AUG 19 1985

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

LBL--19527

DE85 016561

Presented at the 1985 Particle Accelerator Conference,
Vancouver, B.C., Canada, May 13-16, 1985

THE EFFECT OF FOCUSING FIELD NONLINEARITIES IN MBE-4 ON TRANSVERSE BEAM DYNAMICS

C.M. Celata, V.O. Brady, L.J. Laslett and L. Smith

May 1985

THE EFFECT OF FOCUSING FIELD NONLINEARITIES IN MBE-4
ON TRANSVERSE BEAM DYNAMICS*

C.M. Celata, V.O. Brady, L.J. Laslett, L. Smith
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

I. Haber
Naval Research Laboratory
Washington, DC 20375

May 1985

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

*This was supported by the Office of Energy Research, Office of Basic Energy Sciences, U.S. Department of Energy under Contract DE-AC03-76SF00098.

THE EFFECT OF FOCUSING FIELD NONLINEARITIES IN MBE-4 ON TRANSVERSE BEAM DYNAMICS*

C. M. Celati, V.O. Brady, L.J. Laslett, L. Smith
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

I. Haber
Naval Research Laboratory
Washington, DC 20375

Abstract

A particle simulation code was used to study the effect on transverse beam dynamics of nonlinearities of the focusing field in a linear accelerator transporting a multiple beam array. Nonlinear field strengths for various multiple-beam design geometries were calculated by relaxation codes for use in the simulation calculation. Nonlinearities due to asymmetry of the electrode array with respect to a single beam were found to be negligible. Electrode end effect nonlinearities led to emittance growth for off-axis beams, though for the geometry of MBE-4, this was negligible. For misaligned beams, a dodecapole field caused significant emittance growth. This was not seen in single particle tracking calculations. Fields due to induced charge on the electrodes can reduce this effect, or the dodecapole field can be eliminated by proper choice of the electrode radius.

1. Introduction

An array of quadrupoles can be used to transport multiple charged particle beams through a linear accelerator.¹ The geometry of one such array, that of the 4 beam Multiple Beam Experiment (MBE-4) presently under construction at Lawrence Berkeley Laboratory, is shown in Fig. 1. This experiment will accelerate high current heavy ion beams through an alternating gradient electrostatic focusing system.² We examine here the effect of focusing field nonlinearities on the transverse dynamics of the transported beams. Such field nonlinearities result from a

variety of sources of field error. Those studied here can be divided into three classes: (1) imperfections in the field of a single quadrupole, (2) nonlinearities due to the asymmetry of the complete electrode array with respect to the beam of interest, and (3) end effects of the quadrupoles.

Of particular interest is the effect of these nonlinearities on the space-charge-dominated beams needed to transport the required power for heavy ion fusion applications. Therefore beam stability has been explored using a two-dimensional (transverse plane) particle-in-cell (PIC) computer simulation code, as well as an "envelope-type" code, which tracks single particles through the potential of the beam, generated (non-self-consistently) by an integration of the KV envelope equations.³

2. Imperfections of the Quadrupole Field

The focusing field potential can in general be written as a sum of multipoles:

$$V(r, \theta) = V_0 \sum_{m=1}^{\infty} \left[A_m \left(\frac{r}{d} \right)^m \cos m\theta + B_m \left(\frac{r}{d} \right)^m \sin m\theta \right] \quad (1)$$

where r and θ are polar coordinates about an origin at the center of a focusing channel; d , the aperture radius, is the distance to the surface of the nearest focusing electrodes; and θ is measured from the x axis, which passes through the origin and the center of one of these electrodes. We consider in this section only the field of the single quadrupole centered at the origin. Assuming that the four-fold quadrupole symmetry is maintained, we find that $B_m = 0$ for all m , and $A_m = 0$ unless $m = 4n+2$, with $n = 0, 1, 2, \dots$. Neglecting the terms for $n > 1$, since it is feasible in practice to make the electrodes such that these high order terms are completely negligible, we find that the nonlinearity of concern is due to a dodecapole potential: $V = V_0 A_6 (r/d)^6 \cos 6\theta$. If R is chosen to have the value $1.1457d = R_0$, where R is the electrode radius, then $A_6 = 0$.⁴ However it may be of interest to change this ratio for a number of reasons, e.g., to increase the aperture size, or to decrease the transverse dimensions of the machine.

