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Estimate of the Coupling Impedance
for
the Storage Rings of the NSLS

A. G. Ruggiero

August 1979

NATIONAL SYNCHROTRON LIGHT SOURCE

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BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973

I. Introduction

The most important ingredient to evaluate the stability of a particle beam in a storage ring is the longitudinal coupling impedance Z/n and the transverse impedance $Z_{\perp}$ which is usually associated to the former.

In this paper we calculate these impedances for the two storage rings which are part of the NSLS, namely the Ultra Violet Ring (UVR) and the X-Ray Ring (XRR) - the parameters for these two rings which are used throughout the paper are shown in Table I.

The method we use to estimate the impedance is the one outlined by Faltens and Laslett¹. They emphasize that starting from the lowest cut-off frequency of the vacuum pipe, charged particles are actually radiating in the free space, and therefore for frequencies sufficiently large the impedance is mostly contributed by the radiation in the vacuum. For the special case of the two rings under consideration, the length of the beam bunch is comparable to the lowest cut-off wavelength and a considerable fraction of the bunch current spectrum lies in the range of the cut-off frequencies. Moreover the most serious stability considerations concern the motion within a bunch and henceforth involve the range of frequencies we have just mentioned.

But before to estimate quantitatively the impedances we like first to get an idea of what values can be tolerated. For the longitudinal motion we can take the classical stability criterion which, derived for a coasting², has been proven to be valid also for a bunched beam³

$$\left| \frac{Z}{n} \right| \leq \frac{\alpha E}{e I_p} \left(\frac{\Delta E}{E} \right)^2$$

where α is the momentum compaction factor, E the particle total energy, I_p the peak current that we define as

$$I_p = I_{AV} \frac{2\pi R}{3M\sigma_z}$$

with I_{AV} the average current in M bunches, with rms length σ_z and over a circumference $2\pi R$. Finally $\Delta E/E$ is the FWHM of the relative energy distribution. We shall take $\Delta E/E = 3\sigma_E/E$ where σ_E is the rms spread. With the numbers in Table I we obtain

$$\left| \frac{Z}{n} \right| \leq \begin{array}{ll} 0.6 \text{ ohm} & \text{for UVR} \\ 1.1 \text{ ohm} & \text{for XRR} \end{array} \quad (1)$$

The meaning of n is the ratio of the storage ring circumference to the wavelength of the perturbation travelling around the bunch. It is also an harmonic number, the ratio of an actual frequency to the revolution frequency.

For the transverse motion we can take the following stability criterion as derived by Sacherer⁴ for the case of smooth transverse impedance

$$\Delta\nu \gtrsim \frac{e|Z_{\perp}|I_{AV}R^2}{6\nu\sigma_z EM}$$

where $\Delta\nu$ is the required betatron tune spread by other means than sextupole, for the lowest head-tail mode - higher modes require smaller spreads. In this equation ν is the (vertical) betatron tune.

It is possible to relate the transverse impedance $Z_{\perp}$ to the longitudinal one⁴

$$Z_{\perp} = -i \frac{RZ_o}{\beta^2 \gamma^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) + \frac{2R}{b^2} \frac{Z_n}{n}$$

where a is the average beam radius and b the average pipe radius, and $Z_o = 377$ ohm. Also β and γ are the usually relativistic factors. In the following, we shall always take $\beta = 1$. We obtain

$$Z_{\perp} = \begin{cases} \left(-i 39 + 18 \frac{Z}{n} \right) K\Omega & \text{for UVR} \\ \left(-i 10 + 60 \frac{Z}{n} \right) K\Omega & \text{for XRR} \end{cases}$$

Therefore if the $\frac{Z}{n}$ is kept within the limit (1), the transverse impedance for the UVR is essentially contributed by the space charge term, whereas in the XRR most of the contribution will come from the longitudinal term.

The required tune spread for individual bunch stability is

$$\Delta\nu = \begin{cases} 1.6 \times 10^{-3} & \text{for UVR} \\ 0.7 \times 10^{-3} & \text{for XRR} \end{cases}$$

There is a chance that such tune spread exists already without addition of machine non-linearities, because of the ions produced by scattering on the residual gas⁵.

