FORCE-FREE COIL PRINCIPLES APPLIED TO
HIGH-TEMPERATURE SUPERCONDUCTING MATERIALS

PPPL--2507

DE88 009343

H.P. Furth, S.C. Jardin,
Princeton Plasma Physics Laboratory
Princeton, New Jersey 08544

and

D.B. Montgomery
Massachusetts Institute of Technology
Plasma Fusion Center
Cambridge, Massachusetts 02139

ABSTRACT

Force-free magnetic-field configurations, where the current flows parallel to the magnetic field vector, have the potential to raise the critical magnetic field and current-density limits for high-temperature superconductors.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

In a force-free magnetic-field configuration,¹ the current density flows along the magnetic field ($J \equiv J_{\parallel}$), so that the Lorentz force $J \times \vec{B}$ vanishes. A simple illustration is provided by the force-free cable of Fig. 1. The helical winding pattern generates a B_z -field component on the interior of the cable, thus permitting J to flow parallel to $\vec{B}$, instead of perpendicular to $\vec{B}$, as would be the case in a conventional cable. The exterior magnetic field (B_0) is the same for force-free and conventional cables carrying the same total current I_z .

Force-free cables and coils²⁻⁴ have two advantages that should be important in superconductor applications involving high magnetic fields and current densities: (1) the vanishing of the overall Lorentz force exerted on the conductor greatly relaxes the conditions on the mechanical strength of the material; (2) the vanishing of the Lorentz force on the current-carrying electrons within the conductor is favorable to the achievement of higher critical current densities $J_{\parallel \text{crit}}$ than are likely to be reached with currents flowing perpendicular to the magnetic field ($J_{\perp \text{crit}}$).

In the case of conventional hard superconductors,⁵⁻⁸ where comparative measurements have been made for $J_{\parallel \text{crit}}$ and $J_{\perp \text{crit}}$, one finds $J_{\parallel \text{crit}}$ to be typically larger by an order of magnitude. In the case of the new high-temperature superconductors, anisotropies in critical current density have been reported for different orientations of $\vec{B}$ relative to the crystal planes of the sample^{9,10} -- but these measurements were made by inducing closed current loops within the sample and, therefore, they refer only to $J_{\perp \text{crit}}$. (The measured $J_{\perp \text{crit}}$ for a current loop closed within the favorable crystal plane has been found to be higher than the $J_{\perp \text{crit}}$ for a current loop that passes in and out of the favorable plane.) To measure $J_{\parallel \text{crit}}$, the inductive method is not applicable: one must pass an externally generated current

through the sample, in the direction along B , and measure the voltage drop within the sample.

The advantages of force-free configurations are partly off-set by their greater geometric complexity, and especially by the need for a helical winding pattern, as illustrated in Fig. 1. To estimate the potential benefit of the force-free design approach, we write $r = J_{\parallel \text{crit}}/J_{\perp \text{crit}}$ and assume $B J_{\parallel \text{crit}} = \text{const.}$ (as was found in Ref. 5), up to some critical field B_{crit} , where $J_{\parallel \text{crit}}$ drops to zero. In the simple geometry of Fig. 1, the force-free condition is expressed by

$$B_z dB_z/dr + B_\theta/r d(rB_\theta)/dr = 0 \quad (1)$$

Among the many possible solutions of Eq. (1), we select the particular one that corresponds to a solid cylindrical force-free winding with $B J_{\parallel} = \text{const.}$ The solution is shown in Fig. 2. In comparison with a conventional cable of the same radius a , uniform current density J_z , and the same total longitudinal current $I_z(a)$, we find that for the force-free cable, the required product $B J_{\parallel}$ is 2.2 times greater. The factor by which $I_z(a)$ can be higher in the force-free case is therefore 0.45 r . (By comparison, the maximum factor by which $I_z(a)$ could be increased in a conventional-type cable by optimizing $J_z(r)$ according to $B J_z = \text{const.}$ is limited to 1.33.)

An approximately force-free coil of large R/a (ratio of major to minor radius) can be realized by bending the cable of Figs. 1 and 2 into a closed loop. Coils of this type are potentially useful for inductive energy storage or, conceivably, to form part of the guide field for a high-energy particle accelerator.