The PIC code was used to determine the stability of a high current ion beam to the dodecapole field. In the code, the quadrupole focusing force was applied using a thin lens approximation. Consistent with this, the dodecapole was also applied as a momentum kick to each particle at the axial position of the center of each lens -- i.e., once per lattice half-period. The relaxation code POISSON was used to determine A_6 for a given R . The simulation boundary conditions were periodic in x and y , but the beam was kept far enough from the calculation boundaries that it did not interact with its periodic images. The beam was taken to be uniform in density and elliptical in shape, with radii matched to the focusing system using a KV envelope code. The velocity distribution was Maxwellian, with a uniform temperature throughout the beam. No acceleration was applied, and it was assumed that $v/c \ll 1$.

Fig. 1. Transverse cross section of the MBE-4 experiment.

*This was supported by the Office of Energy Research, Office of Basic Energy Sciences, U.S. Department of Energy under Contract DE-AC03-76SF00098.

Fig. 2. y rms emittance (normalized to its initial value) vs. z , with a dodecapole force acting. Parameters are given in the text. $a = 11$ mm.

The simulation showed no effect of the dodecapole on the beam when the beam was centered on the axis of the transport channel. For misaligned beams, however, emittance growth occurred in both the x and y dimensions. Figure 2 demonstrates this effect. Here R/d was chosen to be 0.573, giving a value of $A_6 = 5.777 \times 10^{-2}$. The beam major radius was $a = 0.523$ d, with $\alpha = 12^\circ$, $\phi_0 = 60^\circ$ (where α and ϕ_0 are the particle phase advance per lattice period with and without the space charge force), and an initial offset of the beam centroid of $\Delta x = a/2$ at the center of a focusing lens. The rms emittance grows, and accompanying the growth is an oscillation of the emittance with frequency slightly greater than the coherent betatron frequency. The growth is linear for ~ 10 periods, then slows or saturates, and subsequently grows again linearly, but with reduced growth rate. Single particle calculations, using the code mentioned above, show no sign of developing stochasticity for the same nonlinearity strength.

To study the effect of the dodecapole a series of simulation runs was done for beams whose centroids were offset in the x direction. The results suggest that the rms emittance growth is roughly proportional to the emittance oscillation amplitude. The growths of emittance in x and y are comparable. As the emittance grows, the amplitude of the centroid motion damps (by $\sim 5\%$ for the case shown in Fig. 2). The oscillation amplitude appears to be linearly proportional to $A_6 h$, where h is the amplitude of the centroid misalignment. It increases with increasing beam radius or ϕ_0 , or decreasing α . Phase plots suggest the occurrence of an even mode of oscillation as the dominant mode (see Figs. 3,4), perhaps the $j = 0$, $m = 4$ mode.⁵ In the beam frame the dodecapole potential has an octupole component whose magnitude is proportional to $h^2 \cos \omega_c t$, where ω_c is the coherent betatron frequency. This component may be driving the $j = 0$, $m = 4$ mode, whose potential ($\sim r^4 \cos 4\theta$) is of the same form, and whose frequency is close to the coherent betatron frequency for low a/ϕ_0 .

The emittance growth due to this mode gives a limit on the acceptable alignment for the accelerator, given the current density one desires to transport. The effect can be reduced, of course, by setting the ratio R/d to reduce the strength of the dodecapole force. We have also found, however, that the effect of the "image force" due to induced charge on the electrodes is to suppress the emittance growth due to the dodecapole. This effect is also discussed in another paper at this conference.⁶ The ratio of $R/d = 0.744$ used in MBE-4 was chosen because for this geometry the simulation predicts that the image force

Fig. 3. Phase space, momentum and density plots for beam at the initialization of the simulation. Parameters are those of Fig. 2, with $a = 14.1$ mm.

Fig. 4. Phase space momentum, and configuration plots at Period 7 for the parameters of Fig. 2, with $a = 14.1$ mm.

suppresses emittance growth due to the dodecapole, and vice versa.

3. Asymmetry Nonlinearities

We next consider nonlinearities which occur due to the asymmetry of the complete electrode array with respect to a single beam. The code POISSON was used to calculate

the strength of the coefficients A_m and B_m for the potential due to the electrode array of MBE-4. The electrode array was surrounded by a grounded cylinder at radius $R_c/d = 5.05$ and $R/d = 0.744$, as in MBE-4, and the electrodes were at potentials of $-V$ and 0 . Results showed that, except for the dodecapole component discussed in Section 2, all nonlinear forces except the dipole were $< 1\%$ of the quadrupole force, for $r < d$. This was assumed to be negligible, and no simulation code studies were done.

The dipole term provides a force which displaces the equilibrium position of the beam. The dipole strength calculated would displace the beam by approximately 0.35 mm.

