in the text. The circle indicates one of a continuous number of possible collision sites along a nucleon trajectory. The exit incidence angle δ and angle of refraction γ will depend upon the position of the collision, and on the scattering angle of the collision. Neither of these quantities is followed in exciton models of the type considered in this work.

energy of the nucleus, which is 30 ± 2 MeV for the parameters used in this work. An angle of refraction is computed for K_{in} corresponding to an interior nucleon energy of $\epsilon_{out} + B_l + E_F^l$ using Snell's law. A single iteration is then performed by calculating the average density of nuclear matter traversed by this refracted trajectory (in planar geometry with the original trajectory and nuclear center). Values of E_F^l and B_l are recalculated based on this result and the angle of refraction is recalculated. The same procedure is followed for an impact parameter of $(l + 1)$, since the partial wave cannot be localized more closely. In Fig. 14 we show examples of incident proton refraction for 45 MeV protons incident on a ^{90}Zr target. The partial cross sections of the various partial waves are shown as are the maximum kinematic single scattering angles for the arbitrary case of 35 MeV emitted neutrons.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

FIG. 14. Relationship between partial reaction cross sections (bottom), angle of refraction (center) and maximum kinematic scattering angle (top) for 45 MeV nucleons incident on ^{90}Zr . The bottom histogram gives partial reaction cross section (ordinate) versus incident proton angular momentum (l , abscissa). The angle of refraction for the entrance channel is shown versus l in the center, calculated by the iterative process described in the text. The upper histogram shows the maximum kinematic single-scattering angle for 35 MeV (laboratory frame) neutrons. The change in kinematic angles results from different Fermi energies resulting from the local density approximation of the geometry dependent hybrid model.

M. BLANN

The kernel for N-N scattering is calculated for each impact parameter based on the value of $E_F^{\frac{1}{2}}$ calculated as described above. This 'bare' kernel is folded between the refractive angles described above for impact parameters l and $(l + 1)$. For additional intranuclear scattering events, a bare N-N kernel based on a nuclear potential averaged over the entire nuclear volume is folded with the distribution for the refraction folded GDH result described above.

Results of including incident wave refraction are shown in Figs. 3 to 6, 8, 9 and 15. This procedure causes the calculated result to go to somewhat larger back angles before the onset of the rapid decrease with increasing angle, beyond which the calculation fails to reproduce experimental results. The change in agreement at forward angles is inconclusive; some results seem in better agreement with data at forward angles, whereas other results seem to predict too flat a distribution at forward angles, giving a result poorer than that from the pure N-N scattering result without the addition of spreading according to Eq. (8). Observing that there is still a major disagreement at very large back angles, we look for possible reasons to permit additional folding of the N-N scattering results. Again we must recognize the very ad-hoc, empirical nature of these procedures. We therefore next discuss refraction in the exit channel as a possible additional candidate for forcing semiclassically calculated results to further back angles, in a search for an empirical recipe to improve the overall success of the 'fit'.

4.3 Entrance and exit channel refraction

As discussed earlier, and as may be seen from Fig. 13, refraction in the exit channel is dependent not only on the entrance channel trajectory, but also on the position of the N-N collision preceding particle emission and on the angle of

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

FIG. 15. As in Fig. 7, but calculated histograms include refraction in the entrance channel as described in the text.

scattering (i.e. the trajectory of the scattered particle). Most assuredly this is the information which is sacrificed in going from an intranuclear cascade representation of the reaction, (which is geometric in nature) to an exciton model representation (which is formulated in an energy space) where we have no geometric information of the type required in application of Snell's law.

M. BLANN

The classical treatment of refraction in the entrance channel is modified by the Coulomb deflection a projectile proton experiences. It has been shown^{1,4} that in first order approximation the deflection is taken into account by replacing Snell's law [Eq. (8b)] by

$$K_{out}^{\infty} \sin \alpha_{\infty} = K_{in} \sin \alpha \quad (8c)$$

Here, K_{out}^{∞} denotes the wave number of the projectile at an infinite distance and α_{∞} the angle between K_{out}^{∞} and the normal of the nuclear surface at the point of incidence.

Coulomb repulsion leads to $\alpha_{\infty} < \alpha$; the refractive index [Eq. (8a)] is reduced by the factor $K_{out}/K_{out}^{\infty} < 1$. These opposite effects in part cancel. At some tens of MeV projectile energy the reduction in angle $\alpha_{\infty} < \alpha$, although small,⁵ dominates. In this sense entrance channel refraction without deflection gives an upper limit, and this is why we do not consider the effect of Coulomb deflection in the remainder of this paper.