II. The Longitudinal Coupling Impedance

A. Smooth Pipe

As suggested by Faltens and Laslett¹ let us start by assuming that the beam is following a circular trajectory in the free space, far away from any environment, without a vacuum chamber. The free space is associated with an impedance¹ which for pure circular orbits we can write

$$\frac{Z}{n} = \frac{354}{n^{2/3}} \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right) \text{ ohm}$$

Such a formula nevertheless must be modified to include the straight sections of the storage rings where the particles do not radiate. For both rings we obtain

$$\frac{Z}{n} = \frac{220}{n^{2/3}} \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right) \text{ ohm} \quad (2)$$

The spectrum of this impedance extends up to some critical harmonic number $n_{\text{crit}} = \frac{3}{2} \gamma^3$, about 4×10^9 for the UVR, and 10^{11} for the XRR. For $n > n_{\text{crit}}$ the impedance is effectively reduced to zero. As we shall see this is the largest frequency range we have to consider. All other items which can give a contribution to Z/n are effective over a much narrower range.

Now let us surround the beam with a vacuum chamber. But first let us assume the pipe is perfectly conductive, perfectly smooth, circular and straight. Let b be the inner radius. With such a pipe there are two contributions to Z/n . One is given by the static inductance and capacitance of the beam

$$\frac{Z}{n} = i \frac{Z_0}{2} \ln \frac{R}{nb} \quad (3)$$

By

which is very negligible since at most is 10^{-3} ohm for the UVR at $n = 1$.

This impedance has obviously a cut-off for $n = R/b$. The second contribution is the usual space charge term

$$\frac{Z}{n} = i \frac{Z_0}{2\beta\gamma^2} \left(1 + 2\ln \frac{b}{a} \right) \quad (4)$$

which rolls off at $n = n_b \approx \gamma R/b$. We have

	<u>UVR</u>	<u>XRR</u>
Z/n	$1. \times 10^{-3}$ ohm	1.2×10^{-4} ohm
n_b	3.8×10^5	3.6×10^6

Also this is a very negligible contribution.

But actually the vacuum pipe, even when it is smooth, perfectly conductive and straight, contributes more than (3) and (4). It has the capability to shield the beam against the "free space" radiation. The "free space" impedance as given by (2) is quite large, 220 ohm for $n = 1$, but it is reduced practically to zero with the presence of the pipe. The only contributions left would be those given by (3) and (4). The pipe shieldings though is not effective at large frequency. It is well known, as one can derive by solving the Maxwell's equations for a circular pipe of radius b , that there are cut-off values of the wavelengths λ given by the equation

$$J_0 \left(2\pi \frac{b}{\lambda} \right) = 0$$

where J_0 is the Bessel function, for which the space occupied by the pipe wall would be at zero (the reference) voltage, whether there is the pipe or

not. The frequencies associated to these wavelengths are therefore sort of windows to the "free space". At these frequencies the particles really radiate undisturbed. Therefore, at the same frequencies the impedance would correspond as given by (2). In terms of harmonic numbers, those which correspond to a cut-off are

$$n_s = \frac{R}{b} j_s \quad s = 1, 2, 3, \dots \quad (5)$$

where j_s are the zeros of the Bessel function J_0

$$j_1 = 2.405$$

$$j_2 = 5.520$$

$$j_3 = 8.654$$

$$j_s = \left(s - \frac{1}{4} \right) \pi \quad \text{for } s \gg 1.$$

By inserting (5) in (2) we can estimate the impedance at the cut-off values. Between these values the impedance should go back to the pipe value, but actually this does not happen quite so; at higher harmonics it becomes more difficult to shield the beam from its own radiation, until when, after few resonating cut-off harmonics the impedance of the "free space" predominates everywhere.

