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**REGIONAL STUDIES—LAKE MICHIGAN  
ENERGY FORECASTS**

**The Econometric Approach  
to Electricity Supply and Demand:  
Review and Analysis**

**by**

**J. G. Asbury**



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**ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS**

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Environmental Statement Project

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May 1974

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ABSTRACT

*Recent econometric analyses of electricity supply and demand have provided insight into the factors and relationships affecting the market for electrical energy. This study reviews the assumptions, methodologies, and results of previous investigations and extends the econometric approach to the study of electricity demand. The results reaffirm earlier findings that electricity demand is price-elastic and that demand is income-inelastic. However, our results indicate that interfuel competition plays a more important role than generally indicated by previous studies. Preliminary results indicate that the elasticity of demand with respect to competing-fuel price is  $-0.30$ ; the cross-elasticity of competing-fuel demand with respect to electricity price is  $-0.40$ . The ratio of the cross-elasticities agrees with the value predicted by the Slutsky-Hotelling reversibility relation.*

*Our analysis of electricity supply and demand for 1959-1970 indicates that electricity price and consumption can be explained as the intersections of a relatively stable supply curve with an upward-shifting demand curve. However, because of expected increases in the real cost of electricity supply, this relationship is unlikely to hold in the future.*

*None of the market models reviewed in this study are adequate for predicting future electricity consumption. The existing dynamic models fail to incorporate the supply relation; the static models are ill-suited to situations involving changes in the long-run supply and demand trends.*

1. INTRODUCTION

Several investigators have reported results of econometric studies of the market for electrical energy.<sup>1-8</sup> The studies have tried to explain temporal and intermarket variations in electricity consumption in terms of variables that, on a priori grounds, might be expected to affect electricity demand. Using multiple-regression techniques, the investigators have examined the dependence of electricity consumption on such variables as electricity price, population, personal income, climate variables, degree of urbanization, the price of electric appliances, and the price of natural gas. Although most of the studies have concentrated on the residential market for electricity, results have been reported for the commercial and industrial markets as well.

Probably the most important result of the studies is the statistical documentation of the strong inverse relation between electricity price and electricity sales. Indeed, the results of a number of investigators indicate that long-run demand for electricity is price-elastic.\* This finding, if true, is of obvious and direct importance to such activities as the forecasting of electricity-demand growth, the regulation of electric-utility rates, and the formulation of energy-conservation policy.

The recent studies, although they disagree on a number of issues, are in substantial agreement with regard to two other important conclusions. First, residential demand for electricity is inelastic with respect to personal income. Second, demand is relatively insensitive to the price of competing fuels. The first conclusion means that actions and policies tending to increase the price of electricity will have a regressive effect on the distribution of income--an important consideration in the design of tax and environmental control programs. The second conclusion means that price-induced substitution of natural gas for electricity (and by implication, inter-fuel competition generally) plays only a minor role in shaping electricity demand.

The present study reexamines the assumptions, methodologies, and results of the previous investigations and, in certain areas, extends the econometric approach to the study of electricity demand. With regard to the results of the previous studies, the most important conclusions are to reaffirm the finding that demand for electricity is (or is nearly) price-elastic and that demand is income-inelastic. However, our results indicate that interfuel substitution plays a more important role than generally indicated by the previous studies.

The importance of substitution effects suggests that a careful analysis of electricity demand should include a simultaneous analysis of competing-fuel demand. Such an analysis would incorporate the Slutsky-Hotelling reversibility relation\*\* as constraint or as a check on the cross-price elasticities. Our preliminary results indicate that the elasticity of electricity demand with respect to competing-fuel price is 0.30, while the cross elasticity of competing-fuel demand with respect to electricity price is about 0.40. The ratio of these cross elasticities is in rough agreement with the value predicted by the Slutsky-Hotelling relation.

Although econometric analyses have provided considerable insight into the economic, demographic, and climatological factors affecting electricity demand, their usefulness as predictive tools for estimating future consumption trends is less well established. Our examination of existing models indicates that they are inadequate as predictors of future consumption. A principal shortcoming in this regard is the failure of dynamic models to handle the supply-demand simultaneity problem. The last section of this report describes the type of model development that may eventually enable the reliable forecasting of electricity consumption.