For purposes of producing maximal magnetic field strength in a small experimental volume, a low- R/a coil of the type shown in Fig. 3 is appropriate. The coil has nested toroidal winding surfaces that coincide with the magnetic surfaces. It generates a poloidal field on its exterior, like a conventional coil, and also generates a toroidal-field component on its interior. The helical pitch of the windings is varied from surface to surface, so that $\mathbf{J}$ points along $\mathbf{B}$ on all surfaces. To be truly force-free, the coil must be supported by an external magnetic field from a conventional force-bearing coil² -- but the maximum pressure $B^2/8\pi$ appearing at the supporting coil can be kept relatively small. A force-free toroid⁴ of low R/a can be used to achieve typical field-enhancement factors $R^* = 2.5$ -3.0, where R^* is defined as the ratio of central magnetic field strength to field strength at the supporting coil, normalized to the same ratio for the supporting coil alone. Reference 4 finds that there is a trade-off between R^* and the maximum diameter of the useful force-free coil bore that can be achieved with a supporting coil of given size.

For the coil design of Fig. 3, the mathematical approach was to solve $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, $\mathbf{J} = \lambda \mathbf{B}$, $\lambda = \psi^{-1/2}$, where ψ is the normalized poloidal magnetic flux, which has a null at the coil surface. The figure uses radial and axial coordinates x and z , normalized to give $r = r_c$ at $x = 1$. In Fig. 3a, we see the poloidal flux surfaces and the variation of $B_z(x)/B_z^s(0)$ along the midplane ($z = 0$), where B_z^s refers to the field of the supporting coil alone. Figure 3b gives the contours of constant toroidal current density J_T and the magnitude of J_T normalized relative to the total toroidal current $I_T = 8\pi r_c B_z^s(0)$ (in units of MA, m, T) on the midplane. Figure 3c shows the contours and normalized magnitude of $x B_T$. This particular coil design gives $R^* = 2.7$, along with a high degree of magnetic-field uniformity $(\partial^2 B_z / \partial x^2) / B_z$

- 0.1 within a moderate-sized coil bore. (The external force-bearing coil is idealized as the single-layer solenoid at $x = 1$ in Fig. 3a.)

Experiments with low-pressure plasmas have demonstrated the formation and stability of virtually force-free configurations^{1,4} much like those of Figs. 1 and 3. By analogy, a tendency of J to flow along $\vec{B}$ within superconducting surfaces would favor the spontaneous emergence of a force-free pattern of current flow -- thus simplifying the construction of force-free cables and coils.

ACKNOWLEDGMENT

This work supported by U.S. Department of Energy Contract No. DE-AC02-76CH03073.

REFERENCES

- ¹ R. Lüst and A. Schlüter, Z. Astrophys. 34, 263 (1954).
- ² H.P. Furth, M.A. Levine, and R.W. Waniek, Rev. Sci. Instrum. 28, 949 (1957).
- ³ H.P. Furth and M.A. Levine, J. Appl. Phys. 33, 747 (1962).
- ⁴ H.P. Furth and S.C. Jardin, Princeton Plasma Physics Laboratory Report PPPL-2465 (1987).
- ⁵ D.B. Montgomery and W. Sampson, Appl. Phys. Lett. 6, 108 (1965).
- ⁶ G.D. Cody and G.W. Cullen, RCA Rev. 25, 466 (1964).

- ⁷ S.T. Sekula, R.W. Boom, and C.J. Bergeron, Appl. Phys. Lett. 2, 102 (1963).
- ⁸ C.J. Bergeron, Appl. Phys. Lett. 3, 63 (1963).
- ⁹ Y. Hidaka et al., Jpn. J. Appl. Phys. 26, (4), L377 (1987).
- ¹⁰ T.R. Dinger et al., Phys. Rev. Lett. 58, 2687 (1987).

FIGURE CAPTIONS

- Fig. 1. Force-free cables use helical windings of varying pitch.
- Fig. 2. For the condition $BJ_\parallel = \text{const.}$, a particular force-free solution is obtained in cylindrical geometry.
- Fig. 3. Insertion of a force-free coil into the force-bearing solenoid at $x = 1$ serves to triple the field strength B_z at $x = 0, z = 0$.