In reality the vacuum chamber in the UVR and in the XRR has a rectangular cross-section with a width $w = 8$ cm and a height $h = 4$ cm. For this geometry the cut-off harmonic numbers are

$$n_s = \frac{2\pi R}{w} s \quad , \quad s = 1, 2, 3, \dots$$

At the first cut-off, for instance, the "free space" impedance is reduced down to

	<u>UVR</u>	<u>XRR</u>
n_1	638	2125
Z/n_1	2.97 ohm	1.33 ohm

Anyway, for $n \geq n_b \sim \gamma n_1$ (9×10^5 for UVR, 8.5×10^6 for XRR) the pipe is completely ineffective and one is definitively in the free space region.

At this time it is interesting to compare the bunch spectrum to the impedance. In terms of harmonic number we can define $n_s = 2\pi R/2\sigma$ as the critical bunch number. We obtain $n_s = 664$ for UVR and $n_s = 2125$ for the XRR. Then in both storage rings the bunch spectrum overlaps with the pipe cut-off region and makes this a very sensitive area for beam stability.

B. Resistive and Bent Pipe

Still preserving the smoothness of the pipe, that is no discontinuities or transitions, but constant cross-sections, let us nevertheless consider a more realistic vacuum chamber with resistive wall and bent over a circumference $2\pi R$.

The vacuum chamber is made of aluminum (ρ , resistivity = $2.83 \mu\Omega \times \text{cm}$ at 20°C) except in few long straight sections where the pipe is circular with 7.3 cm diameter and made of stainless steel ($\rho = 73 \mu\Omega \times \text{cm}$ at 20°C). The contribution to the impedance from the resistive wall is

$$\frac{Z}{n} = \frac{1-i}{\sqrt{n}} \sqrt{\frac{Z_o \rho R}{2b^2}} \quad (6)$$

which is correct for a pipe with circular cross-section and radius b . Taking into account also the long straight sections (with stainless steel) we obtain

$$\begin{aligned}\frac{Z}{n} &= \frac{1-i}{\sqrt{n}} \quad 0.40 \quad \text{ohm for UVR} \\ &= \frac{1-i}{\sqrt{n}} \quad 0.67 \quad \text{ohm for XRR}\end{aligned}$$

Like most of any other pipe impedance this also rolls off at $n_b \sim \gamma n_1$. At the first cut-off harmonic number n_1 the contribution of the resistive wall is 0.015 ohm for both rings, quite negligible compared to the "free space" impedance.

Of quite different concern is the curvature of the pipe. If the pipe were straight the electromagnetic waves would be plane and propagating at the light velocity (assuming again there are no discontinuities or any sort of obstacle). The beam then could never catch up with the wave it radiates and resonances would not be possible. But the picture is different for a bent pipe where for very short wavelengths the waves are not plane. A wave propagating at the outer side of a beam, though speeding at higher velocity, has nevertheless a longer trajectory than the beam which is moving to the inside at slightly less velocity. Therefore, it is possible for the beam to catch up with the wave it radiated the revolution (or few revolutions) before. This is the condition for the "pipe resonance" (or resonances). Faltens and Leslett¹ put the resonance condition as

$$\beta \left(1 + \frac{b}{R} \right) = 1 + 0.80862 n_r^{-2/3} \quad (7)$$

which is valid for a toroidal vacuum chamber with major radius R and minor radius b . When (7) is applied with $b = w/2$ we obtain

$$n_r = \begin{cases} 2100 & \text{for UVR} \\ 12,800 & \text{for XRR} \end{cases}$$

Actually Laslett and Lewish⁶ have calculated the electromagnetic response of a toroidal vacuum chamber with a rectangular cross-section with the inner wall at radius R_i and the outer wall at radius R_o . If the beam is located right in the middle of the vacuum chamber the radius of its orbit would be $R = (R_o + R_i)/2$ and the vacuum chamber width $w = R_o - R_i$.

Define

$$q^2 = \beta^2 \frac{n^2}{R^2} - \frac{(2m+1)^2 \pi^2}{h^2}$$

where m can be any integer, then the resonance condition for the vacuum chamber is

$$J_n(qR_o) N_n(qR_i) - J_n(qR_i) N_n(qR_o) = 0 \quad (8)$$

Here J_n is the Bessel function of order n , and N_n is the Neumann function of order n . For a given vacuum chamber geometry and integer m , there is a value n_r which satisfies (8). This is the resonance harmonic number. The solution of (8) is not expected to be much different from the one that can be obtained from (7).

