SEISMIC DESIGN TECHNOLOGY FOR BREEDER REACTOR STRUCTURES

Volume 4: Special Topics in Piping and Equipment

April 1983

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PREPARED FOR THE UNITED STATES DEPARTMENT OF ENERGY
UNDER SUBCONTRACT TO UNION CARBIDE CORPORATION
SUBCONTRACT NUMBER 41X-40100

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PREFACE

In 1974 Agbabian Associates embarked on a project to develop and update seismic design criteria and analysis methods as part of the national long-range program on Breeder Reactor Seismic Technology. This work was initiated by the Division of Reactor Development and Demonstration of the Energy Research and Development Administration. The work was continued under contract with the Department of Energy (DOE) and under contract with Union Carbide Corporation. The Technical Monitors for the project were C. Chester Bigelow for DOE and Claude Pugh and Richard Gwaltney for Union Carbide. The Project Managers at Agbabian Associates were Dr. George A. Young and Dr. Damoder P. Reddy.

The purpose of the study was to identify the problem areas associated with the application of seismic hazard assessment procedures and to establish guidelines for design and analysis of earthquake-resistant breeder reactor structures, equipment, and piping. The result of these research efforts was a series of thirty topical reports covering various topics related to the overall objective; these reports were produced over the span of years 1975-82.

The present volume is part of a subsequent shorter series that condenses the information from the original volumes into four volumes. There are three reasons for this condensed version. The first is that the original volumes were published on a limited distribution basis, and it is intended that the information published in the present volumes be available for a wide distribution and greater accessibility. Second, the information in the new, shorter form will be easier to use and

therefore will find wider reception and greater utility in this way. Third, since much of the information contained in the topical reports has general application, the text of these four volumes is prepared to serve as a useful reference on many general topics in the analysis and design of earthquake-resistant nuclear facilities.

The general title for the four volumes is Seismic Design Technology for Breeder Reactor Structures, and the volumes are entitled:

- Vol. 1. Special Topics in Earthquake Ground Motion
- Vol. 2. Special Topics in Soil/Structure Interaction
- Vol. 3. Special Topics in Reactor Structures
- Vol. 4. Special Topics in Piping and Equipment

The present volume, Volume 4, consists of five chapters which cover these respective topics pertaining to nuclear power plants:

<u>Chapter</u>	<u>Topic</u>
1	Experimental verification of piping systems
2	Analytical verification of piping restraint systems
3	Seismic analysis techniques for piping systems with multi-support input
4	Development of floor spectra from input response spectra
5	Seismic analysis procedures for in-core components

These chapters are based on six topical reports SAN/1011-116R, SAN/1011-118R, SAN/1011-120, SAN/1011-123, SAN/1011-127, and SAN/1011-131.

Technical contributions to these five topical reports were made by M.S. Agbabian, K.L. Merz, and D.P. Reddy. The four volumes were produced under the general direction of M.S. Agbabian; technical editor was J.W. Athey. While preparing these volumes, Agbabian

Associates received guidance from C. Chester Bigelow and Richard Gwaltney. It is hoped that these volumes will be a useful reference on topics that are essential for the safe design of nuclear facilities.

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CHAPTER 1

EXPERIMENTAL VERIFICATION OF PIPING SYSTEMS

1.1 INTRODUCTION

Verification procedures for the seismic design of Liquid Metal (LM) piping systems for future nuclear power plants are needed. This study is a two-phased program for the development of both analytical and experimental verification procedures. The objective of the first phase is to develop experimental procedures for verifying the seismic response of LM piping systems with mechanical snubbers as seismic restraints. The objective of the second phase is to develop analytical procedures for verifying the seismic response of LM piping restraint systems due to specified design earthquake input motions. The first phase of the study is presented here; the second phase follows in Chapter 2.

1.2 BACKGROUND

1.2.1 SEISMIC DESIGN VERIFICATION

Regulatory requirements pertaining to seismic design verification of nuclear power plant systems are set forth in Code of Federal Regulations 10 CFR 50, Appendix B, which provides quality assurance requirements for nuclear power plants and fuel reprocessing plants. Interpretation of these requirements is provided in Regulatory Guide (RG) 1.64, which, by reference, incorporates the American National Standards Institute Standard N45.2.11 (ANSI, 1974), with some exceptions.

Design is defined in ANSI N45.2.11, as the "technical and management processes which commence with identification of design input and which leads to and includes the issuance of design output documents." Design input is defined as, "those criteria, parameters, bases,

or other design requirements upon which detailed final design is based." Design output is defined as, "documents such as drawings, specifications, and other documents defining technical requirements of structures, systems, and components." Finally, design verification is defined as, "the process of reviewing, confirming, or substantiating the design by one or more methods to provide assurance that the design meets the specified design inputs." Here, it should be noted that one or more methods is indicated rather than any one method.

The extent of the design verification required is indicated in ANSI 45.2.11 to be "a function of the importance of safety of the item under consideration, the complexity of the design, the degree of standardization, the state of the art, and the similarity with previously proven designs. However, the applicability of standardized or previously proven designs, with respect to meeting pertinent design inputs, including environmental conditions, shall be verified for each application."

The methods of providing design verification described in ANSI 45.2.11 are the same three design control methods defined in 10 CFR 50, Appendix B: (1) performance of design reviews, (2) use of alternate calculation methods, or (3) performance of a suitable testing program. As defined in ANSI 45.2.11, design reviews are critical reviews to provide assurance that design documents such as drawings, calculations, analyses, or specifications are correct and satisfactory. Verification by alternate methods of calculation may be achieved by comparison of design analysis results with more rigorous methods of analysis

or in some cases comparison with a simplified method of analysis. The goal of the verification procedure in either case is to provide results consistent with the original calculations or analyses. Design verification for some designs or specific design features can be achieved by suitable qualification testing of a prototype using a test environment that demonstrates the adequacy of performance under the most adverse design conditions. When tests are being performed on models or mock-ups, scaling laws must be established and verified. The test configuration must be clearly defined and documented. Where the test is only intended to verify a specific design feature, the other features of the design shall be verified by other means.

1.2.2 LIQUID METAL PIPING SYSTEMS

In the past, piping systems have been analyzed for design purposes as elastic systems with multiple rigid supports. The selections of support points and snubber locations have been based on dynamic analyses which use support motions determined from independent analysis of the reactor building structure. Often, simplified methods of analysis have been used, coupled with conservative estimates of support motion, to obtain a restrained piping configuration that will maintain stresses within preselected limits. Thus, the resulting piping systems have usually had more restraints than necessary.

Evaluations of piping analysis procedures have demonstrated that the rigid support assumption is in error. Significant flexibility has been noted for structure supports, snubbers, and pipe attachments. Incorporating equivalent linear support stiffnesses into previous rigid support analyses indicates that a redistribution of snubbers may be necessary to maintain the pipe stresses within the selected limits. Further, considering the snubber as a

nonlinear device with gaps or acceleration lockup thresholds alters the response behavior predicted by a linear analysis.

Experience obtained during the construction of the Fast Flux Test Facility (FFTF) and other nuclear facilities indicates that the dynamic characteristics of mechanical snubbers under simulated seismic conditions have not been adequately defined for design (Butler, 1978). Also, mechanical snubbers have experienced field failures under normal operating conditions. Thus, designers are now faced with questions concerning the reliability of these devices to perform the intended function of allowing slow thermal expansion yet providing a restraining support point for the dynamic response of a piping system caused by a seismic event. It should be noted that snubber restraint of thermal expansion can lead to severe overstress (or overstrain) conditions within a piping system and may constitute a more serious concern than the infrequent seismic event for which the snubber has been inplaced.

Seismic design verification of large diameter liquid metal piping is currently provided by using both design reviews and alternate calculations supported by numerous quality control tests of materials, but not by qualification testing of a full-scale prototype system. Because design review and alternate calculation procedures are actually verification of the correctness of the original documents, i.e., of the design assumptions, a test verification is needed to verify seismic design control methods. Therefore, a complete system test should be provided to verify the seismic design of a liquid metal piping system.

1.3 TYPICAL LMFBR PIPING SYSTEM

Since there are no existing Liquid Metal Fast Breeder Reactors (LMFBR), selecting a

prototype piping system for test verification that is typical of future LMFBR's requires a concept of the piping and configuration proposed for such systems. The general arrangement and engineering features of the 1000-MWe power plant developed conceptually as a Prototype Liquid Breeder Reactor (PLBR) by Atomics International and Burns and Roe (AI/B&R, 1977) are selected to represent typical LMFBR systems for test verification. The basic structure and system layout are shown in

Figures 1-1 and 1-2. Note the use of a double containment/confinement building structure.

1.3.1 PLBR LIQUID METAL PIPING SYSTEMS

The PLBR design consists of three independent LM heat transfer loops. The Primary Heat Transfer System (PHTS) of each loop consists of five piping segments connecting the reactor, pump, and Intermediate Heat Exchanger (IHX). The basic layout of each

FIGURE 1-1. PLBR REACTOR CONTAINMENT BUILDING PLAN ABOVE EL. 56'0" (AI/B&R, 1977)

FIGURE 1-2. PLRB REACTOR CONTAINMENT BUILDING SECTION A-A (A1/B6R, 1977)

PHTS loop is shown in Figure 1-3. All of the PHTS piping is designed to meet Class I stress criteria. An independent heat removal system, denoted the Diverse Decay Heat Removal System (DDHRS), is provided by a NaK loop which utilizes natural circulation under adverse operating conditions. The Intermediate Heat Transfer System (IHTS) layout is shown in Figure 1-4. As can be noted, the IHTS loop connects only to the IHX within the containment building. Each IHTS piping loop passes

through wall penetration assemblies into a structurally isolated expansion cell and then into each separate Steam Generator Building. All of the IHTS piping is designed to meet Class II stress criteria. The seismic restraint of the IHTS piping system can be accomplished as required by designers without the detailed stress analysis required of Class I systems. Thus, the PHTS piping segments become the focus of attention as reference configurations for verification studies. The major PHTS

FIGURE 1-3. PRIMARY HEAT TRANSFER SYSTEM (PHTS) LAYOUT (A1/B&R, 1977)

FIGURE 1-4. INTERMEDIATE HEAT TRANSFER SYSTEM (IHTS) LAYOUT STEAM GENERATOR SATELLITE ARRANGEMENT (A1/B&R, 1977)

piping segments are shown in Figures 1-5 through 1-8, where the locations of supports and snubber restraints are indicated. Details of LM piping insulation, support, and snubber restraint are shown in Figures 1-9 through

1-11. The clamp assembly detail shown in Figure 1-11 represents the preferred method of LM piping support. An alternate method is necessary when the pipe is housed within a guard vessel.

FIGURE 1-5. PHTS PUMP SUCTION LINE (36" DIA., 1/2" WALL) (AI/B&R, 1977)

FIGURE 1-7. PHTS IHX DISCHARGE LINE NO. 1 (28" DIA., 1/2" WALL) (AI/B&R, 1977)

FIGURE 1-6. PHTS CROSSOVER LINE (36" DIA., 1" WALL) (AI/B&R, 1977)

FIGURE 1-8. PHTS IHX DISCHARGE LINE NO. 2 (28" DIA., 1/2" WALL) (AI/B&R, 1977)

FIGURE 1-9. TYPICAL LM PIPING SNUBBER RESTRAINT AND SUPPORT CONFIGURATION

FIGURE 1-10. TYPICAL LM PIPING INSULATION DETAIL

FIGURE 1-11. PHTS LM CLAMP ASSEMBLY (AI/B&R, 1977)

1.3.2 CLASS I STRESS REQUIREMENTS

The basic design conditions for PHTS piping are $\sim 930^\circ\text{F}$ and ~ 200 psi. For comparison, basic pressurized water reactor design conditions are $\sim 600^\circ\text{F}$ and ~ 2000 psi. High temperature Class I piping comes under the jurisdiction of ASME Code Case (CC) N-47-17 (also CC 1592), which modifies the ASME Pressure Vessel Code, Section III, Division I stress requirements for temperatures in excess of $\sim 800^\circ\text{F}$. The primary emphasis of high temperature piping component design is to avoid configurations that will require inelastic analysis in order to satisfy the requirements of CC N-47-17. During preliminary analysis, screening rules are often utilized to provide stress checks, which, if satisfied, will ensure that inelastic (and creep) analysis can be avoided. These screening rules indicate which code stress requirements are the most difficult to satisfy and also the importance of stresses due to seismic events. An example of such screening stress checks are given by the following relationships:

$$B_1 \frac{D_o}{2t} P_o + B_2 \frac{D_o}{2I} (M_{DW} + M_{OBE}) \leq \begin{cases} \text{both} \\ 1.5 S_m \\ \text{and} \\ K_t S_t \end{cases} \quad (1-1)$$

$$C_1 \frac{D_o}{2t} P_o + C_2 \frac{D_o}{2I} (M_{OBE} + M_{TE}) + Q_{TH} \leq 3\bar{S}_m \quad (1-2)$$

where

- D_o = Outside diameter of pipe, in.
- t = Nominal wall thickness, in.
- B_i = Primary stress indices for component
- C_i = Secondary stress indices for component
- I = Moment of inertia, in.⁴

- S_m = Allowable design stress intensity value, psi
- S_t = Time dependent stress intensity limit, psi
- M_{OBE} = Resultant moment due to Operating Basis Earthquake, in.-lb
- M_{DW} = Resultant moment due to dead weight, in.-lb
- M_{TE} = Resultant moment due to thermal expansion conditions, in.-lb
- Q_{th} = Thermal transient stresses (assumed to vary linearly through the wall), psi
- K_t = Time-dependent factor as defined in CC N-47-17 (-3223c)

Equations 1-1 and 1-2 represent the primary stress requirements (CC N-47-17, -3223) and primary plus secondary stress range requirements (CC N-47-17, App. T-1325), which are the critical stress checks that must be met if an inelastic analysis is to be avoided for Service Level B (upset) conditions.

In most cases, a configuration may be obtained that will satisfy these screening relationships; however, elbows of loop segments with space restrictions, such as the crossover leg shown in Figure 1-6, often require consideration of inelastic code provisions.

The OBE condition, not the SSE condition, is often the critical seismic design condition. This is due to the fact that although the SSE is twice OBE input levels, the use of increased structure and piping system damping for SSE conditions will give a total response that can easily satisfy the Service Level C (emergency) stress limits. This anomaly is a result of the OBE input level being dependent on the SSE level as required by Appendix A of the Code of Federal Regulations 10 CFR 100. Since the SSE conditions are the critical safety related conditions, the design focus on the OBE conditions is unwarranted and results in additional system costs. The determination of OBE input levels

on a cost-benefit basis has been suggested (Lin and Bohm, 1976) as a means of reducing this design anomaly.

The screening rules may be simplified further by the introduction of the following stress allocations:

$$B_2 \frac{D_o}{2I} M_{DW} = \sigma_{DW}$$

$$C_2 \frac{D_o}{2I} M_{TE} = \sigma_{TE}$$

$$B_2 \frac{D_o}{2I} M_{OBE} = \sigma_{OBE}$$

$$\frac{1}{2} \frac{P_o D_o}{t} = \sigma_P$$

If the OBE stresses are further restricted, $\sigma_{OBE} = 1/3 \sigma_{TE}$, then the application of Equations 1-1 and 1-2 to a given configuration is greatly simplified. Representative values of stress allocations for LM piping are:

$$\begin{aligned}\sigma_{DW} &= 3000 \text{ psi} \\ \sigma_P &= 5000 \text{ psi} \\ \sigma_{TE} &= 20,000 \text{ psi (total stress range)} \\ \sigma_{OBE} &= 6,500 \text{ psi}\end{aligned}$$

1.4 EXPERIMENTAL VERIFICATION

1.4.1 IN-PLACE TESTING OF LM PIPING SYSTEMS

One of the advantages of in-place testing of LM piping systems is that the effects of temperature and pressure can be included. These effects may be significant. For example, LM piping systems undergo considerable thermal expansion (~4 to 6 in.) from ambient conditions to operating temperature levels (~930° F). Thus, the "hot" configuration of a piping loop differs from the "cold" configuration. In addition, the stiffnesses of piping bends and elbows are influenced by pressure, and an 18% decrease in modulus of elasticity occurs for

stainless steel between ambient and operating temperature levels. Also, effective system damping changes can occur due to changes in contact friction caused by thermal expansion. Temperatures of 300° to 400° F are necessary to maintain sodium in a liquid state. All of these factors emphasize the advantage of conducting in-place LM piping tests in a "hot" condition.

Disadvantages to in-place testing of LM piping systems result from the difficulties associated with definition of input, the limitation on input level, and operational restrictions. In general, it is not feasible to excite nuclear reactor structures sufficiently to produce realistic seismic response of the support piping and equipment systems, even though mechanical oscillators and site blasting have been used in the past (AA, 1977; see also Hornbuckle et al., 1973; Bleiweis et al., 1973; Hart et al., 1973; and Morrone, 1974). These methods, at best, produce structure response levels that correspond to about 1/10 the OBE response level. Thus, the piping loops must be excited by the direct application of mechanical oscillators or pulse generators, but often operational considerations will still not permit realistic levels of response. Therefore, in-place testing is generally limited to low levels of excitation. While general response behavior can be obtained from low level testing, the use and extrapolation of this data analytically to higher response levels are difficult.

Because of low system damping (1 to 3%), it is possible to excite in-place piping systems with linear restraints to relatively high levels with attached mechanical or electrohydraulic shakers. The application of this method to a pressurized water reactor primary loop has been demonstrated by Olsen et al. (1973). For a linear system, natural frequencies and mode shapes can be identified using variable frequency steady-state testing with sinusoidal

inputs. Estimates of modal damping can be made from resonance amplitude or bandwidth measurements.

For a system with nonlinear restraints, however, sinusoidal inputs have no particular advantage. To demonstrate the response differences, an example piping system was subjected to a harmonic forcing function placed at midspan as indicated in Figure 1-12a. The computed spring force response for the linear, no-gap case is compared with the spring force response of the gap restraint case in Figures 1-12b and 1-12c. As can be noted, the response of the linear case approaches a steady-state response after the decay of an initial starting transient. However, for the nonlinear case, the response is a beat with a peak response level four times the linear response. The effect of doubling the input force for the gap case is shown in Figure 1-12d. For this case, the response decays to a steady state with an amplitude approximately equivalent to the peak amplitude obtained for the half-force input. The same behavior is indicated in Figure 1-13, which compares the midspan displacement response for the linear and nonlinear cases and the double force case. Thus, definition of the proper input for a nonlinear system and interpretation of the test results may prove difficult.

1.4.2 SURVEY OF EXISTING FACILITIES WITH LM PIPING

A survey was made of existing facilities with LM piping that could be suitable candidates for in-place testing. Five facilities were found, but only two offer the possibility of in-place testing of hot liquid metal filled piping under conditions that are reasonably realistic. There are three LMFBR test facilities and two sodium piping test loop facilities that were considered.

The three LMFBR test facilities considered were FERMI, EBR-II, and the FFTF. FERMI is

a loop-type power reactor that has been decommissioned from service. All sodium has been removed from the plant and testing of "hot" piping is not possible. In addition, all LM piping ($D_o = 30$ in., $t_w = 0.318$ in.) is enclosed within guard pipes that are supported from a common hanger and is not typical of current or future practice. Furthermore, the plant was designed prior to 1960 and seismic requirements were a secondary consideration. The FERMI plant is therefore not recommended for in-place testing. EBR-II is a pool-type reactor with totally immersed primary piping and is also not recommended. The FFTF has LM piping systems with supports, insulation, and mechanical snubber restraints that are representative of current LM piping design practice. The system has also been designed to significant seismic levels. It is, therefore, recommended as a candidate facility.

Sodium piping test loops exist at the Hanford Engineering Design Laboratory and at the Energy Technology Engineering Center (ETEC). Of the two sodium test facilities, only the 70 MWt Sodium Components Test Installation (SCTI) at ETEC should be considered for in-place testing. The other test facilities with LM piping either utilize hydraulic snubbers or are supported without seismic restraints. A general layout of the SCTI main sodium system is shown in Figure 1-14. Most of the system is supported with spring hangers rather than constant effort hangers. Mechanical snubber restraints are provided as indicated. Piping diameters are 10 in. and 14 in. with 5 in. thick calcium silicate insulation. The method of pipe support is not typical of current LM piping design. A large ring assembly is utilized to support the piping by bearing directly on the rigid insulation.

Of the two candidate facilities for in-place testing, each has certain advantages and disadvantages. In-place testing of FFTF piping can

(a) Example piping system

(b) Linear response

(c) Nonlinear response for 500-lb input load

(d) Nonlinear response for 1000-lb input load

FIGURE 1-12. SPRING FORCE RESPONSE OF EXAMPLE PIPING SYSTEM TO MID-SPAN HARMONIC (5 Hz) FORCING FUNCTION

(a) Example piping system

(b) Linear response

(c) Nonlinear response for $F_0 = 500 \text{ lb}$

(d) Nonlinear response for $F_0 = 1000 \text{ lb}$

FIGURE 1-13. MID-SPAN DISPLACEMENT RESPONSE OF EXAMPLE PIPING SYSTEM TO MID-SPAN HARMONIC (5 Hz) FORCING FUNCTION

FIGURE 1-14. 70MWt SCTI MAIN SODIUM SYSTEM AT ETEC

provide useful data concerning system damping and general response behavior of LM systems restrained with mechanical snubbers. However, the FFTF will be an operational reactor, and considerable space conflict will limit access for in-place testing. The selection of piping loops and the method of testing will, in general, depend upon the access and operating restrictions imposed. In this regard, the SCTI has an advantage. All SCTI LM piping is directly accessible. However, the support of piping within SCTI is atypical and the method of

support for portions of the SCTI loop would need to be changed to correspond to current practice.

Both facilities provide the opportunity for in-place testing under operating temperatures and pressures. It is postulated that the SCTI is the more logical facility to consider since there will be less operational interference and the piping is more accessible. However, either facility could be used, depending on their availability.

1 4 3 EXISTING SEISMIC TEST FACILITIES

Since in-place testing is restricted to low-level excitations and access to candidate facilities is limited, laboratory-type testing will be required to accomplish experimental verification of LM piping system response. A survey of existing seismic test facilities indicates that the specimen weight and spatial limitations of the U S facilities (20 ft by 20 ft maximum) restrict the use of these seismic test facilities to the testing of relatively small scale models of "typical" LM piping (i e , PHTS loop segments). A list of the major U S testing facilities as of 1980 is given in Table 1-1. The list is limited to university, government, and commercial testing laboratories with demonstrated capability of testing items with physical dimensions exceeding 6 ft by 6 ft. The performance capabilities of the various test machines and configurations within each facility

are also tabulated for comparison. Only test machines with simultaneous vertical and horizontal motion capabilities are listed. It should be noted that numerous machines exist in the listed facilities and within the test laboratories of other organizations with single axis motion capability.

All of the facilities listed in Table 1-1 provide a test bed or table which allows rigid attachment of a test specimen and any necessary additional support frames or fixtures. The table motion for all facilities is controlled by electrohydraulic servocontrol systems that are capable of duplicating earthquake motions or structure floor motions (Kao, 1975). Several of the listed facilities have the capability of providing simultaneous vertical and horizontal motions which are phase incoherent. For three-dimensional configurations, such as piping systems that are equally influenced by vertical

TABLE 1-1. MAJOR U S SEISMIC TEST FACILITIES

Organization	Machine (Facility) Designation	Maximum Specimen Dimensions, ft	Maximum Weight (g-limit), lb	Maximum Table Input			Mode of Operation
				Accel , g	Veloc , in /sec	Displ , in	
Wyle Laboratories Huntsville, AL	Machine C	17 x 11 x 40	60,000 (2 g)	8	35	6	Simultaneous Biaxial
	Machine D	12 x 12 x 12	10,000 (1 5 g)	3	25	8	Simultaneous Biaxial
Wyle Laboratories Norco, CA	Machine G	8 x 12 x 12	10,000 (2 g)	7 (h) 8 (v)	46 (h) 33 (v)	12 (h) 9 (v)	Simultaneous Biaxial
U S Army Construction Eng Research Lab Champaign, IL	CERL Biaxial Shock-Test Machine	12 x 12 x 12*	15,000 (30 g)	20* (h) 30* (v)	32 (h) 27 (v)	6 (h) 3 (v)	Simultaneous Biaxial
University of Calif Berkeley, CA	Earthquake Eng Res Center Earthquake Sim Lab	20 x 20 x 40	100,000 (1 0 g)	1 5 (h) 1 0 (v)	25 (h) 15 (v)	10 (h) 4 (v)	Simultaneous Biaxial
Westinghouse Advanced Energy Systems Div Large, PA	Seismic Test Lab	9 x 9 x 30	40,000	1 4	--	20	Vector Biaxial (air bag suspension, simultaneous biaxial)
Applied Nucleonics Co Santa Monica, CA	Cable Tray and Conduit Raceway Test Fixture	40 x 13 x 13	10,000	1 2	30	6	Vector Biaxial
Approved Eng Test Labs Los Angeles, CA	System 2	6 x 6 x 12*	10,000	6	30*	12	Simultaneous Biaxial

* Estimated Value

h Horizontal

v Vertical

and horizontal motion, simultaneous biaxial input should be considered as a testing requirement (IEEE, 1975). The use of unidirectional and vector biaxial test facilities is not recommended for verification testing of model piping loops, but should be limited to preliminary investigation of restrained piping dynamic behavior.

In general, only water-filled piping at ambient temperatures can be tested at the listed facilities. Use of heaters and substitute fluid mass, such as steel shot, would be necessary for testing of LM piping systems at operational temperature levels.

Similar to in-place testing, the selection of test input motions for shake table testing requires careful consideration. Since the response of nonlinear systems is input dependent, the test inputs should be realistic estimates of vertical and horizontal structure floor motions. Additional study is required to adequately define representative instructure motions for use in model verification studies. The difficulty of interpreting the response of nonlinear systems for a given input can be demonstrated with the following figures. First, an example piping system is shown in Figure 1-15; then Figures 1-16 and 1-17 compare the linear and nonlinear response of the example piping system subjected to a simple harmonic (5 Hz) floor acceleration input. Figure 1-16 compares the spring force response for the linear, no-gap case with the spring force response of the gap restraint case. As can be noted, the response of the linear case approaches a steady-state response after the decay of an initial starting transient. However, for the nonlinear case, the transient response does not decay to a steady state. The same behavior is indicated in Figure 1-17, which compares the midspan displacement response for the linear and nonlinear cases. Thus, the use of input motions that are not representative of

floor motion will produce system responses which, in general, cannot be correlated with analytical studies of system response to structure motion.

1.4.4 SPECIAL TEST FACILITIES

Since typical piping systems have multiple supports at different levels and different inputs at each level of support, properly phased input cannot be provided with normal laboratory shake table equipment. This suggests that special test facilities would be required to provide properly phased input to model or full-scale configurations of this category. A test frame can be designed that would allow the motion of support points for a test piping configuration to be controlled by electrohydraulic actuators. Several organizations have test facilities (such as airframe test facilities of major aircraft firms) that would allow the setup and conduct of such tests.

The major disadvantage of a special test setup is cost. An estimate of \$300,000 to \$400,000 (1975 dollars) has been suggested (Roberts and Shipway, 1976) for the cost of seismic qualification of a single large item of equipment, using a special test system. Allowing for an average annual cost escalation rate of 8.5%, the cost in 1980 dollars would be in the range of \$450,000 to \$600,000. The cost of a single test using the commercially available facilities would be substantially less.

The use of model piping configurations would allow several tests using the available facilities to be conducted within a given test budget. Thus, on a cost-benefit basis, model tests using available test facilities are recommended. However, the use of scale models would require the validation of model scaling relationships prior to testing in order to remove uncertainties associated with the nonlinear behavior of snubbers.

D_o = Outside pipe diameter = 24 in.

t_w = Pipe wall thickness = 1/2 in.

$(D_o/t_w = 48)$

$l = 57$ ft, $\mu l = 22$ ft.

I = Pipe cross-section moment of inertia = 2549.4 in.⁴

A = Pipe cross-section area = 36.9 in.²

Material: 316 SS

Design Conditions: $T = 930$ °F

$P_o = 200$ psi

$E(@ 930 \text{ °F}) = 23.1 \times 10^6$ psi

Weights: Pipe 126 lb/ft

Sodium at 52 lb/ft³ 150 lb/ft

Heaters, insulation,
jackets, and supports 44 lb/ft

Total $w = 320$ lb/ft

FIGURE 1-15. EXAMPLE PIPING ANALYSIS CASE

1.4.5 SCALING RELATIONSHIPS

Model similitude relationships must be carefully evaluated in order to scale test results from nonlinear systems. In general, model scaling laws are valid for both linear and nonlinear cases provided both model and prototype are fabricated from the same materials. The basis of dimensional analysis and the theory of models has been fully discussed in the literature (Langhaar, 1951; ASME, 1963) and applied

to nuclear pumps and valves (Kao et al., 1976). The dimensionless products, or Pi-terms, of the physical variables are not unique for a given system. If geometric or dynamic nonlinearities are present, then it is often necessary to define different forms of the Pi-terms than would be commonly used for a linear system. As an example, consider the simple piping span with an intermediate gap restraint as shown in Figure 1-15. Let 18 physical variables be

(a) Linear response (no gap)

(b) Nonlinear response (gap = 0.030 in.)

FIGURE 1-16. SPRING FORCE RESPONSE OF EXAMPLE PIPING SYSTEM TO HARMONIC (5 Hz) SUPPORT MOTION

(a) Linear response (no gap)

(b) Nonlinear response (gap = 0.030 in.)

FIGURE 1-17. MID-SPAN DISPLACEMENT RESPONSE OF EXAMPLE PIPING SYSTEM TO HARMONIC (5 Hz) SUPPORT MOTION

identified as parameters to express the system response in the function form:

$$\emptyset = \emptyset \left\{ \varepsilon, \sigma, E, K, I, \ell, D_o, t_w, G, \mu, A_{cs}, \bar{\rho}, M, \ddot{y}_s, g, \alpha, \beta, T \right\}$$

where $E, K, I, T, \ddot{y}_s, g, \bar{\rho}$, and M are defined in Table 1-2; μ, α , and β in Table 1-3; and

ε = Pipe strain

σ = Pipe stress

A_{cs} = Enclosed pipe area $\left(= \pi D_o^2/4 \right)$

D_o = Outside pipe diameter

ℓ = Span length of pipe

t_w = Pipe wall thickness

G = Snubber gap

Since there are three fundamental units (time, mass, length), fifteen Pi-terms may be identified among the eighteen physical variables. Thus, the system response can be expressed in the form:

$$\emptyset = \emptyset \left\{ \varepsilon, \frac{\sigma}{E}, \mu, \frac{D_o}{t_w}, \frac{D_o}{\ell}, \frac{G}{\ell}, \frac{I}{\ell^4}, \frac{A_{cs}}{\ell^2}, \frac{K\ell^3}{EI}, \frac{EI\ell^2}{\bar{\rho}A_{cs}\ell^4}, \frac{M}{E\ell^3}, \frac{\beta}{T}, \alpha T, T(g/\ell)^{1/2}, \frac{\bar{\rho}A_{cs}\ell\ddot{y}_s}{KG} \right\}$$

For a beam in flexure, the usual governing Pi-term is identified in the form $P\ell^2/EI$, where P is the transverse loading. For a beam with gap restraint, this Pi-term may be expressed as:

$$\frac{P\ell^2}{EI} = \frac{P}{KG} \times \frac{G}{\ell} \times \frac{\ell^3 K}{EI}$$

Thus, given that the terms G/ℓ and $\ell^3 K/EI$ have been identified as Pi-terms, only the term P/KG is available as an independent dimensionless loading term. With the identification of the transverse loading in the form $P = \bar{\rho}A_{cs}\ell\ddot{y}_s$, then the source of the Pi-term $\bar{\rho}A_{cs}\ell\ddot{y}_s/KG$ is apparent.

The average mass density is

$$\bar{\rho} = w_{ave}/gA_{cs}$$

where w_{ave} is the pipe system weight per unit length including pipe (p), sodium (Na), and insulation (i), which may be expressed as:

$$\begin{aligned} w_{ave} &= g\rho_p A_p + g\rho_{Na} A_{Na} + g\rho_i A_i \\ &= gA_{cs} \left\{ \frac{4\rho_p}{D_o/t_w} + \rho_{Na} \left(1 - 2 \frac{t_w}{D_o} \right)^2 \right. \\ &\quad \left. + 4\rho_i \left(1 + \frac{a}{D_o} + \frac{t_i}{D_o} \right) \frac{t_i}{D_o} \right\} \end{aligned}$$

Thus, $\bar{\rho}$ may be expressed as

$$\begin{aligned} \bar{\rho} &= \frac{4\rho_p}{D_o/t_w} + \rho_{Na} \left(1 - 2 \frac{t_w}{D_o} \right)^2 \\ &\quad + 4\rho_i \left(1 + \frac{a}{D_o} + \frac{t_i}{D_o} \right) \frac{t_i}{D_o} \end{aligned} \quad (1-3)$$

where

ρ_p = Pipe material mass density

ρ_{Na} = Sodium mass density

ρ_i = Insulation mass density

a = Pipe-insulation annulus clearance

t_i = Insulation thickness

This definition of average mass density is valid for a model given that the nonstructural model materials (i.e., ρ_{Na} and ρ_i) are chosen such that the ratio of the model and prototype average mass densities given by Equation 1-3 satisfies the appropriate scaling factor.

The applicable model scaling relationships are summarized in Table 1-2. Since the response is expressed in three fundamental units, only three scale factors may be chosen. Given that geometric scaling (λ) is required and that both model and prototype have the same materials ($\lambda_E = 1$), then only one additional scaling factor may be chosen. Two cases, depending upon the choice of parameter scaled,

TABLE 1-2. MODEL SCALING RELATIONSHIPS

System Parameters	Scale Factor	Case 1	Case 2
		λ $\lambda_E = 1$ $\lambda_{\bar{\rho}} = 1$	λ $\lambda_E = 1$ $\lambda_g = 1$
Geometric Dimensions D_o, t_w, u, ℓ, G	λ	λ	λ
Areas A, A_{cs}, A_1	λ^2	λ^2	λ^2
Moment of Inertia, I	λ^4	λ^4	λ^4
Modulus of Elasticity, E	λ_E	1	1
Average Mass Density, $\bar{\rho}$	$\lambda_{\bar{\rho}}$	1	$1/\lambda$
Acceleration of Gravity, g	λ_g	$1/\lambda$	1
Time, T	λ_T	λ	$\sqrt{\lambda}$
Support Acceleration, y_s	λ_{ys}	$1/\lambda$	1
Support Stiffness, K	λ_K	λ	λ
Elastic System Frequency, f	$1/\lambda_T$	$1/\lambda$	$1/\sqrt{\lambda}$
Resultant Moment, M	λ^3/λ_E	λ^3	λ^3
Restraint Spring Force, F	λ^2/λ_E	λ^2	λ^2
<p>Frequency Scaling $\frac{\lambda_E \lambda_T^2}{\lambda_{\bar{\rho}} \lambda^2} = 1$</p> <p>Gravity Scaling $\frac{\lambda_T^2 \lambda_g}{\lambda} = 1$</p> <p>Spring Stiffness Scaling $\frac{\lambda_K}{\lambda_E \lambda} = 1$</p> <p>Support Acceleration Scaling $\frac{\lambda_{\bar{\rho}} \lambda^2 \lambda_{ys}}{\lambda_K} = 1$</p>			

are shown in Table 1-2. If model and prototype are fabricated from the same materials ($\lambda_{\bar{\rho}} = 1$), then the acceleration of gravity must be scaled. If gravity cannot be scaled (as is usually the case) and gravity loading effects are important, then the mass density of the model and prototype must be scaled. Note that the gravity or mass scaling factor is not arbitrary but specified for each case. Thus, if the gravity dependent hanger pendular effects are to be simulated in a model, the model mass must

be scaled. If these effects are assumed to be negligible, then dynamic response stresses must be independent from static gravity stresses.

The effect of scaling on input motions should also be noted. If mass density is scaled, then input acceleration remains unscaled. However, if gravity scaling is assumed, then the input acceleration must be scaled by the inverse of the geometric scaling factor. Thus, mass scaling would allow the

testing of a model piping configuration for SSE input acceleration levels at most of the listed facilities while gravity scaling would exclude the use of some facilities for testing to scaled (increased) SSE input acceleration levels.

To demonstrate the validity of the scaling relationships, a 1/3-scale model of the example piping system (Fig. 1-12a) is defined in Table 1-3. This model was chosen in accordance with the Case 1 scale factors given in Table 1-2. To verify the model scaling factors, the response of the 1/3-scale model was computed using a scaled floor motion (Fig. 1-18) and the scaled response parameters given in

Table 1-3. The response of the scaled example problem is shown in Figure 1-19. Comparison of the prototype spring force response (Fig. 1-20) and the midspan moment response (Fig. 1-21) with the model responses indicates (within numerical accuracy) that the scaling relationships are correct.

Thus, given that a mechanical snubber/support behaves approximately as a linear spring with gap, the response of a scale model to transient inputs should correspond to a full-scale prototype. However, exact correspondence between a model and prototype is difficult to maintain. Distortion may be caused

TABLE 1-3. EXAMPLE PROTOTYPE AND MODEL PIPING SYSTEM PARAMETERS

Parameter	Prototype	Model
Pipe Outside Diameter, D_o	24 in.	8 in.
Pipe Wall Thickness, t_w	0.5 in.	0.167 in.
Support Location Factor, μ	0.386	0.386
Pipe Span, ℓ	57 ft	19 ft
Snubber Gap, G	0.030 in.	0.010 in.
Enclosed Pipe Area, A_{cs}	452.4 in. ²	50.3 in. ²
Moment of Inertia, I	2549.4 in. ⁴	31.5 in. ⁴
Modulus of Elasticity, E	23.1 x 10 ⁶ psi	23.1 x 10 ⁶ psi
Average Mass Density, $\bar{\rho}$	0.000153 lb sec ² /in. ⁴	0.000153 lb sec ² /in. ⁴
Restraint Stiffness, K	57 x 10 ³ lb/in.	19 x 10 ³ lb/in.
Peak OBE Ground Acceleration, $\ddot{x}_g$	96.5 in./sec ²	289.5 in./sec ²
Gravity Acceleration, g	386 in./sec ²	1158 in./sec ²
Response Analysis Parameters		
Pipe Mass Lumping, $\bar{M}_1$	9.14 lb sec ² /in.	0.3385 lb sec ² /in.
Pipe Mass Lumping, $\bar{M}_2$	14.53 lb sec ² /in.	0.5385 lb sec ² /in.
Time Step, Δt	0.005 sec	0.00167 sec
Time Duration of Floor Motion	2.5 sec	0.833 sec
Mass Proportional Damping Factor, α	0.5944 $\frac{1}{\text{sec}}$	1.7832 $\frac{1}{\text{sec}}$
Stiffness Proportional Damping Factor, β	0.0004876 sec	0.0001625 sec
Scale Factors: $\lambda = 1/3$, $\lambda_E = 1$, $\lambda_{\bar{\rho}} = 1$

FIGURE 1-18. PLRB FLOOR ACCELERATION HORIZONTAL TIME HISTORY 2.5 s

(a) Spring force

(b) Mid-span moment

FIGURE 1-19. SPRING FORCE AND MID-SPAN MOMENT RESPONSE OF 1/3-SCALE EXAMPLE PIPING SYSTEM MODEL FOR OBE (x3) INPUT

by a number of factors. The geometric scaling of elbow and bend elements is often difficult. The effective restraint spring rate will be difficult to scale. Damping, however, is the critical parameter that requires very careful attention to limit distortion. The modeling of

joints and other frictional interfaces can be achieved if similar materials, joint details, and surface conditions are maintained (ASME, 1963). The behavior of a mechanical snubber as a dynamic subsystem must be clearly understood if modeling of a LM piping system is to be a

FIGURE 1-20. PROTOTYPE SPRING FORCE

FIGURE 1-21. PROTOTYPE MID-SPAN MOMENT

valid procedure. Thus, the development of a mathematical model of a snubber as a mechanical subsystem is a key element for development of both analytical and experimental methods of piping system design verification. The general dynamic behavior of a mechanical snubber is

similar to a mechanical servocontrol system with response feedback. Thus, the modeling of such systems should be successful, given proper attention to clearances, backlash, and mandrel-spring interface conditions.

1.4.6 DEFINITION OF PROTOTYPE TEST CONFIGURATION

To properly model the behavior of a large diameter piping segment, the model configuration must include anchor points. While the behavior of a small diameter piping loop may be approximated by considering only an unanchored portion of a segment supported by

pendular hangers, a larger diameter loop (with spatial dimensions typical of a PHTS loop) requires the inclusion of anchor points to account for the unrestrained behavior of the piping configuration. The effect of hanger pendular behavior on piping system frequency is illustrated in Figure 1-22. Comparison of the frequency estimates for the example piping

FIGURE 1-22. EFFECT OF PENDULAR BEHAVIOR ON UNRESTRAINED EXAMPLE PIPING SYSTEM FREQUENCY

systems shown in Figure 1-22 indicates that the spacing of anchor points is the critical parameter for frequency determination. The influence of hanger pendular restraint on system frequency becomes effective as the anchor-to-anchor span is increased; however, for large diameter piping, this effect is minor.

A proposed piping configuration suitable for conduct of model tests is shown in Figure 1-23. This configuration is an adaptation of the classic Hovgaard bend, which is also used as a NRC benchmark problem for verification of piping analysis computer codes (Reich et al., 1976). This prototype test configuration is the simplest configuration that can be defined that incorporates the general spatial characteristics of actual LM piping segments. If the snubber/support stiffness is idealized as a linear spring, then the distributions of modal system frequencies for both small diameter and large diameter configurations are quite similar

for the spatial envelopes indicated in Figure 1-24. As can be noted by comparison of the test facility envelopes given in Table 1-1 with the spatial dimensions of the prototype test configurations given in Figure 1-24, a 4-in. dia., schedule 40 pipe is the largest full-scale system that can be tested. A 2/3-scale model would be required for the 24-in. prototype configuration indicated in Figure 1-24. The resulting scale model (16-in. dia.) fundamental system frequency (linear) would then be 8.9 Hz (mass density scaling). Given a mathematical model for the snubber, then the appropriate characteristics of a 2/3-scale snubber can be defined and the snubber manufacturer consulted to determine if special units can be fabricated with the desired characteristics. If such a snubber unit cannot be manufactured, the distortion effect of using the closest available standard snubber unit would need to be determined. It is anticipated that this effect would be small in the event that a special snubber

FIGURE 1-23. PROTOTYPE TEST CONFIGURATION

$$D_o = 24 \text{ in.}, t = 0.500 \text{ in.}$$

MODE	FREQUENCY
1	7.24 Hz
2	10.2
3	10.3
4	11.2
5	12.9
6	14.6
7	16.9
8	18.4
9	21.7
10	24.9
11	28.7
12	33.2

(a) Large diameter pipe

$$D_o = 4 \text{ in.}, t = 0.237 \text{ in. (SCH. 40)}$$

MODE	FREQUENCY
1	8.15 Hz
2	8.47
3	9.06
4	10.0
5	10.9
6	14.5
7	16.7
8	16.8
9	19.1
10	20.9
11	21.3
12	22.9
13	24.9
14	28.0
15	35.5

(b) Small diameter pipe

FIGURE 1-24. PROTOTYPE CONFIGURATION MODAL (LINEAR) SYSTEM FREQUENCIES

unit is not available. The use of mass density scaling can be achieved by using a fluid or water/slurry with an equivalent specific gravity sufficient to account for the scaled average mass density.