We note in Fig. 12 that the minimum localization angles for particles in the entrance channel are roughly comparable to the angles of refraction as calculated by Snell's law [compare Fig. (14)]. We therefore recomputed the angular distributions calculated with Snell's law for the entrance channel only, using the $\Delta\theta$ of Fig. 12 in place of the result of Snell's law. The two results were very similar for all systems compared. We therefore used Fig. 12 results [i.e., the relationship of Eq. (6)] to fold once more the angular distribution of particles in the exit channel. Some improvement resulted over the 'entrance channel only' results, but some back angle discrepancy remained. We therefore arbitrarily reduced the radius of Eq. (7) from the 50% density value to the 90% density value (by reducing the R of Eq. (5) by 1.1fm) to give some additional improvement at back angles.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

Results of calculations treating 'refraction' in the entrance and exit channels (via the relationship of Eq. 6 which might better be called a diffraction result) may be seen in Figs. 16-18. In all cases we observe that this results in an over-correction at forward angles with too flat a shape and too low a cross section. At the lower incident and exit channel energies there is some improvement in fitting back angles, but for the case of $^{90}\text{Zr}(p,n)$ with 70 MeV exit channel energy, for which semiclassical approaches should be most valid, we see that an over correction results at the back angles as well. It is our opinion, based on the results presented herein, that our attempts to add semiclassical approximations to account for quantal phenomena fails to give a uniform improvement in the angular distribution calculation based on the nucleon-nucleon (N-N) scattering approach.

Having made perturbations to the results of the N-N scattering angular distribution by our semiclassically added estimates of quantal effects, it is valuable to see if the modified angular distributions result mainly from the N-N scattering kernel, mainly from the additions to approximate quantal phenomena, or if they are somewhat sensitive to both.

In order to get a feeling for the answer to this question we re-computed several angular distributions with 'refraction' in the entrance and exit channels as before, but we set the N-N scattering kernel to zero at all angles but 45° in one set of calculations (Figs. 16, 17), and in yet another set of calculations we set the N-N scattering kernel to zero at all angles except 0° , i.e., the assumption that there is no N-N scattering (Figs. 16-18). From this exercise we can make several observations. First, there is little difference between the refraction folded result using the full K-K scattering kernel of Eqs. 4-5, and the result assuming that all N-N scattering is at 45° . Sensitivity to the

M. BLANN

details of the N-N scattering kernel may be seen to be mostly lost when the 'refraction' is added to the calculation.

FIG. 16. Angular distributions for $^{90}\text{Zr}(p,n)$ for 25 MeV incident protons, 9 and 14 MeV exit channel neutrons. Data are as in Fig. 5. The solid histogram is the folded N-N scattering calculation which includes folding for refraction in both entrance and exit channels as described in the text. The dot-dash histogram is as the solid histogram, except that the N-N scattering kernel is replaced by one which is zero at all angles except 45° . The dotted histogram is as for the solid histogram with the N-N kernel zero at all angles except 0° , i.e., only the ad-hoc 'refractive' effects are represented in the folded result.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

FIG. 17. As in Fig. 16 for 45 MeV incident proton energy.

Considering the result where we assume no N-N scattering, we get a better overall fit to data for $^{90}\text{Zr}(p,n)$ at 25 MeV incident energy (Fig. 16) and 9 MeV exit energy over the result using the full K-K kernel. At 45 and 90 MeV incident energy a stronger case may be made for the necessity of some additional mechanism to broaden the angular distribution - which we of course believe is the N-N scattering. There are different interpretations which may be placed on these observations. One we would keep in mind is that the main message from these and earlier observations in this work (see Figs. 10 and 12

FIG. 18. As in Fig. 15, but an additional folding is performed for refraction in the exit channel. Results are shown at 50 and 70 MeV exit channel energy using a N-N kernel which is zero at all angles except 0°.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

and discussions) is that at the lower bombarding energies the angular distributions owe their form more to quantal considerations than to the results of the N-N scattering process. If this is so it may be that semiclassical modifications are unable to approximate these wave-mechanical results, or alternatively that an adequate semiclassical approximation has not as yet emerged. We stress that these are our conclusions based on the analyses of this work. Broader analyses of better data sets could cause a re-evaluation of these conclusions, and other authors doubtless do not share these viewpoints.