The figure of quality Q of the vacuum chamber is estimated in ref.[6]. In the approximation that the wavelength which corresponds to n_r , namely $\lambda_r = 2\pi R/n_r$, is smaller than the two dimensions w and h of the pipe, one has the following formula

$$Q = \frac{4\pi^3 Z_o w}{\lambda_r^3 q^2 R_{\text{surf.}}}$$

where

$$R_{\text{surf}} = \frac{\text{resistivity}}{\text{skin depth}} = \sqrt{\frac{n_r Z_o \rho}{2R}}$$

Inserting numbers one obtains

$$Q = \begin{cases} 1.2 \times 10^5 & \text{for UVR} \\ 1.5 \times 10^5 & \text{for XRR} \end{cases}$$

In terms of the chamber Q the impedance at the resonance is given by

$$\frac{Z}{n} = \frac{2Z_o Q \lambda_r}{n_r^2 n^2 h R w q^2}$$

where $\eta = w/2R$. For our rings this gives

$$\begin{aligned} Z/n &= 0.14 \text{ ohm} & \text{for} & \text{ UVR} \\ &= 0.22 \text{ ohm} & \text{for} & \text{ XRR} \end{aligned}$$

Though the resonance is very narrow, yet the amount involved is very large, a couple of kilo-ohm for the XRR, to cause enough worry about the beam stability. Moreover, as pointed out, there are several resonances, and we have looked only at the first one. From differentiation of (7) one can see that successive resonance harmonics are spaced by 0.0013 cm in the UVR and by 0.00021 cm in the XRR. As put it by Faltens and Laslett this "may warrant reexamination of the usual relations between the peak value $|Z_n/n|$ and longitudinal stability".

Our advice is that the impedance from the curvature of the vacuum chamber deserves much more careful analysis than done here.

C. Discontinuities

A realistic pipe is made of discontinuities and obstacles. These will have the effect to scatter the radiated waves and brake the electromagnetic shielding of the vacuum chamber, until the "free space" impedance is restored and even exceeded.

The major items that are included in storage rings like UVR and XRR are the following:

- i. Pick-up Plates
- ii. Clearing Electrodes
- iii. Bellows

iv. Steps and Box-like objects, Cavities

v. RF Cavities

vi. Vacuum Ports

Slits for synchrotron radiation extraction.

Holes for distributed pumping.

Slits for beam injection, extraction, dumping.

Each of these items requires a careful examination, and we have indeed estimated their contribution to the impedance as we shall see later. There are of course other elements, like ceramic pipes with kicker magnets, but they are not included in this analysis. Otherwise we believe the above list is quite comprehensive.

i. Bellows

There are 8 bellows in the UVR and 16 in the XRR all with the same cross-section of $8 \times 4 \text{ cm}^2$. They are made of copper ($\rho = 1.7 \mu\Omega \times \text{cm}$ at 20°), each with 6 wiggles for a total length of 7.4 cm. The bellow height is 0.6 cm.

In the low frequency range, bellows cause just an inductance⁷

$$\frac{Z}{n} = -i Z_0 \left(\frac{Ml}{2\pi R} \right) \ln \frac{\text{outer radius}}{\text{inner radius}}$$

where Ml is the total length. We obtain

$$\frac{Z}{n} = -i \begin{cases} 0.45 \text{ ohm} & \text{for UVR} \\ 0.26 \text{ ohm} & \text{for XRR} \end{cases}$$

A more serious problem nevertheless exists in the large frequency range when each convolution thought of as a cavity, starts to resonate. The calculation is done by Keil and Zotter⁸ but we also applied the technique of the radial transmission line model⁹ and we obtained about the same result. The lowest resonating frequency is around 4 GHz; for copper the corresponding Q is about 10^4 , and (unfortunately) the impedance Z about 10 k Ω per convolution. Since there are about 100 convolutions in the XRR this makes a total of 1M Ω , quite a large number that is balanced only by the narrow width of the resonance. Also bellows are slightly different from each other and will resonate at different frequency. This will make a cumulative broader resonance with a lower impedance. The impedance for the higher order resonance decreases like $f_r^{-3/2}$.