---

\*Mount et al.,<sup>1,3</sup> Halvorsen,<sup>4</sup> and Wilson<sup>5</sup> find price elasticities approximately equal to -1.25. Anderson's value (-0.86) indicates that demand is "almost" price elastic.

\*\*See Refs. 21 and 22 later in this report.

## 2. RELATIONSHIPS AMONG THE MARKET VARIABLES

Before discussing the formal techniques for analyzing the residential market for electricity, we first discuss several key relationships among the market variables. These relationships can be most directly illustrated by a few tables and plots of the market data.

### 2.1 Price versus Quantity: Cross-section Data

As discussed in Sec. 1, a number of investigators, on the basis of statistical analyses, have concluded that there is a strong inverse relationship between electricity consumption and electricity price. Actually, this conclusion follows more or less directly from the simple fact that electricity sales are generally higher in those areas of the country where electricity prices are low. This is illustrated in Fig. 1, where per-capita residential sales for 1970 are plotted against average residential price for the 48 coterminous states. (Average residential price equals total revenues divided by total sales. Sales and revenue data were taken from Ref. 9.) The strong inverse correlation between quantity and price evident in Fig. 1 is one of the most conspicuous features of the market for electric energy.

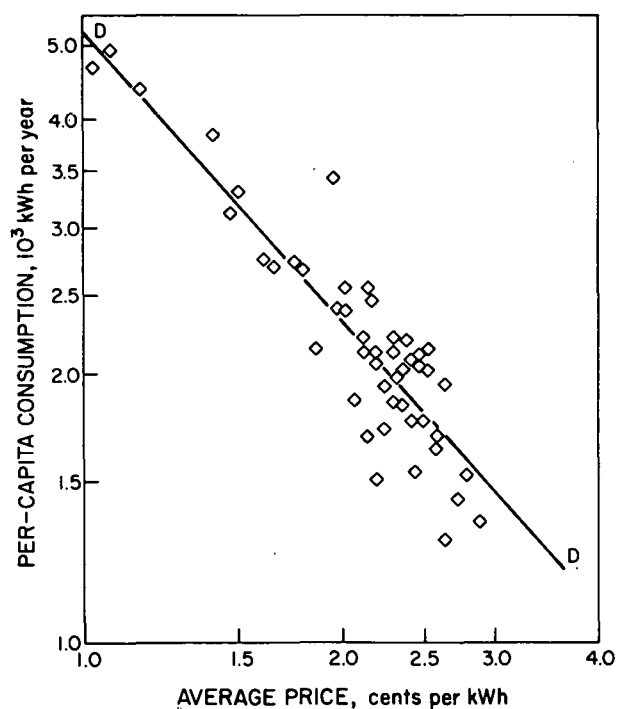


Fig. 1

Residential Electricity Consumption vs Average Electricity Price: 1970 State Data.

As a means of characterizing the distribution of points in Fig. 1, we have added the curve DD. This curve represents the results of a simple least-squares fit to an equation of the form

$$Q = AP^{\alpha}, \quad (1)$$

where  $Q$  = consumption,  $P$  = price, and  $A$  and  $\alpha$  are constants. The best-fit value for  $\alpha$  is  $-1.15$ . As discussed in Sec. 3 below, the curve DD, under certain assumptions, can be interpreted as an electricity-demand curve.

### 2.2 Price versus Quantity: Time-series Data

Another way of illustrating the relationship between price and quantity is by comparing timewise changes in price with timewise changes in consumption. This is done in Fig. 2, where the ratio of 1970/1959 sales is plotted against the 1970/1959 price ratio for the 48 mainland states. By way of "explaining" part of the variation in the sales increases, we have

identified the points according to the states' average July temperatures. The higher percentage increase in sales for states with high summer temperatures can be attributed to a number of factors. For the time period in question, these include (1) the higher rates of installation of air-conditioning and

electric-space-heating equipment in the warmer states and (2) the higher rates of income growth in the southern part of the United States.