Having emphasized a very cautious approach in concluding what we have learned about the physics involved in the continuous angular distributions under discussion, we should also re-emphasize the positive side: the semiclassical approach reproduces the forward part of the angular distribution (and therefore the major part of the cross section) quite well, using only a folded N-N scattering kernel. This success is achieved by combining the work of Goldberger⁹⁻¹² and of Hayakawa *et al.*,^{2,14} with the concept due to Mantzouranis *et al.*⁸ of using an exciton model approach to fold successive kernels. This result is easily calculable and useful for nuclear technology.

While we feel that the answer to the back angle yields requires quantum mechanical treatments, we have seen no convincing evidence that such approaches are as yet at a stage that accurate a-priori predictions of angular distributions can be made. For example, the approach of Tamura *et al.* has been shown to work well for incident proton energies up to 62 MeV. However in Fig. 19 we show results of the first step only calculation for the angular distribution of neutrons following 90 MeV proton bombardment of ^{90}Zr .²⁰ The 20 MeV result required significantly different parameters than the 50 and 70 MeV results. The 20 MeV calculated result of Tamura is

M. BLANN

plotted multiplied by 165, whereas the 50 and 70 MeV results are plotted multiplied by five. The results of the semiclassical calculations presented in this work are also repeated in Fig. 19, all values plotted without renormalization. These results are at least as good as the quantum mechanical results of Tamura with respect to shape of the angular distributions, and are better in predicting the absolute magnitude of the experimental results. The theory of Tamura requires, at the least, fitting to experimentally measured discrete transitions before magnitudes of continuum angular distributions may be calculated.

In Fig. 20 we show results of INC calculations which include reflection and refraction in the exit channel.²⁹ Here the back angle agreement is improved over results without reflection/refraction, but back angle yields are still inadequate. This figure serves to illustrate that the exciton model with ad-hoc addition of refraction gives results qualitatively consistent with the INC model.

4.4 Additional possible causes of back angle scattering in the semiclassical framework.

A model for precompound angular distributions due to Feshbach *et al.*⁷ (FKK) suggests that two classes of exciton configurations should be considered during the equilibration process, multistep direct (MSD) and multistep compound (MSC). In the former, one (or more) scattered particles are always in the unbound continuum. In the latter, all particles are bound for at least one exciton configuration preceding emission. For the MSD component a quantum mechanical calculation is performed predicting a forward peaked angular distribution. For the MSC component it is assumed that the angular distribution will be

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

FIG. 19. Angular distributions for the $^{90}\text{Zr}(p,n)$ at 90 MeV incident proton energy. The histograms are results of the folded N-N scattering calculation as in Fig. 7. The heavy solid, heavy dotted, and heavy dot-dash curves are results of the one step scattering calculation using the theory of Tamura *et al.** The 50 MeV and 70 MeV results are shown increased fivefold over the calculated result; the 20 MeV result has been multiplied by 165. These adjustments were made to facilitate comparisons between calculated and experimental shapes. All semiclassical results presented in this work were plotted without renormalization.

FIG. 20. Calculated (INC) and experimental angular distributions for (p,p') reactions on ^{12}C , ^{54}Fe , and ^{209}Bi targets. These results are from Ref. 29. The solid histogram (ORNL) is from an INC code without refraction; the dashed histogram (BNL) is from a code which includes reflection and refraction.

symmetric about 90° in the center of mass due to random relative phases of matrix elements involving different total angular momenta.

To the extent that the MSC contribution is treated as a free parameter in calculations with the FKK model, a fit to the back angle data is guaranteed. Indeed if such a component were invoked in the semiclassical N-N scattering results (without refraction) a reasonable fit could be obtained for all spectra shown in this work.

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

Fortunately or unfortunately we may use the exciton model energy partition functions to estimate the fraction of configurations for which all particles are bound. If the least tightly bound particle has binding energy B , and the excitation energy is E , then the result for the 3 exciton configuration is

$$F_{\text{MSC}} = (B/E)^2 . \quad (9)$$

This expression could easily be modified for finite well depth, however the limit given in Eq. (9) is sufficient for our purposes. Particle emission for subsequent configurations clearly cannot exceed an energy of $2B + E_f$ for MSC components following the 3 exciton configuration. A higher limit applies to higher exciton numbers, but their contribution decreases so rapidly that they may safely be ignored.