1.4.7 TEST PLAN OUTLINE

An essential feature of the test program is the prior analytical prediction of system response to various input motions. A major objective of the test will be to note the deviation of the test response from the analytical predictions. A brief outline of the test configurations, test facility, measurement requirements, and test parameters is given here.

1.4.7.1 Test Configurations

The test specimen will be a scaled model of a prototype LM piping configuration. The piping segment shall be anchored at each terminal end to a fixture that simulates attachment to a pressure vessel. The deadweight of the piping shall be supported by constant force hangers in addition to the anchor points. Snubber/support reactions shall be provided by scaled models of snubber units and typical structural support components. A test frame of sufficient stiffness to preclude dynamic interaction with the test specimen shall be attached to the test table and shall provide the necessary structure support for the piping configuration.

A desirable feature of the test frame is a capability to change the stiffness of frame by addition (removal) of stiffening elements and joints to allow phased horizontal input between the table motion and the upper (support) level of the frame. The method of attachment of the test frame to the test table shall be specified in the test plan.

1.4.7.2 Test Facility

Only those facilities with large table areas and high-bay building enclosures are suitable for testing scaled piping configurations and the necessary support fixtures. The test table shall have the capability of providing simultaneous vertical and horizontal motion which is phase incoherent. The table motion shall be controlled by an electrohydraulic servocontrol system with frequency response characteristics that enable floor spectra centered at $5/\sqrt{\lambda}$ Hz, where λ is the scale factor, to be approximated by the table motion. Further, the control system should be capable of duplicating specified complex waveforms similar to a filtered ground motion scaled in time.

1.4.7.3 Measurement Requirements

Sufficient instrumentation shall be provided to enable real-time data acquisition for direct comparison of the predicted response of the test configuration. Motion instrumentation shall consist of both accelerometers and LVDT units to be located as specified in the test plan. Load cells shall be provided in series with all constant hangers and snubber units. Strain gages (rosette) shall be located on the pipe configuration at critical locations to be designated in the test plan. The adhesive and gage specifications shall be suitable for dynamic stress applications and have high-elongation characteristics ($\pm 5\%$ strain). A preliminary estimate indicates that at least 200 data

channels will be required. Motion photography of certain portions of the testing program will be required. Formal quality assurance requirements shall be specified in the test plan.

1.4.7.4 Test Conduct and Test Parameters

The type (duration) and sequence of input excitations shall be fully detailed in the test plan. All test parameters shall be identified, and the probable range of values expected during the test program shall be specified.

1.5 CONCLUSIONS

The following conclusions are derived from this pilot study:

- a. Two candidate facilities have been identified where LM piping systems exist that are adequate for in-place testing. These are LM piping loops associated with the FFTF and the 70 MWt SCTI Main Sodium System at ETEC.
- b. Problems associated with in-place testing of these LM piping systems include:
 1. Potential interference with or from normal operating functions of the facilities.
 2. In-place operational systems can generally be tested only to relatively low levels of seismic response.
 3. Definition of the input and interpretation of the test results are complex and will require nonlinear analyses and prior experimental determination of the dynamic properties of snubbers, supports, and other elements of the piping system.

4. Availability for in-place testing may be restricted due to operational requirements.
- c. Existing seismic test facilities are restricted in size and can only be used to test scale models of prototype LM piping systems. Model tests on existing test facilities may experience scheduling delays but will not incur the operational interference associated with in-place testing. Also, model tests can be conducted at relatively high levels of dynamic response. However, the model piping systems will require prior definition of the input, and interpretation of the test results will be complex, requiring nonlinear analyses of test configurations prior to test conduct to ensure accomplishment of test objectives.
- d. Fabrication of a special test facility would permit testing of LM piping (model) systems to a relatively high level of dynamic response, and phased input could be provided for piping systems having multiple supports at different levels. Interpretation of the test results would require prior nonlinear analyses. The primary disadvantage of a special test facility would be the cost of constructing the test facility.
- e. Scale model piping configuration tests using an existing test facility are recommended as the most cost-effective approach for providing experimental verification of LM piping systems. Preliminary studies indicate that the response of a scale model LM piping configuration with nonlinear restraints will correspond to a full-scale prototype system, with minor distortion.
- f. The key element for development of experimental verification methods is the development of a mathematical model for a snubber viewed as a mechanical subsystem. (This applies to analytical verification methods, too; see Chapter 2). The proper characterization of snubber behavior is necessary in order to define a scale model piping configuration with nonlinear restraints that will yield a system response with minimum distortion.
- g. To properly model the behavior of a large diameter piping segment, a prototype test configuration must include anchor points. For small diameter pipe, the influence of hanger pendular restraint on system frequency becomes effective as the anchor-to-anchor span is increased; however, for large diameter pipe this effect is minor.
- h. An adaptation of the classic Hovgaard bend is suggested as the basic prototype test configuration to be used in the experimental verification program. A prototype system with a 4-in.-dia. pipe is the largest full-scale configuration that can be defined for testing at existing test facilities. A 2/3-scale model (16-in. dia.) of a prototype system with a 24-in.-dia. pipe is the largest recommended model configuration that will fit into the spatial envelope imposed by existing test facilities.

CHAPTER 2

ANALYTICAL VERIFICATION OF PIPING RESTRAINT SYSTEMS

2.1 INTRODUCTION

Recommended procedures for design verification analyses of liquid metal piping systems that utilize mechanical snubbers as a seismic restraint component are presented here. These analyses are part of a two-phase verification procedure that also includes validation through testing (see Chapter 1).

Seismic design verification of nuclear plant liquid metal piping is currently provided by using both design reviews and alternate calculations supported by numerous quality control tests of materials. However, the seismic design control methods required for nuclear plant quality assurance are such that the design review and alternate calculation procedure are actually a verification of the correctness of documents, and this procedure is reliable only when the design assumptions are correct and have been previously verified. These seismic design assumptions need additional verification, and hence the need for testing and analytical procedures.

In order to limit the seismic stresses and nozzle loads caused by the piping system response to structure motion induced by earthquake, a piping configuration must be restrained during the seismic event. However, this restraint must not prevent thermal expansion of the piping system due to normal operating conditions. Thus, snubbers are utilized as seismic restraint components. For liquid metal piping systems, mechanical snubbers are used due to the thermal and radiation environment. In the past, seismic restraints have been considered as rigid support points for seismic analyses of piping systems. Only recently has the flexibility of

the support structure, snubber, and piping attachment been considered in the dynamic analysis of piping systems. The interface seismic loads applied to the pressure boundary of the piping system may be separated into three categories:

1. Component nozzle load analysis
2. Pipe support equipment analysis
3. Hanger and support civil/structural analysis

The effect of nozzle compliance and civil/structure support stiffness can be represented by simple linear axial or rotational springs in an analytical model of a piping system. The representation of the pipe support components, however, is associated with a greater uncertainty since both the clamp/pipe interaction and mechanical snubber exhibit nonlinear behavior. Several alternative idealizations for the behavior of clamp/pipe interaction and mechanical snubbers can be utilized in an analytical model of a piping system. The present study is concerned with the idealization of mechanical snubbers in piping design verification analyses.

Mechanical snubbers are nonlinear dynamic subsystems whose response characteristics for various types of complex input are not known a priori. In order to establish the applicability and limitations associated with various snubber idealizations, it is necessary to have an accurate reference model of the actual mechanisms within the snubber unit. Such a model, correlated with available test data, could be incorporated into an analytical model of a piping system and the system response computed for complex seismic input motions. The resulting solution could then be used directly for design

verification or as a reference solution for comparison of the results obtained using other snubber idealizations.

2.2 DEVELOPMENT OF SNUBBER SUBSYSTEM MATHEMATICAL MODEL

2.2.1 SNUBBER PHYSICAL DESCRIPTION

Mechanical snubbers that are currently used for liquid metal piping restraint employ a ball screw or acme screw to convert relative linear motion between support points to an absolute rotary motion. This rotary motion is, in turn, an input to a kinematic mechanism. The type of mechanism considered here employs a ball screw, torque drum, coupling torsion spring, and inertia mass, with designed system backlash and contact friction. A cut-away sectional view of the snubber body and internal mechanism is shown in Figure 2-1a. The coupling torsion spring, denoted as the capstan spring, is loosely fitted over a mandrel formed by an extension of the snubber housing and is coupled between the torque drum and inertia mass without positive attachment to either the torque drum or inertia mass. Slots in the torque drum provide a rotational backlash. An additional complication in the system arrangement is introduced by a clip spring which provides the torque transfer between the torsion spring and inertia mass. This slip spring, denoted as the clutch spring, is held within the inertia mass by preload friction alone and slips when the net friction resistance is overcome.

Given that a snubber unit is included as a component link in the support of a piping system, the ball screw of the snubber unit will be forced to rotate whenever the relative displacement of the snubber end points exceed any axial freeplay present due to backlash in either the ball screw nut, thrust bearing, or spherical end bearing. The rotation of the ball screw

drives the torque drum which engages the capstan spring tang after partial rotation through the angular backlash (slot). The capstan spring, in turn, engages the tang of the clutch spring (clip spring) after complete rotation through the backlash. Given that the clutch spring does not slip, the inertia mass rotates and the mechanism is then equivalent to a springmass system with displacement input. The system will behave as a single-degree-of-freedom system until the diametric contraction of the capstan spring (torsion spring) causes friction contact between the capstan spring and fixed mandrel. This friction contact "locks" the mechanism and retards the linear-to-rotary conversion of motion. The ends of the snubber unit will then impose an axial force carried by the snubber body parts and transferred by the ball screw acting in tension or compression.

2.2.2 SNUBBER MATHEMATICAL MODEL

Under an oscillating motion of the snubber end points, the motion of the snubber mechanism can be quite complex. The system may be modeled by considering the force transfer characteristics and constitutive relations for each component part of the mechanism and then considering each kinematic state which interconnects these parts. Equations of motion can then be written for each kinematic state. The component parts of the snubber mechanism are identified in the idealized subsystem model shown in Figure 2-1b. The model is subdivided into two models, denoted as external and internal. The external model deals with the linear-to-rotary conversion of snubber end point displacement and axial snubber force. The axial backlash due to the ball screw nut and thrust bearing must also be considered in the external model kinematics. The internal model deals with rotational motion of the internal snubber parts. The angular backlash due to the torque drum slot, the frictional mandrel

(a) Snubber design utilizing capstan spring mechanism (HEDL, 1980)

(b) Subsystem model of snubber mechanism

FIGURE 2-1. SNUBBER PHYSICAL DESIGN AND CORRESPONDING SUBSYSTEM MODEL

contact due to diametrical contraction of the capstan spring, and the slip of the clutch spring are the basic considerations of the internal model kinematics.

The descriptive parameters of the system are:

- μ , Value of mandrel/spring friction coefficient
- K_{bs} , Ball screw torsional stiffness
- B_{ℓ} , Ball screw total axial backlash
- K^* , Overall axial stiffness of locked snubber unit
- C , Ball screw transformation constant (screw lead $\div 2\pi$)
- r^* , Force-torque transformation constant of ball screw (0.9 C)
- K_{ϕ} , Capstan spring stiffness
- K'_{ϕ} , Locked capstan spring stiffness
- $\Delta\theta_{cr}$, Relative angular displacement of capstan spring when diametrical contraction limit (mandrel contact) is reached
- $T_{cr} = K_{\phi}(\Delta\theta_{cr})$, Locking torque of capstan spring
- N , Number of capstan spring turns
- $\Delta\theta_{bl}$, Clutch spring angular backlash
- K_{cs} , Clutch spring preslip stiffness
- $\Delta\theta_{cscr}$, Relative angular displacement of clutch spring (including clutch spring backlash) when slipping torque is reached
- $T_{cscr} = K_{cs}(\Delta\theta_{cscr} - \Delta\theta_{bl})$, Clutch spring slipping torque
- J_{td} , Torque drum inertia
- J_i , Mass moment of inertia of inertia mass

Internal force variables necessary for the description of system kinematics and the governing equations of motion are:

- T_{bs} , Torque applied to ball screw shaft by ball screw nut

- T_r , Resisting torque of snubber mechanism
- T_{td} , Effective torque applied to torque drum by capstan spring tang
- T_{cs} , Effective torque applied to clutch spring by capstan spring tang
- T_i , Resultant torque applied to inertia mass by clutch spring

The input and output variables are:

- $\delta = \delta_1 - \delta_2$, Relative displacement of snubber end points

- F , Axial snubber reaction force

The state variables are:

- θ_{bs} , Ball screw input rotation
- θ_{td} , Torque drum rotation
- θ_{cs} , Clutch spring rotation
- θ_i , Inertia mass rotation

Details on the kinematics of the snubber model are given in AA (1981). The governing equations of motion for each model segment are outlined below, using the state variables and parameters identified here.

2.2.2.1 Formulation of Governing State Equations

The governing equations of motion take the form:

$$[\ddot{J}]\{\ddot{\theta}\} + [\ddot{K}]\{\dot{\theta}\} + \{\ddot{N}\} = \{\ddot{F}\} \quad (2-1)$$

where the vector of state variables is

$$\{\theta\} = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} \theta_{bs} \\ \theta_{td} \\ \theta_{cs} \\ \theta_i \end{Bmatrix}$$

the inertia array is

$$[\bar{J}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \bar{J}_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{J}_{44} \end{bmatrix}$$

the effective stiffness array is

$$[\bar{K}] = \begin{bmatrix} \bar{K}_{11} & \bar{K}_{12} & 0 & 0 \\ \bar{K}_{21} & \bar{K}_{22} & \bar{K}_{23} & 0 \\ 0 & \bar{K}_{32} & \bar{K}_{33} & \bar{K}_{34} \\ 0 & 0 & \bar{K}_{43} & \bar{K}_{44} \end{bmatrix}$$

the internal force components that are independent of the magnitude of the state variables are

$$\{\bar{N}\} = \begin{pmatrix} 0 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix}$$

and the effective input forces to the subsystem are

$$\{\bar{F}\} = \begin{pmatrix} \bar{F}_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In general, the elements $\bar{K}_{ij}$, $\bar{N}_i$, and $\bar{F}_i$ are functions of the state variables and thus dependent upon the current kinematic state of the system. Each of the possible system kinematic states is summarized in Table 2-1 in terms of the state variable notation and the associated physical description. Only the inertia array (J_{ij}) and two stiffness elements ($\bar{K}_{12} = \bar{K}_{21}$) are independent of the system kinematics.

2 2 2 2 Contraction of Governing Equations

Since the idealized system model contains only two mass degrees of freedom, the governing state variables are the torque drum rotation, θ_{td} , and the inertia mass rotation, θ_1 .

TABLE 2-1 SUMMARY OF SYSTEM KINEMATIC STATES

State	State Condition	Physical Description
1	$ \delta - C\theta_{bs} \leq \frac{1}{2} B_\ell$	Ball screw axial backlash
2	$ \delta - C\theta_{bs} > \frac{1}{2} B_\ell$	Ball screw engagement
3	$ \theta_{td} - \theta_{cs} \leq \Delta\theta_{cr}$	Capstan spring free from mandrel
4 ⁺	$ \theta_{td} - \theta_{cs} > \Delta\theta_{cr}$	Capstan spring engaged on mandrel
5	$\text{Sgn}(\dot{\theta}_{td}) = \text{Sgn}(\dot{\theta}_{cs})$	Unidirectional motion of torque drum and clutch spring coordinate
6	$\text{Sgn}(\dot{\theta}_{td}) = -\text{Sgn}(\dot{\theta}_{cs})$	Opposing motion of torque drum and clutch spring coordinates
7 ⁺	$ \theta_{cs} - \theta_1 - \theta_o \leq \Delta\theta_{bl}$	Clutch spring backlash (torque drum slot angular backlash)
8	$\Delta\theta_{bl} < \theta_{cs} - \theta_1 - \theta_o \leq \Delta\theta_{cscr}$	Clutch spring engagement with no slip
9	$ \theta_{cs} - \theta_1 - \theta_o > \Delta\theta_{cscr}$	Clutch spring engagement with slip

*Combination of States 4 and 7 represents a physically incompatible condition

Thus the governing equations of the system may be condensed to a series of two-degree-of-freedom systems representing the motion of each kinematic state. The state equations may be shown to be equal to (AA, 1981):

$$\theta_{bs} = \frac{1}{\bar{K}_{11}} (\bar{F}_1 - \bar{K}_{12}\theta_{td})$$

and (2-2)

$$\theta_{cs} = \frac{1}{\bar{K}_{33}} \left(-\hat{N}_3 - \hat{f}_3 - \hat{K}_{32}\theta_{td} - \hat{K}_{34}\theta_i \right)$$

where $\hat{K}_{ij}$, $\hat{N}_i$, and $\hat{f}_i$ are modified forms of $\bar{K}_{ij}$, $\bar{N}_i$, and $\bar{f}_i$, respectively. The governing equation (Eq. 2-1) can now be written in the contracted form as:

$$[J]\{\ddot{\theta}\} + [K]\{\dot{\theta}\} + \{\tilde{N}\} = \{\tilde{F}\} \quad (2-3)$$

where the governing state variables are

$$\{\tilde{\theta}\} = \begin{Bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \end{Bmatrix} = \begin{Bmatrix} \theta_{td} \\ \theta_i \end{Bmatrix}$$

the contracted inertia array is

$$[J] = \begin{bmatrix} \tilde{J}_{11} & 0 \\ 0 & \tilde{J}_{22} \end{bmatrix}$$

with

$$\begin{aligned} \tilde{J}_{11} &= \tilde{J}_{22} = J_{td} \\ \tilde{J}_{22} &= \tilde{J}_{44} = J_i \end{aligned}$$

the contracted stiffness array is

$$[K] = \begin{bmatrix} \tilde{K}_{11} & \tilde{K}_{12} \\ \tilde{K}_{21} & \tilde{K}_{22} \end{bmatrix}$$

with

$$\begin{aligned} \tilde{K}_{11} &= \hat{K}_{22} - \bar{K}_{21}\bar{K}_{12}/\bar{K}_{11} - \hat{K}_{23}\hat{K}_{32}/\hat{K}_{33} \\ \tilde{K}_{12} &= -\hat{K}_{23}\bar{K}_{34}/\bar{K}_{33} \\ \tilde{K}_{21} &= -\bar{K}_{43}\bar{K}_{34}/\bar{K}_{33} \\ \tilde{K}_{22} &= \bar{K}_{44} - \bar{K}_{43}\bar{K}_{34}/\bar{K}_{33} \end{aligned}$$

the contracted form of the internal force vector is

$$\{\tilde{N}\} = \begin{Bmatrix} \tilde{N}_1 \\ \tilde{N}_2 \end{Bmatrix}$$

with

$$\tilde{N}_1 = \hat{N}_2 - \frac{\hat{K}_{23}}{\bar{K}_{33}} \hat{N}_3$$

$$\tilde{N}_2 = \hat{N}_4 - \frac{\hat{K}_{43}}{\bar{K}_{33}} \hat{N}_3$$

and the contracted force vector (including frictional pseudoforces) is

$$\{\tilde{F}\} = \begin{Bmatrix} \tilde{F}_1 \\ \tilde{F}_2 \end{Bmatrix}$$

with

$$\tilde{F}_1 = -\hat{f}_2 - \frac{\bar{K}_{21}}{\bar{K}_{11}} \bar{F}_1 - \frac{\hat{K}_{23}}{\bar{K}_{33}} \left(-\hat{f}_3 \right)$$

$$\tilde{F}_2 = -\frac{\bar{K}_{43}}{\bar{K}_{33}} \left(-\hat{f}_3 \right)$$

AA (1981) discusses physical constraints on the snubber model and their effect on the governing equations of motion given by Equation 2-3.

2.2.3 PROCESS SOLUTION

2.2.3.1 Incremental Process Formulation

Let the governing process equations for the snubber subsystem, given in the contracted form of Equations 2-3, be represented in the form

$$[\tilde{J}]\{\ddot{\tilde{\theta}}\} + \{\tilde{\eta}\} = \{\tilde{F}\} \quad (2-4)$$

where the vector $\{\tilde{\eta}\}$ represents the total internal force vector

$$\{\tilde{\eta}\} = [\tilde{K}]\{\tilde{\theta}\} + \{\tilde{N}\}$$

If at time $t+\Delta t$, the internal forces are given in the incremental form

$$\{\tilde{\eta}\}_{t+\Delta t} = \{\tilde{\eta}\}_t + \{\Delta\tilde{\eta}\}$$

where the i th element, $\Delta\tilde{\eta}_i$, is given by the total differential form

$$\Delta\tilde{\eta}_i = \sum_j \frac{\partial \tilde{\eta}_i}{\partial \tilde{\theta}_j} \bigg|_t \Delta\tilde{\theta}_j$$

then

$$\{\Delta\tilde{\eta}\} \approx [\tilde{K}]_t (\{\tilde{\theta}\}_{t+\Delta t} - \{\tilde{\theta}\}_t)$$

where the elements of the tangent stiffness array are given by the elements of $[\tilde{K}]$ evaluated at time t . The governing Equations 2-4 may be expressed at time $t+\Delta t$ in the total response form with equilibrium check as

$$\begin{aligned} [\tilde{J}]\{\ddot{\tilde{\theta}}\}_{t+\Delta t} + [\tilde{K}]_t\{\tilde{\theta}\}_{t+\Delta t} &= \{\tilde{F}\}_{t+\Delta t} \\ &+ [\tilde{K}]_t\{\tilde{\theta}\}_t - \{\tilde{\eta}\}_t \end{aligned} \quad (2-5)$$

The corrective terms which appear on the right-hand side of this equation may be identified as

$$[\tilde{K}]_t\{\tilde{\theta}\}_t - \{\tilde{\eta}\}_t = -\{\tilde{N}\}_t$$

thus Equation 2-5 becomes

$$[\tilde{J}]\{\ddot{\tilde{\theta}}\}_{t+\Delta t} + [\tilde{K}]_t\{\tilde{\theta}\}_{t+\Delta t} = \{\tilde{F}\}_{t+\Delta t} - \{\tilde{N}\}_t \quad (2-6)$$

The incremental process represented by Equation 2-6 is illustrated in Figure 2-2 for a one-dimensional static case with bilinear stiffness. The frictional pseudoforce components of the contracted force vector, $\{\tilde{F}\}$, are not known a priori at time $t+\Delta t$; thus a suitable extrapolation technique must be selected for estimation of the frictional contact forces, $\{\hat{f}\}$, at time $t+\Delta t$. For the process considered herein, the frictional forces were assumed to be constant over the time interval,

FIGURE 2-2. INCREMENTAL PROCESS SOLUTION TECHNIQUE

Δt . Thus, the frictional contact forces were estimated as

$$\{\hat{f}\}_{t+\Delta t} \approx \{\hat{f}\}_t$$

unless a backlash condition occurred which invoked the process constraint,

$$\{\hat{f}\}_{t+\Delta t} = \{0\}$$

2.2.3.2 Process Equilibrium Iteration

Due to the discontinuous nature of some process functions, an iterative solution procedure must be utilized to determine the appropriate system states at a given time in the numerical process solution. Thus, having established a k th approximation to the process state variables at time $t+\Delta t$, a vector of out-of-balance forces, $\{R\}^{(k)}$, was estimated as

$$\{R\}_{t+\Delta t}^{(k)} = \{\tilde{F}\}_{t+\Delta t} - [\tilde{J}]\{\tilde{\theta}\}_{t+\Delta t}^{(k)} - \{\tilde{\eta}\}_{t+\Delta t}^{(k)} \quad (2-7)$$

where

$$\{\tilde{\eta}\}_{t+\Delta t}^{(k)} = [\tilde{K}]_{t+\Delta t}^{(k)} \{\tilde{\theta}\}_{t+\Delta t}^{(k)} + \{\tilde{N}\}_{t+\Delta t}^{(k)}$$

The elements of the array $[\tilde{K}]$ and the vector $\{\tilde{N}\}$ were evaluated using the k th estimate of the process state variables at time $t+\Delta t$. The k th estimate of the state variables were taken as an acceptable approximation of the process solution at time $t+\Delta t$, if the absolute norm of the out-of-balance force vector was less than a specified numerical tolerance, or

$$\left\| \{R\}_{t+\Delta t}^{(k)} \right\| \leq \text{TOL} \quad (2-8)$$

where

$$\left\| \{R\}_{t+\Delta t}^{(k)} \right\| = \sum_i \left| (R_i)^k \right|_{t+\Delta t}$$

and $\text{TOL} = 10^{-2}$ (e.g.)

Given that the k th estimate of the state variables did not satisfy the inequality given by

Equation 2-8, then an updated estimate ($k+1$) of the state variables was sought by expressing the internal forces in terms of an incremental correction:

$$\{\tilde{\eta}\}_{t+\Delta t}^{(k+1)} = \{\tilde{\eta}\}_{t+\Delta t}^{(k)} + \{\Delta\tilde{\eta}\}_{t+\Delta t}^{(k)} \quad (2-9)$$

where

$$\{\Delta\tilde{\eta}\}_{t+\Delta t}^{(k)} \approx [\tilde{K}]_{t+\Delta t}^{(k)} \left(\{\tilde{\theta}\}_{t+\Delta t}^{(k+1)} - \{\tilde{\theta}\}_{t+\Delta t}^{(k)} \right)$$

Thus, the governing equations for the $k+1$ iteration were obtained by using Equation 2-9 to express Equation 2-4 as

$$[\tilde{J}]\{\ddot{\tilde{\theta}}\}_{t+\Delta t}^{(k+1)} + [\tilde{K}]_{t+\Delta t}^{(k)} \{\tilde{\theta}\}_{t+\Delta t}^{(k+1)} = \{\tilde{F}\}_{t+\Delta t}^{(k)} - \{\tilde{N}\}_{t+\Delta t}^{(k)} \quad (2-10)$$

Equations 2-10 may be recognized as an application of the Newton-Raphson iteration technique to the process solution at time $t+\Delta t$. A recursive solution of Equations 2-10 was continued until convergence was indicated by satisfaction of the inequality in Equation 2-8. If convergence was not obtained in a specified number of iterative steps (e.g., 10 iterations), then the final estimate of the state variables was accepted as the solution at time $t+\Delta t$, and the process advanced to the next time step.

2.2.3.3 Numerical Integration of Incremental Process

To achieve a numerical solution for the incremental process represented by Equation 2-6, a one-step implicit integration algorithm is taken in the form

$$\begin{aligned} \ddot{\tilde{\theta}}_{t+\Delta t} &= c_1 \ddot{\tilde{\theta}}_{t+\Delta t} - \ddot{\tilde{\theta}}_t + c_2 \dot{\tilde{\theta}}_t + c_3 \ddot{\tilde{\theta}}_t \\ \dot{\tilde{\theta}}_{t+\Delta t} &= c_4 \dot{\tilde{\theta}}_{t+\Delta t} - \dot{\tilde{\theta}}_t + c_5 \ddot{\tilde{\theta}}_t + c_6 \dot{\tilde{\theta}}_t \end{aligned} \quad (2-11)$$

where the coefficients c_i are functions of the time step Δt and the interpolation parameters of

the particular implicit scheme chosen (e.g., Newmark, 1959; Hilber-Hughes, 1978).

Introduction of Equations 2-11 into Equation 2-6 allows the incremental process to be represented by the system of linear equations

$$[K^*]_{t+\Delta t} \{\theta^*\}_{t+\Delta t} = \{F^*\}_{t+\Delta t} \quad (2-12)$$

where

$$[K^*]_{t+\Delta t} = [\tilde{K}]_t + a_0[\tilde{J}]$$

$$\begin{aligned} \{F^*\}_{t+\Delta t} = & \{\tilde{F}\}_{t+\Delta t} - \{\tilde{N}\}_t \\ & + [\tilde{J}] a_1\{\tilde{\theta}\}_t + a_2\{\dot{\tilde{\theta}}\}_t + a_3\{\ddot{\tilde{\theta}}\}_t \end{aligned}$$

with

$$\begin{aligned} a_0 &= c_1 \\ a_1 &= c_1 \\ a_2 &= -c_2 \\ a_3 &= -c_3 \end{aligned}$$

The solution of Equation 2-12 is given by

$$\{\theta^*\}_{t+\Delta t} = [K^*]_{t+\Delta t}^{-1} \{F^*\}_{t+\Delta t} \quad (2-13)$$

where

$$[K^*]_{t+\Delta t}^{-1} = \frac{1}{(K_{11}^* K_{22}^* - K_{12}^* K_{21}^*)} \begin{bmatrix} K_{22}^* & -K_{12}^* \\ -K_{21}^* & K_{11}^* \end{bmatrix}$$

since the form of $[K^*]$ is the 2 x 2 array

$$[K^*]_{t+\Delta t} = \begin{bmatrix} K_{11}^* & K_{12}^* \\ K_{21}^* & K_{22}^* \end{bmatrix}$$

Now the process solution at time $t+\Delta t$ is given by

$$\begin{aligned} \{\ddot{\theta}\}_{t+\Delta t} &= a_4\{\theta^*\}_{t+\Delta t} + a_5\{\tilde{\theta}\}_t + a_6\{\dot{\tilde{\theta}}\}_t + a_7\{\ddot{\tilde{\theta}}\}_t \\ \{\dot{\theta}\}_{t+\Delta t} &= b_0\{\theta^*\}_{t+\Delta t} - b_1\{\tilde{\theta}\}_t - b_2\{\dot{\tilde{\theta}}\}_t - b_3\{\ddot{\tilde{\theta}}\}_t \\ \{\theta\}_{t+\Delta t} &= \{\theta^*\}_{t+\Delta t} \end{aligned} \quad (2-14)$$

where

$$\begin{aligned} a_4 &= c_1 & b_0 &= c_4 \\ a_5 &= -c_1 & b_1 &= c_4 \\ a_6 &= c_2 & b_2 &= -c_5 \\ a_7 &= c_3 & b_3 &= -c_6 \end{aligned}$$

The system state variables at time $t+\Delta t$ may be established by Equations 2-2 and the governing state variables from Equation 2-3.

2.2.4 SNUBBER DYNAMIC PROCESS SUMMARY

The process formulation just presented was implemented in FORTRAN V on a Data General Eclipse S/130 minicomputer for conduct of correlation studies with snubber test data. The coded process descriptions for the snubber system model are given in Table 2-2. The logic

TABLE 2-2. PROCESS DESCRIPTIONS

Process	Description
SNUBBER	This process controls the main functions of the process, including subprocess calls and data storage.
STIFF	This process evaluates the inertia matrix ($\underline{J}$), damping matrix ($\underline{C}$), tangent stiffness matrix ($\underline{K}$), internal force vector ($\underline{N}$), and friction force vector ($\underline{f}$) for the snubber at a given time of the forcing history. The evaluation is based on the kinematics of the snubber system at a given instant of time and previous values of stiffness.
HYSTBL	HYSTBL is a subprocess of STIFF. This process evaluates the current behavior of the clutch spring based on previous histories of the spring and current values of rotational displacement.
SOLVE	This process integrates the equations $\underline{J}\ddot{\theta} + \underline{C}\dot{\theta} + \underline{K}\theta = \underline{F} - \underline{N} - \underline{f}$ using the Newmark method at a given time step. During a time step, if the stiffness changes, iteration is performed using a Newton-Raphson technique.
COND	COND is a subprocess of SOLVE. This process condenses the 4 DOF process formulation to the 2 DOF contracted process formulation, $\underline{K}$ and $\underline{F}$.
SOLVE2	SOLVE2 is a subprocess of SOLVE. This process solves a 2 x 2 system of linear equations.

flow diagrams of the model are given in Figures 2-3 to 2-6. The FORTRAN code statements for the process are given in AA (1981) along with a sample correlation study problem.

2.3 COMPARISON OF ANALYTICAL MODEL WITH TEST RESULTS

2.3.1 REVIEW OF SNUBBER COMPONENT TESTING RESULTS

A total of twenty-one mechanical snubbers (PSC shock arrestors), encompassing the general range of available sizes, have been subjected to a sequence of tests to establish the dynamic characteristics of mechanical snubbers as a function of input amplitude (reaction force) and frequency (HEDL, 1980a). The full dynamic characterization sequence included four nominal load levels (25, 50, 75, and 100% rated load for snubbers with rated loads below 2500 lb) at nine discrete frequencies from 2 to

30 Hz. Each snubber unit was subjected to a quasi-harmonic input displacement of the snubber end points. The input displacement amplitude at each frequency was increased until the required reaction force level (25%, 50%, etc.) was reached. Force and displacement data were recorded at each force level for either a maximum of 100 cycles or a total time of 10 sec. Data samples were presented for two cycles of test input at each force level in the form of force and displacement time-history plots (HEDL, 1980b). The corresponding Lissajous pattern (displacement vs. force) was also presented. The latter form of data presentation was utilized as the primary source of data for the correlation studies presented herein.

A representative snubber unit (PSA01; 100% rated load = 1500 lb) was chosen as the reference unit for the correlation study. An example Lissajous pattern (reaction force vs.

FIGURE 2-3. MAIN PROCESS SNUBBER

FIGURE 2-4. PROCESS STIFF

FIGURE 2-5. PROCESS HYSTBL

† CLUTCH SPRING BACKLASH CONDITION DETERMINED
IN PROCESS HYSTBL (CALLED BY PROCESS STIFF)

FIGURE 2-6. PROCESS SOLVE

overall displacement) obtained from the test data is shown in Figure 2-7. A piecewise linear representation of the example force-displacement path is also shown in the figure. This approximate representation was then utilized to obtain estimates of the average peak force amplitude, F^* , average peak displacement amplitude, U_{TOT}^* , and average backlash amplitude or gap, G . The average cyclic energy loss, D_s , was estimated by computing the enclosed area of the approximate force-displacement path. The reduced test data for the reference snubber unit are tabulated at each input frequency for three force levels (50%, 75%, 100%) in Table 2-3. A plot of the axial displacement amplitude of the snubber end points versus frequency is shown in Figure 2-8, where it is seen that the incremental change in input displacement between force levels is small. A line of constant acceleration, $\ddot{u} = 0.02$ g, is also shown in Figure 2-8 to indicate the acceleration threshold often discussed in the manufacturer's descriptive literature for the snubber units. A plot of the cyclic energy loss versus frequency is shown in Figure 2-9.

The process model requires that fifteen descriptive parameters be established for the reference snubber unit. Based upon physical measurements obtained from a disassembled snubber unit (PSA01), the required parameters were estimated and denoted as the base model parameters (see Table 2-4). For example, the capstan spring stiffness, K_ϕ , may be computed using equations developed by Ancker and Goodier (1958) for helical torsion springs.

2.3.2 CORRELATION OF ANALYTICAL SNUBBER MODEL WITH AVAILABLE TEST DATA

Based on a qualitative review of a perturbation study and a check of the parameter estimates obtained or calculated using the physical measurements of the disassembled

reference snubber unit, the base model descriptive parameters were revised, as indicated in Table 2-4, and denoted as the PSA01 model parameters. Only one parameter, K^* , the locked axial snubber stiffness, required iterative process simulation until an acceptable value was found that yielded frequency dependence of the input displacement amplitudes comparable to the test amplitudes.

TABLE 2-3. PSA01 TEST DATA REDUCTION

Test f, Hz	U_{TOT}^* , mil	G, mil	$(U_{TOT}^* - G)$, mil	F^* , lb	D_s , in-lb
(a) $F_{NOM} = 700$ lb					
2	49.2	4.4	44.8	728	88.8
3	26.0	4.1	21.9	715	33.9
4.5	20.4	5.1	15.3	694	25.1
6	19.1	5.7	13.4	721	21.5
9	19.0	5.5	13.5	720	18.7
12	23.4	5.6	17.8	720	23.2
15	23.0	5.2	17.8	678	29.4
20	20.8	5.6	15.2	647	23.2
30	11.0	3.0	8.0	645	12.4
(b) $F_{NOM} = 1000$ lb					
2	42.5	5.7	36.8	1081	105.9
3	30.6	4.0	26.6	1081	62.8
4.5	21.9	5.4	16.5	1034	39.0
6	21.5	5.0	16.5	979	30.9
9	24.4	5.9	18.5	985	27.7
12	26.1	5.2	20.9	997	34.8
15	26.7	5.0	21.7	1020	45.0
20	24.5	6.0	18.5	967	44.8
30	18.7	3.6	15.1	1034	34.8
(c) $F_{NOM} = 1400$ lb					
2	37.7	10.2	27.5	1416	95.8
3	29.2	4.0†	25.2	1445	80.4
4.5	24.6	3.8	20.8	1457	57.1
6	26.1	5.8	20.3	1430	43.8
9	27.6	5.1	22.5	1426	44.6
12	30.8	4.9	25.9	1456	57.0
15	29.7	4.3	25.4	1410	68.3
20	30.4	5.9	24.5	1459	84.8
30	21.5	3.4	18.1	1350	60.9

† $G/2$

(a) Lissajous pattern obtained from test data (HEDL, 1980b)

(b) Approximate representation of force — displacement path

FIGURE 2-7. REDUCTION OF DYNAMIC CHARACTERIZATION TEST DATA

FIGURE 2-8. AXIAL DISPLACEMENT AMPLITUDE OF SNUBBER UNIT (PSA01)

FIGURE 2-9. AVERAGE CYCLIC ENERGY LOSS OF SNUBBER UNIT (PSA01)

TABLE 2-4. DESCRIPTIVE PARAMETER ESTIMATES
(See Sec. 2.2.2 for Symbols)

Parameter	Base Model	Revised Model, PSA01 Model
μ	0.1	0.1
K_{bs} (in-lb/rad)	1250	1250
B_l (in)	0.008	0.010
K^* (lb/in)	40,000	80,000
r^* (in)	0.02	0.018
C (rad/in)	0.02	0.02
K_ϕ (in-lb/rad)	7.0	5.4
K'_ϕ (in-lb/rad)	300,000	270,000
T_{cr} (in-lb)	1.56	2.00
N	6.667	6.667
K_{cs} (in-lb/rad)	100,000	42,500
$\Delta\theta_{cscr}$ (rad)	0.133	0.133
T_{cscr} (in-lb)	7.0	7.0
J_{td} (in-lb-s ² /rad)	0.002*	0.001
J_1 (in-lb-s ² /rad)	0.005	0.005

*An error was made in the calculation of this value. The correct value was used for the revised model.

The process response of the PSA01 model is compared to the process response of the base model in Figure 2-10. The data for the PSA01 model are shown in Figures 2-8 and 2-9 (open-circle symbols). As can be noted from Figure 2-8, the frequency dependence of the input displacement amplitudes is quite similar to the test amplitudes. Figure 2-9 indicates that the magnitude and frequency dependence of the cyclic energy loss of the PSA01 process model is in general qualitative agreement with the energy loss behavior of the reference snubber unit.

An example of the dynamic characterization test data for the reference snubber unit is shown in Figure 2-11 for two nominal reaction force levels. Comparison of the corresponding process model (9 Hz) force-displacement plots

given in Figure 2-10 with the test force-displacement plot given in Figure 2-11, indicates that the process model response is qualitatively similar to the reference snubber unit test response (allowing for the scale difference). However, the displacement time-history plot shown in Figure 2-11 indicates that the input displacement across the snubber end points was not a sinusoidal function of time during the dynamic characterization testing. To assess the behavior of the process model for nonsinusoidal input, the test displacement function was digitized and scaled to the same amplitude used for the sinusoidal input function (9 Hz, ± 0.027 in). The process response obtained for the digitized test displacement input is shown in Figure 2-12. Comparison of the time-history response of the reference snubber unit shown in Figure 2-11 with the process time history shown in Figure 2-12 indicates (allowing for scale differences) that both reaction force time histories lead the input displacement and compare qualitatively in general form. Additional comparison of the force-displacement plot shown in Figure 2-12 with the corresponding plot (9 Hz) shown in Figure 2-10 for sinusoidal input, indicates that the nonsinusoidal input yields a significantly different hysteresis loop and that the response process loop (Fig. 2-12) with nonsinusoidal input has better agreement with the test response loops. It should be noted that to properly characterize the behavior of a snubber unit, the force output should be determined for a given harmonic (sinusoidal) displacement amplitude. If the input displacement is not harmonic, the parameter identification for an equivalent analytical representation of the snubber and/or process model correlation may be difficult. Care should be exercised in the design of test feedback control systems to ensure that the "purity" of the relative displacement input waveform is maintained.

(a) Base model

(b) PSA01 model

FIGURE 2-10. COMPARISON OF PROCESS RESPONSE FOR BASE AND PSA01 MODELS

(a) $F_{nom} = 1000$ lb

(b) $F_{nom} = 1400$ lb

FIGURE 2-11. EXAMPLE OF DYNAMIC CHARACTERIZATION TEST DATA (HEDL, 1980b)

FIGURE 2-12. EXAMPLE RESPONSE OF SNUBBER PROCESS MODEL (PSA01) FOR DIGITIZED TEST DISPLACEMENT INPUT

Both the process model and reference snubber unit test response indicate that the frequency response behavior of the snubber mechanism begins to disengage for displacement input frequencies less than 3 Hz. To assess the behavior of the process model for input waveforms with frequency content less than 3 Hz, an 0.8 Hz, 0.1 in. amplitude waveform was added to the 9.0 Hz, 0.027 in. amplitude waveform used for process correlation at 9 Hz input frequency, resulting in the modulated input waveform shown in Figure 2-13a. Using this waveform as input for the process model PSA01, the response shown in Figures 2-13b and c was computed. As can be noted from Figure 2-13, the process model is "transparent" to the low-frequency input component, resulting in the drift behavior noted. (The discontinuity in the input waveform is an error, but results only in a local response perturbation and does not affect the overall process response.) Such behavior of a snubber unit is to be expected, since the purpose of a snubber is to allow slowly varying thermal expansion, yet provide restraint for more rapidly varying motion of the piping system. Only limited low-frequency testing of snubber units has been performed,

thus correlation of the process model for input frequencies less than 2 Hz is not possible at this time. Additional testing of snubber units should be conducted using complex input displacement waveforms (i.e., sum of harmonics with varying amplitudes) to allow further correlation of the process model.

2.3.3 CONCLUSIONS

The process model correlation studies lead to the following conclusions:

- a. The computer process code yields a state variable response that is sufficiently smooth, indicating that the process algorithms formulated are numerically stable.
- b. The response of the detailed process model correlates qualitatively with the test response of a representative snubber unit. Specifically, the frequency dependence and magnitude of both the input displacement and cyclic energy loss of the process model are in general agreement with the behavior of the representative snubber unit.

