DOPPLER MODEL FOR ADIABATIC
TOROIDAL COMPRESSOR DRIFTWAVE
FREQUENCY PROFILES

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Doppler Model for Adiabatic Toroidal Compressor Driftwave
Frequency Profiles

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Microwave and CO_2 laser scattering experiments on ATC (Adiabatic Toroidal Compressor) Tokamak have shown that there exists a spectrum of fluctuations with $|f| = |\omega/2\pi| \lesssim 1 \text{ MHz}$ at wavelengths $\lambda \sim 0.28 \text{ cm}$ and with $|f| \lesssim 400 \text{ kHz}$ for $\lambda \sim 1 \text{ cm}$. However, a typical maximum frequency calculated for given experimental density and temperature profiles is $f \lesssim 150 \text{ kHz}$ for the longer wavelengths and differs with the experimental results by roughly a factor of 2 or 3. At shorter wavelengths the calculated maximum $f \lesssim 100 \text{ kHz}$, which differs quite substantially with the experimental result. We interpret this discrepancy as evidence of an $\vec{E} \times \vec{B}$ Doppler shift from a radial electric field.

Fluctuation spectra associated with drift waves have been observed on ATC Tokamak using microwave¹ and CO₂ laser² scattering techniques. In the microwave scattering experiment, fluctuations with wavelengths $0.7 \text{ cm} \leq \lambda \leq 1.0 \text{ cm}$ were found to have spectral components with maximum frequencies, f , in the range $300 \text{ kHz} \leq f = (\omega/2\pi) \leq 400 \text{ kHz}$. In the same experiment the frequencies were observed to change sign, while the spectrum maintains the same shape, though inverted, when the toroidal magnetic field direction is changed. In the laser scattering experiment, fluctuations with wavelengths $0.28 \text{ cm} \leq \lambda \leq 2 \text{ cm}$ were observed. At longer wavelengths the maximum frequency is in the same range as in the microwave experiment, but can be as large as 1 MHz at the shorter wavelengths.

For a tokamak plasma of electrons and singly-charged ions the electron and ion collisionality are given by

$$\nu_{*e} = \frac{4(2\pi)^{1/2}}{3} \frac{r B_o}{B_p} \left(\frac{R}{r} \right)^{3/2} \frac{n_i e^4 \ln \Lambda}{T_e^2}$$

$$\nu_{*i} = 2^{-1/2} (T_e/T_i)^2 \nu_{*e}$$

where R is the distance to the major axis, r is the distance to the minor axis, $B_o = B_T(r=0)$ is the toroidal magnetic field at $r=0$, B_p is the poloidal magnetic field, n_i is the ion density, T_e (T_i) is the electron (ion) temperature, and the Coulomb logarithm is

$$\ln \Lambda = \begin{cases} 23.4 - 1.15 \log_{10} n_e + 3.45 \log_{10} T_e & T_e < 50 \text{ eV} \\ 25.3 - 1.15 \log_{10} n_e + 2.3 \log_{10} T_e & T_e > 50 \text{ eV} \end{cases}$$

For $\nu_{*e} \geq 0.5$ characteristic of the experiment, dissipative trapped-electron and collisionless drift modes should be present over a large part of the minor radius. The scattering volumes in both the microwave and CO_2 laser experiments also intersect the plasma over a range of minor radii and poloidal angles depending on the plasma region selected for observation. Hence, it would be expected that the observed spectra should contain a range of frequencies and amplitudes for given λ corresponding to the variation of the density and temperature profiles and of the scattering angle over the scattering volume. We have written a simple computer code which calculates, using profiles³ typical of these experiments, the radial variation of $f_{\text{DW}} = \omega_{\text{DW}}/2\pi$, where

$$\omega_{\text{DW}} = \omega_{*e} S \quad (1)$$

$$\omega_{*e} = - \frac{ck_{\perp} T_e}{eB_T} \frac{1}{n_e} \frac{dn_e}{dr}$$

$$S = \frac{\Gamma_o + \eta_i b (\Gamma_l - \Gamma_o)}{1 + \tau (1 - \Gamma_o)}$$