We have used Eq. (9) to estimate a MSC component under the assumption that Eq. (9) gives the fraction of the precompound cross section at each channel energy which is emitted isotropically. The addition of this component greatly improves the agreement of calculated and experimental spectra in Figs. 6-12; however the MSC component is still too small to fully reproduce the back angle data. More importantly though, an additional two body interaction is required to direct sufficient energy on a particle exciton for emission, so that only a very small fraction of our estimated upper limit should actually be realized in the exit channel. Given the cross section as a free parameter, as in some applications of the model of Ref. 7, we can reproduce the data very well. Making more realistic estimates we feel that this component cannot reasonably explain the general back angle deficiency.

Recent work by Bonetti and Colli-Milazzo,³⁰ using the theory of Ref. 7 has however made a good case for the

M. BLANN

importance of the multi-step compound contribution in some regions of the spectra. These calculations use a two body force which is consistent with that used for the multi-step direct component. This result seems to be an encouraging, large step forward in removing some of the objections stated in this subsection. However the point that the semi-classically calculated back angle deficiency at higher energies cannot be reproduced via the MSC component is reinforced by the work of Bonetti and Colli-Milazzo.

4.5 Possible improvements of semiclassical model for fitting back angle data.

There are several factors not included in the calculations of this work which might modify the back angle part of the semiclassical N-N scattering calculation which should be stated, some of which warrant further investigation:

(1) Isotropic N-N scattering angular distributions were assumed in the work of Hayakawa *et al.*¹⁴ as used in this work. Experimentally there is considerable forward-backward peaking over much of the energy range we have considered.³¹

(2) A maximum value of the Fermi energy of 40 MeV was used in the N-N scattering calculations in this work. A higher momentum component would increase back angle scattering (this effect may be seen in Fig. 14, where (going in the opposite direction) the maximum scattering angle decreases as the average E_f decreases with increasing impact parameter).

(3) The kinematics assumed in the N-N scattering calculation are two-body. A small admixture of the projectile nucleon interacting with e.g., a nucleon pair would grossly alter the back-angle distribution of the kernel we used which includes only two body scattering processes. Nucleon interactions with

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

larger clusters would have a similar result; the limit of this effect is nuclear scattering with the entire target nucleus, which was discussed in previous subsections.

Let us consider these items in order. The first could result in up to factors of three in intensity at extreme forward and backward angles, (before Pauli principle reduction) but would not modify the kinematic back angle limit for a single N-N scattering process. Much of the failure to reproduce back angle results comes from this kinematic limit. A more realistic N-N angular distribution might therefore give better agreement to slightly larger angles, but the main precipitate decrease in calculated cross sections at large angles would not change in angle.

Item (2) should probably be classified as extremely unlikely to be relevant. Rough estimates of high nucleon momentum components suggest that significant contributions to the N-N scattering result due to this cause would be unrealistic.

Item (3) is well worth consideration. The assumption of two body scattering is dependent upon the N-N collision period being short with respect to the period between collisions. This problem was recently considered by Kawai and Weidenmüller.^{3,2} Their estimates showed that while the N-N collision period was less than the inter-collision period for the energies considered in this work, it was not so much shorter as to render multiple body collision processes negligible. An admixture of 10-20% of such processes would not significantly alter the predicted angle integrated spectra, yet they could very considerably modify the assumed intranuclear scattering kernel with respect to kinematic back angle limits. This is probably an important class of processes to consider if one is to adequately explore the limitations of the semiclassical approach in reproducing back angle scattering results.^{1,6}

5. CONCLUSIONS

In this work we have applied the analytic solution of the angular distribution of nucleons following the scattering of a projectile nucleon with nucleons of a Fermi gas. An isotropic nucleon-nucleon scattering angular distribution was assumed in deriving the analytic expression, and the Pauli exclusion principle was included. While the equations used were taken from the work of Kikuchi and Kawai², earlier expositions are to be found in the work of Goldberger^{1,3} and of Hayakawa, Kikuchi and Kawai^{1,4}. One important observation is that the shape of the single scattering kernel resulting from the approach is in much better agreement with experimental precompound angular distributions than results using the stationary target nucleon approximation, as shown by Costa *et al.*^{1,2}

Following the example of Mantzouranis *et al.*,⁶ we have used an exciton model formulation to give the spectral contributions as a function of the number of intranuclear scattering events (the so-called exciton hierarchy). We have however used the geometry dependent hybrid model¹ which retains somewhat more of the reaction information which may be relevant to angular distributions than pure exciton formulations (such as the hybrid model) which are averaged over the additional geometric details, making local density dependent treatments more tenuous.