Similar calculations were done with the electrode centers at the same positions as in MBE-4, but with larger R , so that $R/d = 1.1457$. In this case for $r < d$ and $m > 2$ all nonlinear forces were less than 0.06% of the quadrupole force. The beam displacement due to the dipole force was $7 \mu\text{m}$. This configuration therefore shows much less nonlinearity than the version with the smaller electrode radius (and therefore larger aperture) discussed above. This is due to the greater shielding of the beam by the electrodes of its focusing channel. However, neither geometry appears to present nonlinearity of sufficient strength to cause collective oscillations of the beam or significant beam misalignment.

4. End Effects

In order to study quadrupole end effects, a 3-D relaxation code was used to determine the multipole expansion of the associated nonlinearity. The fringe field was found to consist mainly of an octupole component, with a z dependence about the center of the lens which was odd in z . (The force is defocusing at the entrance to a focusing lens.) This was applied in the simulation as momentum kicks of alternating sign one eighth of a lattice period before and after each lens, approximating an occupancy factor of 0.5 . The strength of the kick was determined by the integration over z of the octupole component of the fringe field as calculated by the relaxation code, and therefore was of the form $V = V_0 A_4' (r/d)^4 \delta(z) \cos 4\theta$, where the potentials at the quadrupoles were $\pm V_0$. For $R = R_0 = 1$ inch and half periods of $6, 9$, and 12.5 inches, the values of A_4' were $1.7 \times 10^{-3} \text{ m}^{-1}$, $1.8 \times 10^{-3} \text{ m}^{-1}$, and $1.9 \times 10^{-3} \text{ m}^{-1}$, respectively.

The simulation showed no change in the beam for the nonlinearity strengths calculated for MBE-4, which has a half-period of 9 inches. For values of A_4' approximately twice as large, an oscillation of the rms emittance was seen, which was approximately 5% for a beam misalignment of 5 mm. For higher nonlinearity strengths, for misaligned beams, some rms emittance growth occurred in beams with low α , along with an oscillation of the emittance. An example is shown in Fig. 5. (A_4' for this figure is 6.4 times its value for MBE-4.) The emittance growth increased with A_4' and the misalignment magnitude, and decreased with increasing α/α_0 . The rms momenta increased, while the rms beam radii remained approximately constant. As with the dodecapole nonlinearity discussed above and the image force nonlinearity discussed elsewhere,⁶ the oscillation of the rms emittance occurred at a frequency of approximately twice the coherent betatron frequency. As the rms emittance grew, the amplitude of the coherent betatron oscillation increased -- by approximately 25% for the case shown in Fig. 5. This occurred for the MBE-4 design parameters also, but was of negligible magnitude for the 30 lattice period length of the experiment.

Fig. 5. y emittance (normalized to its initial value) vs. z for $A_4' = 1.16 \times 10^{-2} \text{ m}^{-1}$, $\alpha_0 = 60^\circ$, $\alpha = 12^\circ$, $a = 11$ mm, $h = 5$ mm, $\theta = 45^\circ$.

Summary

A particle simulation code has been used to study the transverse dynamics of a space-charge dominated beam in the presence of various focusing field nonlinearities. A dodecapole potential was found to cause an oscillation and growth of the rms emittance for misaligned beams and low particle phase advance, though the simulation predicts that in MBE-4 this will be suppressed by image forces. Nonlinearity due to the asymmetry of the electrode array as seen by a single beam was calculated to be of negligible magnitude. Finally, quadrupole end effects caused no significant beam changes for the parameters of the MBE-4 experiment, but appear to amplify coherent betatron motion and lead to rms emittance growth for much higher nonlinearity strengths and longer accelerators.

References

- [1] A.W. Maschke, "Megalac A New Approach to Low Beta Acceleration," BNL 51029, UC-28, 1 June 1979.
- [2] R.T. Avery et al., "MBE-4, A Heavy Ion Multiple-Beam Experiment," also at this conference.
- [3] I.M. Kapchinskiy and V.V. Vladimirskiy, Proc. 1959 Internat. Conf. High Energy Accelerators, p. 274 (CERN, Geneva, Switzerland; 1959).
- [4] I.E. Dayton, C. Shoemaker, and R.F. Mozley, *Rev. Sci. Instr.*, **25**, 488 (1954).
- [5] R.L. Gluckstern, Proc. 1970 Nat. Accelerator Lab. Linear Accelerator Conf. (M.R. Tracy, Ed.), vol. 2, p. 811 (FNAL, Batavia, Ill., 1970).
- [6] C.M. Celata, I. Haber, L.J. Laslett, L. Smith, and M.G. Tiefenbach, "The Effect of Induced Charge at Boundaries on Transverse Dynamics of a Space-Charge-Dominated Beam," also at this conference.

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.