B7A0053

Fig. 1

87A0052

Fig. 2

87A0054

EXTERNAL DISTRIBUTION IN ADDITION TO UC-20

Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
 Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
 Plasma Research Lab., Australian Nat. Univ., AUSTRALIA
 Prof. I.R. Jones, Flinders Univ., AUSTRALIA
 Prof. F. Cao, Inst Theo Phys, AUSTRIA
 Prof. M. Heindler, Institut für Theoretische Physik, AUSTRIA
 M. Goossens, Astronomisch Instituut, BELGIUM
 Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
 Commission-European, Dg-XII Fusion Prog, BELGIUM
 Prof. R. Boucique, Laboratorium voor Natuurkunde, BELGIUM
 Dr. P.M. Sakanaka, Instituto Fisica, BRAZIL
 Instituto De Pesquisas Espaciais-INPE, BRAZIL
 Documents Office, Atomic Energy of Canada Limited, CANADA
 Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
 Dr. H.M. Skarsgard, University of Saskatchewan, CANADA
 Dr. H. Barnard, University of British Columbia, CANADA
 Prof. J. Teichmann, Univ. of Montreal, CANADA
 Prof. S.R. Sreenivasan, University of Calgary, CANADA
 Prof. Tudor W. Johnston, INRS-Energie, CANADA
 Dr. C.R. James, Univ. of Alberta, CANADA
 Dr. Peter Lukac, Komenského Univ, CZECHOSLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 The Librarian, Rutherford Appleton Laboratory, ENGLAND
 Mrs. S.A. Hutchinson, JET Library, ENGLAND
 C. Mouttet, Lab. de Physique des Milieux Ionisés, FRANCE
 J. Radet, CEN/CADARACHE - Bat 506, FRANCE
 Univ. of Ioannina, Library of Physics Dept. GREECE
 Dr. Tom Muul, Academy Bibliographic Ser., HONG KONG
 Preprint Library, Hungarian Academy of Sciences, HUNGARY
 Dr. B. Dasgupta, Saha Inst of Nucl. Phys., INDIA
 Dr. P. Kaw, Institute for Plasma Research, INDIA
 Dr. Philip Rosenau, Israel Inst. Tech, ISRAEL
 Librarian, Int'l Ctr Theo Phys, ITALY
 Prof. G. Rostagni, Univ Di Padova, ITALY
 Miss Clelia De Palo, Assoc EURATOM-ENEA, ITALY
 Biblioteca, Instituto di Fisica del Plasma, ITALY
 Dr. H. Yamato, Toshiba Res & Dev, JAPAN
 Prof. I. Kawakami, Atomic Energy Res. Institute, JAPAN
 Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
 Direc. Dept. Large Tokamak Res. JAERI, JAPAN
 Prof. Satoshi Itoh, Kyushu University, JAPAN
 Research Info Center, Nagoya University, JAPAN
 Prof. S. Tanaka, Kyoto University, JAPAN
 Library, Kyoto University, JAPAN
 Prof. Nobuyuki Inoue, University of Tokyo, JAPAN
 S. Mori, JAERI, JAPAN
 Librarian, Korea Advanced Energy Res. Institute, KOREA
 Prof. D.I. Choi, Adv. Inst Sci & Tech, KOREA
 Prof. B.S. Liley, University of Waikato, NEW ZEALAND
 Institute of Plasma Physics, PEOPLE'S REPUBLIC OF CHINA
 Librarian, Institute of Phys., PEOPLE'S REPUBLIC OF CHINA
 Library, Tsing Hua University, PEOPLE'S REPUBLIC OF CHINA
 Z. Li, Southwest Inst. Physics, PEOPLE'S REPUBLIC OF CHINA
 Prof. J.A.C. Cabral, Inst Superior Tecnico, PORTUGAL
 Dr. Octavian Petrus, AL I CUZA University, ROMANIA
 Dr. Johan de Villiers, Fusion Studies, AEC, SO AFRICA
 Prof. M.A. Hellberg, University of Natal, SO AFRICA
 C.I.E.M.A.T., Fusion Div. Library, SPAIN
 Dr. Lennart Stenflo, University of UMEA, SWEDEN
 Library, Royal Inst Tech, SWEDEN
 Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
 Centre Phys des Plasmas, Ecole Polytech Fed, SWITZERLAND
 Bibliotheek, Fom-Inst Voor Plasma-Fysica, THE NETHERLANDS
 Dr. D.D. Ryutov, Siberian Acad Sci, USSR
 Dr. G.A. Eliseev, Kurchatov Institute, USSR
 Dr. V.A. Glukhikh, Inst Electrophysical Apparatus, USSR
 Dr. V.T. Tolok, Inst. Phys. Tech. USSR
 Dr. L.M. Kovrizhnykh, Institute Gen. Physics, USSR
 Nuclear Res. Establishment, Julich Ltd., W. GERMANY
 Bibliothek, Fom-Inst Voor Plasma-Fysica, W. GERMANY
 Dr. K. Schindler, Ruhr Universität Bochum, W. GERMANY
 ASDEX Reading Rm, IPP/Max-Planck-Institut für
 Plasmaphysik, W. GERMANY
 Librarian, Max-Planck Institut, W. GERMANY
 Prof. R.K. Janev, Inst Phys, YUGOSLAVIA