We have used this approach to compare with data spanning a fairly broad range of incident nucleon energies (14.7 to 90 MeV) and exit channel energies (9 to 70 MeV). The folded N-N scattering kernel (which is mostly the single scattering result when exit channel energy is more than half the entrance channel energy) reproduces this broad range of results very well at angles less than $\sim 90^\circ$ and greater than 20° ; this region includes the major part of the cross section. As with earlier related

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

treatments such as the INC, angles in excess of 90° are, for many of our examples, underestimated. The GDH model results are in better agreement with data than results of the hybrid model.

We have compared the single scattering result of Tamura *et al.*⁷ with our semiclassical result and see that quantum mechanical treatments provide back angle cross sections which are kinematically forbidden in the semiclassical N-N scattering kernel. This result and a simple comparison with angular localization constraints due to Eq. (6) lead us to the conclusion that understanding of the back angle yields requires quantum mechanical formulations, a conclusion in agreement with Tamura *et al.*⁷ and with Feshbach *et al.*⁸ However the comparisons of Fig. 19 indicate that these approaches may still be poorer than the much simpler semiclassical treatments when a-priori predictions are required, and for applications in nuclear technology the semiclassical approach is surely vastly easier to perform. We should, however, emphasize the excellent results being obtained by Bonetti using the formalism of Feshbach *et al.*; note for example Fig. 21 from a recent work by Scobel *et al.*^{3,3}

We have investigated ad-hoc methods of trying to join quantum mechanics with classical mechanics via 'refraction' and/or folding due to Heisenberg constraints. While we can force a fit to back angle yields at lower projectile energies by this method, we find that at the higher bombarding energies where the assumptions of refraction should be best, the results show an over correction at forward angles, and poorer shapes overall. We therefore are skeptical of even the empirical efficacy of such approaches, while agreeing that quantal effects are the major missing physics in using the semiclassical N-N scattering kernel. If there is to be a legitimate method of increasing calculated back angle yields in the semiclassical approach it is likely that some

M. BLANN

contribution from multiple excitation modes (other than pure nucleon-nucleon interactions) should be considered together with nucleon-nucleon interactions.

FIG. 21. Experimental (solid points) and calculated angular distributions for proton (26.7 MeV) induced reactions on ^{65}Cu . The dashed curve is the prediction by Bonetti of the multi-step direct component of the theory of Ref. 7; the solid line is the sum of calculated multi-step direct plus multi-step compound reactions.

M. BLANN

Finally the experimental results should not escape scrutiny. Many of the data cited in this work did not include adequate background measurements at back angles, since the purpose of the experiments did not include special focus on back angle yields. If we are to focus great attention on fitting the very small portion of the cross sections at far back angles, some data sets extending as close to 180° as possible, with careful attention to minimizing and correcting for background at these back angles, would be valuable and indeed indispensable to final evaluation of possible models. Additional measurements extending toward 0° would similarly be valuable, as the comparisons of this and other works show this to be another region where the pure N-N scattering kernel may be inadequate in reproducing details of the angular distributions.

ACKNOWLEDGEMENTS

One of the authors (M.B.) appreciates valuable discussions with Profs. A. Kerman, T. Tamura, and F. Petrovich. We appreciate also the calculations provided us for Fig. 19 by Prof. Tamura and by Mr. M. Benhamow. We appreciate a critical reading of the manuscript and helpful suggestions from Drs. H. Gruppelaar and H. Akkermans. Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

REFERENCES

1. M. BLANN, *Phys. Rev. Lett* **28**, 757 (1972).
2. K. KIKUCHI and M. KAWAI, "Nuclear Matter and Nuclear Reactions," North Holland, Amsterdam (1968).
3. R. SERBER, *Phys. Rev.* **72**, 1114 (1947); N. METROPOLIS et al., *Phys. Rev.* **110**, 185, 204 (1958).
4. H. W. BERTINI, *Phys. Rev.* **131**, 1801 (1963).