(a) Relative displacement of snubber end points (δ)

(b) Axial snubber reaction force (F)

(c) Process Response

FIGURE 2-13. LOW-FREQUENCY DRIFT BEHAVIOR OF SNUBBER PROCESS MODEL (PSA01)

c. The response of the detailed process model is sensitive to variations in the shape of the input displacement waveform. The waveform used in characterization testing may differ from a sinusoid, thus process model correlation may require the use of a digitized input test waveform if quantitative correlation is desired.

2.4 SIMPLIFIED SNUBBER MODELS FOR PIPING SYSTEM ANALYSES

2.4.1 RESPONSE OF VISCOELASTIC COMPONENT REPRESENTATIONS

Consider the behavior of a structural element or mechanical link under harmonically varying cyclic deformation with constant amplitude. If the load-deformation curve is a closed elliptical loop, the element or link may be represented by a network of linear elastic springs and viscous dashpots. Such a representation is denoted as a viscoelastic model. For a viscoelastic model, a very convenient and simple functional expression for the peak force amplitude and damping (energy loss) characteristics is possible in terms of complex moduli. Consider an example elliptical hysteresis loop, such as shown in Figure 2-14. The equation of this ellipse may be expressed as

$$P = F \left(\cos \delta \frac{X}{A} + \sin \delta \left(1 - \frac{X^2}{A^2} \right)^{1/2} \right) \quad (2-15)$$

where F is the peak force amplitude, A is the peak displacement amplitude, and δ is the phase angle between the displacement, $X = A \sin(\omega t)$, and force, $P = F \sin(\omega t + \delta)$. Equation 2-15 may also be expressed as

$$P = F \left(\cos \delta \frac{X}{A} + \sin \delta \frac{\dot{X}}{\omega A} \right) \quad (2-15a)$$

Using the complex form for the harmonic input, $X = Ae^{i\omega t}$, we obtain

FIGURE 2-14. ELLIPTICAL HYSTERETIC LOOP CHARACTERISTIC OF VISCOELASTIC ELEMENT WITH SINUSOIDAL DISPLACEMENT INPUT

$$P = \left(\frac{F}{A} \cos \delta + i \frac{F}{A} \sin \delta \right) X$$

or

$$P = (K'_S + i K''_S) X \quad (2-16)$$

where $K'_S = F \cos \delta / A$ is the storage modulus and $K''_S = F \sin \delta / A$ is the loss modulus. The ratio of the loss modulus and storage modulus,

$$\eta_S = K''_S / K'_S = \tan \delta \quad (2-17)$$

is denoted as the loss tangent or loss coefficient.

The enclosed area within the elliptical hysteresis loop, or the cyclic energy loss, is given by

$$D_S = \pi F A \sin \delta \quad (2-18)$$

2.4.2 TEST DATA LINEARIZATION

Since Equation 2-15a is a linear differential equation, we may attempt to linearize the behavior of a structural element or mechanical link by idealizing the arbitrary force-displacement loop as an equivalent ellipse with equal area and the same peak force and displacement

amplitude, as shown in Figure 2-15. For mechanical links with backlash or gaps, two alternate methods of linearization are possible as is indicated in Figure 2-16. The gap-inclusive model ignores the gap effect and represents the hysteresis loop in an "average" sense. The gap-exclusive model represents the hysteresis loop as a "split" ellipse or piecewise linear (nonlinear) system.

From the test data (HEDL, 1980b), the equivalent loss tangent may be computed for each test frequency using Equation 2-17 as indicated in Table 2-5 for the gap-inclusive case. The loss tangent is a suitable parameter for qualitative identification of viscoelastic element type. The frequency dependence of the loss tangent for the four basic types of viscoelastic elements is shown in Figure 2-17 (log-log presentation) for an arbitrary choice of model parameters. The loss tangent data for the reference snubber unit are tabulated in Table 2-5 and plotted in Figure 2-18. Comparison of Figures 2-17 and 2-18 indicates that the frequency dependence exhibited by the test

TABLE 2-5. PSA01 TEST DATA LINEARIZATION,
GAP INCLUSIVE (HEDL, 1980b)

Test f, Hz	$\sin \bar{\delta},$ $D_s/\pi U_{TOT}^* F^*$	$\bar{\eta}_s,$ $\tan \bar{\delta}$	$\cos \bar{\delta},$ $\sin \bar{\delta}/\tan \bar{\delta}$
(a) $F_{NOM} = 700$ lb			
2	0.789	1.28	0.616
3	0.580	0.713	0.813
4.5	0.564	0.684	0.825
6	0.497	0.573	0.867
9	0.435	0.483	0.901
12	0.438	0.488	0.898
15	0.600	0.750	0.800
20	0.549	0.656	0.837
30	0.556	0.669	0.831
(b) $F_{NOM} = 1000$ lb			
2	0.734	1.080	0.680
3	0.604	0.758	0.797
4.5	0.548	0.655	0.837
6	0.467	0.529	0.883
9	0.367	0.394	0.931
12	0.426	0.470	0.906
15	0.526	0.618	0.851
20	0.602	0.754	0.798
30	0.573	0.699	0.820
(c) $F_{NOM} = 1400$ lb			
2	0.571	0.696	0.820
3	0.607	0.763	0.796
4.5	0.507	0.588	0.862
6	0.374	0.403	0.928
9	0.361	0.387	0.933
12	0.405	0.442	0.916
15	0.518	0.606	0.855
20	0.609	0.767	0.794
30	0.668	0.897	0.745

FIGURE 2-15. LINEARIZATION OF TEST DATA BY EQUIVALENT
ELLIPSE WITH EQUAL AREA AND EQUAL MAXIMUM
FORCE AND AXIAL DISPLACEMENT

(a) Gap-inclusive model

(b) Gap-exclusive model (nonlinear)

FIGURE 2-16. ALTERNATE LINEARIZATION OF TEST DATA INCLUDING AND EXCLUDING GAP

FIGURE 2-17. IDENTIFICATION OF VISCOELASTIC ELEMENT FROM LOSS COEFFICIENT CURVES

FIGURE 2-18. EQUIVALENT LOSS TANGENT FOR SNUBBER UNIT (PSA01)

data is best represented by the three-parameter fluid viscoelastic element (series dashpot and Voigt element). It should also be noted that the loss tangent data are only a weak function of reaction force level, indicating that a viscoelastic element idealization would be a valid approximate representation of the snubber unit dynamic test behavior.

Next, for reduced test data and the computed loss tangent, the storage modulus and loss modulus may be computed and tabulated, as shown in Table 2-6 for the gap-inclusive case. Plots of both moduli are given in Figure 2-19. To provide an additional comparison of the process model response with the reference snubber test behavior, the loss tangent, storage modulus, and loss modulus for both process models are tabulated in Table 2-7 and included in Figures 2-18 and 2-19 for reference. As can be noted, the revised process model (PSA01) yields moduli that are in good agreement with the moduli determined from the test data.

2.4.3 VISCOELASTIC PARAMETER IDENTIFICATION

After establishing the frequency dependence of the experimentally determined moduli,

the next step is to identify and assign numerical values to the parameters of the viscoelastic element that best represents the frequency dependence of the equivalent linear moduli. Since the three-parameter fluid element was tentatively identified as the simplest viscoelastic element that has the same qualitative frequency dependence, the parameter identification procedure will be demonstrated with this element. For the schematic three-parameter fluid element shown in Figure 2-20, the governing differential equation is

$$F + \tau_{\epsilon} \dot{F} = D_R(\dot{u} + \tau_O \ddot{u}) \quad (2-19)$$

where the coefficients of the differential equation are

$$\begin{aligned} D_R &= C_2 \\ \tau_{\epsilon} &= (C_1 + C_2)/K_1 \\ \tau_O &= C_1/K_1 \end{aligned}$$

Taking the solution in the complex harmonic form

$$\begin{aligned} F &= F_0 e^{i(\omega t + \delta)} \\ u &= u_0 e^{i\omega t} \end{aligned}$$

where F_0 is the reaction force amplitude and u_0 is the relative displacement amplitude, we may

TABLE 2-6. EQUIVALENT VISCOELASTIC MODULI FOR PSA01 SNUBBER UNIT (GAP INCLUSIVE)

Test, f Hz	$F_{NOM} = 1400 \text{ lb}$		$F_{NOM} = 1000 \text{ lb}$		$F_{NOM} = 700 \text{ lb}$	
	$\bar{K}'_s$, lb/in $\times 10^3$	$\bar{K}''_s$, lb/in $\times 10^3$	$\bar{K}'_s$, lb/in $\times 10^3$	$\bar{K}''_s$, lb/in $\times 10^3$	$\bar{K}'_s$, lb/in $\times 10^3$	$\bar{K}''_s$, lb/in $\times 10^3$
2	30.8	21.4	9.1	18.7	17.3	11.6
3	39.4	30.1	22.4	21.3	28.1	16.0
4.5	51.1	30.0	28.1	25.9	39.5	19.2
6	50.8	20.5	32.7	21.3	40.2	18.7
9	48.2	18.7	34.1	14.8	37.6	16.5
12	43.3	19.1	27.6	16.3	34.6	13.5
15	40.6	24.6	23.6	20.1	32.5	17.7
20	38.1	29.2	26.0	23.8	31.5	17.1
30	46.8	42.0	48.7	31.7	45.3	32.6

$$\bar{K}'_s = F^* \cos \delta / U_{TOT}^* \quad \bar{K}''_s = \bar{\eta}_s \bar{K}'_s$$

(a) Storage modulus

(b) Loss modulus

FIGURE 2-19. EQUIVALENT MODULI FOR SNUBBER UNIT (PSA01)

Viscoelastic Element	Governing Equation	Moduli	Parameter Estimation Form
<p>MAXWELL</p>	$F + \tau_e \dot{F} = D_R \dot{u}$ $\tau_e = C_1/K_2, \quad D_R = C_1$	$K'_S = \frac{\omega^2 D_R \tau_e}{1 + \omega^2 \tau_e^2}$ $K''_S = \frac{\omega D_R}{1 + \omega^2 \tau_e^2}$	$D_R - \tau_e K'_S = K''_S / \omega$ $\tau_e K''_S = K'_S / \omega$
<p>VOIGT (KELVIN)</p>	$F = E_R u + D_R \dot{u}$ $E_R = K_1, \quad D_R = C_1$	$K'_S = E_R$ $K''_S = D_R$	$D_R = K''_S / \omega$ $E_R = K'_S$
<p>3-PARAMETER FLUID</p>	$F + \tau_e \dot{F} = D_R (\dot{u} + \tau_\sigma \ddot{u})$ $D_R = C_2, \quad \tau_e = \frac{C_1 + C_2}{K_1}$ $\tau_\sigma = C_1/K_1$	$K'_S = \frac{D_R \omega^2 (\tau_e - \tau_\sigma)}{1 + \omega^2 \tau_e^2}$ $K''_S = \frac{(1 + \omega^2 \tau_\sigma \tau_e) D_R \omega}{1 + \omega^2 \tau_e^2}$	$D_R - K'_S \tau_e = K''_S / \omega$ $K''_S \tau_e - \omega D_R \tau_\sigma = K'_S / \omega$

FIGURE 2-20. VISCOELASTIC ELEMENT REPRESENTATION FOR SINUSOIDAL INPUT

TABLE 2-7. EQUIVALENT VISCOELASTIC MODULI FOR PROCESS MODELS (GAP INCLUSIVE)

f, Hz	$\bar{\eta}_s$	$\bar{K}'_s$, lb/in x 10 ³	$\bar{K}''_s$, lb/in x 10 ³
(a) Base Model			
3	-	-	-
3 5	3 065	17 4	53 3
6	0 363	39 6	14 4
9	0 387	35 4	13 7
12	0 280	33 7	9 4
15	0 241	29 1	7 0
20	0 225	32 9	7 4
30	0 211	37 6	7 9
(b) PSA01 Model			
3	0 995 0 662	32 2 26 1	32 0 17 3
3 5	1 29 1 13	35 4 37 1	45 7 41 9
4 5	1 19	52 4	62 4
6	0 471	57 4	27 0
9	0 524 0 555	47 2 51 2	24 7 28 4
12	0 805	44 6	35 9
15	0 807	46 3	37 4
17 5	0 481	40 0	19 2
20	0 465	47 7	20 8
30	0 511	57 0	29 1

$$\delta = \sin^{-1} (D_s / \pi U^*_{TOT} F^*)$$

$$\bar{\eta}_s = \tan \delta$$

$$\bar{K}'_s = F^* \cos \delta / U^*_{TOT}$$

$$\bar{K}''_s = \bar{\eta}_s \bar{K}'_s$$

write Equation 2-19 in the form

$$F/u = K'_s + iK''_s \quad (2-20)$$

where the moduli are

$$K'_s = \frac{D_R \omega^2 (\tau_\epsilon - \tau_\sigma)}{1 + \omega^2 \tau_\epsilon^2}$$

$$K''_s = \frac{(1 + \omega^2 \tau_\sigma \tau_\epsilon) D_R \omega}{1 + \omega^2 \tau_\epsilon^2}$$

These may be rearranged to yield the parameter estimation form

$$D_R - K'_s \tau_\epsilon = K''_s / \omega \quad (2-21)$$

$$K''_s \tau_\epsilon - \omega D_R \tau_\sigma = K'_s / \omega$$

If we identify the three coefficients of the differential Equation 2-19 as

$$a = D_R$$

$$b = \tau_\epsilon$$

$$c = D_R \tau_\sigma$$

then a set of Equations 2-21 may be written for each input frequency, ω_k , as

$$[A_k] \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \{B_k\}$$

where

$$[A_k] = \begin{bmatrix} 1 & -K'_s & 0 \\ 0 & K''_s & -\omega \end{bmatrix}_k$$

$$\{B_k\} = \begin{Bmatrix} K''_s / \omega \\ K'_s / \omega \end{Bmatrix}_k$$

Thus, given K sets of viscoelastic moduli representing the equivalent linear moduli obtained from the reduced test data for K input frequencies, we may form 2 x K linear equations in three unknowns represented by

$$[A] \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \{B\} \quad (2-22)$$

where

$$[A] = \begin{bmatrix} A_1 \\ \vdots \\ A_J \\ \vdots \\ A_K \end{bmatrix} \text{ and } \{B\} = \begin{Bmatrix} B_1 \\ \vdots \\ B_J \\ \vdots \\ B_K \end{Bmatrix}$$

The least-squares solution (Noble, 1969) to Equation 2-22 is given by the 3 x 3 set of

equations

$$[A]^T[A] \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = [A]^T\{B\} \quad (2-23)$$

where $[A]^T$ is the transpose of the array $[A]$.

Given an estimate of the three coefficients in Equation 2-19, the moduli for Equation 2-20 may be determined as a function of frequency. The least squares gap-inclusive moduli and corresponding loss tangent determined for the reference snubber unit using the test data for the $F = 1400$ -lb reaction load level are plotted compared to the experimentally determined moduli in Figures 2-21 and 2-22.

The least squares gap-inclusive moduli for the Voigt element (2-9 Hz) and the Maxwell element (9-30 Hz) were also determined using

the corresponding moduli and parameter estimation forms given in Figure 2-20. These moduli are included in Figures 2-21 and 2-22 for comparison to both the test moduli and the equivalent moduli obtained for the process model.

For completeness, the gap-exclusive experimental moduli were computed (Table 2-8) and the least squares three-parameter fluid moduli determined as shown in Figure 2-23. The viscoelastic snubber model parameters corresponding to a least-squares fit to the reference snubber unit test data for a reaction force level of 1400 lb are summarized in Table 2-9. It should be noted that while other optimized methods of parameter identification can be applied to estimate the viscoelastic

FIGURE 2-21. LEAST SQUARES LOSS TANGENT FIT FOR LINEAR VISCOELASTIC SNUBBER (PSA01) MODEL

(a) Storage modulus

(b) Loss modulus

FIGURE 2-22. LEAST SQUARES MODULI FIT FOR LINEAR VISCOELASTIC SNUBBER (PSA01) MODEL (GAP INCLUSIVE)

TABLE 2-8. EQUIVALENT VISCOELASTIC MODULI FOR
PSA01 SNUBBER UNIT (GAP EXCLUSIVE)

Test, f Hz	F _{NOM} = 1400 lb		F _{NOM} = 1000 lb		F _{NOM} = 700 lb	
	K' _s , lb/in x 10 ³	K'' _s , lb/in x 10 ³	K' _s , lb/in x 10 ³	K'' _s , lb/in x 10 ³	K' _s , lb/in x 10 ³	K'' _s , lb/in x 10 ³
2	32 0	40 3	15 6	24 9	8 1	14 1
3	40 8	40 3	29 2	28 2	23 7	22 5
4 5	56 1	42 0	43 0	45 6	29 9	34 1
6	61 7	28 3	47 0	36 1	37 9	37 5
9	56 9	28 1	46 6	25 8	42 1	32 6
12	49 2	27 0	40 4	25 4	33 0	23 3
15	44 2	33 8	35 8	30 4	24 0	29 5
20	39 0	44 9	31 6	41 7	28 1	31 9
30	45 4	59 2	48 2	48 5	51 9	61 7

$$K'_s = F^* \cos \delta / U^*$$

$$K''_s = \eta_s K'_s$$

TABLE 2-9. VISCOELASTIC SNUBBER MODEL (PSA01) PARAMETERS FOR
LEAST SQUARES FIT TO TEST DATA (F_{NOM} = 1400 LB)

Element	Governing Equation Coefficients				Element Parameters		
	E _R , lb/in x 10 ³	D _R , lb-s/in	τ _ε , s	D _R τ _σ , lb-s ² /in	C ₁ , lb-s/in	C ₂ , lb-s/in	K ₂ , lb/in x 10 ³
(a) Gap Inclusive							
Maxwell (2-9 Hz)	--	4095	--	--	4095	--	59 3
Voigt (9-30 Hz)	43 4	260	0 0691	--	--	260	43 4
3-parameter Fluid (2-30 Hz)	--	4240	0 0821	16 14	4240	206	54 2
(b) Gap Exclusive							
3-parameter Fluid (2-30 Hz)	--	4035	0 0628	17 96	4035	308	69 2

FIGURE 2-23. LEAST SQUARES MODULI FIT FOR NONLINEAR VISCOELASTIC SNUBBER (PSA01) MODEL (GAP EXCLUSIVE)

parameters (Bland, 1960), the least-square method used herein is basic and can be directly applied without difficulty. The least-squares estimation of the differential equation coefficients followed by the indirect computation of the viscoelastic element constants is the preferred procedure rather than direct estimation of the element constants since the coefficients of the differential equation control the viscoelastic element linear behavior.

2.4.4 CONCLUSIONS AND RECOMMENDATIONS

The consideration of viscoelastic snubber element representations leads to the following conclusions:

- a. The frequency dependence of equivalent linear viscoelastic moduli, determined from the reduced test data for the reference snubber unit, indicates that a three-parameter fluid (series dashpot and Voigt element) element is the simplest viscoelastic representation for a snubber unit within the input frequency range, 2 to 30 Hz.
- b. Viscoelastic element parameter estimates can be obtained by least-squares fit of an overdetermined set of linear equations representing the coefficients of the governing differential equation for the viscoelastic element.

2.5 EXAMPLE PIPING SYSTEM

Consider the example case shown in Figure 2-24. An 8-in. nominal dia., schedule 40 pipe (316 stainless steel) was chosen as being compatible with the representative snubber unit (PSA01) load rating (1500 lb). The placement of the hanger support was chosen to limit the deadweight stress within the 36-ft span to a stress allocation of 3000 psi. If the

pressure stress allocation is 5000 psi and the range of stress for thermal expansion and anchor movement is limited to 20,000 psi, then preliminary code screening stress checks (see AA, 1981) will be satisfied if the resultant seismic (OBE) stress is limited to 6500 psi. For seismic response due to a single input direction, the stress component is limited to $6500/\sqrt{2} \cong 4600$ psi. For the 8.625-in. dia., 0.322-in. wall thickness pipe considered, this stress is equivalent to a moment limit of 77.3×10^3 in.-lb. Thus, the basic problem posed is to determine the effect of various snubber characterizations on the internal moment reactions of the example piping system of Figure 2-24 for an OBE event. Also of importance is the magnitude of the peak snubber reaction force since it establishes the size (load rating) of the device to be utilized.

For this example piping system, the problem was discretized by lumping the mass of a half-span at the quarterspan points as suggested by Lin (1974) for simple piping spans between support points. The total seismic restraint path between the pipe and the point of structure motion input was idealized as three colinear elements: (1) clamp/pipe interaction spring, (2) idealized snubber unit, and (3) structure support spring.

The clamp/pipe interaction stiffness was estimated by considering the diametrical contraction/extension due to diametrical compression/tension of a ring section of pipe as suggested by Mello and Pollono (1979). For a 16-in. ring segment of schedule 40, 8-in. dia. pipe, the clamp/pipe interaction stiffness was estimated to be $K_{CLP} = 250 \times 10^3$ lb/in. The structure support stiffness was estimated as $K_{STR} = 90 \times 10^3$ lb/in. This value is representative of the cantilever beam stiffness of a 6-ft, 12W50 steel wide flange section plus an additional transition tube element.

SCHEDULE 40, 8 IN. NOMINAL DIAMETER

D_o = Outside pipe diameter = 8.625 in.

t_w = Pipe wall thickness = 0.322 in.

$(D_o/t_w = 26.8)$

$l = 36$ ft, $\mu l = 14$ ft

I = Pipe cross-section moment of inertia = 72.5 in.⁴

A = Pipe cross-section area = 8.4 in.²

Material: 316 SS

Design Conditions: $T = 930$ °F

$P_o = 200$ psi

$E(@ 930$ °F) = 23.1×10^6 psi

Weights: Pipe 28.6 lb/ft

Sodium at 52 lb/ft³ 18.1 lb/ft

Heaters, insulation, jackets, and supports 21.9 lb/ft

Total $w = 68.6$ lb/ft

FIGURE 2-24. EXAMPLE PIPING ANALYSIS CASE

The unrestrained natural frequency (36-ft span, fundamental mode) of the example piping system is $f_s = 2.8$ Hz, while the fixed support natural frequency (14-ft and 22-ft spans, fundamental mode) of the example piping system is $f = 9.5$ Hz. Linear system damping was specified by selecting values of α and β that would give 2% effective modal damping for both a 3-Hz and 10-Hz oscillator according to the relation

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \quad (2-24)$$

where ω_i is the oscillator frequency. The minimum damping provided by this relationship is 1.7% critical damping.

2.5.1 SEISMIC INPUT MOTIONS

In order to provide a realistic estimate of structure motion (i.e., piping system support motion) during a seismic event, the prototype large breeder reactor (PLBR) 1000-MWe power plant design, developed conceptually by Atomics International and Burns and Roe (AI/B&R, 1977), was selected as a guideline for the definition of a structure model (see Chapter 1 for basic structure description and system layout). The PLBR conceptual design utilizes a double containment/confinement building structure and provides a combined base mat for the major nuclear system structures. The common base mat and the monolithic character of the structures (all are structurally tied) greatly reduce the rocking response of the nuclear island. Thus, for simplicity, the effects of the balance-of-plant structures and rock/ or soil/ structure interaction were not included, and the PLBR containment/confinement structure was modeled as a free-standing, fixed-base, double cantilever beam. The PLBR structure is approximately symmetric in any two orthogonal horizontal directions; thus, the horizontal model was used to represent the PLBR structure in

both principal horizontal directions, N-S and E-W.

The model description and the properties assigned to each beam member and joint of the model are given in detail in AA (1981). The resulting mode shapes and frequencies for the model are also shown there.

The time-history response of the horizontal model was computed using TRI/SAC (AA, 1976b) for two artificial ground acceleration input time histories scaled to an OBE peak acceleration of 0.25 g. The response spectra of these artificial time records with 12-sec duration closely approximate the Regulatory Guide 1.60 spectra (see App. D of AA, 1981). A structure damping of 4% of critical was utilized in the computation of the structure response. The acceleration response during the first three seconds of motion for the model node representing the PLBR operating floor is shown in Figure 6-5 for both input directions. Longer duration floor response time histories and the corresponding floor response spectra are provided in AA (1981).

2.5.2 RESPONSE ANALYSES

The analytical model of the example piping system shown in Figure 2-25a was utilized to compare the system response for the alternative snubber unit idealizations in part b. The example case using an effective linear spring as a snubber idealization was established as the base case for comparison of the other three idealizations. First, the effect of integration time step on system response was addressed for the base case using an effective linear snubber idealization. Second, the system response obtained using a nonlinear gap-spring idealization was compared to the base linear response case. Next, the system response using a viscoelastic element as a snubber idealization was compared to the base linear response case

(a) Analytical model

(b) Alternative snubber unit idealization

FIGURE 2-25. ANALYTICAL MODEL OF EXAMPLE ANALYSIS CASE

and, finally, the system response using the process model (PSA01) was compared to the base linear response case.

All of the response case comparisons were conducted using the 3-sec duration floor acceleration time history, denoted N-S OBE Response (FL3) in Figure 2-26a, as input support motion for the example piping system (common input for all supports). An additional response study was conducted for the process snubber idealization case using the input support motion denoted E-W OBE Response (FL3) in Figure 2-26b. A description of selected response analyses and results follows. Additional details for all of the response analyses can be found in AA (1981).

The computed time histories of the snubber reaction force response, midspan moment response (Node 11, see Fig. 2-25), and beam displacement response at the restraint attachment point are compared to the corresponding time histories computed for the linear composite case in Figures 2-27, 2-28, and 2-29. It should be noted that both response cases utilize the same integration scheme and time step ($\Delta t = 0.0005s$), thus the additional algorithmic damping in either case is comparable. Comparison of the peak response values indicates that the process model has 16-30% less force response and 35-37% less moment response than the linear restraint case and that very little time correspondence exists between the two cases. Comparison of the peak displacement response values indicates that the process restraint case has 100-300% greater displacement than the linear case and that no time correspondence exists between the two cases due to the process model drift behavior. This drift behavior is caused by the low-frequency "transparency" of the process model, which is further evident in the hysteresis plot shown in Figure 2-30.

An additional example was also computed using the 3 sec of floor motion shown in Figure 2-26b. The time-history responses obtained for this alternate input motion are shown in Figures 2-31 and 2-32 for the restraint force and moment response. Comparison of the peak response values indicates that the process restraint case has $\pm 9\%$ difference in force response and 6-28% less moment response than the linear restraint case used in Figures 2-27 and 2-28. Again, the low-frequency drift behavior is evident in the displacement response and process hysteresis plot shown in Figures 2-33 and 2-34.

This example shows the importance of considering several different input motions with varying frequency content in verification analyses. Comparison of the process restraint displacement response with the viscoelastic restraint displacement response shows that the viscoelastic model is not capable of simulating the behavior of a snubber unit for low-frequency components of input motion.

2.5.3 CONCLUSIONS AND RECOMMENDATIONS

The response comparison studies conducted resulted in the following conclusions:

- a. The choice of integration scheme and selection of integration time step is a critical preliminary step in the conduct of verification analyses. Since the equivalent damping of a piping system is often specified as design criteria to be within the range 2-3% critical damping, an integration time step must be carefully selected to ensure that additional algorithmic damping is not (indirectly) included in the response analyses.
- b. A viscoelastic representation (three-parameter fluid) of a mechanical snubber unit is satisfactory for linear

(a) PLBR floor (F1 3) acceleration time history
for N-S OBE input motion (3 sec)

(b) PLBR floor (F1 3) acceleration time history
for E-W OBE input motion (3 sec)

FIGURE 2-26. PLBR OPERATING FLOOR ACCELERATION TIME HISTORIES

(a) Restraint force response for process snubber definition

(b) Restraint force response for linear composite restraint definition

FIGURE 2-27. COMPARISON OF RESTRAINT FORCE RESPONSE FOR LINEAR COMPOSITE AND PROCESS SNUBBER RESTRAINT DEFINITIONS

(a) Moment response (node 7) for process snubber definition

(b) Moment response (node 7) for linear composite restraint definition

FIGURE 2-28. COMPARISON OF MIDSPAN MOMENT RESPONSE FOR LINEAR COMPOSITE AND PROCESS SNUBBER DEFINITION

(a) Beam displacement response (node 4) for process snubber definition

(b) Beam displacement response (node 4) for linear composite restraint definition

FIGURE 2-29. COMPARISON OF BEAM RESTRAINT DISPLACEMENT FOR LINEAR COMPOSITE PROCESS SNUBBER DEFINITION

FIGURE 2-30. HYSTERESIS OF PROCESS SNUBBER FOR N-S OBE INPUT

FIGURE 2-31. RESTRAINT FORCE RESPONSE FOR PROCESS SNUBBER DEFINITION WITH E-W OBE INPUT

FIGURE 2-32. MIDSPAN MOMENT (NODE 7) RESPONSE FOR PROCESS SNUBBER DEFINITION WITH E-W OBE INPUT

FIGURE 2-33. BEAM DISPLACEMENT RESPONSE (NODE 4) FOR PROCESS SNUBBER DEFINITION WITH E-W OBE INPUT

FIGURE 2-34. HYSTERESIS OF PROCESS SNUBBER FOR E-W OBE INPUT

verification analyses for inputs with frequency content greater than 3 Hz.

- c. Snubber units can be modeled as a process subsystem and utilized in nonlinear verification analyses. Verification analyses using a snubber process simulation can serve as a gage to compare analysis results obtained using simple snubber models.

The following are recommendations for additional study.

- a. A viscoelastic snubber model would allow the use of response spectra techniques to be utilized for piping verification analyses. An issue for further study is the definition of composite modal damping to represent the effect of dashpots in the snubber models. Comparative time-history and response spectra studies (including multiple support input) are recommended.
- b. The response studies indicate that a snubber unit is transparent to input motion with frequency content less than 3 Hz. Additional study and correlation with test data are required to validate this behavior. The study results suggest that a piping analysis using simple snubber models should be conducted in two parts: (1) using a system model including all snubber representations and an input motion that has been highpass filtered at a cutoff frequency of 3 Hz, and (2) using a system model without snubber restraints (but including the pendular restraint effect of hangers) and an input that has been low-pass filtered at a cutoff frequency of 3 Hz. The proper combination of the stress resultants obtained from the two-part analysis would require further study.

CHAPTER 3

SEISMIC ANALYSIS TECHNIQUES FOR PIPING SYSTEMS WITH MULTISUPPORT INPUT

3.1 INTRODUCTION

Piping systems for both water-cooled and liquid metal-cooled nuclear reactors traditionally have been designed for seismic effects based on response spectrum procedures using floor response spectra generated from time-history analyses of the structure. Since piping systems have multiple supports that are frequently located at different floor levels, the input to the systems is defined by multiple sets of response spectra for each orthogonal direction of input motion. Currently, response spectra analyses of piping systems are conducted on the assumption that all anchor and support points have a common base (floor) motion defined by the envelope (umbrella) spectra of all floor spectra in a given direction. The system stresses due to differential anchor or support displacement (considered to be statically applied) are then added to the stresses computed from the common base motion response spectrum analysis in the manner specified in the ASME Pressure Vessel Code, Section III, Division I requirements for piping stresses. Often, very conservative, bounding type computations are used to estimate the stresses due to differential support motion (such as assuming that the peak floor displacements of the structure are 180° out of phase).

Thus, the application of current design procedures gives an upper bound estimate of peak seismic stress which, in most cases, leads to placement of more seismic restraints than actually required. For high temperature piping configurations that must accommodate considerable thermal expansion, each seismic restraint employs a snubber in series with a structural support. A snubber is a device that allows

slow movement, such as thermal expansion, to occur but prevents sudden movements, such as seismic motion, by a locking or braking mechanism. Due to field failure of snubbers under normal operating conditions (Butler, 1978) and an incomplete definition of the dynamic characteristics of snubbers under seismic conditions, designers are now faced with questions concerning the reliability of these devices to perform the intended function of allowing slow thermal expansion, yet providing a restraining support point for the dynamic response of a piping system caused by a seismic event. It should be noted that snubber restraint of thermal expansion can lead to severe overstress (or overstrain) conditions within a piping system and may constitute a more serious concern than the infrequent seismic event for which the snubber has been in place. Thus, in addition to the conventional motivation of cost reduction, there is a great incentive to minimize the number of seismic restraints required for a given design in order to increase system reliability. For high temperature liquid metal systems that undergo considerable thermal expansion and yet require a large number of seismic restraints, this design conflict is particularly acute.

To eliminate any excessive design conservatism due to implicit assumptions in the analysis procedures, piping analysts now consider the use of spectral analysis techniques that account for different input spectra at various support levels. Several authors (Amin et al, 1971; Lin, 1973; Singh and Ang, 1974) have considered the application of spectral methods to the problem of multisupport motion of piping systems. The most concise presentations are the methods proposed by Vashi (1975) and

Thaler (1976). Thus, there is a current need for a study which evaluates the proposed multi-support spectral methods and provides a comparison of the analysis results of a response spectrum solution with a corresponding time-history solution for a typical liquid metal piping system.

3.2 FORMULATION OF GOVERNING EQUATIONS

3.2.1 INTRODUCTION

The formulation of the governing equations of motion for a linear elastic system in the context of the normal mode method of dynamic response analysis is the basic preliminary step for approximate dynamic analysis techniques that employ response spectra for input definition. For systems that are subjected only to motion of the points of support, two basic cases are the common base support motion and the multiple support motion. For systems with all points of support located on a common base or floor, either an absolute formulation based on a global motion reference, or a relative formulation based on the common base motion as reference may be used. For systems with differing multiple support motion, three formulations are possible. Again, a global motion reference may be used to obtain an absolute formulation, but two relative motion formulations may be developed. The first relative formulation uses an arbitrarily selected support point as the motion reference, while the second relative formulation uses a static displacement pattern of the system as a motion reference.

A general derivation for each normal mode formulation is presented here. While the modal decomposition of the governing equations for each formulation is similar, the specification of the input support motion requires a different format for each formulation. For time-history analyses, each formulation must yield the same

system response for a given support motion. However, for analyses using response spectra for input specification, each formulation will not yield the same response due to the loss of phase information between the different response components and time derivatives of support motion required for input to each formulation. The presentation of the basis of each formulation is necessary in order to allow comparison of the response obtained by alternate formulations that use response spectra to define the input motion.

The term "system" will be used here to represent a piping configuration supported from L different floors within a structure. The mathematical model of the piping system is considered to be uncoupled from the model of the building used to obtain the floor motions (resulting from a base excitation of the building structure). The input motion of each floor is assumed to be translational motion only, specified in three orthogonal directions. The mass distribution of the piping system is lumped at discrete points of the mathematical model, in accordance with current practice, resulting in a diagonal mass matrix. Both lumped translational and/or rotary inertia may be represented. The use of a "consistent" mass definition, resulting in a nondiagonal mass matrix, is not recommended (Clough, 1969) due to the extra computational effort and additional machine storage required. The assembled equations of motion for the mathematical model of the piping system are defined in a global reference frame chosen to represent the principal axes of the building structure. The mathematical model of the piping system has a total of $N + S$ degrees of freedom.

A "shorthand" matrix notation is used in the following sections. A vector, say $\{U\}$, is represented in shorthand notation as $\underline{U}$. A two-dimensional array, say $[K]$, is represented by $\underline{\underline{K}}$. The summation of vectors (or scalars) is

represented as $\sum_k \underline{U}_k$ where the symbol $\sum_k$ implies summation over the range of index k. A vector or scalar with an absolute (global) base of reference is denoted as $\bar{\underline{U}}$ or $\bar{\underline{U}}_i$, while the vector or scalar with a relative base of reference is denoted as $\underline{U}$ or $\underline{U}_i$. The use of subscripts (i, k, j, l, h) to identify the global directions, floors with input motion, modal response components, and matrix indices is a notational complication that is necessary in order to implement computer programming for each formulation.

3.2.2 EQUATIONS OF MOTION

Consider a mathematical model of a piping system with a three-dimensional configuration represented by discrete elastic finite elements and lumped translational inertia. The equations of motion of this system (in a global reference frame) for a set of applied forcing functions, $\{P(t)\}$, may be written in the general form

$$[M]\{\ddot{\bar{\underline{U}}}\} + [C^*]\{\dot{\bar{\underline{U}}}\} + [K]\{\bar{\underline{U}}\} = \{P(t)\} \quad (3-1)$$

where the vectors $\{\ddot{\bar{\underline{U}}}\}$, $\{\dot{\bar{\underline{U}}}\}$, $\{\bar{\underline{U}}\}$ represent the absolute acceleration, absolute velocity, and absolute displacement of the model nodes, while the arrays $[M]$, $[C^*]$, $[K]$ represent the corresponding mass matrix, damping matrix, and stiffness matrix. Each component $\bar{\underline{U}}_n$ of vector $\{\bar{\underline{U}}\}$ (also $\{\dot{\bar{\underline{U}}}\}$ and $\{\ddot{\bar{\underline{U}}}\}$) represents the three-dimensional motion of model node n and is, in general, given by the six-component subvector

$$\bar{\underline{U}}_n = \begin{Bmatrix} \bar{\underline{U}}_{xn} \\ \bar{\underline{U}}_{yn} \\ \bar{\underline{U}}_{zn} \\ \bar{\theta}_{xn} \\ \bar{\theta}_{yn} \\ \bar{\theta}_{zn} \end{Bmatrix}$$

where $\bar{\underline{U}}_{in}$ represents the translational degrees of freedom and $\bar{\theta}_{in}$ represents the rotational degrees of freedom associated with node n.

If the mechanisms of energy dissipation within the system are to be represented by the concept of composite modal damping, then the formulation of the governing equations for the case of support motion may be developed for an undamped system without loss of generality. Thus, deleting the damping terms of Equation 3-1, we may rearrange the equations of motion to yield the partitioned form

$$\begin{bmatrix} \underline{M}_{pp} & \vdots & \underline{0} \\ \vdots & \ddots & \vdots \\ \underline{0} & \vdots & \underline{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{U}}_p \\ \vdots \\ \ddot{\underline{U}}_s \end{Bmatrix} + \begin{bmatrix} \underline{K}_{pp} & \vdots & \underline{K}_{ps} \\ \vdots & \ddots & \vdots \\ \underline{K}_{sp} & \vdots & \underline{K}_{ss} \end{bmatrix} \begin{Bmatrix} \underline{U}_p \\ \vdots \\ \underline{U}_s \end{Bmatrix} = \begin{Bmatrix} \underline{0} \\ \vdots \\ \underline{P}_s \end{Bmatrix} \quad (3-2)$$

where the subscript p denotes the n nodes of the piping system without input motion, and the subscript s denotes the m support nodes of the system with input motion. The rotational degrees of freedom associated with each support node are, in general, identified as fixed boundary conditions, and thus the corresponding equation is not retained in Equations 3-2. For systems with elastic rotational restraints at support nodes (including the case of no restraint or simple support), the corresponding rotational degrees of freedom are included in the total N degrees of freedom of vector $\bar{\underline{U}}_p$. In any case, the vector $\bar{\underline{U}}_s$ represents only the S translational degrees of freedom associated with each support node.

Since the values of the support mass array, $\underline{M}_{ss}$, and support forcing function

vector, $\underline{P}_s$, are in general unknown, we define the support reactions, $\underline{R}_s$, as

$$\underline{R}_s = -\underline{M}_{ss} \ddot{\underline{U}}_s + \underline{P}_s$$

With this, the equations of motion expressed by Equation 3-2 may be modified to yield

$$\begin{bmatrix} \underline{M}_{pp} & \vdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \vdots & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\underline{U}}_p \\ \vdots \\ \ddot{\underline{U}}_s \end{Bmatrix} + \begin{bmatrix} \underline{K}_{pp} & \vdots & \underline{K}_{ps} \\ \vdots & \ddots & \vdots \\ \underline{K}_{sp} & \vdots & \underline{K}_{ss} \end{bmatrix} \begin{Bmatrix} \underline{\bar{U}}_p \\ \vdots \\ \underline{\bar{U}}_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ \underline{\bar{R}}_s \end{Bmatrix} \quad (3-3)$$

Since the support degrees of freedom are included in the formulation of the equations of motion, the system model is an unrestrained structure for translational motion and thus rigid body translation of the system cannot generate internal forces. The stiffness matrix of an unconstrained structure is singular, which may be demonstrated by the rigid body translation constraints

$$\begin{bmatrix} \underline{K}_{pp} & \vdots & \underline{K}_{ps} \\ \vdots & \ddots & \vdots \\ \underline{K}_{sp} & \vdots & \underline{K}_{ss} \end{bmatrix} \begin{bmatrix} \underline{1}_{px} & \vdots & \underline{1}_{py} & \vdots & \underline{1}_{pz} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{1}_{sx} & \vdots & \underline{1}_{sy} & \vdots & \underline{1}_{sz} \end{bmatrix} = \begin{bmatrix} 0 & \vdots & 0 & \vdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \vdots & 0 & \vdots & 0 \end{bmatrix} \quad (3-4)$$

where $\underline{1}_{px}$, $\underline{1}_{sx}$; $\underline{1}_{py}$, $\underline{1}_{sy}$; $\underline{1}_{pz}$, and $\underline{1}_{sz}$ are vectors that have a value of unity at translational degrees of freedom corresponding to x,

y, and z, respectively, and zeros for the rest of the degrees of freedom.

Equation 3-3 may be expanded to yield the set of equations

$$\underline{M}_{pp} \ddot{\underline{U}}_p + \underline{K}_{pp} \underline{\bar{U}}_p + \underline{K}_{ps} \underline{\bar{U}}_s = 0 \quad (3-5)$$

$$\underline{K}_{sp} \underline{\bar{U}}_p + \underline{K}_{ss} \underline{\bar{U}}_s = \underline{R}_s \quad (3-5a)$$

Equation 3-5a represents the equilibrium condition for support reactions which may be computed given the solution, $\underline{\bar{U}}_p$, of Equation 3-5 for a specified support motion time history, $\underline{\bar{U}}_s$. Thus, the solution of Equation 3-5 is the subject of further consideration.

3.2.3 COMMON BASE SUPPORT MOTION

The response of systems with all points of support upon a common base may be computed given the specification of the three components of motion for the common support base. If the support base motion is specified as an orthogonal set of absolute displacement time histories, then the solution of the equations of motion is sought using a formulation based on an absolute motion reference. If the support base motion is specified as an orthogonal set of absolute acceleration time histories, then a solution is sought using a formulation based on a relative motion reference.

3.2.3.1 Absolute Formulation

For the governing equations of motion (Eq. 3-5), let

$$\begin{aligned} \underline{\bar{U}}_s &= \underline{\bar{U}}_{cx} \underline{1}_{sx} + \underline{\bar{U}}_{cy} \underline{1}_{sy} + \underline{\bar{U}}_{cz} \underline{1}_{sz} \\ &= \sum_i \underline{\bar{U}}_{ci} \underline{1}_{si} \end{aligned} \quad (3-6)$$

where $\underline{\bar{U}}_{ci}$ is the scalar displacement time history of the common support base in direction

i ; $\underline{1}_{si}$ is a vector that identifies the translational degree of freedom that has support motion in direction i ; and the summation indicated is over the index range $i = x, y, z$.