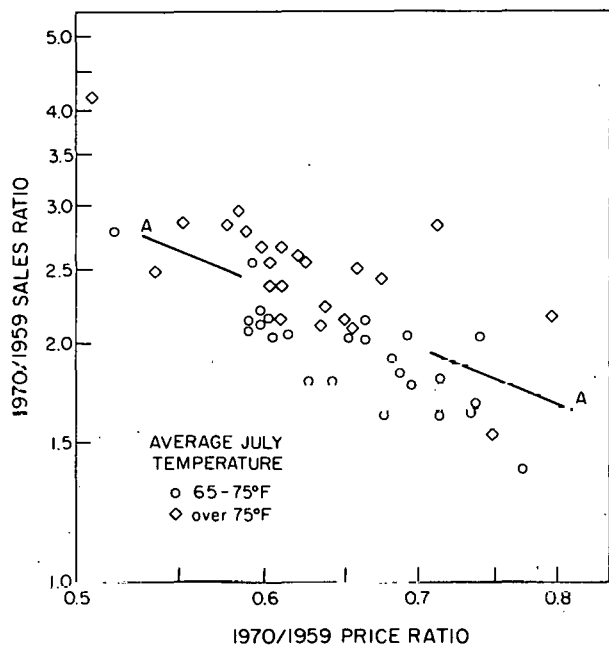


Fig. 2

1970/1959 Electricity Sales Ratio vs  
1970/1959 Electricity Price Ratio:  
State Data, Residential Market.

Perhaps more important than illustrating a systematic difference in demand growth, Fig. 2 indicates that overall demand growth in the United States has been such as to tend to preserve the price-quantity relation in Fig. 1. Curve AA, having the same slope as DD in Fig. 1, has been drawn in to illustrate this point.

### 2.3 Interfuel Substitution

As discussed in Sec. 1, several recent studies have concluded that the price of natural gas and (by implication) the prices of competing fuels generally have had little effect on consumption of electricity. This result is somewhat surprising, because electricity sales are highest in precisely those areas of the country where electricity competes successfully in such energy-intensive applications as space heating, water heating, and cooking.

Table I shows electricity-use data for the two states ranking highest and the two ranking lowest in per-capita consumption of electricity. The more-than-a-factor-of-three difference between the highest- and lowest-ranking states is largely explained by the extent to which electricity has replaced and continues to replace competing fuels in the space- and water-heating markets. In Tennessee, for example, fully 25% of

TABLE I. Residential Electricity-use Data<sup>12</sup>

State	1970 Per-capita Consumption 10 <sup>3</sup> kWh	Urban/ Rural	Electricity Use (by % of occupied dwelling units)					
			Space Heating		Water Heating		Cooking	
			1960	1970	1960	1970	1960	1970
Tennessee	4.90	Urban	22	39	50	62	59	71
		Rural	18	42	49	75	74	86
Washington	4.75	Urban	11	27	82	83	85	90
		Rural	14	39	85	91	82	90
New York	1.39	Urban	0.14	1.6	3.3	4.3	12	17
		Rural	0.34	3.1	37	39	41	47
New Mexico	1.34	Urban	0.40	2.7	9.2	8.6	26	38
		Rural	0.77	2.3	12	12	12	19
U.S. Average	2.20	Urban	1.5	7.1	14	18	26	37
		Rural	2.4	9.4	37	47	44	53
		Total	1.8	7.7	20	25	31	41

residential consumption is for space heating (based on an annual average requirement of 10,000 kWh per space-heating system<sup>10</sup> and an average customer family size of 3.24<sup>11</sup>). Another 22% is consumed for water heating.

In some states, the penetration of electricity into markets normally served by heating oil and natural gas is so great that it can be expected to show up in the sales data for these competing fuels. Figure 3 shows total sales of No. 2 fuel oil plus residential sales of natural gas plotted against heating degree days. (Natural-gas and heating-oil sales are from Refs. 12 and 13.) In the figure, each data point has been identified according to the state's average electricity price. The effect of low electricity prices to reduce demand for heating oil and natural gas is evident in the figure. The effect is more pronounced in states having milder winters. In these states, electric heating systems with their relatively high running costs, but low capital costs, enjoy their best competitive position.