$$b = (k_{\perp} \rho_i)^2 / 2$$

$$\rho_i = \frac{v_i}{\Omega_i} = \left(\frac{2 T_i}{m_i} \right)^{1/2} \bigg/ \left(\frac{e B_T}{m_i c} \right)$$

$$\Gamma_{0,1} = e^{-b} I_{0,1}(b)$$

$$\eta_{i,e} = \frac{d \ln T_{i,e}}{d \ln n_{i,e}}$$

$$\tau = T_e / T_i$$

and $I_{0,1}$ are modified Bessel functions. In this calculation we take $k_{\perp} = k = 2\pi/\lambda$, although we note that generally^{1,2} $k_{\perp} \leq k$ because there will be a finite $k_{\parallel}$ and k_r, k_{θ} decomposition associated with the fluctuations. The maximum value of f obtained from this calculation for $\lambda \sim 1$ cm was less than 150 kHz with $k_{\perp} \rho_i \leq 1$ over most of the cross section. For $\lambda \sim 0.28$ cm $\max \{f_{DW}\} \leq 100$ kHz with $k_{\perp} \rho_i \geq 1$ over most of the cross section. Typical profile values at $r=0$ for these calculations were $T_e \sim 800$ eV, $T_i \sim 200$ eV, and $n_e \sim n_i \sim 2 \times 10^{13}$ cm⁻³. Also, the generally broader ion temperature profiles also allowed $0.2 \leq \tau \leq 5.0$. It appears that the maximum frequency observed for the fluctuations is 2-10 times the maximum value calculated using Eq. (1) for the given experimental profiles. In particular, this discrepancy is quite pronounced at shorter wavelengths.

To explain the discrepancy we consider the possibility of there being a Doppler shift included in the observed frequency. Such an effect has been observed⁴⁻⁶ previously in a variety of experiments. Thus, we take⁷ for the observed frequency

$$\omega = \omega_{DW} + \omega_{DOPP} \quad (2)$$

with

$$\omega_{DOPP} = \frac{ck_{\perp}}{B_T} \frac{d\Phi}{dr} \quad (3)$$

where Φ is the lowest order electrostatic potential. In order to explain the observed frequency spectrum this model ω_{DOPP} must also have radial variation on the same scale as ω_{DW} . The value of $d\Phi/dr$ was not measured directly for the ATC Tokamak and must be calculated. According to neoclassical theory the time asymptotic value of this field depends upon the fastest nonambipolar diffusion process. In a completely ionized electron and single ion species plasma, neoclassical diffusion is ambipolar, and the steady state potential is determined at higher order in the gyroradius. This problem has been considered in the banana⁸ and Pfirsch-Schlüter⁹ limits. Neither of these is appropriate because in the ATC Tokamak the ions are a predominately plateau regime species and because the time scale on which the potential is established is much longer than the duration of the experiment. Thus, we consider nonambipolar processes capable of establishing the potential on shorter time scales. For ORMAK (Oak Ridge Tokamak Device) it has been proposed that one such process is charge exchange.¹⁰ For the ATC Tokamak the same conclusion appears to be valid.¹¹ The potential is determined by the condition that there is no ion toroidal momentum on any magnetic surface. The basis for this condition is that in the steady state there is no net momentum transfer between ions

and neutrals via charge exchange while the neutral particles transfer all of their momentum to the walls. The radial ion flux driven by any net ion-neutral momentum exchange has no electron counterpart and thereby provides the freedom for the nonambipolar change in the potential necessary to satisfy the condition on the ion toroidal momentum. Thus, for $t \gg \nu_{cx}^{-1}$, the potential is then determined¹⁰ by

$$0 = R_{\parallel cx} \propto u_{\parallel i} + \begin{cases} O\left(\sqrt{\frac{r}{R}}\right) & \text{banana} \\ O\left(\frac{r}{R}\right) & \text{plateau} \end{cases} \quad (4)$$

where¹²

$$u_{i\parallel} \approx \frac{cT_i}{eB_p} \left[\left(\frac{5}{2} - y \right) \frac{d \ln T_i}{dr} - \frac{d \ln p_i}{dr} - \frac{e}{T_i} \frac{d\Phi}{dr} \right] \quad (5)$$

with

$$\frac{5}{2} - y = \frac{1.17 - 0.35 \nu_{*i}^{1/2}}{1 + 0.7 \nu_{*i}^{1/2}} \quad (6)$$