Note that if the bellows were made of stainless steel, though Q would be lowered by an order of magnitude making the resonance broader, the impedance would also be reduced by the same amount.

ii. Pick-ups

There are 16 p.u. devices in the UVR and 32 in the XRR. Each device is made of four round buttons with a diameter of 1.8 cm, made of stainless steel and with a termination to the centre. The capacitance of a button to ground is 6pF which corresponds to a characteristic impedance of 10 ohm. Each button is terminated to a 50 ohm resistor.

Obviously one can apply the theory of clearing electrodes¹⁰ to the pick-up plates, but we have to approximate the circular geometry with a rectangular one, as it was formulated in the original calculations¹⁰. The adjustment will cause a correction factor of 0.5 in the impedance.

In the low frequency range the impedance is inductive again

$$\frac{Z}{n} = - 2\pi i Z_c \left(\frac{4Ml}{2\pi R} \right) \left(\frac{\phi_o}{\pi} \right)^2$$

where Z_c is the characteristic impedance and ϕ_o an effective semi-angular aperture. We obtain

$$Z/n = -i \begin{cases} 0.036 \text{ ohm} & \text{for UVR} \\ 0.022 \text{ ohm} & \text{for XRR} \end{cases}$$

In the large frequency range pick-up plates resonate. The technique used is the model of transmission lines¹⁰. There are two kind of resonances: The loaded resonances which depend on the ratio Z_T/Z_c of the termination impedance Z_T to the characteristic impedance Z_c , and the unloaded resonances with a wavelength that has a knot in correspondance of the termination and therefore make the plate resonate as floating. The latter resonances are eventually damped by the resistivity of the plates. For the two storage rings UVR and XRR, because of the large value of γ and because of the material (stainless steel) involved, the two kind of resonances merge to the same kind. They are rather broad-band, resonate at frequencies

$$f_K = \frac{c}{l} \frac{2K-1}{2} \sim 8(2K-1) \text{ GHz}, \quad K = 1, 2, 3, \dots$$

and have an impedance

$$Z_K = Z_C M \left(\frac{\phi_O}{\pi} \right)^2 \left(\frac{Z_C}{Z_T} \right) = \begin{cases} 32 \text{ ohm} & \text{for UVR} \\ 65 \text{ ohm} & \text{for XRR} \end{cases}$$

Though they extend over a quite large frequency range, these resonances do not involve very large impedances.

iii. Clearing Electrodes

There might be some need of clearing excessive quantities of ions trapped by the electron beam after ionization of the residual gas⁵. Then we conceive a situation where there are 16 pairs of clearing electrodes in the UVR and 32 in the XRR. Each electrode can be made round with 5 cm diameter. The expected characteristic impedance could be 5 ohm, whereas the termination can be a standard 50 ohm. The plates made of stainless steel will have the termination to the centre. The same procedure outlined for the pick-ups can also be applied here and all the considerations we made there still hold.

In the low frequency range we estimate the following inductance

$$\frac{Z}{n} = -i \begin{cases} 0.38 \text{ ohm} & \text{for UVR} \\ 0.23 \text{ ohm} & \text{for XRR} \end{cases}$$

Again the two kind of resonances (loaded and unlaoded) could not be

distinguished from one another. The resonating frequencies are

$$f_K = 3 \times (2K-1) \text{ GHz} \quad K = 1, 2, 3, \dots$$

and the impedance is

$$Z_K = \begin{cases} 65 \text{ ohm} & \text{for UVR} \\ 125 \text{ ohm} & \text{for XRR} \end{cases} \quad \text{for any } K$$

quite a manageable amount.

The impedance from both pick-ups and clearing electrodes are eventually cut-off for $n \geq \gamma \frac{2\pi R}{w}$.