Now Equation 3-5 becomes

$$\underline{M}_{pp} \ddot{\underline{U}}_p + \underline{K}_{pp} \underline{U}_p = \underline{F}_a \quad (3-7)$$

where the forcing function is

$$\underline{F}_a = - \sum_i \bar{U}_{ci} \underline{K}_{ps} \underline{1}_{si}$$

The modes, ϕ_j , and frequencies, ω_j , of the eigenvalue problem

$$\left(-\underline{M}_{pp} + \frac{1}{\omega_j^2} \underline{K}_{pp} \right) \phi_j = \underline{0} \quad (3-8)$$

represent the free vibration behavior of a fixed based system. Given that the modes are orthonormalized such that

$$\underline{\phi}^T \underline{M}_{pp} \underline{\phi} = \underline{I} = \begin{bmatrix} 1 \\ \vdots \end{bmatrix} \quad (3-9)$$

then

$$\underline{\phi}^T \underline{K}_{pp} \underline{\phi} = \underline{\omega}^2 = \begin{bmatrix} \omega_1^2 \\ \vdots \end{bmatrix} \quad (3-9a)$$

where $\underline{\phi}$ is the array of modal columns, ϕ_j , and $\underline{\phi}^T$ represents the transpose of $\underline{\phi}$.

If we let

$$\bar{\underline{U}}_p = \underline{\phi} \bar{\underline{Q}} \quad (3-10)$$

then Equation 3-7 may be transformed by the modal decomposition

$$\underline{\phi}^T \underline{M}_{pp} \underline{\phi} \ddot{\bar{\underline{Q}}} + \underline{\phi}^T \underline{K}_{pp} \underline{\phi} \bar{\underline{Q}} = \underline{\phi}^T \underline{F}_a \quad (3-11)$$

Noting the orthogonality relations given by Equations 3-9, we obtain the decoupled set of equations

$$\ddot{\bar{q}} + \omega_j^2 \bar{q} = - \sum_i \bar{U}_{ci} \Gamma_{ai} \quad (3-12)$$

where the vector of modal participation factors is

$$\Gamma_{ai} = \underline{\phi}^T \underline{K}_{ps} \underline{1}_{si} \quad (3-13)$$

Due to the constraints given by Equation 3-4, we may write

$$\underline{K}_{pp} \underline{1}_{pi} + \underline{K}_{ps} \underline{1}_{si} = \underline{0} \quad (3-14)$$

thus the modal participation factors may also be written as

$$\Gamma_{ai} = - \underline{\phi}^T \underline{K}_{pp} \underline{1}_{pi}$$

Equation 3-12 represents the scalar set of equations

$$\ddot{\bar{q}}_j + \omega_j^2 \bar{q}_j = \sum_i \Gamma_{aij} \bar{U}_{ci} \quad (3-15)$$

where the index j ranges over the N modes given previously (Eq. 3-8) and the scalar modal participation factors are

$$\begin{aligned} \Gamma_{aij} &= \underline{\phi}_j^T \underline{K}_{ps} \underline{1}_{si} = - \underline{\phi}_j^T \underline{K}_{pp} \underline{1}_{pi} \\ &= \sum_{\ell=1}^N \sum_{h=1}^N \phi_{\ell j} \left(\underline{K}_{pp} \right)_{\ell k} \left(\underline{1}_{pi} \right)_h \end{aligned} \quad (3-16)$$

where the indices ℓ and h range over the N degrees of freedom associated with subscript p . Given the solution to each modal Equation 3-15, then the system response is found directly from the modal transformation represented by Equation 3-10.

3.2.3.2 Relative Formulation

Again, for Equation 3-5, let

$$\bar{U}_p = \bar{U}_p + \sum_i \bar{U}_{ci} \bar{1}_{si} \quad (3-17)$$

and

$$\bar{U}_s = \sum_i \bar{U}_{ci} \bar{1}_{si} \quad (3-17a)$$

where $\bar{U}_p$ is the motion of the system relative to the common base support. Now Equation 3-5 can be rewritten

$$\bar{M}_{pp} \ddot{\bar{U}}_p + \bar{K}_{pp} \bar{U}_p = \bar{F}_r \quad (3-18)$$

where the forcing function is

$$\begin{aligned} \bar{F}_r = & - \sum_i \ddot{\bar{U}}_{ci} \bar{M}_{pp} \bar{1}_{pi} \\ & - \sum_i \bar{U}_{ci} (\bar{K}_{pp} \bar{1}_{pi} + \bar{K}_{ps} \bar{1}_{si}) \end{aligned}$$

Due to the rigid body translation constraints given by Equations 3-14, the second term of the forcing function vanishes and we obtain

$$\bar{F}_r = - \sum_i \ddot{\bar{U}}_{ci} \bar{M}_{pp} \bar{1}_{pi}$$

Again, the modes of Equation 3-8 allow the modal decomposition of Equation 3-18 if

$$\bar{U}_p = \underline{\Phi} \underline{q} \quad (3-19)$$

Thus, we obtain

$$\underline{\Phi}^T \bar{M}_{pp} \underline{\Phi} \ddot{\underline{q}} + \underline{\Phi}^T \bar{K}_{pp} \underline{\Phi} \underline{q} = \underline{\Phi}^T \bar{F}_r \quad (3-20)$$

Due to the orthogonality relations given by Equations 3-9, this equation may be written in the decoupled form

$$\ddot{\underline{q}} + \underline{\omega}^2 \underline{q} = - \sum_i \ddot{\bar{U}}_{ci} \underline{\Gamma}_{ri} \quad (3-21)$$

where the vector of modal participation factors is

$$\underline{\Gamma}_{ri} = \underline{\Phi}^T \bar{M}_{pp} \bar{1}_{pi}$$

The orthogonality relations given by Equations 3-9 may also be written as

$$\underline{\Phi}^T \bar{M}_{pp} \underline{\Phi} = \underline{I}$$

and

$$(\underline{\omega}^2)^{-1} \underline{\Phi}^T \bar{K}_{pp} \underline{\Phi} = \underline{I}$$

which means

$$\underline{\Gamma}_{ri} = -(\underline{\omega}^2)^{-1} \underline{\Gamma}_{ai}$$

Each of Equations 3-21 represents the scalar equation

$$\ddot{q}_j + \omega_j^2 q_j = - \sum_i \Gamma_{rij} \ddot{\bar{U}}_{ci} \quad (3-22)$$

where

$$\begin{aligned} \Gamma_{rij} &= \underline{\Phi}_j^T \bar{M}_{pp} \bar{1}_{pi} \\ &= \sum_{\ell=1}^N \phi_{\ell j} (\bar{M}_{pp})_{\ell\ell} (\bar{1}_{pi})_{\ell} \end{aligned}$$

So now,

$$\Gamma_{rij} = - \Gamma_{aij} / \omega_j^2 \quad (3-23)$$

Thus the participation factors of the absolute formulation given by Equation 3-16 may be obtained from this equation without prior knowledge of the array $\bar{K}_{ps}$.

Now, given the solution to each of Equations 3-22, the system response relative to the common base is found from the transformation represented by Equation 3-19, and the total

system response is obtained using the first of Equations 3-17.

3.2.4 MULTIPLE SUPPORT MOTION

The response of systems with differing input motion at the points of supports (or groups of support) requires the specification of three components of motion for each support point with differing motion. In general, specification of an orthogonal set of absolute displacement time histories for each support point with differing motion is required regardless of the basic formulation of the governing equations of motion. If only absolute displacement of the support points is specified, then the solution is sought using a formulation based on an absolute motion reference. If an orthogonal set of absolute acceleration time histories is specified at each support point in addition to absolute displacement time histories, then a solution may be sought using either of two formulations based on a relative motion reference.

If the first relative motion formulation is utilized, then a solution is sought using an arbitrarily selected support point as the base of reference. The alternative relative formulation seeks a solution using a quasi-static displacement pattern of the system as the base of reference.

3.2.4.1 Absolute Formulation

For the governing equations of motion (Eq. 3-5), let the motion of the support points be given by

$$\begin{aligned}\bar{U}_s &= \underline{C} \bar{U}_f = \begin{bmatrix} \underline{C}_x & \underline{C}_y & \underline{C}_z \end{bmatrix} \begin{Bmatrix} \bar{U}_{fx} \\ \bar{U}_{fy} \\ \bar{U}_{fz} \end{Bmatrix} \\ &= \underline{C}_x \bar{U}_{fx} + \underline{C}_y \bar{U}_{fy} + \underline{C}_z \bar{U}_{fz}\end{aligned}\quad (3-24)$$

where $\bar{U}_{fi}$ is a vector of absolute floor displacements in direction i and $\underline{C}_i$ is a connectivity array that indicates which support degree of freedom is attached to a given floor.

Equation 3-24 may be written in expanded notation as

$$\begin{aligned}\bar{U}_s &= \sum_{k=1}^L \underline{C}_{xk} \bar{U}_{f_{kx}} + \sum_{k=1}^L \underline{C}_{yk} \bar{U}_{f_{ky}} \\ &\quad + \sum_{k=1}^L \underline{C}_{zk} \bar{U}_{f_{kz}} \\ &= \sum_i \sum_k \underline{C}_{ik} \bar{U}_{f_{ki}}\end{aligned}\quad (3-25)$$

where $\bar{U}_{f_{ki}}$ is the scalar displacement time history of floor k in direction i and $\underline{C}_{ik}$ is a column vector of the connectivity array $\underline{C}_i$ which has a value of unity for each support degree of freedom attached to floor k and a value of zero otherwise. The index k of the summation indicated in Equation 3-25 ranges over the L floors with specified motion. Since Equation 3-25 must also be valid for the case of all support points attached to the same floor (i.e., common base support), the vectors $\underline{C}_{ik}$ must satisfy the relationship

$$\sum_k \underline{C}_{ik} = \underline{1}_{si} \quad (3-26)$$

Now we rewrite Equation 3-5 using Equation 3-25:

$$\underline{M}_{pp} \ddot{\bar{U}}_p + \underline{K}_{pp} \bar{U}_p = \underline{F}_a \quad (3-27)$$

where the forcing function is

$$\underline{F}_a = - \sum_i \sum_k \underline{K}_{ps} \underline{C}_{ik} \bar{U}_{f_{ki}}$$

The modes of Equation 3-8 may be used to define the coordinate transformation

$$\bar{\underline{U}}_p = \underline{\phi} \underline{q} \quad (3-28)$$

which allows the decomposition of Equation 3-27 in the form

$$\underline{\phi}_{=pp}^T \underline{\ddot{q}} + \underline{\phi}_{=pp}^T \underline{\ddot{q}} + \underline{\phi}_{=pp}^T \underline{\ddot{q}} = \underline{\phi}_{=a}^T \underline{F}_a \quad (3-29)$$

Noting the orthogonality relations given by Equations 3-9, we may write Equation 3-29 in the decoupled form

$$\ddot{\underline{q}} + \underline{\omega}^2 \underline{q} = - \sum_i \sum_k \Gamma_{aik} \bar{U}_{fki} \quad (3-30)$$

where the vector of modal participation factors is defined for both a given floor k and direction i as

$$\Gamma_{aik} = \underline{\phi}_{=ps}^T \underline{C}_{ik} \quad (3-31)$$

Due to Equation 3-26, we note the identity

$$\begin{aligned} \sum_k \Gamma_{aik} &= \underline{\phi}_{=ps}^T \sum_k \underline{C}_{ik} \\ &= \underline{\phi}_{=ps}^T \underline{1}_{si} = \Gamma_{ai} \end{aligned} \quad (3-32)$$

Equations 3-30 may be written in the scalar form

$$\ddot{q}_j + \omega_j^2 \bar{q}_j = - \sum_i \sum_k \Gamma_{aikj} \bar{U}_{fki} \quad (3-33)$$

where

$$\begin{aligned} \Gamma_{aikj} &= \underline{\phi}_j^T \underline{K}_{ps} \underline{C}_{ik} \\ &= \sum_{\ell=1}^N \sum_{h=1}^S \phi_{\ell j} (K_{ps})_{\ell h} (C_{ik})_h \end{aligned}$$

These summations are over the range of N degrees of freedom for index ℓ and S support degrees of freedom for index h .

Given the solution to each modal Equation 3-33, the total system response may be found directly from the modal coordinate transformation given by Equation 3-28.

3.2.4.2 Relative Formulation

Two alternate formulations of the governing equations may be derived using a relative motion reference. The first formulation is denoted the selected floor reference formulation and is essentially the formulation presented by Thaler (1976). The second formulation is denoted the quasi-static displacement reference formulation and is the basic formulation presented by Vashi (1975).

Selected Floor Reference Formulation

For the basic governing equations of motion (Eq. 3-5), let

$$\bar{\underline{U}}_p = \underline{U}_{pr} + \sum_i \bar{U}_{fri} \underline{1}_{pi} \quad (3-34)$$

and

$$\bar{\underline{U}}_s = \underline{U}_{sr} + \sum_i \bar{U}_{fri} \underline{1}_{si} \quad (3-34a)$$

where $\bar{U}_{fri}$ is the absolute displacement of an arbitrarily chosen floor r in direction i and $\underline{U}_{pr}$ and $\underline{U}_{sr}$ are the motions of the system relative to floor r .

Now Equation 3-5 can be rewritten as

$$\underline{M}_{pp} \ddot{\underline{U}}_{pr} + \underline{K}_{pp} \underline{U}_{pr} = \underline{F}_{rr} \quad (3-35)$$

where the forcing function is

$$\begin{aligned} \underline{F}_{rr} &= \sum_i \ddot{\bar{U}}_{fri} \underline{M}_{pp} \underline{1}_{pi} \\ &\quad - \sum_i \bar{U}_{fri} \left(\underline{K}_{pp} \underline{1}_{pi} + \underline{K}_{ps} \underline{1}_{si} \right) - \underline{K}_{ps} \underline{U}_{sr} \end{aligned}$$

If we rewrite Equation 3-34a as

$$\underline{u}_{sr} = \underline{\bar{u}}_s - \sum_i \underline{\bar{u}}_{fri} \underline{1}_{si} \quad (3-36)$$

and let the support degrees of freedom be related to the floor motions by the connectivity transformation given by Equation 3-25 and note Equation 3-26, then Equation 3-36 becomes

$$\underline{u}_{sr} = \sum_i \sum_k \underline{c}_{ik} (\underline{\bar{u}}_{fki} - \underline{\bar{u}}_{fri}) \quad (3-37)$$

or

$$\underline{u}_{sr} = \sum_i \sum_{k \neq r} \underline{c}_{ik} \underline{u}_{fki} \quad (3-38)$$

where $\underline{u}_{fki} = \underline{\bar{u}}_{fki} - \underline{\bar{u}}_{fri}$ is the motion of floor k relative to the motion of floor r in direction i .

Using Equation 3-38 and noting that the second term of $\underline{F}_{rr}$ (see Eq. 3-35) vanishes due to the rigid body translation constraints expressed by Equations 3-14, we obtain the forcing function in the form

$$\begin{aligned} \underline{F}_{rr} &= \sum_i \ddot{\underline{\bar{u}}}_{fri} \underline{M}_{pp} \underline{1}_{pi} \\ &- \sum_i \sum_{k \neq r} \underline{K}_{ps} \underline{c}_{ik} \underline{u}_{fki} \end{aligned} \quad (3-39)$$

Using the modes of Equation 3-8 and the modal transformation $\underline{U}_{pr} = \underline{\phi} \underline{q}$, we obtain the modal decomposition of Equation 3-35 in the form

$$\underline{\phi}^T \underline{M}_{pp} \ddot{\underline{\phi} \underline{q}} + \underline{\phi}^T \underline{K}_{pp} \underline{\phi} \underline{q} = \underline{\phi}^T \underline{F}_{rr} \quad (3-40)$$

which, due to the orthogonality relations given by Equations 3-9, yields the following decoupled set of equations

$$\begin{aligned} \ddot{\underline{q}} + \underline{\omega}^2 \underline{q} &= - \sum_i \ddot{\underline{\bar{u}}}_{fri} \underline{\Gamma}_{ri} \\ &- \sum_i \sum_{k \neq r} \underline{u}_{fki} \underline{\Gamma}_{aki} \end{aligned} \quad (3-41)$$

where the modal participation factors are the same vector forms given previously:

$$\underline{\Gamma}_{ri} = \underline{\phi}^T \underline{M}_{pp} \underline{1}_{pi}$$

$$\underline{\Gamma}_{aik} = \underline{\phi}^T \underline{K}_{ps} \underline{c}_{ik}$$

The scalar form of Equations 3-41 is

$$\begin{aligned} \ddot{q}_j + \omega_j^2 q_j &= - \sum_i \ddot{\bar{u}}_{fri} \Gamma_{rij} \\ &- \sum_i \sum_{k \neq r} u_{fki} \Gamma_{aikj} \end{aligned} \quad (3-42)$$

where Γ_{rij} and Γ_{aikj} have been given previously (see Eqs. 3-22 and 3-33).

Now, given the solution to each modal Equation 3-42, the relative response is obtained from the modal transformation $\underline{U}_{pr} = \underline{\phi} \underline{q}$ and the total response obtained from Equation 3-34.

Quasi-Static Displacement Reference Formulation

Again, for Equation 3-5, let

$$\underline{\bar{u}}_p = \underline{u}_d + \underline{\bar{u}}_{qs} \quad (3-43)$$

where the vector $\underline{\bar{u}}_{qs}$ is defined by

$$\underline{K}_{pp} \underline{\bar{u}}_{qs} + \underline{K}_{ps} \underline{\bar{u}}_s = \underline{0}$$

or

$$\ddot{\underline{U}}_{qs} = - \underline{K}_{pp}^{-1} \underline{K}_{ps} \ddot{\underline{U}}_s \quad (3-44)$$

Since Equation 3-44 represents the static (i.e., $\ddot{\underline{U}}_p = 0$) solution of Equation 3-5 for a specified support displacement pattern, $\ddot{\underline{U}}_s$, the vector $\ddot{\underline{U}}_{qs}$ is denoted the quasi-static (Clough, 1969) displacement of the system. The preface quasi- (or pseudo-; Clough and Penzien, 1975) indicates that the vector $\ddot{\underline{U}}_{qs}$ represents time histories or time-dependent functions. The array $(\underline{K}_{pp}^{-1} \underline{K}_{ps})$ may be viewed as a matrix of influence coefficients, which indicate the system displacements due to an independent unit displacement of each support degree of freedom.

The vector $\underline{U}_d$ in Equation 3-43 is the inertial (or dynamic) response of the system relative to the quasi-static displacement of the system. The use of the quasi-static system displacement as a reference state is a computational artifice that has been used by several authors (Bisplinghoff, 1962) to compute the dynamic stress within unrestrained structures. The quasi-static reference state is simply a convenient choice that allows the form of the governing equations to be simplified as indicated in the following development.

Equation 3-5 may be rewritten as

$$\underline{M}_{pp} \ddot{\underline{U}}_d + \underline{K}_{pp} \underline{U}_d = \underline{F}_{rd} \quad (3-45)$$

where the forcing function is

$$\underline{F}_{rd} = - \underline{M}_{pp} \ddot{\underline{U}}_{qs} - (\underline{K}_{pp} \ddot{\underline{U}}_{qs} + \underline{K}_{ps} \ddot{\underline{U}}_s)$$

Noting both Equation 3-44 and the corresponding second time derivative form

$$\ddot{\underline{U}}_{qs} = - \underline{K}_{pp}^{-1} \underline{K}_{ps} \ddot{\underline{U}}_s$$

we obtain the simplified form of the forcing function as

$$\underline{F}_{rd} = \underline{M}_{pp} \underline{K}_{pp}^{-1} \underline{K}_{ps} \ddot{\underline{U}}_s \quad (3-46)$$

Now, let the support degrees of freedom be related to the floor motions by the second time derivative form of the connectivity transformation given by Equation 3-25:

$$\ddot{\underline{U}}_s = \underline{C} \ddot{\underline{U}}_f = \sum_i \sum_k \underline{C}_{ik} \ddot{\underline{U}}_{fki} \quad (3-47)$$

thus the forcing function becomes

$$\underline{F}_{rd} = \sum_i \sum_k \underline{M}_{pp} \underline{K}_{pp}^{-1} \underline{K}_{ps} \underline{C}_{ik} \ddot{\underline{U}}_{fki} \quad (3-48)$$

The modes of Equation 3-8 may be used to define the modal transformation

$$\underline{U}_d = \underline{\phi} \underline{q}$$

and obtain the modal decomposition of Equation 3-45 in the form

$$\underline{\phi}^T \underline{M}_{pp} \underline{\phi} \ddot{\underline{q}} + \underline{\phi}^T \underline{K}_{pp} \underline{\phi} \underline{q} = \underline{\phi}^T \underline{F}_{rd} \quad (3-49)$$

Noting the orthogonality relations given by Equations 3-9, the decoupled form of Equation 3-49 may be written as

$$\ddot{\underline{q}} + \underline{\omega}^2 \underline{q} = - \sum_i \sum_k \underline{\Gamma}_{rik} \ddot{\underline{U}}_{fki} \quad (3-50)$$

where the participation factors are

$$\underline{\Gamma}_{rik} = - \underline{\phi}^T \underline{M}_{pp} \underline{K}_{pp}^{-1} \underline{K}_{ps} \underline{C}_{ik}$$

The orthogonality relations are as given previously,

$$\underline{\phi}^T \underline{M}_{pp} \underline{\phi} = \underline{I}$$

and

$$(\underline{\omega}^2)^{-1} \underline{\phi}^T = \underline{\phi}^{-1} \underline{K}_{pp}^{-1}$$

Thus,

$$\Gamma_{rik} = - (\underline{\omega}^2)^{-1} \underline{\phi}_j^T \underline{K}_{ps} \underline{c}_{ik}$$

and the inversion of the array $\underline{K}_{pp}$ is avoided. Also,

$$\Gamma_{rik} = - (\underline{\omega}^2)^{-1} \Gamma_{aik}$$

so we can deduce the identity

$$\begin{aligned} \Gamma_{ri} &= \sum_k \Gamma_{rik} = - (\underline{\omega}^2)^{-1} \sum_k \Gamma_{aik} \\ &= - (\underline{\omega}^2)^{-1} \Gamma_{ai} \end{aligned}$$

The scalar form of Equations 3-50 may be written as

$$\ddot{q}_j + \omega_j^2 q_j = - \sum_i \sum_k \Gamma_{rikj} \ddot{u}_{fki} \quad (3-51)$$

where

$$\Gamma_{rikj} = - \Gamma_{aikj} / \omega_j^2$$

and

$$\begin{aligned} \Gamma_{aikj} &= \underline{\phi}_j^T \underline{K}_{ps} \underline{c}_{ik} \\ &= \sum_{\ell=1}^N \sum_{h=1}^S \phi_{\ell j} (K_{ps})_{\ell h} (c_{ik})_h \end{aligned}$$

Given the solution to each modal Equation 3-51, the inertial relative response $\underline{U}_d$ is obtained from the transformation equation $\underline{U}_d = \underline{\phi} \underline{q}$. The quasi-static displacement $\bar{\underline{U}}_{qs}$ is obtained for a sequence of solutions of the linear set of algebraic Equations 3-44 corresponding to the time history $\bar{\underline{U}}_s$ given by Equation 3-25. The total solution is then the sum of the relative inertial and quasi-static response as indicated in Equation 3-43.

3.2.5 EFFECT OF DAMPING ON FORMULATION

The foregoing development of each formulation assumed that the motion of the piping system was undamped. If viscous damping is chosen to represent the mechanism of energy dissipation within the system, then the governing equations of motion are given by Equation 3-1. The damping terms may be rearranged to yield the partitioned form

$$[C^*] \{\dot{\underline{U}}\} = \begin{bmatrix} \underline{C}_{pp}^* & \vdots & \underline{C}_{ps}^* \\ \vdots & \ddots & \vdots \\ \underline{C}_{sp}^* & \vdots & \underline{C}_{ss}^* \end{bmatrix} \begin{Bmatrix} \dot{\underline{U}}_{ps} \\ \vdots \\ \dot{\underline{U}}_s \end{Bmatrix}$$

which is consistent with the notation of Equation 3-2. Since rigid body translation of the system cannot generate internal forces, the damping matrix must also satisfy the rigid body translation constraints

$$\begin{bmatrix} \underline{C}_{pp}^* & \vdots & \underline{C}_{ps}^* \\ \vdots & \ddots & \vdots \\ \underline{C}_{sp}^* & \vdots & \underline{C}_{ss}^* \end{bmatrix} \begin{bmatrix} 1_{px} & \vdots & 1_{py} & \vdots & 1_{pz} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 1_{sx} & \vdots & 1_{sy} & \vdots & 1_{sz} \end{bmatrix} = \begin{bmatrix} 0 & \vdots & 0 & \vdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \vdots & 0 & \vdots & 0 \end{bmatrix} \quad (3-52)$$

in the same manner as required for the stiffness matrix of an unrestrained structure (Eq. 3-4).

Thus, the basic governing equations for each formulation are given by

$$\begin{aligned} \underline{M}_{pp} \ddot{\underline{U}}_p + \left[\underline{C}_{pp}^* \dot{\underline{U}}_p + \underline{C}_{ps}^* \bar{\underline{U}}_s \right] \\ + \underline{K}_{pp} \bar{\underline{U}}_p + \underline{K}_{ps} \bar{\underline{U}}_s = 0 \end{aligned} \quad (3-53)$$

where the additional damping terms added to Equation 3-5 are enclosed within the dashed lines.

To allow the modal decomposition of Equation 3-53 for each formulation, the damping array $\underline{C}_{pp}^*$ must be defined such that the modal transformation yields a diagonal array, or

$$\underline{\Phi}^T \underline{C}_{pp}^* \underline{\Phi} = 2\underline{\omega\xi} = \begin{bmatrix} 2\underline{\omega_j} & \underline{\xi_j} \end{bmatrix} \quad (3-54)$$

where ξ_j is the modal damping factor associated with mode j .

Now, given that the motion of the support points is represented by

$$\bar{U}_s = \underline{C} \bar{U}_f = \sum_i \sum_k \underline{C}_{ij} \bar{U}_{fki}$$

and

$$\dot{\bar{U}}_s = \underline{C} \dot{\bar{U}}_f = \sum_i \sum_k \underline{C}_{ik} \dot{\bar{U}}_{fki} \quad (3-55)$$

where $\bar{U}_{fki}$ and $\dot{\bar{U}}_{fki}$ are the scalar displacement and velocity time histories of floor k in direction i and $\underline{C}_{ik}$ is a column vector of the connectivity array, $\underline{C}_i$, defined by Equation 3-24, we can obtain the corresponding normal mode formulation for each choice of motion reference. The application of Equations 3-52 through 3-55 to the same formulation development presented previously yields the following formulations for each case:

1. Common Base Support Motion

$$(\bar{U}_{fki} = \bar{U}_{ci}, \quad \sum_k \underline{C}_{ik} = \underline{1}_{si})$$

a. Absolute Formulation

$$\begin{aligned} \bar{U}_p &= \underline{\Phi} \bar{q} \\ \ddot{\bar{q}} + 2\underline{\omega\xi} \dot{\bar{q}} + \underline{\omega}^2 \bar{q} &= - \sum_i \underline{\Gamma}_{ai} \bar{U}_{ci} - \sum_i \underline{\Gamma}_{ai}^* \dot{\bar{U}}_{ci} \end{aligned}$$

$$\underline{\Gamma}_{ai} = \underline{\Phi}^T \underline{K}_{ps} \underline{1}_{si}$$

$$\underline{\Gamma}_{ai}^* = \underline{\Phi}^T \underline{C}_{ps}^* \underline{1}_{si}$$

(3-56)

b. Relative Formulation

$$\underline{U}_p = \underline{\Phi} \underline{q}$$

$$\ddot{\underline{q}} + 2\underline{\omega\xi} \dot{\underline{q}} + \underline{\omega}^2 \underline{q} = - \sum \underline{\Gamma}_{ri} \ddot{\bar{U}}_{ci}$$

$$\underline{\Gamma}_{ri} = \underline{\Phi}^T \underline{M}_{pp} \underline{1}_{pi}$$

(3-57)

2. Multiple Support Motion

a. Absolute Formulation

$$\begin{aligned} \bar{U}_p &= \underline{\Phi} \bar{q} \\ \ddot{\bar{q}} + 2\underline{\omega\xi} \dot{\bar{q}} + \underline{\omega}^2 \bar{q} &= - \sum_i \sum_k \underline{\Gamma}_{aik} \bar{U}_{fki} \\ &\quad - \sum_i \sum_k \underline{\Gamma}_{aik}^* \dot{\bar{U}}_{fki} \end{aligned}$$

$$\underline{\Gamma}_{aik} = \underline{\Phi}^T \underline{K}_{ps} \underline{C}_{ik}$$

$$\underline{\Gamma}_{aik}^* = \underline{\Phi}^T \underline{C}_{ps}^* \underline{C}_{ik}$$

(3-58)

b. Relative Formulation

(1) Selected Floor Reference Formulation

(r ≡ reference floor,
 $U_{fki} = \bar{U}_{fki} - \bar{U}_{fri}$)

$$\underline{U}_{pr} = \underline{\phi} \underline{q}$$

$$\begin{aligned} \ddot{\underline{q}} + 2\underline{\omega}\underline{\xi}\dot{\underline{q}} + \underline{\omega}^2\underline{q} &= - \sum_i \underline{\Gamma}_{ri} \ddot{\underline{U}}_{fri} \\ &- \sum_i \sum_{k \neq r} \underline{\Gamma}_{aik} \underline{U}_{fki} \\ &- \sum_i \sum_{k \neq r} \underline{\Gamma}_{aik}^* \dot{\underline{U}}_{fki} \end{aligned}$$

$$\underline{\Gamma}_{ri} = \underline{\phi}^T \underline{M}_{pp} \underline{1}_{pi}$$

$$\underline{\Gamma}_{aik} = \underline{\phi}^T \underline{K}_{ps} \underline{c}_{ik}$$

$$\underline{\Gamma}_{aik}^* = \underline{\phi}^T \underline{C}_{ps}^* \underline{c}_{ik}$$

(3-59)

(2) Quasi-Static Displacement Reference Formulation

$$(\bar{\underline{U}}_p = \underline{U}_d + \bar{\underline{U}}_{qs}, \bar{\underline{U}}_{qs}, \underline{U}_{qs} = \underline{K}_{pp}^{-1} \underline{K}_{ps} \bar{\underline{U}}_s)$$

$$\underline{U}_d = \underline{\phi} \underline{q}$$

$$\begin{aligned} \ddot{\underline{q}} + 2\underline{\omega}\underline{\xi}\dot{\underline{q}} + \underline{\omega}^2\underline{q} &= - \sum_i \sum_k \ddot{\underline{U}}_{fki} \underline{\Gamma}_{rik} \\ &- \sum_i \sum_k \dot{\underline{U}}_{fki} \underline{\Gamma}_{rik}^* \end{aligned}$$

$$\underline{\Gamma}_{rik} = - \underline{\phi}^T \underline{M}_{pp} \underline{K}_{pp}^{-1} \underline{K}_{ps} \underline{c}_{ik}$$

$$= - (\underline{\omega}^2)^{-1} \underline{\phi}^T \underline{K}_{ps} \underline{c}_{ik}$$

$$= - (\underline{\omega}^2)^{-1} \underline{\Gamma}_{aik}$$

$$\underline{\Gamma}_{rik}^* = - \underline{\phi}^T \left(\underline{C}_{pp}^* \underline{K}_{pp}^{-1} \underline{K}_{ps} - \underline{C}_{ps}^* \right) \underline{c}_{ik}$$

(3-60)

The formulations of Equations 3-59 and 3-60 are, respectively, the formulations obtained by Thaler and Vashi. All formulations (except the common base relative formulation, Eq. 3-57) appear to require the definition of an array, $\underline{C}_{ps}^*$, of damping coefficients which couple the system motion to the support motion. $\underline{C}_{ps}^*$ is specified by the modal transformation (see Eq. 3-54). The array $\underline{C}_{ps}^*$ cannot be specified independently due to the rigid body constraints given by Equations 3-52 and 3-4. This may be demonstrated by writing the rigid body constraints in the form

$$\underline{C}_{pp}^* \underline{1}_{pi} + \underline{C}_{ps}^* \underline{1}_{si} = 0 \quad (3-61)$$

$$\underline{K}_{pp} \underline{1}_{pi} + \underline{K}_{ps} \underline{1}_{si} = 0 \quad (3-61a)$$

Equation 3-61a may be written also as

$$\underline{1}_{pi} = - \underline{K}_{pp}^{-1} \underline{K}_{ps} \underline{1}_{si}$$

and substituted into Equation 3-61 to yield

$$\left(- \underline{C}_{pp}^* \underline{K}_{pp}^{-1} \underline{K}_{ps} + \underline{C}_{ps}^* \right) \underline{1}_{si} = 0$$

which can, in general, be satisfied only by the identity

$$\underline{C}_{ps}^* = \underline{C}_{pp}^* \underline{K}_{pp}^{-1} \underline{K}_{ps} \quad (3-62)$$

Thus, the vector of participation factors, $\underline{\Gamma}_{rik}^*$, given by the last of Equations 3-60, must be a null vector, or

$$\underline{\Gamma}_{rik}^* = \underline{0}$$

Thus, the quasi-static displacement reference multisupport formulation (Eq. 3-60) may be written as

$$\ddot{\underline{q}} + 2\underline{\omega}\underline{\xi}\dot{\underline{q}} + \underline{\omega}^2 \underline{q} = - \sum_i \sum_k \ddot{\underline{U}}_{fki} \underline{\Gamma}_{rik} \quad (3-63)$$

If, in addition, we note the identity

$$\underline{K}_{pp}^{-1} = \underline{\Phi} \left(\underline{\omega}^2 \right)^{-1} \underline{\Phi}^T$$

then, the vector of participation factors, $\underline{\Gamma}_{aik}$, given in Equation 3-59, becomes

$$\begin{aligned} \underline{\Gamma}_{aik} &= \underline{\Phi}^T \underline{C}_{pp} \underline{K}_{pp}^{-1} \underline{K}_{ps} \underline{1}_{ps} \\ &= \underline{\Phi}^T \underline{C}_{pp} \underline{\Phi} \left(\underline{\omega}^2 \right)^{-1} \underline{\Phi}^T \underline{K}_{ps} \underline{1}_{ps} \\ &= 2\underline{\omega}\underline{\xi} \left(\underline{\omega}^2 \right)^{-1} \underline{\Gamma}_{ai} \end{aligned}$$

and in a similar manner the vector of participation factors, $\underline{\Gamma}_{aik}^*$, given by the last of Equations 3-59, becomes

$$\underline{\Gamma}_{aik}^* = 2\underline{\omega}\underline{\xi} \left(\underline{\omega}^2 \right)^{-1} \underline{\Gamma}_{aik}$$

Thus, the modal equations of the common base absolute formulation (Eq. 3-56) may be written as

$$\ddot{\underline{q}} + 2\underline{\omega}\underline{\xi}\dot{\underline{q}} + \underline{\omega}^2 \underline{q} = - \sum_i \left(\bar{\underline{U}}_{ci} \underline{\Gamma}_{ai} + 2\dot{\underline{U}}_{ci} \underline{\xi}\underline{\omega}^{-1} \underline{\Gamma}_{ai} \right) \quad (3-64)$$

and, then the modal equations of the selected floor reference multisupport formulation (Eq. 3-59) may be written as

$$\begin{aligned} \ddot{\underline{q}} + 2\underline{\omega}\underline{\xi}\dot{\underline{q}} + \underline{\omega}^2 \underline{q} &= - \sum_i \underline{\Gamma}_{ri} \ddot{\underline{U}}_{fri} \\ &- \sum_i \sum_{k \neq r} \left(\underline{U}_{fki} \underline{\Gamma}_{aik} + 2\dot{\underline{U}}_{fki} \underline{\xi}\underline{\omega}^{-1} \underline{\Gamma}_{aik} \right) \end{aligned} \quad (3-65)$$

Both Thailer and Vashi maintain the notation of the coupled damping array, $\underline{C}_{ps}^*$, in their presentations. Vashi concludes (correctly) for the quasi-static reference form via a circuitous argument (incorrect) that the forcing terms associated with the absolute velocity of each floor, $\dot{\underline{U}}_{fki}$, should be neglected. However, as shown by Equation 3-63, they in fact must vanish due to the definition of modal damping. Thailer does not discuss an alternate form of Equations 3-59, such as Equation 3-65, but suggests that the forcing terms associated with the relative velocity and displacement are small compared to the absolute acceleration forcing term.

3.2.6 CONCLUSIONS

The systematic presentation of the alternate formulations for the governing equations of systems with multisupport input resulted in the following conclusions:

- a. The formulations proposed by Vashi and Thailer represent alternate choices of relative motion reference for the governing equations. The essential difference between the two formulations is that the system response due to relative floor motion is implicitly contained in the response solution for the Thailer method, while

the Vashi method requires the superposition of the stresses due to the response solution with the stresses due to the quasi-static reference state (time varying).

- b. The Thailer method requires specification of three orthogonal components of absolute acceleration (time history) for the selected reference floor and six additional components (three velocity and three displacement) of motion (time histories) relative to the selected reference point for each additional structure floor or support location. The Vashi method requires specification of three orthogonal components of absolute acceleration (time history) for each structure floor or support location. In addition, the Vashi method requires specification of the three components of absolute displacement (or the displacement relative to a chosen floor such as the base) for each point of support in order to determine the quasi-static state of stress.
- c. Both formulations require the specification of a connectivity array (transformation matrix) that indicates which support translation degree of freedom is attached to a given floor or point of known structure motion.
- d. Both formulations require the definition of a set of modal participation factors for each floor that are distinct from the conventional participation factors associated with the common base relative formulation. While not apparent from the presentations of Vashi and Thailer, the modal participation factors of each alternative multisupport formulation differ only

by a constant factor. Thus, both methods require the same computational effort to compute the corresponding modal participation factors.

- e. Both Vashi and Thailer maintain the notation of coupling terms in the mass and damping arrays. For piping analysis, the use of mass coupling resulting from a mass definition "consistent" with the finite element representation of the system is not warranted. The use of a lumped mass representation is recommended. The coupling coefficients of the damping array are not arbitrary but instead are specified by the definition of modal damping. The consideration of these specified coupling terms leads directly to simplifications in the governing equations for both formulations regardless of any arguments presented by either Vashi or Thailer.

3.3 RESPONSE SPECTRA ANALYSIS OF SYSTEMS WITH MULTIPLE SUPPORT INPUT MOTION

3.3.1 INTRODUCTION

Response spectrum techniques have been the dominant analysis tools utilized for seismic analysis of piping systems in the nuclear industry. To avoid the computational problems associated with multiple support motion, the common practice has been to assume that all support points of a piping system are attached to the same common base that has a motion defined by the envelope (umbrella) spectra of all floor spectra. Given this assumption, the relative normal mode formulation for common base motion is utilized as the basis for response spectra analysis. The necessary first step of the analysis procedure is to obtain the frequencies, ω_j , and mode shape vectors, ϕ_j , of the

eigenvalue problem defined by the mass and stiffness properties of a discrete mathematical model (fixed base) for the piping system being analyzed. This eigenvalue problem may be represented in symbolic form as

$$\frac{1}{\omega_j^2} K_{pp} \phi_j = M_{pp} \phi_j \quad (3-66)$$

where K_{pp} is the stiffness matrix and M_{pp} is the mass matrix for the mathematical model. A convenient choice for orthonormalization of each mode shape is given by the scalar norm

$$\phi_j^T M_{pp} \phi_j = 1 \quad (3-67)$$

Now, given that the motion of the system relative to the motion of the common base is denoted by $\underline{U}_p$, the modal transformation defined by

$$\underline{U}_p = \sum_i \phi_j q_j \quad (3-68)$$

allows the decomposition of the governing equations to normal form. The resulting scalar governing equation for each modal response component, q_j , is given by (see Eq. 3-57)

$$\begin{aligned} \ddot{q}_j + 2\xi_j \omega_j \dot{q}_j + \omega_j^2 q_j \\ = -\Gamma_{rxj} \ddot{U}_{cx} - \Gamma_{ryj} \ddot{U}_{cy} - \Gamma_{rzj} \ddot{U}_{cz} \end{aligned} \quad (3-69)$$

where ξ_j is the modal damping factor; $\ddot{U}_{cx}$, $\ddot{U}_{cy}$, $\ddot{U}_{cz}$ are the absolute accelerations of the common base in the global x, y, and z directions; and Γ_{rij} are the modal participation factors for the jth mode in the ith direction, defined by the equation

$$\Gamma_{rij} = \sum_{\ell=1}^N \phi_{\ell j} (M_{pp})_{\ell\ell} (1_{pi})_{\ell} \quad (3-70)$$

where the summation index, ℓ , is over the range of N degrees of freedom of the fixed base mathematical model, $\phi_{\ell j}$ is the ℓ th component of the jth modal vector, $(M_{pp})_{\ell\ell}$ is the inertia of the ℓ th degree of freedom, and $(1_{pi})_{\ell}$ is a vector component that has a value of unity for a translational degree of freedom corresponding to the ith global direction and a value of zero otherwise.

The response spectrum for the ith direction associated with the modal damping factor, ξ_j , is defined by the peak response of a single-degree-of-freedom oscillator with frequency ω_n , or the set of peak solutions obtained from the equations

$$\ddot{R}_i + 2\xi_j \omega_n \dot{R}_i + \omega_n^2 R_i = -\ddot{U}_{ci} \quad (3-71)$$

over the spectrum range of oscillator frequency.

If we let $\omega_D = \omega_n \sqrt{1 - \xi_1^2}$, then the solutions to Equation 3-71 may be expressed as

$$\begin{aligned} R_i^n(t) = -\frac{1}{\omega_D} \int_0^t \ddot{U}_{ci}(\tau) \\ \exp\left[-\omega_n \xi_j(t - \tau)\right] \sin \omega_D(t - \tau) d\tau \end{aligned} \quad (3-72)$$

and the peak responses given by

$$D_i^n \equiv \max_t |R_i^n(t)| \quad (3-73)$$

The resulting response spectrum is plotted as a continuous function using D_i^n as ordinate and ω_n as abscissa. More commonly, the ordinate value of peak response is scaled by ω_n^2 , and the resulting plot is presented using $\omega_n^2 D_i^n$ as ordinate versus ω_n as abscissa.

In a conventional time-history analysis, the total modal response would be given by

$$q_j(t) = \sum_i \Gamma_{rij} R_i^j(t) \quad (3-74)$$

and the system response given by

$$u_p(t) = \sum_j \phi_j q_j(t) \quad (3-75)$$

However, for a spectral analysis the phase relationship between the spatial and modal response components, R_i^j , is lost due to the definition of peak response, D_i^j , given by Equation 3-73. Thus, the peak value of modal response for the j th mode with input in the i th direction is given by the product of the response spectrum ordinate, D_i^j , associated with modal frequency ω_j , and the absolute value of the modal participation factor, Γ_{rij} , or

$$q_{ji} = |\Gamma_{rij}| D_i^j \quad (3-76)$$

The total (relative) response of the system is then bounded by the inequality

$$u_p \leq \sum_i \sum_j |\phi_j| q_{ji} \quad (3-77)$$

where the summations indicated are over the three global directions ($i = x, y, z$) and over the $j = 1, N^*$ modes where $N^* (< N)$ is the number of modes considered in the analysis.