Thus, to lowest order in the small aspect ratio limit,

$$\omega_{DOPP} \approx \tau \omega_{*e} \{ 1 + \eta_i [1 - (5/2 - y)] \} , \quad (7)$$

and in the plateau regime

$$\frac{\omega_{DOPP}}{\omega_{DW}} \approx \frac{\tau [1 + 3/2 \eta_i]}{S} \quad (8)$$

For $\eta \sim \tau \sim 1$ we obtain $\omega_{\text{DOPP}}/\omega_{\text{DW}} \sim 10$ when $k_{\perp} \rho_i \sim 1$. When the experimental profiles are used to calculate $\max [f = (\omega_{\text{DW}} + \omega_{\text{DPP}})/2\pi]$ we obtain values ~ 500 kHz for $\lambda = 1$ cm and ~ 1.3 MHz for $\lambda \sim 0.28$ cm in reasonable agreement with the scattering results.

We consider now the applicability of our model to the fluctuation spectra for other experiments. First, microwave scattering on TFR¹³ (Fontenay-Aux-Roses Tokamak) indicates $f = (\omega/2\pi) \sim (3 - 5)(\omega_{\text{DW}}/2\pi)$. The Doppler shifted frequencies of our model are in quantitative agreement with this result. However, for the PLT Tokamak the maximum frequencies observed appear to be above $\max(\omega_{\text{DW}}/2\pi)$, but below $\max[(\omega_{\text{DW}} + \omega_{\text{DOPP}})/2\pi]$. This result may be a consequence of the fact that the neutral particle mean free paths are shorter than the PLT Tokamak minor radius¹⁵ so that $u_{i\parallel} = 0$ is no longer a good approximation. In other words, momentum transfer from ions to neutrals via charge exchange does not imply $u_{i\parallel} = 0$ unless the neutrals also lose momentum, e.g., to the walls. The solution for $u_{i\parallel}$ in this regime remains open to investigation. For ALCATOR (Massachusetts Institute of Technology Tokamak), we expect at lower densities results similar to those in the ATC Tokamak. At higher densities in ALCATOR, a situation similar to that in PLT Tokamak regarding the neutral particle mean free path may arise. A possible complicating factor in the latter regime is the field ripple associated with diagnostic ports on ALCATOR. The ripples may be sufficiently large to produce non-ambipolar diffusion¹⁶ without charge exchange effects and give qualitatively the same results for ω_{DOPP} as discussed above.

At this point we note that it has recently been proposed¹⁷ that nonlinear effects could account for the observed frequency spectrum. The result is a frequency spread around $\omega = 0$ proportional to $k^3/(1+k^2)$, where k is normalized by c_s/Ω_i . This theory requires for its validity the existence of a $k=0$ potential with $e\phi/T_e \sim 0.2$ to give the observed frequency spread. If both exist, the nonlinear effect and the Doppler shift would presumably be additive. This means that the $k \approx 0$ potential need not be as large in order to give the observed spread. It should also be noted that this nonlinear theory¹⁷ seems to predict spectra symmetric about $\omega = 0$ and hence invariant when $\vec{B}$ changes direction. This, however, is in contradiction with the experimental observation.¹

In conclusion the Doppler shifted drift wave frequency model appears to account for the frequencies $\omega > \omega_{DW}$ observed in ATC. With regard to the nonlinear theory of drift waves our result provides support for the weak turbulence approach. In particular, our calculation assumes that the observed frequencies are approximately the mode frequencies of linear theory (though Doppler shifted) consistent with a basic requirement for the validity of weak turbulence theory. We note that the observed^{1, 2} relative fluctuation amplitudes $|\tilde{n}/\bar{n}| \sim 0.03$ are also consistent with the weak turbulence assumption. However, this fact alone would be insufficient basis to invoke a weak turbulence description if substantial nonlinear broadening of the mode frequencies, associated with strong turbulence, was necessary to account for the observation of frequencies with $\omega > \omega_{DW}$.

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