PRECOMPOUND NUCLEON ANGULAR DISTRIBUTIONS

5. K. CHEN *et al.*, Phys. Rev. 166, 949 (1968); K. CHEN, G. FRIEDLANDER and J. M. MILLER, Phys. Rev. 176, 1208 (1968); K. CHEN, G. FRIEDLANDER, G. D. HARP and J. M. MILLER, Phys. Rev. C 4, 2234 (1971).
6. T. TAMURA, T. UDAGAWA and H. LENSKE, Phys. Rev. C 26, 379 (1982).
7. H. FESHBACH, A. KERMAN and S. E. KOONIN, Ann. Phys. (NY) 125, 429 (1980); for use of this theory see R. BONETTI, M. CAMNASIO, L. COLLI-MILAZZO and P. E. HODGSON, Phys. Rev. C 24, 71 (1981).
8. C. MANTZOURANIS, H. A. WEIDENMULLER and D. AGASSI, Z. Phys. A276, 145 (1976).
9. Sun ZIYANG, Wang SHUNUAN, Zhang JINGSHANG and Zhuo YIZHONG, Z. Phys. A305, 61 (1982).
10. J. M. AKKERMANS, H. GRUPPELAAR and G. REFFO, Phys. Rev. C 22, 73 (1980).
11. J. M. AKKERMANS, Phys. Lett. 82B, 20 (1979).
12. C. COSTA, H. GRUPPELAAR, and J. M. AKKERMANS, Phys. Rev. C 28, 587 (1983).
13. M. L. GOLDBERGER, Phys. Rev. 74, 1269 (1948).
14. S. HAYAKAWA, M. KAWAI, and K. KIKUCHI, Prog. Theor. Phys. 13, 415 (1955).
15. T. UDAGAWA, K. S. LOW, and T. TAMURA, Phys. Rev. C 28, 1033 (1983); T. TAMURA, private communication (1983).
16. A. KERMAN, private communication (1983).
17. M. BLANN and H. K. VONACH, Phys. Rev. C 28, 1475 (1983).
18. W. D. MYERS, Droplet Model of Atomic Nuclei, (IFI Plenum, New York, 1977).
19. M. BLANN, Nucl. Phys. A213, 570 (1973).
20. G. MANTZOURANIS, Phys. Lett. 63B, 25 (1976).
21. *loc. cit.* Ref. 10; in this work (n,n') spectra from 14.7 MeV n induced reactions were summed for a bin between 2 and 11 MeV. The spectra below 7 MeV, which dominate the cross sections, correspond to 'slow' particles. Refs. 10 and 20 are only a few of many examples where the 'fast' particle requirement of Ref. 8 has been relaxed or ignored.
22. A. GALONSKY, R. R. DOERING, D. M. PATTERSON, and H. W. BERTINI, Phys. Rev. C 14, 748 (1976).
23. D. HERMSDORF *et al.*, Report ZfK-277 (Rossendorf bei Dresden, 1974); A. TAKAHASHI *et al.*, Osaka Univ. Oktavian Report A-83-01 (1983) unpublished; J. L. KAMMERDIENER, Lawrence Livermore National Laboratory, Livermore, CA, UCRL-51232 (1972) unpublished.
24. A. MARCINKOWSKI, *et al.*, Nucl. Sci. Eng. 83, 13 (1983); A. MARCINKOWSKI *et al.*, unpublished.

M. BLANN

25. M. BLANN, R. R. DOERING, A. GALONSKY, D. M. PATTERSON, and F. E. SERR, *Nucl. Phys.* A257, 15 (1976).
26. W. SCOBEL *et al.*, *Phys. Rev. C* 30, 1480 (1984).
27. A. M. KALEND *et al.*, *Phys. Rev. C* 28, 105 (1983).
28. T. TAMURA and M. BENHAMOW, private communication (1983).
29. H. W. BERTINI, G. D. HARP, and F. E. BERTRAND, *Phys. Rev. C* 10, 2472 (1974).
30. R. BONETTI, private communication (1984); R. BONETTI and L. COLLI-MILAZZO, reported at workshop on Coincident Particle Emission From Continuum States; Bad Honnef, F.R.G. (1984).
31. R. WILSON, "The Nucleon-Nucleon Interaction", *Interscience Tracts on Physics and Astronomy* no. 16, Ed. R. E. Marshals, Interscience (New York) 1963.
32. M. KAWAI and H. A. WEIDENMULLER, *Z. für Physik A* 311, 117 (1983).