The equality in Equation 3-75 would be satisfied only if all peak modal responses occurred at the same instant in time. Since the modal responses will, in general, be uncorrelated time functions, the total system response is often estimated by the Square Root-Sum-of-Squares (SRSS) method, or

$$\begin{aligned} (u_{pij})_\ell &= \phi_{j\ell} q_{ji} \\ (u_{pi})_\ell &= \sqrt{\sum_j (u_{pij})_\ell^2} \quad (\text{modal SRSS}) \\ (u_p)_\ell &= \sqrt{\sum_i (u_{pi})_\ell^2} \quad (\text{spatial SRSS}) \end{aligned} \quad (3-78)$$

where $(u_p)_\ell$ is the total system response of the ℓ th degree of freedom. The SRSS method of modal response combination is accepted by R.G. 1.92 for systems with modes that are not closely spaced. Two consecutive modes are defined as closely spaced if their frequencies differ from each other by 10% or less of the lower frequency. For systems that have closely spaced modes (as do most piping systems), three alternate methods are given in R.G. 1.92 as acceptable. The simplest method to apply is denoted as the Ten Percent Method, which may be expressed as

$$\begin{aligned} (u_{pij})_\ell &= \phi_j q_{ij} \\ (u_{pi})_\ell &= \sqrt{\sum_j (u_{pij})_\ell^2 + \sum_{m=1}^{N^*} \sum_{n=1}^{N^*} C_{mn} (u_{pim})_\ell (u_{pin})_\ell} \\ (u_p)_\ell &= \sqrt{\sum_i (u_{pi})_\ell^2} \end{aligned} \quad (3-79)$$

where $C_{mn} = 1$ if and only if mode m is closely coupled to mode n ($C_{mm} = C_{nn} = 0$ by definition).

It should be noted that although Equations 3-78 and 3-79 are expressed in terms of system relative displacement response, the SRSS and Ten Percent methods are also valid for modal response combination of any total response or stress component (i.e., velocity, acceleration, moment, shear, etc.). While it is generally accepted in the nuclear industry that piping system modes with frequencies in excess of 33 Hz may be ignored for seismic analysis, the question of whether the "static" effect of peak floor accelerations is properly accounted

for in a spectral analysis using the first N^* modes ($f^* \leq 33$ Hz) is often encountered. Some authors have suggested that a static (i.e., constant acceleration) analysis be included by assuming that the static system deformations and stresses be considered as an additional mode in the SRSS combination of modal response (Munson et al., 1974). Duff and Biswas (1978) presented a more rational method of accounting for this static effect by defining residual terms, which are added to the SRSS or Ten Percent response estimates.

The following sections summarize the application of response spectrum methods for systems with differing multiple support motion. Both the selected floor reference formulation (Thaler, 1976) and the quasi-static displacement reference formulation (Vashi, 1975) are considered, and the additional computational effort required for spectral analysis is indicated for both formulations. Also discussed are the requirements for computer program implementation of the two multiple support spectral methods into existing software based on the relative formulation for common base motion.

3.3.2 SELECTED FLOOR REFERENCE SPECTRAL METHOD

Given that the motion of the system relative to a selected reference floor is denoted by $\underline{U}_r$, the modal transformation defined by

$$\underline{U}_r = \sum_i \phi_j q_{Tj} \quad (3-80)$$

allows the decomposition of the governing equations to normal form if ϕ_j are the fixed base mode shape vectors that satisfy Equations 3-66 and 3-67. The resulting scalar governing equation for each modal response component, q_{Tj} , is given by (see Eq. 3-65)

$$\begin{aligned} \ddot{q}_{Tj} + 2\xi_j \omega_j \dot{q}_{Tj} + \omega_j^2 q_{Tj} = & \\ & - \Gamma_{rxj} \ddot{U}_{frx} - \Gamma_{ryj} \ddot{U}_{fry} - \Gamma_{rzj} \ddot{U}_{frz} \\ & - \sum_{k \neq r} \left\{ \Gamma_{axkj} \left(U_{fkx} + 2\xi_j \dot{U}_{fkx}/\omega_j \right) \right. \\ & \quad + \Gamma_{aykj} \left(U_{fky} + 2\xi_j \dot{U}_{fky}/\omega_j \right) \\ & \quad \left. + \Gamma_{azkj} \left(U_{fkz} + 2\xi_j \dot{U}_{fkz}/\omega_j \right) \right\} \end{aligned} \quad (3-81)$$

where

$$U_{fki} = \bar{U}_{fki} - \bar{U}_{frk}, \quad \dot{U}_{fk} = \dot{\bar{U}}_{fki} - \dot{\bar{U}}_{fri}$$

$$\Gamma_{aikj} = \sum_{\ell=1}^N \sum_{h=1}^S \phi_{\ell j} \left(K_{ps} \right)_{\ell n} \left(C_{ik} \right)_n$$

$$\Gamma_{rij} = \sum_{\ell=1}^N \phi_{\ell j} \left(M_{pp} \right)_{\ell \ell} \left(1_{pi} \right)_\ell$$

For small damping ($\xi < 0.05$), the forcing terms associated with $\dot{U}_{fki}$ in Equation 3-81 have only a second order effect on the amplitude of modal response, q_{Tj} , when compared to the response component due to U_{fki} . Thus, the velocity terms of the effective forcing function given by the right-hand side of Equation 3-81 may be neglected. With this simplification, the selected floor reference spectral method requires the definition of "traditional" acceleration spectra associated with the absolute acceleration components of the selected reference point, $\ddot{U}_{fri}$, and the definition of "new" relative displacement spectra associated with the relative displacement of each support with respect to the selected reference point, U_{fri} .

The acceleration response spectrum is defined by Equations 3-71 through 3-73 with the term $\ddot{U}_{fri}$ replacing the term $\ddot{U}_{ci}$. The relative displacement spectrum for the i th direction associated with the modal damping factor, ξ_j , is defined by the peak response of a single-degree-of-freedom oscillator with frequency ω_n , or the set of peak solutions obtained from the equations

$$\ddot{H}_i + 2\xi_j\omega_n\dot{H}_i + \omega_n^2 H_i = -U_{fki} \quad (3-82)$$

over the spectrum range of oscillator frequency, ω_n . The solutions to Equation 3-82 may be expressed as

$$H_{ki}^n(t) = \frac{-1}{\omega_D} \int_0^t \left\{ U_{fki}(\tau) \exp \left[-\omega_n \xi_j (t - \tau) \right] \sin \omega_D (t - \tau) \right\} d\tau \quad (3-83)$$

and the peak responses given by

$$E_{ki}^n = \max_t \left| H_{ki}^n(t) \right| \quad (3-84)$$

The resulting response spectrum is plotted as a continuous function using E_{ki}^n as ordinate and ω_n as abscissa. To make the relative displacement spectra consistent with the usual presentation of acceleration spectra, it is suggested that the ordinate value of peak response be scaled by ω_n^2 , and the resulting plot presented using $\omega_n^2 E_{ki}^n$ as ordinate versus ω_n as abscissa.

In a conventional time-history analysis, the total modal response would be given by

$$q_{Tj}(t) = \sum_i \Gamma_{rij} R_i^j(t) + \sum_i \sum_{k \neq r} \Gamma_{aikj} H_{ki}^j(t) \quad (3-85)$$

But as noted previously, the phase relationship between the spatial and modal components is

lost due to the definition of peak response given by Equations 3-73 and 3-84. Thus, the peak value of modal response for the j th mode with input in the i th direction is given by

$$q_{Tji} = \left| \Gamma_{rij} \right| D_i^j + \sum_{k \neq r} \left| \Gamma_{aikj} \right| E_{ki}^j \quad (3-86)$$

The spectral method outlined above requires, in general, the definition of three acceleration response spectra and $3 \times (L - 1)$ relative displacement spectra, where L is the number of structure floors or levels. It should be noted that the notation used in Equations 3-84 and 3-86 differs from that used by Thaler (1976). The notation change is necessary to clarify that the absolute values of the participation factors, Γ_{rij} and Γ_{aikj} , are utilized in the computation of the peak modal response. Given that the total model response components are expressed as

$$\left(U_{rij} \right)_\ell = \phi_{j\ell} q_{Tji} \quad (3-87)$$

then the total system response may be estimated using the SRSS method or the Ten Percent method given by Equations 3-78 and 3-79, respectively.

3.3.3 QUASI-STATIC DISPLACEMENT SPECTRAL METHOD

Now, given that the motion of the system relative to a quasi-static reference state is denoted by $\underline{U}_d$, the modal transformation defined by

$$\underline{U}_d = \sum_i \phi_j q_{vj} \quad (3-88)$$

also allows the decomposition of the governing equations to normal form if ϕ_j are the fixed base mode shape vectors that are defined by Equations 3-66 and 3-67. The resulting scalar governing equation for each modal response component, q_{vj} , is given by (see Eq. 3-63):

$$\ddot{q}_{vj} + 2\xi_j \omega_j \dot{q}_{vj} + \omega_j^2 q_{vj} = - \sum_k \left(\Gamma_{rxkj} \ddot{U}_{f kx} + \Gamma_{rykj} \ddot{U}_{f ky} + \Gamma_{rzkj} \ddot{U}_{f kz} \right) \quad (3-89)$$

where

$$\Gamma_{rikj} = - \Gamma_{aikj} / \omega_j^2$$

Thus, the quasi-static displacement reference spectral methods require only "traditional" acceleration spectra for each floor or support level with input $\ddot{U}_{fki}$ defined. The acceleration response spectrum for each floor, k, is defined by Equations 3-71 through 3-73 with the term $\ddot{U}_{fki}$ replacing the term $\ddot{U}_{ci}$. The corresponding peak response is denoted by

$$D_{ki}^n = \max_t \left| R_{ki}^n(t) \right| \quad (3-90)$$

and the resulting response spectrum plotted using $\omega_n^2 D_{ki}^n$ as ordinate versus ω_n as abscissa.

In a conventional time-history analysis, the modal response would be given by

$$q_{vj} = \sum_i \sum_k \Gamma_{rikj} R_{ki}^j(t) \quad (3-91)$$

However, since the spectral method uses only peak response values without consideration of phase relationships, the peak value of modal response for the jth mode with input in the ith direction is given by

$$q_{vji} = \sum_k \Gamma_{rikj} D_{ki}^j \quad (3-92)$$

The spectral method outlined above requires definition of $3 \times L$ acceleration response spectra. Now, given that the total modal response components are expressed as

$$u_{dij} = \phi_{jl} q_{vji} \quad (3-93)$$

then the SRSS or Ten Percent methods, as expressed by Equations 3-78 and 3-79, may be utilized to obtain an estimate of the total system response. To determine the total stress state, the deformation pattern imposed by the quasi-static reference state must be determined and the associated stresses evaluated. Referring to the definition of the quasi-static reference,

$$\underline{K}_{pp} \underline{\bar{U}}_{gs} = - \underline{K}_{ps} \underline{\bar{U}}_s \quad (3-94)$$

we note that this relationship represents static equilibrium of the system under the imposed support displacements. The quasi-static reference state is not determined by spectral methods, but is determined instead by any number of available techniques for static solution. For example, a unit displacement analysis could be conducted to determine the influence coefficients for any desired stress due to a unit support displacement. Knowledge of the peak structure (i.e., support) motion would allow computation of the quasi-static stresses at a given location that would then be added (absolutely) to the SRSS or Ten Percent response estimate.

3.3.4 COMPUTER PROGRAM IMPLEMENTATION

To implement either (or both) of the multisupport spectral methods into existing computer software, which is based on common base spectral analysis procedures, the major programming effort is associated with the computation of the additional participation factors required for multisupport spectral analysis. Reference to the modal participation factors in Equation 3-81 and 3-89 or the general development of Section 3.2 will indicate that both methods require the computation of the set of participation factors given by

$$\Gamma_{aikj} = \phi_{jl}^T \underline{K}_{ps} \underline{C}_{ik} \quad (3-95)$$

where ϕ_j is the j th mode shape vector, $\underline{K}_{ps}$ is the coupled stiffness matrix and $\underline{C}_{ik}$ is the connectivity vector for floor k in the i th direction. In general, the physical rearrangement of the governing equations into the partitioned form, used for the formulation development in Section 3.2, is unnecessary for computer implementation. The computational artifice of adding a boundary spring with an extremely large arbitrary stiffness value to each support degree of freedom allows the computation of the fixed base modes and associated frequencies without computer code modification and also allows the coupled stiffness terms desired for computation of the modal participation factors to be identified. This procedure is illustrated by the following example. Given the governing equations expressed as

$$\underline{M} \ddot{\underline{X}} + \underline{K} \underline{X} = \underline{R} \quad (3-96)$$

in which $\underline{K} \underline{X}$ is shown in expanded form as

$$\begin{bmatrix} K_{11} & & K_{1s} & & \\ & \ddots & & & \\ & & (K_{ss} + T_r) & & \\ & & & K_{sn} & \\ & & & & K_{nn} \end{bmatrix} \begin{Bmatrix} \bar{X}_s \\ \bar{X}_s \\ \bar{X}_s \\ \bar{X}_s \\ \bar{X}_n \end{Bmatrix} \quad (3-97)$$

symmetric

where s identifies a support degree of freedom and T_r is an arbitrary large stiffness value, say $T_r = 10^{15} \times \max |K_{ij}|$. Given that the first NF modes and frequencies of Equation 3-96 ($\underline{R} = \underline{0}$) are extracted by inverse iteration, the j th mode will have the form

$$\phi_j = \begin{Bmatrix} \phi_{1j} \\ \vdots \\ \phi_{sj} \approx 0 \\ \vdots \\ \phi_{nj} \end{Bmatrix} \quad (3-98)$$

Next, we search the diagonal of the stiffness matrix and identify the terms associated with the support degrees of freedom by noting if $K_{ii} > T_r$. Thus, we may form the column of the stiffness array associated with the support degree of freedom from the column above the diagonal term and the row to the right of the diagonal term, or

$$\underline{K}_s = \begin{Bmatrix} K_{1s} \\ \vdots \\ (K_{ss} + T_r) \\ \vdots \\ K_{sn} \end{Bmatrix} \quad (3-99)$$

Since the components of the modal column associated with the support degree of freedom are approximately equal to zero, we note that

$$\phi_j^T \underline{K}_s = \phi_j^T \underline{K}_{ps} \quad (3-100)$$

Additional input of the connectivity information that identifies which support nodes are attached to each structure floor or level is required for both multisupport spectral methods. A useful way to implement this data is to form an index array, $IC(NS, K) = I$, which allows both the connectivity and global direction information to be transferred. This is accomplished by letting $I = 1, 2, 3$, or 0 for each NS support degree of freedom and K floor. If $I > 0$, then NS is an input degree of freedom

associated with the K floor in the I global direction, while if $I = 0$, input degree of freedom NS is not associated with floor K. Now, given the array $IC(NS,K)$, the array of mode shapes $PI(J,N)$, and the effective coupled stiffness array $KS(NS,N)$ formed by the column vectors $\underline{K}_s$, we may compute the array of participation factors, $PF(J,I)$, associated with each floor K using the simple FORTRAN algorithm:

```

Step 1.  Choose K
Step 2.  Initialize PF (I,J) = 0.0
Step 3.  DO 1 J = 1,NF
          DO 1 L = 1,NS
          DO 1 N = 1,NEQ
            I = IC (L,K)
            IF (I.EQ.0) GO TO 1
            PF (I,J) = PF (I,J)
                      + PI (J,N)*KS (L,N)
          1 CONTINUE

```

where

NF $\equiv$ Number of modes
 NS $\equiv$ Number of support degrees of freedom
 NEQ $\equiv$ Number of equations

Existing spectral analysis programs are usually set up to compute the combined response (SRSS) for each independent spatial direction as a separate problem. For multisupport input problems, $3 \times K$ independent (i.e., one for each floor and spatial direction) problems are required; however, all modal component data must be stored and a postprocessing program written to combine the response components associated with floor input in an absolute manner (see Eq. 3-86 and 3-92) and then combine the modal response components according to the SRSS or associated method. For large order problems, considerable storage will be required to keep and process all response parameters. To alleviate this problem, the modal response components may be combined

in the following manner. Using the quasi-static reference or Vashi method as an example, let

$$q_{vjik} = \Gamma_{rikj} D_{ki}^j \quad (3-101)$$

and

$$\left(u_{dijk}\right)_\ell = \phi_{j\ell} q_{vjik} \quad (3-102)$$

Now, use Equation 3-102 to obtain an SRSS combination of modal components

$$\left(u_{dik}\right)_\ell = \sqrt{\sum_j \left(u_{dijk}\right)_\ell^2} \quad (3-103)$$

and then take an absolute sum of the SRSS results given by this equation, that is,

$$\left(u_{di}\right)_\ell = \sum_k \left(u_{dik}\right)_\ell$$

and then compute the spatial SRSS bound as

$$\left(u_d\right)_\ell = \sqrt{\sum_i \left(u_{di}\right)_\ell^2} \quad (3-104)$$

This alternate method of combining the modal response components is considerably simpler than the rigorous method and allows the existing program procedure of computing the combined response for each spectrum input (i.e., each floor and direction) as a complete independent problem. Besides, the absolute sum of SRSS components is always greater than the SRSS of the absolute sum of components due to Hölder's Inequality. Thus, the alternate method provides a conservative response bound.

3.3.5 CONCLUSIONS

The presentation of the spectral analysis procedures for each multisupport input formulation resulted in the following conclusions:

- a. For systems with small damping ($\xi < 0.05$), the forcing term in the selected floor reference formulation (Thailer) associated with velocity may be neglected. With this simplification, the Thailer method requires the definition of "traditional" acceleration spectra associated with the absolute acceleration components (time history) of the selected reference point and "new" relative displacement spectra associated with the relative displacement components (time history) associated with each additional point of system support.
- b. For the Thailer method, different response bounds are obtained for each choice of floor reference. While Thailer provides some guidance for reference floor selection, the selection of the reference point which gives a minimum response bound cannot, in general, be determined a priori. Thus, additional spectral analyses using alternate selected reference points will usually be required.
- c. The quasi-static displacement reference formulation (Vashi) requires the definition of only "traditional" acceleration spectra for each support level or floor. Since these spectra are required for equipment specifications, no additional effort in spectra definition is required. However, additional static analyses of the piping system are required for the Vashi method in order to determine the stresses due to the relative support displacements (quasi-static system displacements).
- d. The implementation of either the Thailer (selected reference) or Vashi (quasi-static reference) spectral

analysis methods into existing computer software, which is based on common base spectral analysis procedures, can be accomplished with a minimum of programming effort. Both methods require the computation of additional modal participation factors that involve the coupled stiffness terms, mode shapes, and floor connectivity information. The physical arrangement of the governing equations into the partitioned form used for the analytical development is not necessary for computer implementation. The computational artifice of adding a boundary spring with an extremely large arbitrary stiffness value to each support degree of freedom allows the computation of the fixed base modes and associated frequencies without computer code modification and also allows the coupled stiffness terms desired for computation of the modal participation factors to be identified. Additional input of the connectivity information that identifies which support nodes are attached to each structure floor or level is required for both spectral methods. Given this connectivity data, the computation for the appropriate modal participation factor may be accomplished by minor changes in computer coding.

- e. Both methods require an increase in storage requirements for modal stress components and considerable data manipulation if a rigorous combination of modal response components is desired. However, an upper bound estimate of the total response can be obtained with a modest increase in storage requirements and minimal data

handling if a simplified method of modal combination is used.

3.4 COMPUTER PROGRAM

The above techniques for performing seismic analyses of piping systems with multisupport input are incorporated into the SAP IV computer program (Bathe et al., 1974), which is a public domain program.

SAP IV is a general purpose finite element structural analysis program for static and dynamic response of linear three-dimensional systems. The program includes a library of one-, two-, and three-dimensional finite elements to model any complex three-dimensional structural system. The static analysis options include dead load, live load, thermal load, and support settlement. The program includes two different methods for calculating mode shapes and frequencies. Three different options are available to perform a dynamic analysis: (1) response time-history analysis using mode shapes and frequencies, (2) response spectrum analysis, and (3) response time-history analysis using step-by-step integration technique.

The standard SAP IV program can solve piping problems with multisupport input in the time domain if the input is available in the form of time histories at each support point. However, time-history analysis is cumbersome and expensive. For design of piping systems in nuclear power plants, the input is usually available only in the form of response spectra at the support points. Therefore, modifications to SAP IV are undertaken to enable the program to solve piping problems with multisupport input in the form of response spectra using the techniques by Vashi and Thailer described above.

3.4.1 DESCRIPTION OF PROGRAM CHANGES

To incorporate the multisupport input

capability, some existing subroutines in SAP IV code had to be modified and some new subroutines added.

The existing subroutines involving changes are SAPIV, MODES, SECNTD, STRSR, and SPECTR. The changes in these subroutines are identified in Appendix A of AA (1982).

The newly introduced subroutines are DYNspe, RESPMS, SPTID, EMIDMS, CPSTMx, and SPCTMS. The listings of these new subroutines are also included in Appendix A of AA (1982).

The relationship among these modified and newly introduced subroutines is depicted in Figure 3-1. Basically, the code first goes to subroutine SOLEIG to extract the designated number of frequencies and mode shapes. It then calls DYNspe to determine if the problem is specified as a conventional common-base response spectrum problem (NDYN.EQ.3) or as the new multisupport input problem (NDYN.EQ.5). In the former case, the code reverts to the existing part of the code to handle the traditional response spectrum problem. However, in the latter case, the new section of the code is executed.

The code first goes to the control subroutine RESPMS to call SPTID to establish a one-dimensional array, IA, which identifies all the support points and the floors they are on. The code then checks the value of MS, which is a numerical flag specifying the method of analysis to be used. If the value is 0 (zero), what the user has opted is the traditional common-base method despite the fact that the value of NDYN has been specified as 5 instead of 3. In this case the code calls subroutine EMIDR and SPECTR the way they are called when NDYN is specified as 3 to get the problem solved in the traditional manner.

However, if the value of MS is 1 or 2, which indicates that Thailer's method or Vashi's

AA150

FIGURE 3-1. RELATIONSHIP AMONG SUBROUTINES CONNECTED WITH THE RESPONSE SPECTRUM ANALYSIS OPTION

method, respectively, has been specified, the part of the control program RESPMS for multisupport problems is executed. First, RESPMS calls EMIDMS to establish a two-dimensional array, IC, which relates each support degree of freedom (DOF) to a floor number and a global coordinate direction. Subroutine CPSTMX is then called to retrieve the global stiffness matrix and save all the off-diagonal stiffness terms related to all the input degrees of freedom. The resulting data block, in the form of a two-dimensional array, is called RKSA. Subroutine SPCTMS then performs matrix multiplication between the mode shapes and RKSA to establish the modal participation factors, PX. PX is then multiplied by the input spectral ordinates to obtain the modified spectral ordinates which are in turn multiplied by the mode shapes to obtain the modal displacements. Finally, the modal displacements are printed by PRINTD and the corresponding element stresses are calculated and printed in STRESR.

3.4.2 SPECIAL MODELING PROBLEM ASSOCIATED WITH MULTISUPPORT INPUT ANALYSES

To implement the multisupport input analysis method, not only the "input points" but also the "input DOF's" at the individual input points must be identified. For coding convenience, node points are identified as input points by use of large stiffnesses (of the order of 10^{15} lb/in., for example) provided by boundary spring elements. Since each linear degree of freedom in a direction at an input point represents the existence of an earthquake input in that direction, an input point must be properly constrained to transmit the correct number of earthquake components through that point.

This necessitates the use of some truss elements in the modeling as illustrated in Figure 3-2. Shown in Figure 3-2a is a segment

of a pipe that is supported by a roller in the x-direction at Point A, and supported by a suspender in the y-direction at Point B. This means that the pipe can physically receive earthquake motion in the horizontal (x) direction only at A and in the vertical (y) direction only at B. In this case it is necessary to model the roller support and the suspender as shown in Figure 3-2b using a truss element and a boundary spring connected in series at each point. Points C and D then constitute two "input points" that the code recognizes by virtue of the fact that the stiffness at each of these two points is greater than the very large common spring constant assigned to the boundary springs in the input.

It is important to note that Points C and D must be properly constrained as shown in Figure 3-2b so that the correct component of the input motion gets through each of the two points. It would not be correct to attach the boundary springs directly to the pipe as shown in Figure 3-2c because this arrangement introduces additional constraints in two directions both at A and B (y and z at A; x and z at B), whereas no such constraints are present in the physical problem.

At a point that is completely fixed, however, the three boundary springs may be attached directly to the pipe without employing any truss element, as can be seen in Figures 3-5 and 3-8. This is because all three components of the input motion are input at such a point so that no additional constraint need be imposed at such a point. And with the presence of the three very stiff springs, the point is effectively fixed as called for in the original problem.

Henceforth, these truss elements will be called "support elements." In the actual modeling, the user must strive to give each of the support elements the correct stiffness of the

(a) A supported segment of a pipe

(b) Correct modeling of supports

(c) Incorrect modeling of supports

FIGURE 3-2. MODELING OF SUPPORTS FOR MULTISUPPORT INPUT ANALYSIS

actual support it represents. Where the user chooses to consider a support infinitely rigid and unyielding, he can assign a large stiffness to the support element. Care must be taken, however, that this stiffness does not exceed in order of magnitude the stiffness assigned to the boundary springs to identify support input points. For, otherwise, a point like A in Figure 3-2b may be mistaken by the code as another input point. To avoid such an outcome, one can simply assign a stiffness of 10^{13} or 10^{14} lb/in. to the support elements when he assigns a value of 10^{15} lb/in., for example, to the boundary springs that identify

input points.

3.4.3 CHANGES TO SAP IV USER'S GUIDE

The changes to SAP IV User's Guide consist of the introduction of new control flags in the master control card (Card Group II) and the creation of a new section, D, in the Dynamic Analysis part to parallel the old Section C for response spectrum analysis using the traditional common-base approach. The new section will follow Section C on Page VII.23 of the User's Guide. This section is applicable when the parameter NDYN has a value of 5.

CARD GROUP II MASTER CONTROL CARD

Format: Old: (8I5) New: (8I5, 5X, 2I5)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
1-20	(No Change)	Same as in SAP IV User's Guide.
21-25	NDYN	Addition of the value 5 for multisupport input problems. As before, a negative sign may be affixed to the number 5 to signify a restart problem.
46-50	ITFLG	A flag to signify whether all the solutions to be obtained are to be written on TAPE12 for postprocessing. EQ.0 Do not write TAPE12. EQ.1 Write TAPE12.
51-55	NRSP	Total number of individual response spectrum analysis problems included in this run. If, for example, a pipe system is supported on three floors and each floor has x, y, and z input, then $NRSP = 3 \times 3 = 9$. This number is for dynamic allocation only. Therefore, declaring a value larger than real does not matter, but declaring a smaller value will result in a code error. When $ITFLG = 0$, this field may be ignored.

D. Multisupport Response Spectrum Analysis (NDYN.EQ.5)

This option combines all (NF) mode shapes and frequencies computed during the eigenvalue solution (Sec. VII.A) to calculate R.M.S. stresses/deflections due to each individual input displacement (or acceleration) spectrum. The proper modal, spatial, and interfloor combination of these individual solutions to get the overall solution is left up to the user. A

suitable postprocessor may be written to read all the solutions on TAPE12 to effect the desired combination to obtain the final solution.

Any number of individual response spectrum problems (representing the x, y, and/or z component at various floor levels) may be stacked up in any order in one run. However, for the convenience of the postprocessor to read TAPE12, which contains the solutions in the order the corresponding problems are specified in the input, the user may want to stack up the problems in one particular order - the x components of all the floors, followed by the y components and then the z components, for example.

1. Control Card (2I5, E10.2)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
1-5	NNPI	Total number of support points with input.
6-10	NFLR	Total number of floor levels with different inputs.
11-20	TOL	This is a stiffness value which is to be used to identify a node point as a support point with input. Since a support point is to be always connected to a boundary spring or springs which have a common, large spring constant, a node point may be identified as a support point if its stiffness value is greater than this spring constant. Therefore, for convenience, TOL may be given the same value assigned to the boundary springs in C.G.IV.7C.

2. Floor Assignment (5X, 2I5)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
6-10	L	Node point number of a support point.
11-15	K	Floor number the support point is on.

This group of cards may be arranged in any order. The total number of cards must be equal to NNPI, of course.

In the following card groups, C.G.3 through C.G.7 represent a response spectrum problem involving a particular component at a particular floor level. Any number of such card groups may be stacked up in any order as has been mentioned. What signals the program of another oncoming spectrum problem is the asterisk in Column 1 of C.G.3. Therefore, any card not containing an asterisk in Column 1 (including a blank card) may be used to terminate the run.

3. Problem Heading Card (A1, 12A6, 17)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
1	ISTR	Asterisk.
2-73	HED1	Heading information for the response spectrum problem.
74-80	MS	Identification flag for the three analysis methods at the user's option. EQ.1 Thailer's Method. EQ.2 Vashi's Method. EQ.0 Conventional common-base method. The same result can be obtained by using the NDYN = 3 option.

4. Control Card (3F10.0, 2I5)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
1-10	DIR(1)	NE.0 if the input is in the x direction.
11-20	DIR(2)	NE.0 if the input is in the y direction.
21-30	DIR(3)	NE.0 if the input is in the z direction. It is convenient to put a 1 (one) in the appropriate field.
31-35	IND	Input spectrum type. EQ.0 Displacement vs. period. EQ.1 Acceleration vs. period.
36-40	NFL	The floor number this input is associated with.

5. Spectrum Heading Card (12A6)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
1-72	HED	Heading information for the input spectrum.

6. Control Card (I5, F10.0)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
1-5	NPTS	Number of points defining the spectrum curve.
6-15	SFTR	Scaling factor to be used to adjust the ordinates (displacements or accelerations) in the spectrum table. EQ.1.0 No adjustment

7. Spectrum Data (2F10.0)

<u>Columns</u>	<u>Variable</u>	<u>Entry</u>
1-10	T	Period.
11-20	S	Value of displacement (IND.EQ.0) or acceleration (IND.EQ.1). It is very important that these cards be arranged in ascending values of the period.

To terminate the input deck as defined above, merely add a blank card, as has been mentioned.

In the case of a run that is performed in two parts, the input deck setup is as follows:

Deck 1: ("Start Deck") All the card groups prepared as usual up to and including C.G.VIIA. Add a blank card to terminate the first deck.

If all the solutions are to be written on TAPE12, specify the proper value of ITFLG and NRSP in C.G.II in the second deck as described below rather than in the first deck.

Deck 2: ("Restart Deck")

C.G.I Heading card.

C.G.II Master control card. Specify the value of NDYN as -5. Define the values of ITFLG and NRSP if TAPE12 is to be written.

These two cards are then followed by Card Groups D.1 through D.7 prepared in the usual fashion as described above.

3.4.4 EXAMPLE PROBLEMS

3.4.4.1 Example Problem No. 1

For the first example problem for the new multisupport input option, the pipe assemblage shown in Figure 3-3a has been selected from the User's Manual for the PIPDYN computer program (CSC). The assemblage is fixed at Point 1 and supported vertically (constrained in the y-direction) at Points 7, 10, and 14. All the pipes are assumed to be made of carbon steel, with an outer diameter of 24 in. and a wall thickness of 0.687 in. from Point 1 to Point 12, and an outer diameter of 12.75 in. and a wall thickness of 0.406 in. from Point 8 to Point 14. The Young's Modulus is taken as 29.0×10^6 psi and the Poisson's ratio as 0.3. There is a concentrated mass of $18.116 \text{ lb-sec}^2/\text{in.}$ at Point 2.

The problem is first solved as a traditional common-base response spectrum problem with the horizontal (x) input specified as shown in Figure 3-4a and the vertical (y) input specified as two-thirds of it. There is no input in the z-direction. The eight lowest frequencies computed by the code are given in Table 3-1,

and the moments M_z for end 1 of all the members are listed in Table 3-2 together with the corresponding values found by PIPDYN. The comparison is seen to be excellent.

Next, the problem is solved as a multi-support input problem, with different inputs specified at Points 1, 7, 10, and 14. For this purpose, the spectrum shown in Figure 3-4a is arbitrarily modified into what is shown in Figure 3-4b. The hump introduced on the left is centered around the lowest frequency and is used for the support at Point 7. The center hump and the remaining one on the right are centered around the next two lowest frequencies and are used for the supports at 10 and 14, respectively. Since the inputs at Supports 7, 10, and 14 consist of only the vertical y-component, the input spectra actually used in the run are taken as two-thirds of what are shown in Figure 3-4b. The input at Point 1 is taken as the same as that used in the common-base solution.

The model used in this multisupport solution is shown in Figure 3-3b. It is noted that only two boundary springs are introduced at

(a) Original problem

(b) Model for multisupport input analysis

FIGURE 3-3. EXAMPLE PROBLEM NO. 1 FOR MULTISUPPORT INPUT ANALYSIS

(a) Input spectrum (x-component)
for common-base solution

(b) Input spectrum (x-component) for
multisupport input solution

FIGURE 3-4. INPUT SPECTRA FOR EXAMPLE PROBLEM NO. 1

TABLE 3-1. FIRST EIGHT NATURAL FREQUENCIES
(Example No. 1)

Natural Mode Number	Frequency, Hz
1	1 454
2	4 410
3	7 997
4	10 34
5	11 85
6	17 59
7	20 69
8	28 86

TABLE 3-2. MOMENT M_z AT END I OF ALL PIPE ELEMENTS FOR COMMON BASE INPUT (Example No. 1)

Element Number	Moment M_z^* at End I, k-in	
	SAP IV	PIPDYN
1	376 9	377 0
2	30 67	30 68
3	152 9	152 9
4	100 6	100 6
5	83 27	83 27
6	46 17	46 19
7	1 081	1 082
8	21 59	21 81
9	7 052	7 038
10	7 537	7 571
11	160 3	160 4
12	78 07	78 09
13	26 08	25 80

* z is a local coordinate

Point 1, because the input motions are to be applied only in x and y directions at this point, and that Point 1 is constrained in the z direction. The "input points" in this problem are 1, 17, 19, and 21. Although not shown because of the desire to keep the figure simple, it is understood that Points 17, 19, and 21 are all constrained in the x and z directions

Vashi's method is used in this run so that the value of MS is specified as 2 The run

consists of a stack of five individual response spectrum inputs because five input components are involved with the four input points. The entire input data are listed in Appendix B of AA (1982). As can be seen, the solution has been obtained in two parts: the initial run to extract the natural modes and frequencies of the piping system and the second (restart) run to perform the response spectrum analysis. The second run utilized all the 15 mode shapes computed in the first run.

The output from the second run is listed in Appendix C of AA (1982). Table 3-3 gives the values of the five sets of moments obtained from this run for End I of three selected elements, which consist of two tangents and one bend. The subscripts x , y , and z for the moments in this table are referred to the local coordinates and not the global coordinates indicated in Figure 3-3. It is to be noted that these are obtained by the code by combining the moments calculated for the 15 modes specified in the run using the SRSS method. The overall results obtained by further combining these five sets of moments by the SRSS method are also given in the table. In this simple example problem, the final SRSS combination is done by hand. In a more complex problem, the user may choose to write a small postprocessor of his own to effect this combination. A sample postprocessor of this type is provided in the next example, Example 2

3.4.4.2 Example Problem No. 2

The pipe network shown in Figure 3-5 is similar to the one which is used as a static example in the User's Manual for the ADLPIPE computer program (Little, 1971). The ADLPIPE example was modified by introducing support stiffnesses in lieu of external forces at Node Points 3, 4, and 8. The pipe leading from Point 12 to Point 13 represents the primary

FIGURE 3-5. EXAMPLE PROBLEM NO. 2 FOR MULTISUPPORT INPUT ANALYSIS

TABLE 3-3. SOME CALCULATED MOMENTS IN THREE SELECTED PIPE ELEMENTS FOR MULTISUPPORT INPUT (Example No. 1)

Spectral Analysis Case		Mx (torsion), k-in	My, k-in	Mz*, k-in
Input Point	Component			
Moments in Element No 1, End I				
1	x	28 0	141 4	246 2
1	y	13 3	68 0	153 5
17	y	35 8	175 5	29 1
19	y	13 9	67 1	39 9
21	y	13 4	58 1	4 9
SRSS Combination		51 1	251 6	294 3
Moments in Element No 4, End I				
1	x	8 3	49 3	65 5
1	y	6 6	19 6	40 8
17	y	5 9	42 9	6 0
19	y	4 6	16 3	10 6
21	y	4 6	14 0	1 3
SRSS Combination		13 8	71 5	78 2
Moments in Element No 11, End I				
1	x	28 0	43 8	104 4
1	y	13 3	16 2	65 0
17	y	35 8	90 0	9 0
19	y	13 9	33 4	16 8
21	y	13 4	29 1	1 9
SRSS Combination		51 1	110 7	124 5

* x, y, and z are local coordinates

system, while that going from Point 5 to Point 8 represents the secondary system. The short segment from 3 to 4, having a different cross-sectional property, simulates a valve. The pipe used in the primary system is assumed to have an outer diameter 10-3/4 in. and a wall thickness of 1.125 in., while that used in the secondary system is assumed to have an outer diameter of 8-3/8 in. and a wall thickness of 0.906 in. Assuming carbon steel, the Young's Modulus is taken as 28.3×10^6 psi and the Poisson's ratio as 0.3. All the support stiffnesses not assumed to be infinite are indicated in the figure. Nodes 8, 12, and 13 are fixed in the original example. To provide input at

these nodes, boundary springs are introduced as shown in Figure 3-5.

The problem is first solved as a multisupport input problem with input Point 13 assumed to be on the first or ground floor, input Points 12, 14, and 24 as being on the second floor, and Points 8, 15, and 22 as being on the third floor. All three floors are assumed to have input in all three coordinate directions, and all the input spectra used in the run are plotted in Figure 3-6. These spectra are obtained from AA (1979d).

Vashi's method is used in this run also. The run consists of a jobstream of nine individual response spectrum problems, because there are nine input components associated with the three floors. The entire input, divided as before into two parts for the first and the second (restart) runs, is given in AA (1982, App. B). The output for the second run is also given in AA (App. C-2). Since the output is quite voluminous, only selected pages are shown.

For this and the next example problem, a small demonstration program, named SRSS, has been written to postprocess the results (see App. D, AA, 1982). The program assumes that there are three input components to each floor, so it is not applicable to the results of Example 1. The program reads TAPE12 created by SAP IV to get all the element forces and moments, and then combine them in two different ways. One is to find the SRSS among the floors for each component (x, y, or z), then find the SRSS through the modes, and then finally find the SRSS of the three components (x, y, and z). The other is to find the absolute sum among the floors, and then find the SRSS through the modes, and the SRSS of the three components as before. Naturally, the second combination gives the larger values. The output from postprocessing the results of

(a) Floor 1 design spectrum for horizontal motion

(b) Floor 1 design spectrum for vertical motion

(c) Floor 2 design spectrum for horizontal motion

(d) Floor 2 design spectrum for vertical motion

(e) Floor 3 design spectrum for horizontal motion

(f) Floor 3 design spectrum for vertical motion

FIGURE 3-6. INPUT SPECTRA FOR EXAMPLE PROBLEM NO. 2

this example using this program is given in AA (1982, App. C-3).

For comparison, this example is also solved as a common-base problem, using the third floor spectra as the overall envelope spectra. The resulting three solutions are then combined by hand using the SRSS method. Table 3-4 shows the comparison among the three processed results for four selected pipe elements. Again, the subscripts for the moments in this table are referred to the local coordinates. The user may write his own postprocessor program to combine the modal responses in any way he desires.

TABLE 3-4. SOME CALCULATED MOMENTS IN FOUR SELECTED PIPE ELEMENTS (Example No. 2)

Moments in Element No 3, End I			
Spectral Analysis Case	Mx (torsion), k-in	My, k-in	Mz [†] , k-in
Common-base (MS = 0) (Floor 3 input)	118 6	49 4	157 3
Vashi (MS = 2)			
SRSS-SRSS-SRSS*	100 2	30 5	136 5
ABS-SRSS-SRSS	136 2	42 7	180 6
Moments in Element No 8, End I			
Common-base (MS = 0) (Floor 3 input)	92 2	240 3	117 4
Vashi (MS = 2)			
SRSS-SRSS-SRSS	84 2	220 5	106 5
ABS-SRSS-SRSS	112 5	295 0	141 4
Moments in Element No 10, End I			
Common-base (MS = 0) (Floor 3 input)	258 4	62 2	60 3
Vashi (MS = 2)			
SRSS-SRSS-SRSS	229 3	56 0	48 5
ABS-SRSS-SRSS	297 1	74 1	65 3
Moments in Element No 12, End I			
Common-base (MS = 0) (Floor 3 input)	46 2	61 6	330 0
Vashi (MS = 2)			
SRSS-SRSS-SRSS	41 7	43 6	301 4
ABS-SRSS-SRSS	56 2	56 5	401 5

*Floors-Modes-x,y,z

[†]x, y, and z are local coordinates

3.4.4.3 Example Problem No. 3

The pipe system shown in Figure 3-7 has been obtained from Chapter 2 above. The system is assumed to rest on three floors with three input components at each floor. The two extreme ends are considered to be fixed, and the system is supported at eight intermediate points by sixteen snubbers and three constant pipe supports. The pipe is assumed to have a constant cross section from end to end with an outer diameter of 24 in. and a wall thickness of 1/2 in. The material of the pipe is assumed to be stainless steel (316.SS) with a Young's modulus of 23.0×10^6 psi and a Poisson's ratio of 0.3.

Figure 3-8 shows the finite element model used in the analysis. Since a constant pipe support provides a constant reaction over a wide range of deflection, it may be represented by two orthogonal truss elements with an effective axial stiffness equal to W_t , the dead weight carried by the support hanger, divided by h, the hanger length. These truss elements are labeled "p" in the figure, and some of them are installed parallel to other unlabeled truss elements that represent snubbers.

The problem is first solved as a multisupport input problem. The input points on the first floor are 1, 5, and 7; those on the second floor are 12, 14, 18, 20, 22, 28, 30, 34, 36, 38, 44, 46, 48, 52, and 54; and those of the third floor are 60, 62, and 67. Except for 1 and 67, all the input points have to be appropriately constrained to offer freedom only in the directions of input.

The problem is solved by use of both the Vashi and the Thailer Methods. In the Vashi solution, all the input spectra are taken the same as those used in Example Problem No. 2. In the Thailer solution, the third floor is selected as the reference and hence is given the same input as in the Vashi solution.