Observe that in these devices the termination impedance is not matched to the characteristic one. This will cause some loss of sensitivity in the pick-ups and has no effect in the clearing electrodes which are to be operated d.c. But from the beam point of view there might be the inconvenience of the enhancement of a long range wake field¹¹.

iv. Steps

In the UVR there are four long straight sections and in the XRR there are eight that have a different cross-section from the rest of the ring, a circular pipe with radius of 7.3 cm. The transition is made with steps, which because of the considerable length of the straight sections (several meters), can be regarded as standing alone. We shall estimate the impedance in the worst case the transition is sudden, whereas in practice it is tapered of a length of 8.8 cm.

Define S as the ratio of the outer dimension to the inner dimension b . We take $S = 1.8$. According to Hereward¹² the impedance per step is

$$\frac{Z}{n} = (1-i\pi) \frac{2Z_o}{\pi} (S-1)^2 \frac{b}{2\pi R} = \begin{cases} 0.060 \text{ ohm} & \text{for UVR} \\ 0.018 \text{ ohm} & \text{for XRR} \end{cases}$$

$$\text{for } n < \frac{4\pi R}{4b(S-1)} = n_w$$

where $n_w = 800$ for UVR and 2650 for XRR.

If $n \geq n_w$ then

$$\frac{Z}{n} = \frac{Z_o}{2\pi} \frac{(S-1)}{n} - \frac{50}{n} \text{ ohm per step for both rings.}$$

Also these numbers are quite manageable.

The steps in the two rings are quite apart from each other and therefore we cannot identify any cavity or box-like object which could require a serious consideration.

v. RF Cavities

A major RF cavity can always be approximated with an RLC-resonating circuit. Usually a cavity is defined by a resonating frequency, which in our case is h times the revolution frequency, where h is the harmonic number, a figure of merit Q and a shunt impedance Z_s . The three impedance elements (R , L and C) can then be estimated from these. The impedance at the fundamental harmonic is compensated with proper phase and voltage loops. But for $n < h$ and $n > h$ there is an appreciable impedance which is respectively inductive and capacitive. One has

$$\frac{Z}{n} = \frac{Z_s}{Q^2 n} - i \frac{1}{h} \frac{Z_s}{Q} \quad \text{for } n < h$$

$$\frac{Z}{n} = i \frac{1}{n^2} h \frac{Z_s}{Q} \quad \text{for } n > h$$

The main parameters are

	<u>UVR</u>	<u>XRR</u>
h	9	30
Q	20,000	20,000
Z _s	2MΩ	4MΩ

and one has

$$\left. \begin{aligned} \frac{Z}{n} &= -i \, 11 \, \text{ohm} && \text{for UVR} \\ \frac{Z}{n} &= -i \, 6.7 \, \text{ohm} && \text{for XRR} \end{aligned} \right\} \text{for } n < h$$

and

$$\left. \begin{aligned} \frac{Z}{n} &= i \, \frac{90}{n^2} \, \text{ohm} && \text{for UVR} \\ \frac{Z}{n} &= i \, \frac{6 \times 10^3}{n^2} \, \text{ohm} && \text{for XRR} \end{aligned} \right\} \text{for } n > h$$

There is quite a large inductance for small harmonic number, left-over of the major RF system. Very likely our numbers are overestimated, but still considerably large to cause some concern.

On top of this, RF cavities are responsible for a all host of higher order modes. These are quite dangerous and one should keep control of them. We will not deal with them here.

vi. Slits and Holes in the Vacuum Chamber

The impedance from a slit or hole is eventually caused by the diffraction of the electromagnetic wave through them. Part of the wave is reflected back at the edge of the hole, and another part is lost beyond.

Taking the same approach exposed by M. Sands¹³ but working in the frequency domain instead of the time domain, we have estimated the impedance for a frequency $\omega = n\omega_0$. This is

$$\frac{Z}{n} = \frac{2Z_o \alpha^2}{3\pi^3 R_b^4} \left[n^3 + i \frac{8}{\pi} \left(n^2 n_c + \frac{1}{3} n_c^3 \right) \right] \quad (9)$$

where for a rectangular slit of dimensions ℓ and w ($\ell \gg w$)

$$\alpha = \frac{\pi}{16} \ell w^2$$

and b is the pipe radius. Also $n_c \sim 2\pi R/\ell$. Eq. (9) is valid only for $n \leq n_c$. For $n > n_c$ the wavelength is smaller than the dimension of the hole and the beam is practically radiating in the "free space", though for only the length of the slit.

We have estimated the impedance (9) for several hole geometrics in the two storage rings.

(a) Ports for synchrotron radiation extraction. They are 40 cm long and 0.6 cm wide. There are 16 in the UVR and 32 in the XRR. For them we obtain

$$\frac{Z}{n} = \left[n^3 + i \frac{8}{\pi} \left(n^2 n_c + \frac{1}{3} n_c^3 \right) \right] \times \begin{cases} 1.5 \times 10^{-10} \text{ ohm for UVR} \\ 2.4 \times 10^{-12} \text{ ohm for XRR} \end{cases}$$

with $n_c = 127$ for UVR and 425 for XRR.

(b) There are the major vacuum pump ports. Each pump has seven rows of two slits each with $l = 5.2$ cm and $w = 0.6$ cm.

In this case we multiply the impedance also by a "coherence factor" of seven, because seven holes can resonate coherently at the same time¹³. We obtain

$$\frac{Z}{n} = \left[n^3 + i \frac{8}{\pi} \left(n^2 n_c + \frac{1}{3} n_c^3 \right) \right] \times \begin{cases} 5 \times 10^{-10} \text{ ohm for UVR} \\ 8 \times 10^{-12} \text{ ohm for XRR} \end{cases}$$

and $n_c = 784$ for UVR and 1568 for XRR.

(c) Holes for distributed pumping. For each magnet there are 43 pairs of holes 2.94×0.67 cm². Each pair radiates coherently and therefore the impedance is multiplied by a factor of two.

The contribution from these holes is

$$\frac{Z}{n} = \left[n^3 + i \frac{8}{\pi} \left(n^2 n_c + \frac{1}{3} n_c^3 \right) \right] \times \begin{cases} 1.1 \times 10^{-10} \text{ for UVR} \\ 1.8 \times 10^{-12} \text{ for XRR} \end{cases}$$

with $n_c \sim 1000$ for UVR and 2000 for XRR.

These holes, as the previous ones, can give a considerable contribution when n approaches the limit value n_c . Nevertheless the reactance is

positive, which denotes a capacitance, and can offset some of the large value of the inductance we have accumulated with the other vacuum chamber items.

III. Conclusions

The longitudinal impedance Z/n is plotted in Fig. 1 for the UVR and in Fig. 2 for the XRR. The solid line is our result. The dashed straight line is the impedance in the free space, and the dashed curve the contribution from the very dense resonances of the vacuum chamber curvature, assuming the same impedance for all of them. One can clearly notice the contribution of the "wall resistivity" in the low frequency range, which is followed by a flat plateau which represents the region of the "inductive wall". The wiggles are caused by the lowest cut-off harmonic numbers.

The transition from the "pipe" impedance to the "free space" impedance is obvious.

All the contributions have been included in Figs. 1 and 2 except those from the main RF system and the high resonating modes of the bellow convolutions.

TABLE I

Machine Parameters

	<u>UVR</u>	<u>XRR</u>
E, energy	0.7 GeV	2.0 GeV
I_{AV} , average current	1A	1A
M, no. of bunches	9	30
$2\pi R$, circumference	51m	170m
W_0 , 2π x revol. frequency	$2\pi \times 5.88$ MHz	$2\pi \times 1.76$ MHz
α , mom. comp. factor	0.02345	0.0065
σ_z , rms bunch length	3.84cm	4.0cm
σ_E/E , rms energy spread	4.4×10^{-4}	6.6×10^{-4}
I_p , peak current	49.2A	47.2A
ν , vert. betatron tune	1.4	5.7
a, aver. beam radius	0.2mm	0.25mm
b, aver. pipe radius (circ. approximation)	3cm	3cm
bending radius	1.91m	6.875m
rectan. vac. chamber: wxh	$8 \times 4 \text{ cm}^2$	$8 \times 4 \text{ cm}^2$

Fig. 1 Longitudinal Impedance for UVR

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Fig. 2 Longitudinal Impedance for XRR