FIGURE 3-8. FINITE ELEMENT MODEL FOR EXAMPLE PROBLEM NO. 3

However, according to the theory described in Section 3.3, the remaining two floors have to be given the relative displacement spectra that are plotted in Figure 3-9. The entire inputs for the two runs are listed in AA (1982, App B). The output for both Thailer's method and Vashi's method is also provided in AA (App. C). As in the previous examples, only

selected pages from the output are shown because of the large volume of the output. Since all three floors have three components for inputs, postprocessor SRSS is applicable in this problem also. The results from the postprocessor for the two solutions mentioned above are also included in AA (App. C)

(a) Horizontal relative displacement design spectrum for Floor 1 using Floor 3 as reference

(b) Vertical relative displacement design spectrum for Floor 1 using Floor 3 as reference

(c) Horizontal relative displacement design spectrum for Floor 2 using Floor 3 as reference

(d) Vertical relative displacement design spectrum for Floor 2 using Floor 3 as reference

FIGURE 3-9. RELATIVE DISPLACEMENT SPECTRA FOR FLOORS 1 AND 2 IN EXAMPLE PROBLEM NO. 3

For comparison, this example is also solved as a common-base problem by regarding the third floor spectra as the envelope spectra and using them as the input. The resulting three solutions for the three input components are hand-combined using the SRSS method for comparison.

Tables 3-5 and 3-6 show the comparison among the various solutions mentioned above for the moments in two selected pipe elements and the axial forces in two selected snubbers.

TABLE 3-5. SOME CALCULATED MOMENTS IN TWO SELECTED PIPE ELEMENTS (Example No. 3)

Moments in Element No 15, End J			
Spectral Analysis Case	Mx (torsion), k-in	My, k-in	Mz [†] , k-in
Common-base (MS = 0) -SRSS-SRSS	26 0	670 7	427 6
Vashi (MS = 2) SRSS-SRSS-SRSS*	20 6	551 5	306 3
ABS-SRSS-SRSS	27 1	612 5	401 1
Thaler (MS = 1, F1 3 ref) SRSS-SRSS-SRSS	42 5	708 2	466 0
ABS-SRSS-SRSS	57 3	865 7	548 6
Moments in Element No 28, End J			
Common-base (MS = 0) -SRSS-SRSS	245 6	161 7	718 4
Vashi (MS = 2) SRSS-SRSS-SRSS	212 2	134 2	490 1
ABS-SRSS-SRSS	230 3	175 0	687 9
Thaler (MS = 1, F1 3 ref) SRSS-SRSS-SRSS	253 4	291 7	826 2
ABS-SRSS-SRSS	292 3	390 9	1058 1

*Floors-Modes-x,y,z

[†]x, y, and z are local coordinates

TABLE 3-6. AXIAL FORCE IN TWO SELECTED SNUBBERS (Example No. 3)

Spectral Analysis Case	Force in Snubber 8, kips	Force in Snubber 9, kips
Common-base (MS = 0) -SRSS-SRSS	26 0	27 2
Vashi (MS = 2) SRSS-SRSS-SRSS	22 5	23 0
ABS-SRSS-SRSS	24 3	24 8
Thaler (MS = 1, F1 3 ref) SRSS-SRSS-SRSS	26 3	27 6
ABS-SRSS-SRSS	30 0	32 4

CHAPTER 4

DEVELOPMENT OF FLOOR SPECTRA FROM INPUT RESPONSE SPECTRA

4.1 INTRODUCTION

Appendix A of the Code of Federal Regulations, 10 CFR 100, requires that either suitable dynamic analyses or suitable qualification tests be used to ensure that the required safety functions of all Category I structures, components, and systems of nuclear power plants are maintained during and after the occurrence of the vibratory ground motions resulting from the SSE. The same requirement exists for ensuring that the operational functions of all Category II structures, components, and systems are maintained during the occurrence of the vibratory ground motions associated with the OBE.

Since it is not feasible to subject large structures to seismic qualification tests (AA, 1977), the satisfactory performance of nuclear power plant structures in the SSE and OBE ground motion environments is verifiable only by dynamic analyses. The same constraint is true for verifying the performance of large equipment and piping systems, such as the reactor and the reactor coolant system. The performance of less massive equipment items is frequently verified by seismic qualification tests; nevertheless, the test environment is derived from the response of the equipment support points, which again is determined from dynamic analysis of the supporting structure. Thus, the seismic design and verification of all nuclear power plant piping and equipment are based, either partially or completely, on the results of dynamic analyses. It is therefore essential to evaluate the reliability of currently used methods of dynamic analysis.

Dynamic seismic analyses of nuclear power plant structures, components, and systems are

based on several mathematical models. Requirements and guidelines for formulating mathematical models are provided in the RDT Standards (RDT-F9-2T, Sec. 6.2) and in the Standard Review Plan (SRP) (Secs. 3.7.2 and 3.7.3). Both should be consulted for additional background (ORNL, 1974; USNRC, 1975b). The primary model treats the mass, stiffness, and damping characteristics of the supporting structure and, if necessary, the supporting soil or rock (see AA, 1976a). The usual procedure is to "decouple" lightweight items from the structure model, but to include (i.e., couple) heavy equipment and piping such as the reactor and the reactor coolant system. Secondary mathematical models are then used to predict the response of both the coupled and decoupled piping and equipment, except for those cases that are verified by testing.

Rules for decoupling are usually based on the relative masses of the supporting structure and the supported subsystem. Once an acceptable level of error is chosen, the usual simplified procedure is to assume resonance conditions and then to determine whether the design mass ratio would justify decoupling. Since this approach will lead to the inclusion of subsystems in the coupled model based on their mass ratio only – even if these subsystems are not resonant with the supporting system – this simplified approach is usually conservative. A more appropriate decoupling criterion includes the frequency ratio of the subsystem to the mass of the supporting system. For example, the SRP provides the following guidelines for decoupling, where R_m is the ratio of the total mass of the subsystem to the mass of the supporting system, and R_f is the ratio of the

fundamental frequency of the supported subsystem to the dominant frequency of the support-structure motion:

- a. If $R_m < 0.01$, decoupling can be done for any R_f .
- b. If $0.01 \leq R_m \leq 0.1$, decoupling can be done if $R_f \leq 0.8$ or if $R_f \geq 1.25$.
- c. If $R_m > 0.1$, an approximate model of the subsystem should be included in the system model.

An indication of the error that results through decoupling is illustrated in Figure 4-1. The impact of the frequency ratio restriction of 0.8

to 1.25 (Item b above) on the resulting error in eigenvalues can be noted from Figure 4-1. Another important consideration is the impact of equipment location on system frequencies, shown in Figure 4-2. In this figure, ω_N represents the system frequency with the equipment mass included, and ω_1, ω_2 , etc., represent the indicated system natural frequencies without the mass of the equipment included.

Subsystems may also be decoupled because of diverse damping characteristics that would otherwise result in a single multidegree-of-freedom model. Usually, the equipment subsystems and the structure system have different

FIGURE 4-1. ERROR IN EIGENVALUES AS A FUNCTION OF FREQUENCY RATIO FOR A MASS RATIO OF 0.01 (Pickle, 1972)

FIGURE 4-2. EFFECT OF EQUIPMENT LOCATION ON SYSTEM FREQUENCIES (Hadjian, 1973)

damping characteristics. This requires a model of a coupled system with a nonproportional damping matrix, or the devising of composite damping matrices. The difference in the damping characteristics becomes immaterial if decoupling is adopted, since the structural system and the supported subsystems can be analyzed separately. Like the concept of mass ratio, the concept of damping ratios heavily influences decoupling decisions.

Although not required, the general practice is to provide two dynamic seismic analyses of nuclear power structures. The first is a modal analysis using free-field SSE and OBE

criteria spectra for input. This is usually carried out at an early stage to furnish information for the design of the structure. The second dynamic seismic analysis uses ground-motion time histories as input to generate response time histories for the various floors and for piping and equipment support points. The input time histories are selected so that response spectra generated from them will envelop the input free-field criteria spectra. The mathematical model in this analysis will include approximate models of large coupled equipment and piping items. The response time histories generated from this model for the

support points of the piping and equipment are then input to more detailed mathematical models of the piping and equipment subsystems. Also, floor response spectra are generated from the floor time histories obtained from the structure-system model and are used for the design of lightweight piping and equipment. Such lightweight items are usually decoupled and are excluded from the structure-system model.

Because of the time and expense required to perform seismic dynamic time-history analyses of large structural systems, approximate methods have been developed for obtaining equipment amplification factors directly from the input free-field criteria spectra for use in the design of decoupled piping and equipment. These methods, referred to as approximate methods, require less effort. Such methods are acceptable if the results are conservative even though the procedures are approximate. Evaluation of these methods to determine their degree of conservatism is the primary objective of this study.

It is customary to use only one time-history component of free-field input motion in each of three orthogonal directions (two horizontal and one vertical) when performing seismic dynamic time-history analyses of large structure-system mathematical models. Since the mathematical model is a multidegree-of-freedom system and is subjected to a random input, it is known that different time histories may cause significant variation in the response, even though each time history may closely match the free-field criterion spectrum. Therefore, the free-field criteria spectra may have an established confidence level, but floor response spectra computed from a single time-history analysis will have an undefined level of confidence. Because of the conservative assumptions made in defining the input, it is generally

assumed that the results from a single time-history input are conservative; but the degree of conservatism is unknown.

Floor response spectra having a prescribed confidence level are needed for reliable and economical design of piping and equipment. Procedures for improving the confidence level of present analyses procedures, as well as the development of a complete probabilistic random vibration method of generating floor design response spectra, are presented here.

4.2 METHODS FOR DERIVING SEISMIC INPUT MOTION FOR PIPING AND EQUIPMENT

4.2.1 TIME-HISTORY METHOD

The time-history method is probably the most widely used procedure for obtaining seismic response spectra for the design of piping and equipment. In this method, a synthetic time history is first generated whose response spectrum envelops the free-field criterion spectrum (see Fig. 4-3). Methods and criteria for the generation of spectra-consistent simulated earthquake ground motions have been investigated by Lin (1977), Vanmarcke and Gasparini (1977), Rizzo et al. (1975), Chen (1975), and Ts'ai (1972).

The synthetic time history is then input to a mathematical model of the nuclear power plant structure. The piping and equipment may be included (i.e., coupled) in the structure model, or it may be decoupled. Floor time histories are computed using the structure model, from which floor response spectra can be generated. Either the floor response time histories or the floor response spectra can be used for the design of the piping and equipment. Figure 4-4 provides an example of a typical finite element two-dimensional elastic mathematical model of nuclear power plant structures that includes the supporting soil and rock. (The

FIGURE 4-3. COMPARISON OF HORIZONTAL FREE-FIELD INPUT TIME-HISTORY SPECTRUM WITH COMPUTED FLOOR RESPONSE SPECTRUM AT NODE 450 FOR MODEL IN FIGURE 4-4

FIGURE 4-4. FINITE ELEMENT MODEL OF NUCLEAR POWER PLANT USED TO COMPUTE FLOOR RESPONSE SPECTRUM IN FIGURE 4-3

aboveground portions of the structures are represented by discrete mass models.) The floor response spectrum for Nodal Point 450 in this model, resulting from the synthetic free-field input time history, is compared with the input time-history spectrum and the free-field criterion spectrum in Figure 4-3.

The main advantage of this method is that once an adequate model has been selected to represent the piping/equipment/structure system, the procedure results in an "exact" solution of the governing equations of motion for a particular set of system parameters and excitation characteristics. The word "exact" is used to indicate that no simplifying assumptions have been introduced in deriving the analytical relationships. However, this method suffers from the following limitations and disadvantages:

- a. It is a costly and time-consuming procedure.
- b. Since it is not possible to generate a synthetic free-field time history whose spectrum will exactly match the smoothed criterion spectrum, the resulting free-field response spectrum for the synthetic time history is required to lie above the criterion spectrum at all points. This condition is illustrated in Figure 4-3. The result of this requirement is that the synthetic input motion is conservative for most frequencies.
- c. To consider the effects of damping, different synthetic time histories are needed for each damping value, thus increasing significantly the amount of computational effort needed. An alternative approach is to envelop the design spectra for all damping values with spectra from one time history. This introduces further conservatism

for spectra for all but one damping value.

- d. The currently accepted procedure permits the use of spectra generated from only one time-history input even though it has been shown that two synthetic time histories derived to fit the same smoothed criterion spectrum can result in substantially different peak responses in dynamic systems (Singh et al., 1973). To reduce the conservatism and to provide floor design response spectra with a specified level of confidence, the analyses should be based on an ensemble of time histories rather than on a single time history. This would add greatly to the cost of an analysis procedure that is already costly, and is currently not conventionally done.

4.2.2 DIRECT AMPLIFICATION PROCEDURE

The direct amplification procedure as currently used is an approximate method and has developed because of the expense of the time-history method. This method was proposed by Biggs and Roesset (1970) and requires thorough explanation to be clearly understood. The basic idea behind this method is illustrated by the model shown in Figure 4-5. In this figure, the structure/piping/equipment system is represented by an "equivalent" 2DOF system subject to support excitation. Here, m_1 , c_1 , and k_1 represent the effective mass, damping, and stiffness of the structure (e.g., for a natural mode of the structure), and m_2 , c_2 , and k_2 are the effective mass, damping, and stiffness of the piping or equipment system (e.g., for a natural mode of the piping or equipment system). The support motion due to earthquake excitation is indicated by $s(t)$ and the absolute displacements of m_1 and m_2 by x_1

FIGURE 4-5. EQUIVALENT TWO-DEGREE-OF-FREEDOM STRUCTURE/PIPING/EQUIPMENT SYSTEM

and x_2 , respectively. The equations of motion of the system in Figure 4-5 are:

$$\begin{aligned} \ddot{x}_1 + 2\zeta_1\omega_1(\dot{x}_1 - \dot{s}) + \omega_1^2(x_1 - s) \\ - \mu[2\zeta_2\omega_2(\dot{x}_2 - \dot{x}_1) + \omega_2^2(x_2 - x_1)] = 0 \end{aligned} \quad (4-1)$$

$$\ddot{x}_2 + [2\zeta_2\omega_2(\dot{x}_2 - \dot{x}_1) + \omega_2^2(x_2 - x_1)] = 0 \quad (4-2)$$

where $\mu \equiv m_2/m_1$. Expressing Equation 4-1 in terms of the relative motion of the two masses with respect to ground, i.e., letting

$$z_i = x_i - s, \quad (i = 1, 2)$$

then from Equations 4-1 and 4-2,

$$\begin{aligned} \ddot{z}_1 + 2\zeta_1\omega_1\dot{z}_1 + \omega_1^2z_1 - \mu[2\zeta_2\omega_2(\dot{z}_2 - \dot{z}_1) \\ + \omega_2^2(z_2 - z_1)] = -\ddot{s} \end{aligned} \quad (4-3)$$

$$\ddot{z}_2 + [2\zeta_2\omega_2(\dot{z}_2 - \dot{z}_1) + \omega_2^2(z_2 - z_1)] = -\ddot{s} \quad (4-4)$$

If the equations of motion are expressed in terms of the relative motion between the two masses, i.e.

$$u = x_2 - x_1$$

then Equations 4-1 and 4-2 can be put in the form

$$\begin{aligned} \ddot{x}_1 + 2\zeta_1\omega_1(\dot{x}_1 - \dot{s}) + \omega_1^2(x_1 - s) \\ - \mu[2\zeta_2\omega_2\dot{u} + \omega_2^2u] = 0 \end{aligned} \quad (4-5)$$

$$\ddot{x}_2 + [2\zeta_2\omega_2\dot{u} + \omega_2^2u] = 0 \quad (4-6)$$

So far, no approximations have been introduced into these equations of motion. In other words, once the nature of the earthquake is specified by some time history $s(t)$, the exact solution of Equations 4-5 and 4-6 will give the exact response of the system shown in Figure 4-5. From that, the floor-response spectra, as well as the piping/equipment response spectra can be determined by repeatedly solving Equations 4-5 and 4-6 for various combinations of system parameters ω_1 , ω_2 , ζ_1 , ζ_2 , and μ . However, currently this approach is not followed.

The method proposed by Biggs and Roesset introduces a sequence of simplifying assumptions with the intent of trading inaccuracies and lack of rigor (by making several empirical but judicious assumptions) for a major reduction in the effort needed to derive the floor-response spectra.

The first assumption made is to assume decoupling between the equipment and the structure, i.e., it is assumed that the value of $\mu = m_2/m_1$ is small enough that the response of the structure at the point of equipment support can be accurately determined even if the effect of the equipment is neglected. With this assumption, Equations 4-5 and 4-6 can be written as

$$\ddot{x}_1 + 2\zeta_1\omega_1(\dot{x}_1 - \dot{s}) + \omega_1^2(x_1 - s) \approx 0 \quad (4-7)$$

$$\begin{aligned} \ddot{u} + 2\zeta_2\omega_2\dot{u} + \omega_2^2u &= -\ddot{x}_1 \\ &\approx -[2\zeta_1\omega_1(\dot{x}_1 - \dot{s}) + \omega_1^2(x_1 - s)] \end{aligned} \quad (4-8)$$

Another major assumption is that $\ddot{x}_1$, the absolute acceleration of the structure in a given mode in response to a given earthquake, has the waveform of a damped sine function. That is, $\ddot{x}_1$ is hypothesized to be of the form

$$\ddot{x}_1 = a_1 \exp(-\zeta_1\omega_1 t) \sin \omega_1 t \quad (4-9)$$

where

$$\begin{aligned} a_1 &= S_a(\omega_1, \zeta_1) \Gamma^{(n)} \phi_e^{(n)} \\ S_a &= \text{Peak acceleration response as determined from the ground motion response spectrum at frequency } \omega_1 \text{ and damping ratio } \zeta_1 \\ \Gamma^{(n)} &= \text{Participation factor for structure mode } n \\ \phi_e^{(n)} &= \text{Characteristic shape for mode } n \text{ at the point of equipment support} \end{aligned}$$

The degree of approximation inherent in the assumption of Equation 4-9 is evident through comparison of the time history associated with a damped-sine function exhibited in Figure 4-6 and the actual response of a typical building with a ratio of critical damping in the first few modes between 0.02 to 0.04, as shown in Figure 4-7. The building response in this case is based on acceleration measurements during earthquake excitation.

Making use of Equation 4-9, Biggs and Roesset's procedure is to express Equation 4-8 in the form

$$\begin{aligned} \ddot{u} + 2\zeta_2\omega_2\dot{u} + \omega_2^2u &= -\ddot{x}_1 \\ &= -a_1 \exp(-\zeta_1\omega_1 t) \sin \omega_1 t \end{aligned} \quad (4-10)$$

and then to numerically integrate this equation to obtain $u_{\max}$. From that, the maximum absolute equipment acceleration is computed by

$$\ddot{x}_{2\max} = (\ddot{u} + \ddot{x}_1)_{\max} = \omega_2^2 u_{\max} \quad (4-11)$$

The response of the system shown in Figure 4-5 to decaying sinusoidal excitation, under the same decoupling assumptions made by

FIGURE 4-6. TIME HISTORY OF A DAMPED-SINE FUNCTION

FIGURE 4-7. RESPONSE OF A MULTIDEGREE-OF-FREEDOM SYSTEM TO A TYPICAL EARTHQUAKE (Hudson, 1960)

Biggs and Roesset, has been extensively investigated by Mindlin (1945), Curtis (1959), and Crede et al. (1965). Figure 4-8 shows analog computer results for the time history of the system in Figure 4-5 for various combinations of damping and frequency ratio. Experimental results for the shock spectra of $\ddot{x}_2/\ddot{x}_1$ vs. ω_2/ω_1 with different damping parameters are shown in Figures 4-9 and 4-10. Typical three-dimensional shock spectrum curves for decaying sinusoid excitation are shown in Figure 4-11.

Referring to Equation 4-10, the unknown motion $u(t)$ and its derivatives can be found exactly from

$$u = x_2 - x_1$$

$$= -\frac{1}{\omega_2 \sqrt{1 - \zeta_2^2}} \int_0^t \ddot{x}_1(\tau) \exp[-\omega_2 \zeta_2 (t - \tau)] \sin \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) d\tau \quad (4-12)$$

Differentiating Equation 4-12 with respect to time,

$$\begin{aligned} (\dot{x}_2 - \dot{x}_1) = & - \int_0^t \ddot{x}_1(\tau) \exp[-\omega_2 \zeta_2 (t - \tau)] \\ & \cdot \cos \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) d\tau \\ & + \frac{\zeta_2}{\sqrt{1 - \zeta_2^2}} \int_0^t \ddot{x}_1(\tau) \exp[-\omega_2 \zeta_2 (t - \tau)] \\ & \cdot \sin \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) d\tau \end{aligned}$$

(4-13)

Differentiating Equation 4-13 once more with respect to time,

FIGURE 4-8. RESPONSE OF THE TWO-DEGREE-OF-FREEDOM SYSTEM FOR VARIOUS COMBINATIONS OF DAMPING AND FREQUENCY RATIO (Crede et al., 1965)

FIGURE 4-9. RESPONSE OF THE TWO-DEGREE-OF-FREEDOM SYSTEM FOR VARIOUS COMBINATIONS OF DAMPING AND FREQUENCY RATIO, $\zeta_2 = 0$ (Crede et al., 1965)

FIGURE 4-10. RESPONSE OF THE TWO-DEGREE-OF-FREEDOM SYSTEM FOR VARIOUS COMBINATIONS OF DAMPING AND FREQUENCY RATIO, $\zeta_2 = 0.02$ (Crede et al., 1965)

FIGURE 4-11. TYPICAL THREE-DIMENSIONAL SHOCK SPECTRUM CURVES FOR DECAYING SINUSOID EXCITATION ($\zeta_2 = 0.02$, $\zeta_1 = 0.01$) (Crede et al., 1965)

$$\begin{aligned} \ddot{x}_2 = & \omega_2 \frac{1 - 2\zeta_2^2}{\sqrt{1 - \zeta_2^2}} \int_0^t \ddot{x}_1(\tau) \exp[-\omega_2 \zeta_2 (t - \tau)] \\ & \cdot \sin \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) d\tau \\ & + 2 \omega_2 \zeta_2 \int_0^t \ddot{x}_1(\tau) \exp[-\omega_2 \zeta_2 (t - \tau)] \\ & \cdot \cos \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) d\tau \end{aligned} \quad (4-14)$$

Comparing Equations 4-12 and 4-14, it is seen that, in general, $\ddot{x}_{2\max} \neq \omega_2^2 u_{\max}$. In order for the approximate result in Equation 4-11 to

be valid, the value of ζ_2 corresponding to a particular equipment mode must be small enough so that the term ε is negligible, where

$$\begin{aligned} \varepsilon = & 2\omega_2 \zeta_2 \left\{ \zeta_2 \int_0^t \ddot{x}_1(\tau) \exp[-\omega_2 \zeta_2 (t - \tau)] \right. \\ & \cdot \sin \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) d\tau \\ & - \int_0^t \ddot{x}_1(\tau) \exp[-\omega_2 \zeta_2 (t - \tau)] \\ & \cdot \cos \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) d\tau \left. \right\} \end{aligned} \quad (4-15)$$

Referring to Figure 4-5, if the uncoupled natural frequency ratio is such that $\omega_2/\omega_1 \gg 1$ (i.e., the equipment is very stiff compared to the structure), one would expect that the rigidity of the equipment will cause the equipment acceleration to be identical to that of its supporting structure. It is seen from Figures 4-9 and 4-10 that the asymptotic behavior of Equation 4-10 for increasing frequency ratio ω_2/ω_1 does tend to the correct value of unity:

$$\lim_{\omega_2/\omega_1 \rightarrow \infty} \frac{\ddot{x}_{2\max}}{\ddot{x}_{1\max}} = 1 \quad (4-16)$$

However, the limiting value of the shock spectra for decreasing frequency ratio approaches zero. This limit is at variance with physical reality, since if the equipment is relatively flexible and/or the supporting structure is relatively rigid, then the structure motion is identical to that of the ground (i.e., $\ddot{x}_1 = \ddot{s}$), and it is transmitted directly to the equipment. For this limiting case, it is seen from Equation 4-4 that since $z_1 = x_1 - s$ is now equal to zero, then $z_2 = x_2 - s$ is governed by

$$\ddot{z}_2 + 2\zeta_2 \omega_2 \dot{z}_2 + \omega_2^2 z_2 \approx -\ddot{s}(t) \quad (4-17)$$

provided the structure acts as a rigid body as far as the earthquake spectral contents are concerned (i.e., in response to the particular frequency characteristics of the earthquake).

The solution of Equation 4-17 for a specific earthquake will result in

$$\begin{aligned} z_2 &= (x_2 - s) \\ &= -\frac{1}{\omega_2 \sqrt{1 - \zeta_2^2}} \int_0^t \left\{ \ddot{s}(\tau) \right. \\ &\quad \cdot \exp[-\omega_2 \zeta_2 (t - \tau)] \\ &\quad \cdot \sin \omega_2 \sqrt{1 - \zeta_2^2} (t - \tau) \Big\} d\tau \end{aligned} \quad (4-18)$$

Let the maximum relative displacement of z_2 as given by Equation 4-18 be denoted by

$$z_{2\max} = S_d(\omega_2, \zeta_2)$$

where S_d is the relative displacement spectrum.

Subject to the same limitations discussed in conjunction with Equations 4-12 and 4-14, the maximum absolute modal acceleration of the equipment in a given mode due to direct ground motion can be approximated by

$$S_a(\omega_2, \zeta_2) \approx \omega_2^2 S_d(\omega_2, \zeta_2)$$

In order to correct the asymptotic values of the results predicted for relatively low frequency ratio values, Biggs and Roesset suggest an alternative approach in which it is assumed that, in the low frequency range, the piping or equipment is subjected to magnified ground motion whose value is given by

$$\frac{\ddot{x}_{2\max}}{S_a(\omega_2, \zeta_2)} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_2}{\omega_1}\right)^2\right]^2 + \left(2\zeta_2 \frac{\omega_2}{\omega_1}\right)^2}} \quad (4-19)$$

It is seen from Equation 4-19 that Biggs and Roesset's hypothesis is that the magnification factor is the same as the ratio of the maximum structural response to ground-motion excitation when this input is a pure harmonic with a frequency equal to that of the equipment. The cutoff frequency below which Equation 4-10 ceases to govern the motion of the piping or equipment and is replaced by Equation 4-19 was arbitrarily chosen by Biggs and Roesset to be $\omega_2/\omega_1 = 0.8$.

In summary, Biggs and Roesset's approach consists of two approximate solutions, each to operate within a frequency range assigned according to whether the uncoupled natural

frequency ratio of the equipment to that of the structure, ω_2/ω_1 , is less than or greater than some value $\omega_2/\omega_1 = r_0$, selected arbitrarily to be $r_0 = 0.8$. Figure 4-12 gives a qualitative description of the essential features of the approach. For the range $\omega_2/\omega_1 > r_0$ (Range I in Fig. 4-12), the approximate solution is generated by magnification of the spectral acceleration of the structure in the absence of the equipment; in the range $\omega_2/\omega_1 < r_0$ (Range II in Fig. 4-12), the approximate solution magnifies the ground acceleration spectra directly.

Although the relatively simple approach advocated by Biggs and Roesset is rational in

FIGURE 4-12. EQUIPMENT/STRUCTURE AMPLIFICATION RATIO

that the asymptotic limits of the solution for very stiff equipment or for rigid-structure/flexible-equipment approach the correct limits, it is essentially a collection of formulas for generating an approximate solution of the problem based on empirical results and unverified assumptions. The hypotheses introduced may lead, in general, to unrealistically conservative estimates of the floor response spectra, or even to unconservative estimates in some other cases (see discussion below), depending on system characteristics.

4.2.3 MODIFIED AMPLIFICATION PROCEDURE

The modified amplification procedure, a simple method based on the ground response spectrum and the response spectrum of the supporting structure, was suggested by Biggs (1971) for the seismic analysis of equipment in nuclear power plants, and is an improvement over the direct amplification method. The method was intended to provide an envelope of the floor response spectra and to construct equipment response spectra without a time-history analysis of either the building or the equipment.

For a system in which the mass of the equipment is small in comparison to that of the structure, the maximum acceleration response of the equipment may be considered to be an amplification of either the ground response spectrum or the structural response spectrum at the equipment support. Biggs developed the amplification factors empirically from records of four typical strong earthquakes. The factors are essentially a function of the frequency ratio of the equipment and the structure and the magnitude of damping in the system. Referring again to the "equivalent" 2DOF system as shown in Figure 4-5, Biggs first developed amplification factors for two limiting cases, namely, the case of very flexible equipment ($\omega_2/\omega_1 \ll 1$),

and the case of very rigid equipment ($\omega_2/\omega_1 \gg 1$). In the case of very flexible equipment, the equipment is assumed to respond to the ground motion as though it were supported directly on the ground. For very rigid equipment, the maximum acceleration of the equipment is assumed to be equal to that of the structure at the equipment support.

Figure 4-13 shows Biggs' amplification factors plotted in terms of the frequency ratio ω_2/ω_1 for systems with the values of critical damping ratio of 0.04 and 0.005 in the structure and the equipment, respectively. The curves plotted in Figure 4-13 are the maximum amplification factors generated for values of structural frequency f_1 ranging from 0.4 to 20 Hz. The amplification curves adopted by Biggs are generally upper bounds of the equipment response induced from the four actual earthquake records. Due to some scatter in the response analysis, Biggs further suggested using the amplification of the ground motion for $\omega_2/\omega_1 \leq 1/0.9$ and amplification of the structure's motion for $\omega_2/\omega_1 \geq 2.0$. An interpolation of the results from the two limiting cases was adopted for the range between these two points.

Biggs' method is directly applicable to a single-degree-of-freedom (SDOF) system representing an item of equipment mounted on a SDOF structure. It can also be applied to multidegree-of-freedom (MDOF) structures by utilizing conventional methods of modal analysis. For MDOF systems, the maximum equipment response is determined separately for each mode of the structure. The total equipment response can then be found by taking the root-mean-square of the responses due to the structural modes. The procedure of Biggs' modified amplification method is extremely simple, especially for a SDOF system. The procedure for a SDOF system is outlined as follows:

FIGURE 4-13. EQUIPMENT ACCELERATION AMPLIFICATION FACTOR (Biggs, 1971)

- a. For $\omega_2/\omega_1 \geq 2.0$, the maximum equipment acceleration, A_e , is suggested to be

$$A_e = S_a(\omega_1, \zeta_1) \left[\frac{A_e}{S_a(\omega_1, \zeta_1)} \right] \quad (4-20)$$

where the ratio in the bracket is the amplification of the structure's motion as shown in Figure 4-13; and $S_1(\omega_1/\zeta_1)$ is the peak acceleration response as defined in Equation 4-9.

- b. For $\omega_2/\omega_1 \leq 1/0.9$,

$$A_e = S_a(\omega_2, \zeta_2) \left[\frac{A_e}{S_a(\omega_2, \zeta_2)} \right] \quad (4-21)$$

where the ratio in the bracket is as defined above, and $S_a(\omega_2, \zeta_2)$ is the peak acceleration response as determined from the ground motion

response spectrum at frequency ω_2 and damping ratio ζ_2 .

- c. For $1/0.9 < \omega_2/\omega_1 < 2.0$, the maximum equipment acceleration is obtained by interpolating between two end points in a and b above, i.e.,

$$A_e = A_e \bigg|_{\frac{\omega_2}{\omega_1} = 2.0}$$

$$+ \left[A_e \bigg|_{\frac{\omega_2}{\omega_1} = 1/0.9} - A_e \bigg|_{\frac{\omega_2}{\omega_1} = 2.0} \right] \left[\frac{1 - 0.5 \left(\frac{\omega_2}{\omega_1} \right)}{0.4 \left(\frac{\omega_2}{\omega_1} \right)} \right]$$

(4-22)

Another approximate method for generating floor response spectra was developed by Kapur and Shao (1973). This method shares certain characteristics with the one by Biggs: The ground motion response spectra and the modal analysis of the support structure (building) are required as input, but no time-history calculations are needed.

Except for some minor variations in the hypotheses made to generate the approximate solutions, the basic ideas behind both methods are essentially the same. The slight difference

between the two methods is that Biggs' amplification factors are based on several time-history analyses of a 2DOF system, whereas Kapur's method is based on a hypothetical upper bound to the amplification of the ground motion and/or structure motion.

A study by Arthur et al. (1975) compared results achieved by the time-history method with those obtained by using Biggs' and Kapur's methods, when all methods were used in conjunction with the model shown in Figure 4-14. By comparing the approximate floor spectrum

(a) Analytical model for MOFFP horizontal response

FIGURE 4-14. RATIO OF APPROXIMATE TO TIME-HISTORY FLOOR SPECTRA — HORIZONTAL MOTIONS (Arthur et al., 1975)

to the corresponding time-history spectrum, they obtained the ratio plots exhibited in Figure 4-14. It can be noted in this figure that:

- a. Biggs' floor spectra range from 0.8 to 3.0 times the time-history spectra.
- b. Kapur's spectra range from 0.7 to 4.5 times the time-history spectra.
- c. Both methods tend to underestimate the time-history-derived floor spectrum below the fundamental building frequency, and to overestimate the spectrum at the higher frequencies.

Unfortunately, the lack of rigor in the derivation of this class of approximate methods for generating floor response spectra, and their empirical nature detract from their reliability in the absence of verification. For this reason, a quantitative evaluation of the approximate methods is provided in subsequent paragraphs.

4.2.4 RANDOM VIBRATION TECHNIQUES

Recently, random vibration techniques have been used to obtain instructure response spectra (Singh, 1975; Singh and Chu, 1976). The main features of the approach used by Singh are:

- a. The earthquake motion is assumed to be a zero mean stationary Gaussian random process.
- b. A spectrum-consistent power spectral density function is employed; the response spectrum curve is considered to represent a response value with a specific probability.
- c. The dynamic characteristics of the structure (modal frequencies, participation factors, etc.) are utilized.
- d. The structure is modeled as a MDOF system, but the interaction between the equipment and the supporting

structure is neglected, i.e., the equipment is assumed to have no effect on the structure.

Unlike the other approaches where amplification curves are provided, the approach by Singh involves the use of expressions for amplification factors that are functions of the ratio of equipment and structure frequencies and their damping factors. While the work of Singh is a big step toward improved accuracy in the calculation of equipment/structure response, it suffers from two significant limitations due to the assumptions about the motion of the earthquake excitation and the effects of equipments on their supporting structure. First, the finite duration of the earthquake motion, particularly for lightly damped systems of the type that is likely to be encountered in practical cases, has a significant effect on the dynamic response of systems. Second, even for equipment/structure systems with a very small mass ratio, the effects of equipment/structure interaction are nonnegligible. An "exact" probabilistic approach to the generation of floor response spectra of a 2DOF equipment/structure system is presented later.

Additional studies involving approximate probabilistic approaches to estimate equipment response spectra have been conducted by Peters et al. (1977), Sato et al. (1977), Scanlan (1977), Sundararajan and Gangadharan (1977), Vanmarcke (1977, 1972), Amin and Gunger (1971), and Chakrovorty (1972).

4.3 EVALUATION OF APPROXIMATE DIRECT AMPLIFICATION METHOD

The relative simplicity of the approximate direct amplification method and the resulting economy of effort required to generate the design spectra are the primary justifications for using the method to determine the seismic input motion for the design of piping and equipment.

The simplifying assumptions used to develop this method introduce errors, but it is generally assumed that the method provides conservative results. However, the level of conservatism has never been evaluated, nor is it known whether the method always yields conservative results.

The previous discussion indicates that an analytically correct (i.e., "exact") method can be provided by making parametric analyses of a 2DOF system using the free-field input time history developed for the site. Actually, the results of an ensemble of input records are needed to provide a response of specified confidence levels. Here, the approximate direct amplification method is evaluated by comparing the results obtained using this method with the results obtained using the solutions to the 2DOF system for specific time-history inputs. The study has been limited to a comparison of the results for four excitation time histories. A description of the study parameters and the solution algorithm are given before the results.

4.3.1 STUDY PARAMETERS

The evaluation studies involve the following combinations and permutations of system parameters.

4.3.1.1 Input Time Histories

Four distinct earthquake time histories, both synthetic and real, were used as base excitation $\ddot{s}(t)$. The following time histories were used:

- a. Earthquake A1. This is a simulated earthquake whose characteristics are well documented in a report by Jennings, Housner, and Tsai (1968). It represents upper bound ground motions expected to result for a Magnitude 8+ earthquake. The acceleration, velocity, and displacement

time histories for this earthquake are shown in Figure 4-15. The response spectrum for a damping ratio of 0.05 is compared with the response spectra for the three other time histories in Figure 4-16.

- b. Earthquake B1. This is also a simulated earthquake generated by Jennings, Housner, and Tsai using the same techniques used for Earthquake A1. It represents ground motion characteristics of a Magnitude 7 earthquake. The time histories are shown in Figures 4-17, and the response spectrum for a damping ratio of 0.05 is shown in Figure 4-16.
- c. Earthquake E. The N-S component of the El Centro 1940 earthquake, corrected and digitized in the manner detailed by the California Institute of Technology Report No. 72-80 (CIT, 1972), was used for Earthquake E. The time histories are shown in Figure 4-18, and a response spectrum for a damping ratio of 0.05 is given in Figure 4-16.
- d. Earthquake X. This is a synthetic record generated by conventional techniques used by the engineering profession in the structural analysis of nuclear power plants. The resulting spectrum envelops the Regulatory Guide 1.60 response spectrum. The time histories for this record are shown in Figure 4-19, and a response spectrum for a damping ratio of 0.05 is given in Figure 4-16.

4.3.1.2 Mass Ratio, μ

Two values of $\mu \equiv m_2/m_1$ were selected: $\mu = 0.001$ and $\mu = 0.10$, which represent a very

FIGURE 4-15. ACCELERATION, VELOCITY AND DISPLACEMENT FOR EARTHQUAKE A1 (Jennings et al., 1968)

FIGURE 4-16. COMPARISON OF RESPONSE SPECTRA FOR HORIZONTAL EARTHQUAKE COMPONENTS A1, B1, E AND X

FIGURE 4-17. ACCELERATION, VELOCITY AND DISPLACEMENT FOR EARTHQUAKE B1
(Jennings et al., 1968)

FIGURE 4-18. ACCELERATION, VELOCITY AND DISPLACEMENT FOR EL CENTRO EARTHQUAKE
(CIT, 1972)

FIGURE 4-19. ACCELERATION, VELOCITY AND DISPLACEMENT FOR SYNTHETIC EARTHQUAKE X

light and a moderately light piece of equipment m_2 attached to the structure m_1 .

4.3.1.3 Structure Natural Frequency, f_1

Three values of the primary system natural frequency f_1 were selected: $f_1 = 1/5$ Hz, 1 Hz, and 5 Hz, with corresponding periods $T_1 = 1/f_1$ equal to 5 sec, 1 sec, and $1/5$ sec, respectively.

4.3.1.4 Structure Ratio of Critical Damping, ζ_1

Two values of ζ_1 were used: 0.01 and 0.05, corresponding to a lightly damped and a moderately damped structure, respectively.

4.3.1.5 Equipment Ratio of Critical Damping, ζ_2

Two values of ζ_2 were used: 0.01 and 0.05, corresponding to a lightly damped and a moderately damped equipment system, respectively.

4.3.1.6 Uncoupled Frequency Ratio, ω_2/ω_1

Twenty-one values of the dimensionless frequency ratio ω_2/ω_1 were selected, spanning the range 0.5 to 1.5 in increments of 0.05.

4.3.2 SOLUTION ALGORITHM

For every combination of the four distinct inputs and system parameters that constitutes a complete characterization of a specific case, both approximate and exact direct amplification solutions were provided in the manner described below.

4.3.2.1 Determination of Exact Solutions

"Exact" solutions for the transient response of the 2DOF system shown in Figure 4-5 were obtained by using standard numerical procedures for solving the governing equations of motion (Eqs. 4-3 and 4-4). Once

system characteristics μ , f_1 , ζ_1 , ζ_2 , ω_2/ω_1 and an excitation record $\ddot{s}(t)$ of length $t_{\max}$ have been selected, the relative displacement $z_1(t)$, the relative velocity $\dot{z}_1(t)$, and the absolute acceleration $\ddot{x}_1(t)$ of both components m_1 and m_2 of the 2DOF system were determined incrementally for values of $0 \leq t \leq t_{\max}$, and the peak values of the various response quantities $z_{i\max}$, $\dot{z}_{i\max}$, $\ddot{x}_{i\max}$, $i = 1, 2$ were determined. The resulting peak values constitute the "exact" reference results that were used to evaluate the relative deviation between "exact" and "approximate" results.

4.3.2.2 Determination of Solutions to Approximate Method

Unlike the procedure for the exact solution, the input in this approach is specified in the form of a response spectrum for an earthquake record $\ddot{s}(t)$ of length $t_{\max}$. Given a set of parameters μ , f_1 , ζ_1 , ζ_2 , ω_2/ω_1 and input spectrum $S_a(\omega, \zeta)$, the procedure that was used to generate approximate values for the equipment spectrum was as follows, depending on the magnitude of $r \equiv \omega_2/\omega_1$ compared to a given threshold level r_0 :

Range I

If $r > r_0$, perform the following steps:

- Determine $a_1 = S_a(\omega_1, \zeta_1)$
- Solve the differential equation
$$\ddot{u} + 2\zeta_2\omega_2\dot{u} + \omega_2^2 u = -a_1 \exp(-\zeta_1\omega_1 t) \sin \omega_1 t$$
- Find $u_{\max}$
- Compute $\ddot{x}_{2\max} = \omega_2^2 u_{\max}$

(4-23)

Range II

If $r < r_0$, perform the following steps:

- Determine the amplification ratio

$$A(r, \zeta_2),$$

where $A(r, \zeta)$

$$= \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

- Determine $a_2 = S_a(\omega_2, \zeta_2)$.
- Compute $\ddot{x}_{2\max} = a_2 A(r, \zeta_2)$.

(4-24)

The value of $\ddot{x}_{2\max}$ as computed by Equation 4-23 or 4-24 is referred to as the "approximate solution." Note that if in this procedure the threshold level $r_0 = 0.8$, the procedure becomes essentially the same as the approximate methods discussed in Section 4.2.2.

4.3.3 COMPARISON OF RESULTS

4.3.3.1 Computation of Deviation Error

Let the deviation error between the exact and approximate values of the peak absolute acceleration of the equipment be defined as

$$\varepsilon = \frac{(\ddot{x}_{2\max})_{\text{approx}} - (\ddot{x}_{2\max})_{\text{exact}}}{(\ddot{x}_{2\max})_{\text{exact}}} \quad (4-25)$$

where ε is a function of the specific excitation record considered, as well as the other system parameters:

$$\varepsilon = \varepsilon(E.Q., \mu, f_1, \zeta_1, \zeta_2, \omega_2/\omega_1)$$

Note that from the definition of ε in Equation 4-25, if the approximate value of $\ddot{x}_{2\max}$

exceeds the exact values, then $\varepsilon > 0$ and a design based on $(\ddot{x}_{2\max})_{\text{approx}}$ would be conservative by a percentage value equal to 100ε . Conversely, negative values of ε correspond to cases where $(\ddot{x}_{2\max})_{\text{approx}}$ underestimates the exact equipment acceleration; thus designs based on such values of $(\ddot{x}_{2\max})_{\text{approx}}$ would be unconservative by a percentage value of 100ε , compared to the exact peak acceleration, $(\ddot{x}_{2\max})_{\text{exact}}$.

4.3.3.2 Results

Sample results that indicate the system response for a variety of parameters are shown in Figures 4-20 through 4-23 where comparison is made of exact and approximate responses for various combinations of ζ_1 and ζ_2 . In each figure, part (a) shows $\ddot{x}_{1\max}$, the peak value of the absolute acceleration of the primary system; part (b) compares the exact and approximate value of the equipment acceleration, $\ddot{x}_{2\max}$; and part (c) presents the magnitude of the error ε . In all these figures, the results are plotted as a function of the dimensionless uncoupled frequency ratio ω_2/ω_1 .

For example, it is seen from Figure 4-22 that for values of $\omega_2/\omega_1 = 0.5, 1.0$, and 1.5 , the corresponding values of ε are -0.002 , -0.40 , and $+0.17$, respectively. Thus, the percentage of error is -0.2% , -40% , and $+17\%$, respectively. This means that if a piece of equipment is designed on the basis of the approximate analysis, then for the case under consideration at a frequency ratio of $\omega_2/\omega_1 = 0.5$ the design is unconservative by 0.2% ; at $\omega_2/\omega_1 = 1.0$ it is unconservative by 40% ; and at $\omega_2/\omega_1 = 1.5$ the design is conservative by 17% . Thus, the signs attached to the magnitude of the errors in Figures 4-20 to 4-23 serve as an indication of the unconservatism ($-$ sign) or the conservatism ($+$ sign) in the design. The location of the data points in part (c) for these figures with respect to the zero axis serves as

FIGURE 4-20. COMPARISON OF EXACT AND APPROXIMATE RESPONSE FOR $\zeta_1 = 0.01$, $\zeta_2 = 0.01$

FIGURE 4-21. COMPARISON OF EXACT AND APPROXIMATE RESPONSE FOR $\zeta_1 = 0.01$, $\zeta_2 = 0.05$

FIGURE 4-22. COMPARISON OF EXACT AND APPROXIMATE RESPONSE FOR $\zeta_1 = 0.05$, $\zeta_2 = 0.01$

FIGURE 4-23. COMPARISON OF EXACT AND APPROXIMATE RESPONSE FOR $\zeta_1 = 0.05$, $\zeta_2 = 0.05$

a quick visual marker of the degree of conservatism (or lack of it) inherent in a particular design.

Note from Figures 4-20 through 4-23 that the results shown in each figure represent a summary of 21 separate cases corresponding to 21 different 2DOF systems all subjected to the same earthquake. A collection of figures similar to Figures 4-20 through 4-23 for various combinations and permutations of system parameters discussed in a previous section corresponding to earthquake A1 and B1 inputs are given in AA (1979a).

In general, the results given in AA (1979a) are similar to those indicated in Figures 4-20 and 4-23. A total of 1344 separate solutions are provided in AA. An inspection of the results given in AA provides the following general observations:

- a. The largest positive error found was +15.3, equivalent to 1530% conservatism in the approximate solution as compared to the exact results; and the largest negative error was -0.62, equivalent to 62% unconservatism.
- b. Both the largest positive error and the largest negative error occur at $(\omega_2/\omega_1) = 1$, which corresponds to the case where the equipment is in a state of resonance with its supporting structure (usually resulting in the most severe load on the equipment).
- c. At every one of the 21 distinct values of ω_2/ω_1 considered, the value of ε varied from a large number of negative percentage errors to a large number of positive percentage errors. That is, in the range studied, no frequency ratio value was found for which the approximate solution gave consistently positive errors (conservative) or consistently negative errors

(unconservative).

- d. At every frequency value, the errors fluctuated over a wide range, without any discernible trend. The apparent randomness in the variation of ε did not seem to be correlated with the magnitude of the mass ratio μ , combination of damping ratios ξ_1 and ξ_2 , the frequency f_1 of the structure, or the type of earthquake record used.
- e. The scatter in the values of ε is such that the range of validity of this particular approximate solution technique cannot be ascertained but it is obviously unacceptable. In contrast, the exact solution of the 2DOF system, using an ensemble of input records that approximate the criteria spectra, could be used to generate response values for piping and equipment design that would have the same confidence level as the criteria spectra. This can be accomplished at relatively low cost, and would result in much more reliable input criteria for the design of piping and equipment than the presently used approximate direct amplification method.

4.4 MODIFIED AMPLIFICATION METHOD

The previous results show that the approximate direct amplification method suggested by Biggs and Roesset (1970) does not always yield a conservative solution for the design of piping and equipment for nuclear power plants. A modified amplification method was introduced by Biggs (1971) to provide an envelope of the equipment response spectra which would be produced by all probable strong earthquake records. To evaluate this method, the response spectra obtained from the modified amplification method were directly compared with

those from the approximate direct amplification method and from the 2DOF system exact solutions by using the same four time-history inputs as before (see Sec. 4.3.1.1). The parameters considered and the results are presented below.

4.4.1 STUDY PARAMETERS

4.4.1.1 Input Time Histories

The four time histories, one real and three artificial earthquake records, have different peak accelerations and different frequency contents. The waveforms of the four time histories and their characteristics are presented in Section 4.3.1.1.

4.4.1.2 Mass Ratio, μ

Inasmuch as the modified amplification method is limited to the case of uncoupled systems, one mass ratio value of $\mu = m_2/m_1 = 0.001$ was considered in the studies. This mass ratio is small enough to assure that the motion of the equipment would not materially affect the overall response of the structure. This limitation on the method should not be overlooked (i.e., the modified amplification method is not a function of the mass ratio).

4.4.1.3 Structure Natural Frequency, f_1

The values of the uncoupled, undamped natural frequency of the structure were selected to be 10 Hz, 5 Hz, 1 Hz, and 0.2 Hz. While the lower end of this frequency range is not realistic for nuclear power plants, it was used to illustrate the sensitivity of the structural frequency to the equipment response.

4.4.1.4 Ratios of Critical Damping, ξ_1 and ξ_2

For direct application of the amplification

factors proposed by Biggs as shown in Figure 4-13, the same values of the critical damping ratio were used in the evaluation studies, i.e., the values of critical damping ratio of $\xi_1 = 0.04$ and $\xi_2 = 0.005$ for the structure and the equipment, respectively.

4.4.1.5 Uncoupled Frequency Ratio, ω_2/ω_1

The values of the uncoupled frequency ratio ω_2/ω_1 were selected to cover the range of $\omega_2/\omega_1 = 0.5$ to $\omega_2/\omega_1 = 2.5$ with an increment of $\omega_2/\omega_1 = 0.1$.

4.4.2 COMPARISON OF RESULTS

For each of the four input time histories, the exact solutions and the solutions of the approximate direct amplification method were generated by the procedures presented in Section 4.3.2, while the solutions of the modified amplification method were obtained by the procedure outlined in Section 4.2.3. The results of the equipment response spectra were plotted in Figures 4-24 through 4-27. The values of the peak equipment acceleration for all the plots were normalized to the peak equipment acceleration at $\omega_2/\omega_1 = 1.0$ obtained from the corresponding exact solution. In other words, the ordinates of the plots are the ratios of the peak equipment acceleration obtained by the different methods to that of the exact solution at $\omega_2/\omega_1 = 1.0$.

Comparison of the results reaffirmed the earlier assertion that the approximate direct amplification method indeed does not always provide a conservative solution. In fact, the direction amplification method almost always yields unconservative results in the vicinity of $\omega_2/\omega_1 = 1.0$ for the system with an uncoupled, undamped structural frequency f_1 of 1 Hz or higher. For the cases with $f_1 = 0.2$ Hz, however, the direct amplification method is conservative in the vicinity of $\omega_2/\omega_1 = 1.0$.

FIGURE 4-24. COMPARISONS OF PEAK EQUIPMENT ACCELERATION SUBJECTED TO EARTHQUAKE A1

FIGURE 4-25. COMPARISONS OF PEAK EQUIPMENT ACCELERATION SUBJECTED TO EARTHQUAKE B1

FIGURE 4-26. COMPARISONS OF PEAK EQUIPMENT ACCELERATION SUBJECTED TO EARTHQUAKE E

FIGURE 4-27. COMPARISONS OF PEAK EQUIPMENT ACCELERATION SUBJECTED TO EARTHQUAKE X

For the input time histories studied, the modified amplification method always provides conservative results except for one isolated case for Earthquake E input. As shown in Figure 4-26d, the exact solutions for the $f_1 = 0.2$ Hz case with Earthquake E input produced secondary peaks in the region of $\omega_2/\omega_1 = 1.6$ to $\omega_2/\omega_1 = 1.8$ that lie above the modified amplification method response. Similar phenomena occurred with the remaining three time histories for the $f_1 = 0.2$ Hz cases, but the effect was not severe enough to cause the response from the modified amplification method to be unconservative.

The explanation of the secondary peaks with earthquake input E lies in the fact that for $f_1 = 0.2$ Hz, the structure response (SDOF) is controlled by the peak ground displacement, and by the frequency characteristics of the displacement time history. In reference to the displacement time history in Figure 4-18, the frequency of the displacement during the first 9 sec is about 0.35 Hz. After 25 sec the frequency is about 0.17 Hz. In reference to the E response spectrum in Figure 4-16, peaks will be noted at these frequencies, except greater response occurs at about 0.35 Hz. Since the structure in this example has a frequency of 0.2 Hz, the peak structure response occurs at 0.2 Hz. However, the structure response is also obviously affected by the stronger 0.35 Hz frequency of the displacement input during the first 9 sec. Since the equipment is responding to the structure response as an input, the primary peak occurs at resonance ($\omega_2/\omega_1 = 1.0$) and a secondary peak occurs at $\omega_2/\omega_1 = 1.6$ to 1.8 (i.e., equipment frequencies of 0.32 to 0.36 Hz). However, it has been previously noted that a structural frequency 0.2 Hz is well below the range of the fundamental frequency of nuclear power plant structures. Therefore, the modified amplification method yields conservative results for all structural frequencies

of interest. Unfortunately, the conservatism is excessive in a wide ω_2/ω_1 range for all four input time histories.

4.5 PROBABILISTIC APPROACH TO FLOOR RESPONSE SPECTRA

The evaluation of the approximate direct amplification method indicated that parametric solutions of the 2DOF system using a time history that matches the criteria free-field spectrum would provide input criteria for the design of piping and equipment that involve no approximate assumptions. However, the response should actually be based on an ensemble of input records that approximate the free-field criteria spectra if input is to be determined for a prescribed confidence level. Also, present time-history analysis methods should use an ensemble of input time histories, if floor response spectra are to be derived for a given confidence level. This suggests that a more general probabilistic method is needed. Although some promising work involving probabilistic concepts has been done (Singh, 1975; Sato et al., 1977; Scanlan, 1977; and Vanmarcke, 1977), the validity of each of these studies is limited because of the assumptions made in the mathematical representation of the earthquake, the interaction between equipment and structure, and the mathematical approach and approximations used in the derivation of the floor response spectra. Therefore, a more general probabilistic approach to this overall problem is presented here.

4.5.1 METHODOLOGY

A probabilistic approach is used for determining the expected value of the transient mean-square (m.s.) response of a linear equipment/structure system without making any simplifying assumptions about (1) the degree of interaction between the equipment and the structure, (2) the amount of damping present

in each component of the system, (3) how to combine the contribution of the different degrees of freedom of the system, and (4) the statistical nature of the earthquake. To illustrate the proposed methodology, consider the 2DOF system shown in Figure 4-28, where m_1 , c_1 , k_1 are the generalized mass, damping, and stiffness of the structure in a given mode, and m_2 , c_2 , k_2 represent the corresponding generalized quantities of the equipment in a specific mode. Presumably, the natural frequencies of the structure and equipment in these modes are close together.

It is worth noting that by a proper choice of system parameters, the model shown in Figure 4-28 can be useful in investigating several practical problems:

- a. By setting the spring k_3 and the dashpot c_3 to zero, the system can

be made to resemble an equipment/structure system subjected to base excitation.

- b. If k_2 and c_2 are set equal to zero, then the two masses become uncoupled and the model can be used to determine the relative motion between different components of a structural system or equipment.
- c. The model of the system shown in part (a) is useful in evaluating the response of systems (e.g., piping) to nonuniform support motion. Note that if the form of the excitation $F_2(t)$ is specified as $F_2(t) = u(t - \tau_0) F_1(t - \tau_0)$, it represents a propagating disturbance traveling with wave speed τ_0 (here, $u(\cdot)$ is the unit step function).

FIGURE 4-28. MODEL OF SYSTEM

- d. The model is also of limited use in the area of soil amplification where a soil layer constitutes the primary system and the auxiliary system represents a SDOF structure.

The following probabilistic analysis is based on the work of Masri (1978).

4.5.2 ANALYSIS

Consider the linear 2DOF system shown in Figure 4-28, whose equation of motion is

$$\begin{aligned} a_{11} \ddot{x}_1 + b_{11} \dot{x}_1 + c_{11} x_1 \\ + b_{12} \dot{x}_2 + c_{12} x_2 &= F_1(t) \\ b_{12} \dot{x}_1 + c_{12} x_1 \\ + a_{22} \ddot{x}_2 + b_{22} \dot{x}_2 + c_{22} x_2 &= F_2(t) \end{aligned} \quad (4-26)$$

where

$$\begin{aligned} a_{11} &\equiv m_1 & a_{22} &\equiv m_2 \\ b_{11} &\equiv c_1 + c_2 & b_{22} &\equiv c_2 + c_3 \\ c_{11} &\equiv k_1 + k_2 & c_{22} &\equiv k_2 + k_3 \\ b_{12} &\equiv -c_2 & c_{12} &\equiv k_2 \end{aligned}$$

and where, if the system is excited by a directly applied force, as shown in part (a),

$$\begin{aligned} x_i(t) &= \text{Absolute displacement of mass } m_i \\ F_i(t) &= \text{Exciting force acting on } m_i \end{aligned}$$

whereas if the system is excited by base motion, as shown in part (b),

$$x_i(t) = \text{Relative displacement of } m_i \text{ with respect to the base}$$

$$F_i(t) = -m_i \ddot{S}(t), \text{ the inertia force acting on } m_i$$

Let $h_i^{(j)}(t)$ denote the impulse response of coordinate x_i due to a unit impulse applied at coordinate x_j . Then the general response of the system to an arbitrary excitation can be expressed as

$$x_i(t) = \int_0^t \sum_{j=1}^2 h_i^{(j)}(t - \tau) F_j(\tau) d\tau \quad (i = 1, 2) \quad (4-27)$$

The impulse functions $h_i^{(j)}(t)$ can be obtained by applying appropriate initial conditions to the solution of the homogeneous part of Equation 4-28:

$$\begin{aligned} a_{11} \ddot{x}_1 + b_{11} \dot{x}_1 + c_{11} x_1 \\ + b_{12} \dot{x}_2 + c_{12} x_2 &= 0 \\ b_{12} \dot{x}_1 + c_{12} x_1 \\ + a_{22} \ddot{x}_2 + b_{22} \dot{x}_2 + c_{22} x_2 &= 0 \end{aligned} \quad (4-28)$$

Assume that the solution of Equation 4-28 is of the form

$$x_i(t) = \lambda_i \exp(st) \quad (i = 1, 2) \quad (4-29)$$

On substitution of Equation 4-29 in Equation 4-28,

$$\begin{pmatrix} a_{11} s^2 + b_{11} s + c_{11} & b_{12} s + c_{12} \\ b_{12} s + c_{12} & a_{22} s^2 + b_{22} s + c_{22} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4-30)$$

For a nontrivial solution of Equation 4-30, the determinant of the coefficient matrix must be equal to zero. Thus, the characteristic equation of the system is

$$p_1 s^4 + p_2 s^3 + p_3 s^2 + p_4 s + p_5 = 0 \quad (4-31)$$

where

$$\begin{aligned} p_1 &\equiv c_{11} c_{22} - c_{12}^2 \\ p_2 &\equiv b_{11} c_{22} + c_{11} b_{22} - 2 b_{12} c_{12} \\ p_3 &\equiv a_{11} c_{22} + b_{11} b_{22} + c_{11} a_{22} - b_{12}^2 \\ p_4 &\equiv a_{11} b_{22} + b_{11} a_{22} \\ p_5 &\equiv a_{11} a_{22} \end{aligned}$$

Let the four roots, which are in general complex, of Equation 4-31 be denoted by s_j , $j = 1, 2, 3, 4$. In view of Equation 4-30, for each root s_j the values of λ_1 and λ_2 are related by

$$\begin{aligned} \left(\frac{\lambda_1}{\lambda_2} \right)_j &= - \left(\frac{b_{12}s + c_{12}}{a_{11}s^2 + b_{11}s + c_{11}} \right)_j \\ &= - \left(\frac{a_{22}s^2 + b_{22}s + c_{22}}{b_{12}s + c_{12}} \right)_j \equiv \frac{1}{r_j} \end{aligned} \quad (4-32)$$

Equations 4-30 and 4-32, the free vibrations of the system, can be expressed as

$$\begin{aligned} x_1(t) &= \sum_{j=1}^4 \lambda_{1j} \exp(s_j t) \\ x_2(t) &= \sum_{j=1}^4 \lambda_{2j} \exp(s_j t) \\ &= \sum_{j=1}^4 r_j \lambda_{1j} \exp(s_j t) \end{aligned} \quad (4-33)$$

and the velocities of m_1 and m_2 are given by

$$\begin{aligned} \dot{x}_1(t) &= \sum_{j=1}^4 s_j \lambda_{1j} \exp(s_j t) \\ \dot{x}_2(t) &= \sum_{j=1}^4 r_j s_j \lambda_{1j} \exp(s_j t) \end{aligned} \quad (4-34)$$

Once the initial conditions of the system are specified in the form

$$\begin{aligned} x_1(0) &= x_{10}, \quad \dot{x}_1(0) = \dot{x}_{10}, \\ x_2(0) &= x_{20}, \quad \dot{x}_2(0) = \dot{x}_{20} \end{aligned}$$

the unknown values of λ_{ij} can be found from

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ s_1 & s_2 & s_3 & s_4 \\ r_1 & r_2 & r_3 & r_4 \\ r_1 s_1 & r_2 s_2 & r_3 s_3 & r_4 s_4 \end{pmatrix} \begin{pmatrix} \lambda_{11} \\ \lambda_{12} \\ \lambda_{13} \\ \lambda_{14} \end{pmatrix} = \begin{pmatrix} x_{10} \\ \dot{x}_{10} \\ x_{20} \\ \dot{x}_{20} \end{pmatrix} \quad (4-35)$$

The initial conditions of the system corresponding to the application of a unit impulse at coordinate $j = 1$ are:

$$\begin{pmatrix} x_{10} \\ \dot{x}_{10} \\ x_{20} \\ \dot{x}_{20} \end{pmatrix} = \begin{pmatrix} 0 \\ 1/m_1 \\ 0 \\ 0 \end{pmatrix}$$

and at $j = 2$ are:

$$\begin{pmatrix} x_{10} \\ \dot{x}_{10} \\ x_{20} \\ \dot{x}_{20} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{pmatrix}$$

Let the solution of the linear set of equations (Eq. 4-35) corresponding to these initial conditions be denoted by:

Impulse at x_1 :

$$\lambda_{11}^{(1)} \quad \lambda_{12}^{(1)} \quad \lambda_{13}^{(1)} \quad \lambda_{14}^{(1)}$$

Impulse at x_2 :

$$\lambda_{11}^{(2)} \quad \lambda_{12}^{(2)} \quad \lambda_{13}^{(2)} \quad \lambda_{14}^{(2)}$$

The impulse response of m_1 and m_2 due to a unit impulse at x_1 is

$$\begin{aligned} h_1^{(1)}(t) &= \sum_{j=1}^4 \lambda_{1j}^{(1)} \exp(s_j t) \\ h_2^{(1)}(t) &= \sum_{j=1}^4 r_j \lambda_{1j}^{(1)} \exp(s_j t) \end{aligned} \quad (4-36)$$

Similarly, the impulse response due to a unit impulse at x_2 is

$$\begin{aligned} h_1^{(2)}(t) &= \sum_{j=1}^4 \lambda_{1j}^{(2)} \exp(s_j t) \\ h_2^{(2)}(t) &= \sum_{j=1}^4 r_j \lambda_{1j}^{(2)} \exp(s_j t) \end{aligned} \quad (4-37)$$

The general response of the system due to arbitrary random excitations $F_1(t)$ and $F_2(t)$ is

$$\begin{aligned} x_1(t) &= \int_0^t \left[h_1^{(1)}(t - \tau) F_1(\tau) \right. \\ &\quad \left. + h_1^{(2)}(t - \tau) F_2(\tau) \right] d\tau \\ x_2(t) &= \int_0^t \left[h_2^{(1)}(t - \tau) F_1(\tau) \right. \\ &\quad \left. + h_2^{(2)}(t - \tau) F_2(\tau) \right] d\tau \end{aligned} \quad (4-38)$$

For convenience, assume that the system is initially at rest at $t = t_0$, i.e.,

$$x_{10} = \dot{x}_{10} = x_{20} = \dot{x}_{20} = 0$$

The expected value of $x_i(t)$ is $E[x_i(t)] \equiv m_{x_i}(t)$

$$\begin{aligned} m_{x_i}(t) &= \int_{t_0}^t \sum_{k=1}^2 h_i^{(k)}(t - \tau) m_{F_k}(\tau) d\tau \\ &\quad (i = 1, 2) \end{aligned} \quad (4-39)$$

$$\begin{aligned} &= \int_0^{t-t_0} \sum_{k=1}^2 h_i^{(k)}(\xi) m_{F_k}(t - \xi) d\xi \\ &\quad (i = 1, 2) \end{aligned} \quad (4-39a)$$

where $m_{F_k}(t)$ is the expected (mean) value of excitation $F_k(t)$.

The covariance kernel of the response is

$$K_{x_i x_j}(t_1, t_2) = E \left[\left\{ x_i(t_1) - m_{x_i}(t_1) \right\} \left\{ x_j(t_2) - m_{x_j}(t_2) \right\} \right] \quad (4-40)$$

$$= \int_{t_0}^{t_1} d\tau_1 \int_{t_0}^{t_2} d\tau_2 \sum_{k=1}^2 h_i^{(k)}(t_1 - \tau_1) \cdot \sum_{\ell=1}^2 h_j^{(\ell)}(t_2 - \tau_2) \cdot K_{F_k F_\ell}(\tau_1, \tau_2) \quad (4-40a)$$

$$= \int_0^{t_1-t_0} d\xi_1 \int_0^{t_2-t_0} d\xi_2 \cdot \sum_{k=1}^2 \sum_{\ell=1}^2 h_i^{(k)}(\xi_1) h_j^{(\ell)}(\xi_2) \cdot K_{F_k F_\ell}(t_1 - \xi_1, t_2 - \xi_2) \quad (4-40b)$$

where $K_{F_k F_\ell}(t_1, t_2)$ is the covariance kernel of the excitation defined as

$$K_{F_k F_\ell}(t_1, t_2) = E \left[\left\{ F_k(t_1) - m_{F_k}(t_1) \right\} \left\{ F_\ell(t_2) - m_{F_\ell}(t_2) \right\} \right]$$

Consider a class of excitations that can be expressed as

$$F_k(t) = d_k \ddot{S}(t)$$

$$F_\ell(t) = d_\ell \ddot{S}(t)$$

where

d_i = Amplitude level of excitation F_i

$\ddot{S}(t)$ = Excitation function

Let $\ddot{S}(t)$ be of the form

$$\ddot{S}(t) = g(t) n(t) \quad (4-41)$$

where $g(t)$ is an "envelope" function

$$g(t) = a_1 \exp(a_2 t) + a_3 \exp(a_4 t)$$

and $n(t)$ is Gaussian white noise.

Introduce the following notation:

$$\frac{d^P h_i^{(k)}(t)}{dt^P} = \begin{cases} h_i^{(k)}(t) & \text{if } P = 0 \\ \dot{h}_i^{(k)}(t) & \text{if } P = 1 \end{cases} \quad (4-42)$$

where

$$\dot{h}_1^{(k)} = \sum_{j=1}^4 s_j \lambda_{1j}^{(k)} \exp(s_j t)$$

$$\dot{h}_2^{(k)} = \sum_{j=1}^4 r_j s_j \lambda_{1j}^{(k)} \exp(s_j t)$$

Let parameter $^{(P)}Z_{in}^{(k)}$ be defined as follows:

Values of $^{(P)}Z_{in}^{(k)}$

	$P = 0$	$P = 1$
$i = 1$	$\lambda_{1n}^{(k)}$	$s_n \lambda_{1n}^{(k)}$
$i = 2$	$r_n \lambda_{1n}^{(k)}$	$r_n s_n \lambda_{1n}^{(k)}$

Using the definition of $^{(P)}Z_{in}^{(k)}$, Equation 4-42 can be expressed as

$$\frac{d^P h_i^{(k)}(t)}{dt^P} = \sum_{n=1}^4 {}^{(P)}Z_{in}^{(k)} \exp(s_n t) \quad (4-43)$$

and the covariance kernel of

$$x_i^{(P)} \equiv \frac{d^P x_i(t)}{dt^P} \quad \text{and} \quad x_j^{(Q)} \equiv \frac{d^Q x_j(t)}{dt^Q}$$

becomes

$$\begin{aligned} K_{x_i^{(P)} x_j^{(Q)}}(t_1, t_2) &= E \left[\left. \frac{d^P x_i(t)}{dt^P} \right|_{t_1} \left. \frac{d^Q x_j(t)}{dt^Q} \right|_{t_2} \right] \\ &= \int_0^{t_1-t_0} d\xi_1 \int_0^{t_2-t_0} d\xi_2 \\ &\quad \cdot \sum_{k=1}^2 \sum_{l=1}^2 \frac{d^P h_i^{(k)}(\xi_1)}{dt^P} \frac{d^Q h_j^{(l)}(\xi_2)}{dt^Q} \\ &\quad \cdot K_{F_k F_l}(t_1 - \xi_1, t_2 - \xi_2) \end{aligned} \quad (4-44)$$

$$(4-44a)$$

A typical term in Equation 4-44 is

$$\begin{aligned} T_a &= \int_0^{t_1-t_0} d\xi_1 \int_0^{t_2-t_0} d\xi_2 \frac{d^P h_i^{(k)}(\xi_1)}{dt^P} \\ &\quad \cdot \frac{d^Q h_j^{(l)}(\xi_2)}{dt^Q} K_{F_k F_l}(t_1 - \xi_1, t_2 - \xi_2) \end{aligned} \quad (4-45)$$

From the nature of the excitation,

$$\begin{aligned} K_{F_k F_l}(t_1, t_2) &= d_k d_l u(t_1) u(t_2) K_{SS}^{(k,l)}(t_1, t_2) \\ &= d_k d_l u(t_1) u(t_2) K_{SS}^{(k,l)}(t_1, t_2) \end{aligned} \quad (4-46)$$

where the covariance function for $\ddot{S}$ is

$$\begin{aligned} K_{SS}^{(k,l)}(t_1, t_2) &= g(t_1) g(t_2) E[n(t_1) n(t_2)] \\ &= g(t_1) g(t_2) \delta(t_1 - t_2) \end{aligned}$$

where $\delta(t)$ is the Dirac delta function.

Substituting Equation 4-46 and its covariance function into Equation 4-45 and making use of the definition of the delta function, then

$$\begin{aligned} T_a &= d_k d_l \int_0^{t_1-t_0} d\xi_1 \frac{d^P h_i^{(k)}(\xi_1)}{dt^P} \frac{d^Q h_j^{(l)}(\xi_2')}{dt^Q} \\ &\quad \cdot u(t_1 - \xi_1) g^2(t_1 - \xi_1) \end{aligned} \quad (4-47)$$

where

$$\xi_2' \equiv \xi_1 + t_2 - t_1$$

Making use of Equation 4-43, Equation 4-47 can be expressed as

$$\begin{aligned} T_a &= d_k d_l \sum_{n=1}^4 \sum_{m=1}^4 \int_0^{t_1} d\xi_1 \left[{}^{(P)}Z_{in}^{(k)} \exp(s_n \xi_1) \right] \\ &\quad \cdot \left[{}^{(Q)}Z_{jm}^{(l)} \exp(s_m \xi_2') \right] \\ &\quad \cdot g^2(t_1 - \xi_1) \end{aligned} \quad (4-48)$$

A typical term in Equation 4-48 is of the form

$$\begin{aligned} \tau_b = & \int_0^{t_1} d\xi_1 \quad (p) z_{in}^{(k)} \exp(s_n \xi_1) (q) z_{jm}^{(\ell)} \\ & \cdot \exp(s_m \xi_2) g^2(t_1 - \xi_1) \end{aligned} \quad (4-49)$$

Making use of Equation 4-41, Equation 4-49 becomes

$$\begin{aligned} \tau_b = & b_4 \int_0^{t_1} \left[a_1^2 \exp(b_6 \xi_1 + b_7) \right. \\ & + 2 a_1 a_3 \exp(b_8 \xi_1 + b_9) \\ & \left. + a_3^2 \exp(b_{10} \xi_1 + b_{11}) \right] d\xi_1 \end{aligned} \quad (4-50)$$

where

$$\begin{aligned} b_1 &\equiv t_1 & b_2 &\equiv t_2 \\ b_4 &\equiv \left[(P) z_{in}^{(k)} \right] \left[(q) z_{jm}^{(\ell)} \right] & b_5 &\equiv s_n + s_m \\ b_6 &\equiv b_5 - 2 a_2 & b_7 &\equiv s_m (b_2 - b_1) \\ & & & + 2 a_2 b_1 \\ b_8 &\equiv b_5 - (a_2 + a_4) & b_9 &\equiv s_m (b_2 - b_1) \\ & & & + (a_2 + a_4) b_1 \\ b_{10} &\equiv b_5 - 2 a_4 & b_{11} &\equiv s_m (b_2 - b_1) \\ & & & + 2 a_4 b_1 \end{aligned}$$

A typical term in Equation 4-50 is of the form

$$\begin{aligned} \tau_c = & \int_0^{c_3} \exp(c_1 x + c_2) \cdot dx \quad (4-51) \\ = & \begin{cases} \frac{\exp(c_2)}{c_1} \left(\exp(c_1 c_3) - 1 \right) & c_1 \neq 0 \\ c_3 \exp(c_2) & c_1 = 0 \end{cases} \end{aligned} \quad (4-51a)$$

Thus the evaluation of the general covariance function of Equation 4-44 is reduced to the summation of terms of the form of Equation (4-51a).

4.5.2.1 Mean-Square Response

Assuming for simplicity that the mean of the excitation is zero, the mean-square response of $x_i(t)$ is

$$\begin{aligned} E[x_i^2(t)] &= K_{x_i x_i}(t, t) \\ &= \sum_{k=1}^2 \sum_{\ell=1}^2 \int_0^t d\xi_1 \int_0^t d\xi_2 h_i^{(k)}(\xi_1) \\ &\quad \cdot h_i^{(\ell)}(\xi_2) K_{F_k F_\ell}(t_1 - \xi_1, t_2 - \xi_2) \end{aligned} \quad (4-52)$$

Similarly, the mean-square velocity is

$$\begin{aligned} E[\dot{x}_i^2(t)] &= K_{\dot{x}_i \dot{x}_i}(t, t) \\ &= \sum_{k=1}^2 \sum_{\ell=1}^2 \int_0^t d\xi_1 \int_0^t d\xi_2 \dot{h}_i^{(k)}(\xi_1) \\ &\quad \cdot \dot{h}_i^{(\ell)}(\xi_2) K_{F_k F_\ell}(t_1 - \xi_1, t_2 - \xi_2) \end{aligned} \quad (4-53)$$

4.5.2.2 Covariance Functions for Relative Motion

Let

$$y(t) = x_2(t) - x_1(t)$$

then

$$K_{yy}(t_1, t_2) = E[y(t_1) y(t_2)] \quad (4-54)$$

$$\begin{aligned} &= E\left[\left\{x_2(t_1) - x_1(t_1)\right\}\left\{x_2(t_2) - x_1(t_2)\right\}\right] \\ &= K_{x_2x_2}(t_1, t_2) - K_{x_2x_1}(t_1, t_2) \\ &\quad - K_{x_1x_2}(t_1, t_2) \\ &\quad + K_{x_1x_1}(t_1, t_2) \end{aligned} \quad (4-54a)$$

Therefore, the mean-square relative displacement is

$$\begin{aligned} E[y^2(t)] &= K_{yy}(t, t) \\ &= K_{x_2x_2}(t, t) - 2 K_{x_1x_2}(t, t) \\ &\quad + K_{x_1x_1}(t, t) \end{aligned} \quad (4-55)$$

$$\begin{aligned} &= E[x_2^2(t)] - 2 E[x_1(t) x_2(t)] \\ &\quad + E[x_1^2(t)] \end{aligned} \quad (4-55a)$$

Similarly, the mean-square relative velocity is given by

$$\begin{aligned} E[\dot{y}^2(t)] &= E[\dot{x}_2^2(t)] - 2 E[\dot{x}_1(t) \dot{x}_2(t)] \\ &\quad + E[\dot{x}_1^2(t)] \end{aligned} \quad (4-56)$$

where

$$\dot{y}(t) = \dot{x}_2(t) - \dot{x}_1(t)$$

4.5.3 NUMERICAL EXAMPLE

To illustrate this analysis procedure, consider an example system with the following parameters:

Mass ratio, $\mu \equiv m_2/m_1 = 0.10$

Frequency ratio, $\omega_2/\omega_1 = 1$

Ratio of critical damping in primary system (structure), $\xi_1 = 0.05$

Ratio of critical damping in auxiliary system (equipment), $\xi_2 = 0.05$

Natural frequency of primary system, $\omega_1 = 1$ unit

Mass of primary system, $m_1 = 1$ unit

$k_3 = c_3 = 0$

System is base excited; therefore, excitation (inertia) forces are proportional to the masses: $d_1 = 1$, $d_2 = \mu = 0.1$

The characteristic equation (Eq. 4-31) for the system is

$$\begin{aligned} 0.1 s^4 + 0.02 s^3 + 0.211 s^2 \\ + 0.021 s + 0.1 = 0 \end{aligned}$$

and its four (complex) roots are:

$$\begin{aligned} s_1 &= -0.03649 - 0.8535 i \\ s_2 &= -0.03649 + 0.8535 i \\ s_3 &= -0.06851 - 1.1685 i \\ s_4 &= -0.06851 + 1.1685 i \end{aligned}$$

Note that the real and imaginary parts of the roots correspond to the ratio of critical damping and the frequency, respectively, for

each of the two nonclassical modes of motion of the system.

Figure 4-29 shows the variation of the natural frequencies of the undamped coupled system as a function of system parameters. The four values of $r_j = (\lambda_2/\lambda_1)_j$ appearing in Equation 4-32 are:

$$\begin{aligned} r_1 &= 3.7 - 1.54 \times 10^{-6} i \\ r_2 &= 3.7 + 1.54 \times 10^{-6} i \\ r_3 &= -2.7 + 6.17 \times 10^{-6} i \\ r_4 &= -2.7 - 6.17 \times 10^{-6} i \end{aligned}$$

The values of λ_{ij} corresponding to a unit impulse at m_1 are

$$\begin{aligned} \lambda_{11}^{(1)} &= 1.64 \times 10^{-7} + 0.2472 i \\ \lambda_{12}^{(1)} &= 1.64 \times 10^{-7} - 0.247 i \\ \lambda_{13}^{(1)} &= -1.86 \times 10^{-7} + 0.2474 i \\ \lambda_{14}^{(1)} &= -1.86 \times 10^{-7} - 0.2474 i \end{aligned}$$

Similarly, for an impulse at m_2 ,

$$\begin{aligned} \lambda_{11}^{(2)} &= -8.57 \times 10^{-7} + 0.9149 i \\ \lambda_{12}^{(2)} &= -8.57 \times 10^{-7} - 0.9149 i \\ \lambda_{13}^{(2)} &= 8.57 \times 10^{-7} - 0.6683 i \\ \lambda_{14}^{(2)} &= 8.57 \times 10^{-7} + 0.6683 i \end{aligned}$$

The time histories of the impulse displacement and velocity (i.e., plot of $(P)Z_{in}^{(k)}$ as defined for Eq. 4-42) are shown in Figures 4-30 and 4-31. As a check, note that $\dot{h}_1^{(1)}(0) = \dot{x}_{10} = 1/m_1 = 1$, and $\dot{h}_2^{(2)}(0) = \dot{x}_{20} = 1/m_2 = 10$.

4.5.3.1 Stationary Excitation

Note from Equation 4-41 that, since the assumed form of the excitation is

$$\ddot{S}(t) = g(t) n(t)$$

$$= [a_1 \exp(a_2 t) + a_3 \exp(a_4 t)] n(t)$$

a stationary white-noise excitation with a constant power spectral density can be achieved by selecting the a 's in this equation to be

$$a_1 = a_0, \quad a_2 = a_3 = a_4 = 0$$

With this choice, the covariance function for $\ddot{S}$ becomes (see Eq. 4-46)

$$K_{\ddot{S}\ddot{S}}(t_1, t_2) = a_0^2 \delta(\tau) \quad (4-57)$$

where $\tau = t_1 - t_2$.

In view of the Wiener-Khintchine relations (Lin, 1977), the autocorrelation function $R(\tau) \equiv E[x(t) \cdot x(t + \tau)]$ and the spectral density $S(\omega)$ of a stationary random process are related by

$$R(\tau) = \int_{-\infty}^{\infty} S(\omega) \exp(i\omega\tau) d\omega \quad (4-58)$$

and

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) \exp(-i\omega\tau) d\tau \quad (4-59)$$

For ideal white noise with uniform spectral density S_0 , the autocorrelation function is

$$R(\tau) = 2\pi S_0 \delta(\tau) \quad (4-60)$$

Keeping in mind that for a zero-mean random process, the covariance and correlation functions are identical; then (from Eqs. 4-57 and 4-60),

$$a_0^2 = 2\pi S_0$$

$$\frac{\omega}{\omega_1} = \frac{1}{\sqrt{2}} \sqrt{1 + \left(1 + \frac{m_2}{m_1}\right) \left(\frac{\omega_2}{\omega_1}\right)^2} \pm \sqrt{\left[1 + \left(1 + \frac{m_2}{m_1}\right) \left(\frac{\omega_2}{\omega_1}\right)^2\right]^2 - 4 \left(\frac{\omega_2}{\omega_1}\right)^2}$$

FIGURE 4-29. NATURAL FREQUENCIES OF TWO-DEGREE-OF-FREEDOM SYSTEM (Crede, 1965)

FIGURE 4-30. IMPULSE DISPLACEMENT RESPONSE OF TWO-DEGREE-OF-FREEDOM SYSTEM

FIGURE 4-31. IMPULSE VELOCITY RESPONSE OF TWO-DEGREE-OF-FREEDOM SYSTEM

Response of a Single-Degree-of-Freedom System

The mean square $E[x^2]$ of the stationary response of an SDOF system to ideal white noise is (Lin, 1977)

$$E[x^2] = \sigma_1^2 = \frac{\pi}{2} \frac{S_o}{\zeta \omega_n^3} \quad (4-61)$$

It was determined by Caughey and Stumpf (1961) that the transient mean-square response of an SDOF system to stationary ideal white-noise excitation is given by

$$\sigma_x^2(t) = \frac{\pi S_o}{2 \zeta \omega_o^3} \left\{ 1 - \frac{\exp(-2\omega_o \zeta t)}{\bar{\omega}^2} \cdot \left[\bar{\omega}^2 + \omega_o \bar{\omega} \zeta \sin 2\bar{\omega} t + \frac{(2\omega_o \zeta)^2}{2} \sin^2 \bar{\omega} t \right] \right\} \quad (4-62)$$

where

$$\begin{aligned} \phi(\omega) &= 2 S_o \\ \bar{\omega} &= \omega_o \sqrt{1 - \zeta^2} \\ \omega_o &= \text{Undamped natural frequency} \end{aligned}$$

Figure 4-32 shows the transient response of an SDOF system under random excitation.

Results of the present analysis were used to determine the transient mean-square response of an SDOF system to stationary random excitation whose covariance function is given by Equation 4-57 (i.e., ideal white noise). This response, which is shown in Figure 4-33 for two damping values, is identical to that obtained by Caughey and Stumpf.

Response of Two-Degree-of-Freedom System

Consider a case where the excitation is a stationary random foundation motion with ideal

white-noise characteristics. The stationary value of the mean-square relative displacement of m_1 with respect to the foundation, and the stationary mean-square relative displacement of m_2 with respect to m_1 are shown in AA (1979a) for several combinations of system parameters. The ordinates of the plots have been normalized by σ_1^2 , the stationary mean-square response of m_1 in the absence of m_2 (see Eq. 4-61).

It is to be noted that even for a small mass ratio μ , the coupling effects of the equipment m_2 on the structure m_1 are not insignificant, especially in the region where the uncoupled frequency ratio is ~ 1 .

Results of the analysis agree with the work of Crandall and Mark (1963) and Curtis and Boykin (1961), who investigated the stationary response of a coupled two-degree-of-freedom system. Figures 4-34 and 4-35 provide examples of these displacements for a specific damping ratio.

FIGURE 4-32. TRANSIENT RESPONSE OF A DYNAMIC SYSTEM UNDER RANDOM EXCITATION (Caughey-Stumpf, 1961)

FIGURE 4-33. TRANSIENT MEAN-SQUARE RESPONSE OF AN SDOF SYSTEM TO STATIONARY RANDOM VIBRATION

FIGURE 4-34. STATIONARY MEAN-SQUARE RELATIVE DISPLACEMENT OF m_1 WITH RESPECT TO FOUNDATION WHEN EXCITATION IS IDEAL WHITE NOISE, $\zeta_1 = 0.01$, $\zeta_2 = 0.01$

FIGURE 4-35. STATIONARY MEAN-SQUARE RELATIVE DISPLACEMENT OF m_2 WITH RESPECT TO m_1 WHEN FOUNDATION EXCITATION IS IDEAL WHITE NOISE, $\zeta_1 = 0.01$, $\zeta_2 = 0.01$

4.5.3.2 Nonstationary Excitation

Consider a case where a nonstationary random foundation motion of the form

$$\ddot{S}(t) = g(t) n(t)$$

is applied with an envelope function $g(t)$ defined by

$$g(t) = e^{-0.1 t} - e^{-1.5 t}$$

and $n(t)$ is ideal white noise.

Response of a Single-Degree-of-Freedom System

Figure 4-36 shows the transient mean-square response of an SDOF system to nonstationary random excitation with intensity function as defined by $g(t)$ above.

Response of a Two-Degree-of-Freedom System

The transient mean-square relative displacements and velocities (that is, $E[y_1^2]$ and $E[y_2^2]$ and $E[\dot{y}_1^2]$ and $E[\dot{y}_2^2]$, respectively) of the multidegree-of-freedom system components with respect to the foundation as well as with respect to each other ($E[(y_2 - y_1)^2]$ and $E[(\dot{y}_2 - \dot{y}_1)^2]$) are shown in AA (1979a) for several combinations of system parameters. The shape of the envelope function is also plotted to indicate the amplification effects. Figures 4-37 and 4-38 show these displacements for specific values of system parameters.

The abscissas of the figures have been normalized by dividing the actual time t by the period T_1 of the primary system. The motion in each case was determined for the same length of time t ; consequently, the dimensionless time

(a)

(b)

FIGURE 4-36. TRANSIENT MEAN-SQUARE RESPONSE OF AN SDOF SYSTEM TO NONSTATIONARY RANDOM VIBRATION

FIGURE 4-37. MEAN-SQUARE RESPONSE OF 2 DOF SYSTEM, $\omega_1 = 0.5$

FIGURE 4-38. MEAN-SQUARE RELATIVE RESPONSE OF 2 DOF SYSTEM, $\omega_1 = 0.5$

axes vary by a factor of 3 from cases $\omega_1 = 0.5$ to $\omega_1 = 1.5$.

4.5.4 SUMMARY

The preceding numerical example illustrates the procedure for determining the transient mean-square response of a system when it is subjected to a certain class of nonstationary random excitations characterized by an intensity function $g(t)$ and a random component $n(t)$. If other types of nonstationary excitations are used, then the only effect on this type of analysis is to alter the nature of the covariance function for $\ddot{S}$ and its subsequent integrals.

Once the time history of the mean-square response is determined, the peak values of various response quantities can be found for a given set of parameters. By repeating this procedure with different sets of parameters (e.g., by changing ω_1), the desired response spectra can be constructed. For the reasons pointed out earlier, such spectra are "exact."

Although this probabilistic procedure may require more effort than do conventional techniques for the generation of floor response spectra, the nature (accuracy, margin of safety) and type (complete response time history, coupling, phase effects, etc.) of results more than compensate for the possible additional work. The main intent behind the derivation of the results given here is to illustrate a procedure for employing probabilistic methodologies to generate floor response spectra. Before the approach becomes practical for design engineers to use, more work is needed in the following areas:

- a. Parametric study of dynamic system characteristics
- b. Determining the covariance functions of representative earthquakes
- c. Use of more refined mathematical models for both structures and equipment
- d. Evaluating the range of validity of simplified approximate probabilistic solutions

CHAPTER 5

SEISMIC ANALYSIS PROCEDURES FOR IN-CORE COMPONENTS

5.1 INTRODUCTION

State-of-the-art analysis procedures for in-core components of liquid metal fast breeder reactors (LMFBR) are presented here. There are only a few references that directly relate to this problem, but there are a number of references that discuss analysis procedures for water and gas-cooled reactors. Due to this, a background of these analysis procedures is given before the discussion of procedures currently used for LMFBR in-core components. The objective is to determine whether the problems are comparable and whether a transfer of technology can be accomplished. The review for this study included discussions with technical personnel engaged in design analyses of reactor in-core components.

5.2 STATE-OF-THE-ART PROCEDURES

5.2.1 WATER-COOLED REACTORS

A description of a typical analytical procedure used to predict the loads experienced by fuel elements and their structural supports in a boiling water reactor (BWR) in a seismic environment has been described by Liu (1971). A longitudinal section through such a reactor is shown in Figure 5-1. The weight of the cylindrical pressure vessel and its contents is supported by a vessel skirt that is in turn supported on a concrete pedestal. A system of stabilizer bars near the top of the vessel resists vessel movement in the horizontal direction. Lateral support at the base of the vessel is provided by the skirt and pedestal. The core shroud inside the pressure vessel is a stainless steel cylindrical assembly that provides

FIGURE 5-1. LONGITUDINAL SECTION THROUGH A BWR (Liu, 1971)

a partition to separate the upward flow of coolant through the core from the downward recirculation flow. The shroud head and steam separator assembly is bolted to the top of the shroud that is supported by the pressure vessel through the shroud support assembly.

The steam dryers, which remove moisture from the wet steam exiting from the steam separators, are supported directly by the

pressure vessel. The reactor core, consisting of several hundred fuel bundles, is supported laterally by the top guide and core plate. In the vertical direction, the fuel bundles are supported by control rod guide tubes. The guide tubes, in turn, are supported by the pressure vessel. The control rod guide tubes are designed as lateral guides for the control rods. Attached to the bottom of the pressure vessel are control rod drive housings in which the control rod drives are located. Due to the curvature of the bottom head, the control rod drive housings have variable lengths.

The mathematical model of the BWR is shown in Figure 5-2. This is a lumped mass model in which the concentrated masses are connected by beam and spring elements. The dynamic coupling effect due to fluid structure interaction was taken into account by adding hydrodynamic masses to the mass matrix. The hydrodynamic masses were computed by considering two infinitely long concentric circular cylinders with nonviscous fluid in the annulus between them. In order to ensure that the hydrodynamic mass was positive-definite, an energy approach was used in the calculations. The hydrodynamic mass matrix calculated contained diagonal as well as off-diagonal terms. The BWR internals represent structural members that exhibit nonlinearities due mainly to gaps and frictional forces. However, the nonlinear effects were ignored in this analysis on the assumption that these effects were small and that the linear analysis could be used to predict seismic response. Table 5-1 provides a comparison of basic LMFBR and BWR fuel assemblies.

A typical seismic analysis of the reactor internals of a pressurized water reactor (PWR) has been discussed by Bohm (1971). Figure 5-3 shows the arrangement of the internals for the PWR system. The outer structure is the reactor vessel, which supports and contains the internal support structure and the core.

FIGURE 5-2. MATHEMATICAL MODEL USED IN DYNAMIC ANALYSIS OF BWR INCLUDING INTERNALS (Liu, 1971)

FIGURE 5-3. REACTOR VESSEL AND INTERNALS OF A PWR SYSTEM (Bohm, 1971)

TABLE 5-1. COMPARISON OF BASIC BWR AND LMFBR FUEL ASSEMBLY SEISMIC PARAMETERS

	BWR	LMFBR
Weight, lb	615	400
Length, in	165	168
Bending Rigidity, lb-in ²	1.1×10^8	1.5×10^8
Clearance, in	0.75	0.19
Method of Lateral Support		
Lower End	Conical seat	Two cylindrical seats
Above Core	None	Peripheral former
Top	Upper Grid	Peripheral former

The vessel is characterized by its thick-walled cylindrical shape, its domed lower section, its inlet and outlet nozzles, and its mating flange surface. The core barrel, which is the main core support structure, is shown immediately inside the vessel. The barrel is attached to the vessel at two locations: at the core barrel axial support flange near the top of the vessel, and at the core barrel radial support located just above the lower vessel dome. The lower radial support is a clevis-like arrangement, providing support in the horizontal or tangential direction but allowing relative thermal expansion in the vertical and radial directions. The core barrel is a continuous structure from the vessel flange down to the lower radial support. The thermal shield, a cylindrical shell, is located between the vessel and the core barrel and is fixed to the barrel at discrete points at top and bottom.

The fuel assemblies are located inside the core barrel. These assemblies are attached to the core barrel through the upper and lower core plates which position and support the assemblies. The core plates transmit all the horizontal forces between the assemblies and the barrel, providing the fuel assemblies do not contact each other or the barrel. The horizontal reaction between the barrel and the

vessel is transmitted at the barrel/vessel flange and the core barrel lower radial support. This lower radial support is a custom fitted element that ensures minimum gap and maximum support between the vessel and the barrel.

Figure 5-4 shows the mathematical model used for the dynamic analysis of the internals. This model was used for a linear analysis of the system, which was performed to obtain information about the natural frequencies of the system and about the closing of gaps among various elements. The containment building with the reactor vessel support, the reactor vessel, and the reactor internals were included in the analysis. A horizontal component of the El Centro (1940) earthquake was used as input to the model. The linear analysis indicated that the relative displacements were large enough to close the gaps and that significant impact forces would be generated.

A nonlinear model of the reactor internals was also developed which was similar to the linear model, except that gapped springs were included to simulate impact forces between the elements. The energy losses produced during the impact were simulated by a viscous damping mechanism based on an assumed restitution coefficient. The model shown in Figure 5-5 was subjected to the same El Centro earthquake

FIGURE 5-4. MATHEMATICAL MODEL USED FOR DYNAMIC SEISMIC ANALYSIS OF PWR INTERNALS (Bohm, 1971)

FIGURE 5-5. TYPICAL REACTOR INTERNAL CONFIGURATION USED FOR NONLINEAR ANALYSIS (Bohm, 1971)

input motion. The results of the nonlinear analysis indicated that the response was characterized by high frequency oscillations due to impact. Further, the assemblies tended to oscillate as a group. As expected, below certain maximum ground acceleration values, no impact occurred and the nonlinear response was similar to the linear response.

The nonlinear model was also excited in the vertical direction by an artificial earthquake motion. In the vertical direction, the reactor internals form a highly nonlinear system with gaps and friction devices producing Coulomb type damping. Except for the high frequency oscillations that are superimposed, the response is quite similar to the input excitations. It was concluded from the analysis that the internals behave as a one-degree-of-freedom system under the vertical motion.

5.2.2 GAS-COOLED REACTORS

The core of a high temperature gas cooled reactor (HTGR) is significantly different from that of a LMFBR. Nevertheless, the seismic analysis practice for HTGR cores was examined to determine if there were principles that could be applied to the seismic analysis of a LMFBR. The HTGR core consists of many thousand hexagonal graphite elements. These individual elements are stacked to form several hundred fuel columns. In each column, the stacked elements are constrained by dowels placed at the element-to-element interfaces as shown schematically in Figure 5-6. The restraining effects of the dowels prevent relative horizontal movement, but allow vertical and rocking motions between elements. The fuel columns are separated from one another by gaps. During an earthquake of sufficient magnitude to cause rocking, the motions of the doweled elements interact with those of the adjacent columns or boundary structures through impact.

The dynamic response of an HTGR core has been studied by using an analytical model to characterize the motions of the fuel elements in a vertical plane of the core (Lee and Wesley, 1975). The dynamic model, which is intrinsically nonlinear, consists of a series of interacting fuel columns arranged in a plane with boundary structures at the two ends simulating the reflector columns (see Fig. 5-6). The reflector columns were modeled both as dissimilar columns and as boundary walls having prescribed input motion. The theoretical solution of this problem involves a very large number of degrees of freedom which must be handled in a direct integration process. The required numerical computations were made tractable by using only the rigid-body displacements of each fuel element center of gravity as coordinate variables, thereby minimizing the total number of coupled equations to be integrated. This was done by treating the elements as rigid blocks with only localized surface flexibility represented by discrete spring elements. The locations of the surface springs were determined from the mode of contact during impact. In general, discrete springs at the block corners were required. Springs were also placed at appropriate intermediate locations to improve the stiffness properties for flat-to-flat impacts. All springs had associated viscous dampers that were assumed to be joined in parallel. Unlike conventional spring-mass systems, this model considers the motion of an element due to its own weight, such as rocking on top of another. The nonlinear governing equations were derived, using a Lagrangian formulation, with consideration of large rotational effects.

Analytical results for a single block and a single column were compared with values obtained experimentally. The agreement between the test data and analytical results was indicated to be very good; analytical results

FIGURE 5-6. THE HTGR CORE IN A VERTICAL PLANE (Lee and Wesley, 1975)

were also obtained for the response of a multi-column system to artificial earthquake motions. However, no test data were made available for comparison purposes in the publications.

5.2.3 LIQUID METAL FAST BREEDER REACTORS

Figure 5-7 provides a sectional view of a LMFBR system, and Figure 5-8 shows the general core and core restraint arrangement; the fuel assembly is shown in Figure 5-9. The core consists of hexagonally shaped assemblies, or

elements, made of stainless steel material, each assembly contains a bundle of fuel rods also clad with stainless steel. The assemblies are arranged on a triangular pitch consisting of several rows so that the complete assembly resembles a hexagon. If the center element is considered to be Row 1, then Row 2 consists of six elements surrounding the center element. Row 3 consists of 12 elements and so on. The number of elements in the last, or outermost, row may be reduced either to obtain a desired total number of elements or to provide a nearly circular layout. The total number of assemblies

FIGURE 5-7. LIQUID METAL FAST REACTOR SYSTEM (Morrone et al., 1976)

FIGURE 5-8. CORE RESTRAINT LAYOUT

FIGURE 5-9. TYPICAL FUEL ASSEMBLY

in a typical reactor may be several hundred. The assemblies are placed inside the core barrel. The outside row is supported by two load pads, one at the top and the other at an intermediate point. The assemblies are separated from each other and from the load pads by small gaps that are filled with liquid sodium.

Since there are few LMFBRs in operation throughout the world, the literature on the seismic analysis procedures for LMFBR in-core components is scarce. Morrone et al. (1976) describes the seismic analysis, including in-core components, of a LMFBR, which was performed to determine the effect of the seismic impact

loads on the scram time, and to determine the seismically-induced bending moments, accelerations, and forces developed in the in-core components and on the overall reactor system.

The mathematical model developed by Morrone et al. is shown in Figure 5-10. The model is one dimensional, with two degrees of freedom per node (a horizontal translation and a rotation about the normal to the plane of the figure). The model shown in Figure 5-10 gives a two-dimensional representation for clarification and shows the gaps and springs between nodes. The legend identifies the symbols used to define the types of nodes, elements, gaps, springs, and concentrated masses and rotary inertias. It should be noted that beam elements were used to represent most structural components and that the model does not consider fluid/structure interaction effects, except that an equivalent sodium mass was lumped along with the total inertias of the lower head at Nodes 5 and 8 of the reactor vessel shell.

The response of the system was computed using a special purpose computer program DARI (Dynamic Analysis of Reactor Internals), which is a proprietary two-dimensional finite element program. This program computes the nonlinear response of a structural system idealized by a combination of beam elements, pin joint elements, rocking springs, and impact springs. The horizontal seismic motion was applied in the analysis at Node 1, which represents the reactor vessel head support on the containment building. The input motion was obtained as the response at the reactor vessel supports from a finite element analysis of the containment building and foundation system. The input motion used in the analysis is shown in Figure 5-11. Only the first 10 sec of the original 20-sec record as shown in the figure were input to the program since the accelerations after 10 sec are insignificant. The impact force when a gap closes is computed in the code

FIGURE 5-10. MATHEMATICAL MODEL FOR SEISMIC AND SCRAM ANALYSIS
(Morrone et al., 1976)

FIGURE 5-11. SSE HORIZONTAL ACCELERATION - TIME HISTORY AT REACTOR VESSEL SUPPORTS (Morrone et al., 1976)

by using the following expression

$$F = KX + C\dot{X} \quad (5-1)$$

where

- F = Nodal impact force
- K = Impact spring constant
- C = Impact damping constant
- X = Penetration distance
- $\dot{X}$ = Relative velocity at gap

The penetration distance X is obtained as the difference between the available gap clearance and the relative displacement between the nodes on either side of the gap.

The impact spring constants for most of the components with cylindrical cross section were obtained based on the work of Bijlaard (1954), which considers the impact to occur

over a finite area of the cylinder caused by local deformation. For the components in the core region, which are mostly of hexagonal cross section, the impact spring constants were based on results of load-deflection data and from separate analyses. A coefficient of restitution of 50% was selected for impact damping to obtain the fraction of critical damping at impact. Based on an analytical expression, the critical damping β was computed to be about 10% in DARI. The impact damping constant C was then calculated as

$$C = 2\beta \left[\left(\frac{m_1 m_2}{m_1 + m_2} \right) K \right]^{1/2}$$

where m_1 and m_2 are nodal masses on each side of the gap.

In addition to impact damping, the computer program calculates the nodal damping constants from

$$C_t = 2\beta\omega M$$

$$C_y = 2\beta\omega I$$

where C_t and C_y are the nodal damping constants for translation and rotation, respectively; β is the fraction of critical damping; ω is the component natural frequency; M is the nodal mass; and I is the nodal rotary inertia. For example, the computed relative displacement response and the associated impact forces between the center of driveline (Node 19) and the control rod guide tube (Node 16) were computed for 5-sec duration with an integration time step of 25 μ sec and are shown in Figure 5-12 for the first 2.5 sec.

From the computed seismic response time histories of impact forces for the control rods, scram times were established using a separate computer program specifically developed for scram time calculations. This program employs a numerical solution to calculate the time elapsed, after initiation of control rod scram, for the rod to travel a certain distance. The program considers the effects of retardation forces on free fall such as fluid drag force, friction forces caused by seismic impact, dashpot force, and buoyant force. It accepts directly the necessary information from the tape generated by the DARI program. Figure 5-13 shows an example plot from the program for the 1.3 sec and 0.4 sec scram initiation times. This figure gives the longest time for the rod to travel, as a function of the rod travel distance. The scram time profile curve for normal operating conditions is also shown in the figure to illustrate the influence of the earthquake disturbance on scram time.

Since the above analysis was performed with a proprietary computer program, it was not possible to evaluate the capability of the

FIGURE 5-12. RELATIVE DISPLACEMENTS AND ASSOCIATED IMPACT FORCES AT CENTER OF DRIVELINE (NODE 19). (a) Relative displacement; and (b) impact force. (Morrone et al., 1976)

program based on its formulation procedures. However, some of the simplifying modeling assumptions that should be validated are evident. For example, the mathematical model used in the analysis is one dimensional; therefore, two- and three-dimensional effects were ignored. Also, the mathematical model considers only horizontal motions. Thus, to compute the response from the vertical component of input motion, a separate model is required. As a result, any coupling between the horizontal and vertical response is not considered. The mathematical model represents all of the fuel assemblies as two elements. This implies that half of

FIGURE 5-13. SCRAM TIME VERSUS ROD TRAVEL DISTANCE
(Morrone et al., 1976)

all the fuel assemblies act as a unit and no impact forces are generated between them. As reported later, even though experiments show that the inner assemblies do move together, it is known that the outermost row of elements behave independently. Also, the effect of fluid/structure interaction on the response of the core structure was not considered in the above analysis. While the inertia effect of the coolant between assemblies in an LMFBR may not have a significant effect on the scram time, the inertia effects would make a major contribution to the core dynamics. A study of the vibration of closely spaced rods indicates, when extrapolated to a typical LMFBR core, that this is the dominant inertial effect (Chen, 1975). Finally, the analysis did not consider the effect of Coulomb friction on the load pads. Low quantities of oxygen in the environment leads to

high friction coefficients, which leads to non-linear response. This effect should be considered, or its omission justified by analysis or tests.

A different approach to the seismic analysis of in-core components of a LMFBR is based on the Rayleigh-Ritz method, where a number of modes of deformation of the reactor core under the seismic loads are assumed (Moran, 1976). The assumed modes are then utilized to obtain simplified equations of motion governing the response of the core structure. The advantage of this method is that if the deformation modes of the core structure can be approximated with only a few degrees of freedom, then only these few pertinent degrees of freedom will be used in the analysis, which greatly simplifies the calculations. The assumed mode approach

uses a Lagrangian method in forming the equations of motion for the reactor core subjected to seismic motions. The basis of the method is the assumption that the reactor core deformation can be approximated by N deformation functions w_j . Using these deformation functions, various energy quantities of the core can be formed, including the kinetic and potential energy of the core, the kinetic energy of the fluid, and the potential energy of the core due to local deformations at contact (due to impacting). The various energy expressions permit the formation of the N equations of motion using the Lagrangian method. The generalized forces appearing to one side of these equations will consist of various dissipative forces such as structural damping, viscous damping of fluid, and friction forces, and the input seismic motions.

To illustrate the procedure, a demonstration size LMFBR as a sample problem was considered. The reactor core consisted of 691

elements arranged in sixteen hexagonal rows. The elements were 4.5 m in length, were pinned at the bottom, and had load pads at the top and at 2.7 m from the bottom. A sample problem was solved using two models with different numbers of degrees of freedom (see Fig. 5-14). Both models considered the impact of the elements due to the closing of gaps. The first model considered only a single degree of freedom. A central group of 625 elements was assumed to pivot in unison about their support points while the elements in the outermost row were assumed to contact the inner group at the load pads, and to bend as the inner elements rotated about the pivot. The model was subjected to the artificial horizontal motion given in Figure 5-15. The equations of motion were integrated using a step-by-step procedure. Figure 5-16 gives the displacement response time history for the fuel assembly, and Figures 5-17 and 5-18 show contact force time histories at top load plane (TLP) and at above-core load plane (ACLP) locations, respectively.

FIGURE 5-14. DIAGRAM OF LMFBR CORE MODEL (Moran, 1976)

FIGURE 5-15. CORE SUPPORT INPUT TIME HISTORY (Moran, 1976)

FIGURE 5-16. ONE-DEGREE-OF-FREEDOM DISPLACEMENT RESPONSE (Moran, 1976)

FIGURE 5-17. ONE-DEGREE-OF-FREEDOM CENTRAL SECTOR/RIGHTHAND SECTOR TLP FORCE (Moran, 1976)

FIGURE 5-18. ONE-DEGREE-OF-FREEDOM RIGHTHAND SECTOR/RESTRAINT RING ACLP FORCE (Moran, 1976)

The second model considered a total of five degrees of freedom. These were introduced assuming the center sector of 625 elements to move only by rotating about its base, but the outer row was divided into two sectors of 33 elements, each of which was allowed to move independently of the other elements and of the center sector. The modes of these outer sectors consisted of a rotation about their base and a first bending mode. This model was subjected to the same horizontal input time history given in Figure 5-15. Figures 5-19, 5-20, and 5-21 give the displacement response of the fuel assembly in the first, second, and third modes of vibration, respectively. A comparison of Figure 5-16 and Figures 5-19, 5-20, and 5-21 indicates that the time histories in both cases are similar, which means that the simple one-degree-of-freedom model is a good approximation of the response in this case. Figures 5-22 and 5-23 show impact forces at TLP and ACLP locations. These figures when

compared to Figures 5-17 and 5-18 indicate that the forces generally look similar, except the impact forces for the five-degrees-of-freedom model exhibit high frequency components. This is to be expected as the five-degrees-of-freedom model is capable of transmitting higher frequencies than the single-degree-of-freedom model.

The assumed mode approach by Moran appears to be an effective approach to the problem of seismic analysis of LMFBR in-core structures. The method is flexible and can take into consideration all the important phenomena governing the seismic response of a reactor core structure. As explained above, the method can generate either simple or complex mathematical models, as the problem requires. The method includes in the analysis such important factors as the fluid/structure interaction effects, impact forces, fluid damping, and frictional forces. The method is also capable of

FIGURE 5-19. FIVE-DEGREES-OF-FREEDOM FIRST MODE RESPONSE (Moran, 1976)

FIGURE 5-20. FIVE-DEGREES-OF-FREEDOM SECOND MODE RESPONSE (Moran, 1976)

FIGURE 5-21. FIVE-DEGREES-OF-FREEDOM THIRD MODE RESPONSE (Moran, 1976)

FIGURE 5-22. FIVE-DEGREES-OF-FREEDOM CENTRAL SECTOR/RIGHTHAND SECTOR TLP FORCE (Moran, 1976)

FIGURE 5-23. FIVE-DEGREES-OF-FREEDOM RIGHTHAND SECTOR/RESTRAINT RING ACLP FORCE (Moran, 1976)

constructing models to consider three-dimensional effects. At present the major drawback to this method is that it requires the analyst to derive various energy expressions and to form the governing equations of motion using the Lagrangian approach. This is time consuming and generally leads to errors and omissions, especially by an inexperienced analyst. However, a proposed computer program for seismic analysis of reactor core structures using this method is being developed by Moran at Argonne National Laboratory (see Sec. 5.3.3 below).

5.2.4 RESULTS OF EXPERIMENTAL TEST PROGRAMS

A literature search was made of experimental testing programs that determine or verify the seismic response of in-core components of reactors. Only one of significance was found, and even then, the release of information in the report as well as its identification is restricted. The following paragraph summarizes the conclusions that were drawn from the experimental test program that can be reported.

The results from the experimental tests were compared with analytical calculations based on a single-degree-of-freedom model in which the reactor core composition elements were allowed to rotate at the base. The calculations also considered impact of the elements. The results of the tests and calculations agreed well, especially for the elements at the center of the core. Important conclusions of the study were that the core composition element group moves as one body instead of each element moving independently. Thus, the response of the core is essentially in the fundamental mode and a simple one-degree-of-freedom model may predict the response reasonably well. The response of the core elements was found to be linear for low level input excitations, but was nonlinear for high level excitations. This was

attributed mainly to the fact that impacting of the elements takes place only under higher levels of excitation. The response of the core elements was also found to be dependent on the input wave forms, as would be expected. The response due to the El Centro motions was significantly higher than the response due to the Golden Gate motions when both motions were scaled to the same peak acceleration value. However, the response spectra for the Golden Gate motions are higher in the 4 to 15 Hz frequency range.

5.2.5 SUMMARY

If the analyses by Morrone et al. represent the state of the art for the seismic system analysis of LMFBR in-core components, then the level of analytical sophistication is comparable to that which has been demonstrated for water-cooled reactors (Bohm, 1971). Because of basic differences in the fundamental concepts of the two systems, seismic analysis procedures used for HTGR in-core components are not comparable to those required for the LMFBR, and no transfer of technology is foreseen from past experience with the HTGR.

Both one- and two-dimensional analyses have been provided for water and sodium cooled reactors, with uncoupled horizontal and vertical input being applied to separate models. Non-linear response due to impact between the various elements within an individual fuel assembly has been represented by gapped springs, and the energy loss due to impact has been represented by a viscous damping mechanism based on an assumed coefficient of restitution. Impact between individual fuel assemblies, however, has not been considered nor have the inertia effects of the fluid coolant on the fuel assemblies. It is known that the coolant between full assemblies of a LMFBR has a dominant inertia effect because of the close

spacing of assemblies (Chen, 1975). The effects of Coulomb friction on the load pads has not been considered.

The results of restricted experimental model tests indicate that the response of the core element group is essentially in one fundamental mode, and that a simple one-degree-of-freedom model may predict the response reasonably well. However, the response was found to be nonlinear at higher levels of input excitation due to impacting of the individual fuel assemblies.

A more sophisticated analytical approach for LMFBR in-core components is based on the Rayleigh-Ritz method, in which the modes of deformation of the reactor core under the seismic loads are assumed (Moran, 1976). The assumed modes are then utilized to obtain simplified equations of motion governing the response of the core structure. The method is flexible and can take into consideration all of the important phenomena governing the seismic response of the in-core components, including fluid structure interaction, impact forces, fluid damping, and frictional forces. The method can also include three-dimensional effects.

5.3 COMPUTER PROGRAM CAPABILITY

Since considerable improvement in mathematical modeling procedures are needed in order to predict the seismic response of LMFBR in-core components, the question is whether improved modeling can be accomplished with existing computer programs, or whether improved computer capability is required and must be developed. First, a review of the fundamental problem areas is presented here.

5.3.1 FUNDAMENTAL PROBLEM AREAS

The summary of the previous section indicated that three-dimensional rather than one-dimensional mathematical models are needed.

Important phenomenology includes the impacting of element assemblies with load plates and other element assemblies, the effect of frictional energy losses, and fluid/core element interaction. The impact and functional aspects of the problem induce nonlinear behavior. Hence, nonlinear mathematical models will be required, which also means time step integration.

Impact. Nonlinear dynamic problems involving impact of members are very complex and generally must be solved using computer programs. The computer programs generally simulate the impact phenomena by introducing gapped springs with large stiffnesses. As long as the gaps are open, the springs are inactive, but when the gaps close, large spring forces are suddenly experienced. Because of the large spring stiffness, impact forces are characterized by high-frequency components. To obtain a numerically stable response of the system, a proper choice of the integration time step is necessary. If a large time step is chosen, round-off errors will be amplified and the solution becomes unstable. If the time step selected is small, the computer running time becomes too long and uneconomical. For this type of problem, the integration time step for stable solutions needs to be very small. Computer programs used for these problems should have the capability to vary the integration time step as the solution progresses. Thus, the time step can be increased when there is no impact and the response is moderate. This significantly reduces the computer running time.

Frictional Forces. The seismic analysis of LMFBR in-core components should consider the effect of friction at the load pads on the response. Low quantities of oxygen due to the presence of liquid sodium leads to high friction coefficients between the contacting surfaces. This phenomena can be represented mathematically by Coulomb friction. As with impact,

frictional effects can be included in the analyses only through nonlinear mathematical models of the core. The contact pattern among the elements and the resulting frictional forces form a three-dimensional configuration, so a three-dimensional mathematical model is required for an accurate analysis. However, state-of-the-art analyses reported have not considered the frictional effects. This may be due partly to the fact that the inclusion of frictional effects makes the analysis more complex. Another reason for ignoring friction is the assumption that the frictional effects are not important; however, no investigations have been reported to confirm this assumption. The experimental results reported so far also have not addressed this problem. However, experiments seem to indirectly indicate that the frictional effects may be small. Additional data, either experimental or analytical, are needed to verify this conclusion. Therefore, it should be assumed at this time that an adequate computer code should be able to address this problem.

Fluid Interaction. A study of the vibration of closely spaced rods in fluid indicates that, when extrapolated to a typical LMFBR core, the inertia of the coolant between the assemblies has great influence on the response of the core structure (Chen, 1975). Studies have shown that this influence is proportional to the inverse of the distance between the rods (Fritz, 1972; Moretti and Lowery, 1976). Therefore, to determine the seismic response of the in-core components reliably, the interduct fluid coupling must be taken into account. Any solution that considers the hydrodynamic equations along with the equations of motion for the structure becomes very complex. In the past, some structural problems that include hydrodynamic interaction effects have been solved in this fashion. These include the studies by Chakrabarti and Chopra (1972) for solving hydrodynamic interaction problems involving

dams, and by Liaw and Chopra (1973) for intake towers. Solutions of this type are not feasible for the reactor in-core components. However, the effects of fluid interaction can be approximated by the inclusion of added (apparent, virtual) mass in the mass matrix of the structure. Physically, the added mass effect is due to the force that an accelerating solid body imparts to the fluid in which it is submerged. This force is a function of the acceleration of the body and is different from the drag force, which depends on the velocity of the solid body. Considerable work has been done to define this phenomenon. A brief review follows.

For a single solid body vibrating in a fluid, the response of the solid body can be studied independently by adding a scalar added mass term to the mass of the body. When several structural components are submerged in the same fluid, the coupled response should be treated by extending the concept of the scalar added mass to the added mass matrix. Several investigators have shown that the added mass matrix is not diagonal (e.g., Fritz, 1972). This implies that there exists cross coupling between the structural components as far as the fluid inertia effects are concerned.

For the seismic response of a BWR, the added mass matrix for long concentric circular cylinders with fluid in the annuli was derived by Liu (1971). An energy approach was used in the derivation to ensure that the mass matrix is positive-definite. The results can be extended to cylinders of finite length by applying certain correction factors. However, no discussion was provided to obtain these factors. Levy and Wilkinson (1975) assumed that the motion of the fluid can be represented by the potential flow theory of an incompressible fluid. For this case, the fluid pressure is represented by Laplace's equation, whose solution provides the information for computing the added mass

matrix. The finite element procedure is extended to potential flow problems to solve Laplace's equation for irregular boundaries. The added mass matrices for several configurations, namely two concentric cylinders, two eccentrically placed cylinders, and four square beams in a cylinder, have been computed using the finite element procedure. The finite element results were compared with the closed-form values for the case of the two concentric cylinders, and the results agreed within one percent.

The computer program AMASS was developed for calculating added mass coefficients for a group of circular cylinders based on the potential flow theory (Chen, 1975). Free and forced vibrations of a group of rods including fluid coupling were studied using this program to calculate the added mass matrix. Numerical results were presented for a group of seven rods vibrating in fluid. A series of tests involving a group of tubes in stationary fluid and flowing fluid can verify the theory.

Drag Forces. The added mass discussed above accounts for the inertial effect of the fluid and is a function of the acceleration of the core structure. The fluid also exerts a drag force on the body. However, the drag force depends on the velocity of the structure, the fluid viscosity, and possibly two characteristic distances such as the gap size and the width of the duct flat. An obvious way to account for the drag force in the analysis is to represent it by linear dampers in the mathematical model of the core structure. The existing analyses do not appear to consider drag forces. It is believed that these analyses lead to conservative results.

Structural Damping. The structural damping of the various fuel, control, blanket, and shielding assemblies in a LMFBR probably is

significantly greater than in normal steel structures; hence, higher damping values may have to be used in the analysis. Also, the normal use of "modal critical damping ratios" is not appropriate for nonlinear analyses of reactor cores. Such damping in nuclear reactor structures, including the forms of damping that can be used with each type of analysis, is given by AA (1980), which also provides procedures for finding the equivalent damping values in one form when the given values are in a different form.

5.3.2 ASSESSMENT OF COMPUTER PROGRAMS

Several general purpose finite element computer programs are available for use in seismic analyses of reactor core structures. Some are proprietary, and the user may be required to pay royalties based on the amount of computer time. Representative programs that are judged to be useful in seismic analyses of reactor core structures are shown in Table 5-2. Many of these codes are being updated constantly.

The similarities of general purpose finite element computer programs can be seen in Table 5-2. Of the available general finite element programs, only ANSYS and MARC have been generally used for seismic analyses of reactor in-core components. ANSYS and MARC are general purpose finite element computer programs for large structures. Both programs are proprietary and are available to the general public only on a royalty basis. These programs are capable of solving nonlinear dynamic problems with three-dimensional geometry. These programs provide nonlinear springs for simulating gaps. ANSYS also includes frictional elements to simulate frictional surfaces, but MARC does not appear to have this capability. Both programs use step-by-step integration procedures to solve the nonlinear equations of motion.

TABLE 5-2. THREE-DIMENSIONAL NONLINEAR FINITE ELEMENT COMPUTER PROGRAMS

Items for Comparison	Finite Element Programs			
	ANSYS (Swanson, 1973)	SAGA (AA, 1976c)	MARC (MARC-CDC, 1974)	NONSAP (Bathe et al, 1974)
Static Analyses	Linear, thermal plastic, buckling	Linear, Nonlinear materials and geometry, Thermal, buckling	Linear, Nonlinear materials and geometry, Thermal	Linear, Nonlinear materials and geometry, Thermal
Eigenvalues and Eigenvectors	Yes	No	Yes	No
Dynamic Analysis	Modal Linear Nonlinear Transient Explicit Harmonic response	Step-by-Step Implicit Explicit Linear Nonlinear Damping	Step-by-Step Implicit Linear Nonlinear Modal	Step-by-Step Implicit Linear Nonlinear
Element Types	Truss Beam Plate Shell Solid 2-Dimensional Special	Yes Yes Yes Yes Yes Yes Ship, reinforced concrete	Yes Yes Yes Yes Yes yes	Yes No No No Yes Yes No
Loading	Nodal point Member Gravity	Yes Yes Yes	Yes Yes Yes	Yes Yes Yes
Boundary Conditions	Static Dynamic	Displacement Displacement, velocity, and acceleration	Displacement Displacement, velo and accel Quiet boundary	Displacement Displacement
Max No of Nodal Points or Joints	Dynamic allocation	Dynamic allocation	Dynamic allocation	Dynamic allocation
Max No Elements	Not limited	Dynamic allocation	Not limited	Dynamic allocation
Max 1/2 Bandwidth	Uses wavefront tech	Dynamic allocation	Not limited	Dynamic allocation
Max Load Cases	Not limited	Not limited	Not limited	Not limited
Max No of Materials	Not limited	Not limited	Not limited	Dynamic allocation
Max No of Cross Sections	Not limited	Not limited	Not limited	Dynamic allocation
Graphic Output	Yes Yes Yes Yes Yes	Yes Yes No Yes No Deformed mesh	Yes Contour plots	No
Automatic Mesh Gen	Yes	Yes (sophisticated)	Yes	Yes
Bandwidth Minimization	Uses wavefront tech	Yes	Yes	No
Constrained D O F (Slaving)	Yes	Yes	Yes	Yes
Automatic Design of Steel and Concrete Frames	No	No	No	No
Special Features	Early conversion of Cartesian to cylindrical or polar coordinates Large deflections Extensive choice of elements	Sophisticated mesh generation allows easy user interface		

Fluid inertia effects are currently being treated in one of two ways. The first approach is to neglect the fluid interaction effects on the assumption that this provides a conservative answer. One consequence of ignoring the fluid surrounding the core structure is to amplify the response of the core structure. However, in the absence of the fluid, the fundamental frequencies of the core structure are altered significantly; hence, it is not clear that the final solution is conservative. The second approach is to model the fluid interaction effects by introducing equivalent mass and equivalent damping. Experience with this approach in the seismic analysis of dams with hydrodynamic interaction effects indicates that the equivalent mass and damping values are not constant, but vary with the frequency of the response. The equivalent mass is usually in the form of a diagonal mass matrix. The equivalent mass matrix should be nondiagonal. The equivalent matrix in the form of a diagonal matrix is often selected to make the mass matrix for the whole system diagonal as this leads to savings in computer storage and running times. Studies have not been conducted to indicate that this simplification is conservative.

A modeling procedure for adequately including fluid interaction has still not been demonstrated. One problem is the capacity limitations of the ANSYS and MARC programs. For linear structural problems, these programs are considered to be large capacity programs, and the largest problems solved consisted of about 20,000 degrees of freedom (DOF). For the seismic analysis of a LMFBR core structure, the total number of DOF for a realistic three-dimensional model would be of the order of about 25,000. This is arrived at by noting that the demonstration size LMFBR consists of a total of 721 fuel elements arranged in 15 rows. With a minimum of six nodes per element and six DOF (three translations and three rotations)

per node, a total of about 26,000 DOF is obtained.

To this total, the DOF corresponding to load pads, core barrel, etc., may have to be added. The integration time step for solving this highly nonlinear problem should be very small in comparison to that required for linear problems. For a 5-sec duration seismic motion, the minimum number of time steps for meaningful response is estimated to be about 1000. Even on the largest computer available at present, the solution of the above problem with 25,000 DOF for 1,000 time steps is not feasible. This indicates then that a complete three-dimensional analysis of LMFBR core structures is not possible with current computer software and hardware technology. This explains why present state-of-the-art LMFBR in-core analyses have been limited to one-dimensional analyses and have utilized severe limiting assumptions (i.e., no fluid interaction, no fuel assembly interactions and neglect of frictional forces). Therefore, improved computer capability and/or supporting experimental programs will be required to verify the seismic response of LMFBR in-core components.

5.3.3 SCRAP COMPUTER PROGRAM

The Seismic Core Restraint Analysis Program, SCRAP, is being developed at the Argonne National Laboratory by Moran (1976). This program is based on a Rayleigh-Ritz assumed mode approach and, for the seismic analysis of reactor core structures in a rational manner, appears to include all the important factors that may influence the response. The program will consider in a consistent manner fluid inertia, impact damping, and friction, along with structural stiffness, damping, and inertia. Primary consideration has been given to flexibility in the development of different mathematical models, i.e., assumed modes and

physical properties, of the same reactor core; secondary consideration is given to optimization of numerical algorithms, code exportability, and documentation.

SCRAP consists of three modules: a preprocessor that forms the necessary parameters of the mathematical model from the physical properties and model description, a main program that integrates the differential equations of the mathematical model, and a postprocessor that interprets the mathematical solution in the required form. Figure 5-24 gives the relationship between these three modules and the associated input, output, and storage files.

The preprocessor (Program 1) has two functions. The first function is to generate the undamped natural modes of vibration for the component parts of the reactor. These free vibration modes are used in generating the assumed modes for the reactor core. They are generated by a Rayleigh-Ritz procedure and are stored on disks for possible use in other models of the same reactor. The second function of Program 1 is to generate the assumed modes from the model description and free vibration

modes, calculate the parameters of the mathematical model (e.g., mass, stiffness, and damping matrices, gap/support parameters and modal seismic participation factors), and determine the initial conditions for the mathematical model. This mathematical model information is then stored for use by Program 2, the assumed mode information along with the mathematical model is stored for use by Program 3, and tables and graphs are printed for verification of the model.

Program 2 takes the differential equations generated by Program 1, along with a seismic support time history and solves the equations. The solution, consisting of a time history of the generalized coordinates and velocities, is written on a solution tape. The integration algorithm currently coded is a Newmark implicit method, but other integration algorithms that reduce processing time are under consideration. Because of the time required for solution of the equations, the required computer time becomes quite large as the order of the equations increases. Therefore, a restart capability has been included in Program 2.

FIGURE 5-24. SCRAP FLOW CHART (Moran, 1976)

Program 3, the postprocessor, takes the model description passed from Program 1 and uses the solution history from Program 2 to generate force, displacement, or stress histories for any point in the reactor core. This information can be tabulated or plotted. Currently, coding is complete for simple self-scaling graphs of generalized coordinates and velocities and for forces at impact points. Further development of this program is to continue. In addition, each of the programs features dynamic core allocation and automatic input reporting. While considerable effort is still required to fully develop all the features of SCRAP, preliminary versions of each module have been successfully tested.

5.4 CONCLUSIONS

- a. The present state-of-the-art capability for performing dynamic seismic analyses of LMFBR in-core components has approximately the same level of sophistication as that demonstrated in the past for water-cooled reactors.
- b. Because of basic differences in fundamental concepts between the two reactor systems, seismic analysis procedures used for high temperature gas cooled reactor (HTGR) in-core components are not comparable to those required for the LMFBR, and no transfer of technology is foreseen from HTGR analysis procedures.
- c. In general, one- and two-dimensional analyses are being provided for LMFBR in-core components with uncoupled horizontal and vertical input being applied to separate mathematical models. The nonlinear response resulting from impact of elements within an assembly is being

represented with gapped springs, and the energy loss by a viscous damping mechanism based on assumed coefficients of restitution. Fluid/structure interaction effects, impact between different fuel assemblies, and the effect of Coulomb friction on the load pads are not generally being considered.

- d. It has been demonstrated analytically that the coolant between fuel assemblies of the LMFBR should have a dominant inertia effect because of the close spacing of assemblies. Generally, the fluid motion effects have been completely ignored on the assumption that the resulting analysis is conservative, or its effects have been introduced through an effective mass concept. In the case of the latter, the added mass matrix is usually taken as diagonal, although in principle it should be nondiagonal. Also, a constant added mass matrix is used, but investigations indicate that the added mass matrix is dependent upon the frequency of excitation.
- e. A review of commercially available computer codes indicates that ANSYS and MARC are the two most generally available codes for performing dynamic seismic analyses of in-core reactor components. Both codes have three-dimensional nonlinear capability and can treat impact problems with gapped spring techniques. ANSYS has frictional surfaces, and this capability can probably be incorporated in MARC. However, a review of capacity requirements indicates that complete three-dimensional analyses of LMFBR in-core components is

not economically feasible with current software and hardware technology.

- f. A more sophisticated computer code capability has been demonstrated at the Argonne National Laboratory. The SCRAP code is based on a Rayleigh-Ritz assumed mode approach.

The method is flexible and takes into consideration all of the important phenomena governing the seismic response of the in-core components, including fluid/structure interaction, impact forces, fluid damping, and frictional forces. The method can also include three-dimensional effects.

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