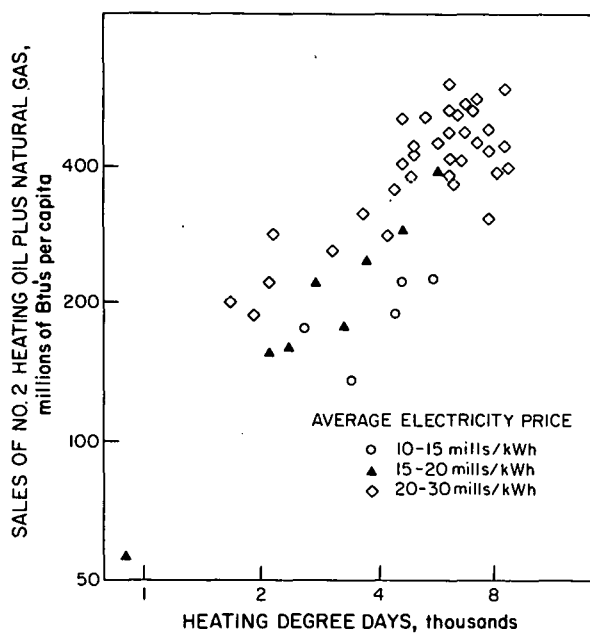


Fig. 3.

Total Sales of No. 2 Heating Oil plus Residential Sales of Natural Gas: 1970 State Data.

competing-fuel prices, degree of urbanization, and climate variables. Another variable that is clearly important, but whose effect has not been shown, is personal income. Direct empirical evidence for the strong relation between family income and household consumption of electricity is provided by a case study of the Los Angeles area by Berman and Hammer.<sup>14</sup>

Another variable of obvious importance is the price of electric appliances. (As used here, the term "electric appliance" refers to any device requiring electricity for its operation.) Services do not flow directly from electricity; rather, electricity is useful only in combination with the appliances it activates. The close complementary relationship between electricity and electric appliances means that electricity consumption can be expected to increase if

The main conclusion to be drawn from the foregoing is that the data suggest that the consumer behaves in a rational, price-conscious manner in selecting from among alternative energy sources. As shown in Sec. 5 below, a number of previous investigators underestimated the role of competing-fuel prices because the studies failed to take into account the fact that electricity's chief competitor in many parts of the country is heating oil, not natural gas. In New England, for example, the coexistence of high natural-gas prices with low electricity sales is largely explained by the extensive use there of relatively low-cost heating oil.

#### 2.4 Other Variables

The foregoing has identified a number of variables that should be considered in any analysis attempting to "explain" electricity demand. These variables include: electricity price,

either the price of appliances or the price of electricity decreases. The effect on total consumption of electricity of a change in the price of an appliance will be in inverse relation with the change in the cost of owning and operating the appliance and in direct relation with the energy requirement of the appliance.

The close dependence of electricity consumption on the existing stock of electric appliances suggests that electricity demand might be understood by analyzing consumer demand for specific appliances. The individual appliance demand functions could then be weighted by their corresponding energy-use factors and then summed to form a total demand function. Symbolically,

$$S_i = S_i(X)$$

and

$$Q = \sum W_i(Z) S_i(X),$$

where  $S_i$  equals the consumer stock of the  $i$ th appliance,  $W_i$  equals the corresponding energy-use factor, and  $X$  and  $Z$  represent the variables affecting appliance ownership and use. In principle, such an analysis could yield detailed information concerning the structure of electricity demand. In practice, the data and analytical requirements are so exacting as to foreclose the attempt.\* For the econometrician, it is much simpler to skip the intermediate step and to try to find  $Q$  directly as a function of the  $X$  and  $Z$ . That is,

$$Q = Q(X, Z).$$

---

\*Wilson estimated relations showing the percentage of homes having individual appliances. However, his explanatory variables did not include appliance price. Nor did Wilson attempt to estimate total electricity consumption as a function of appliance stocks.<sup>5</sup>

### 3. REVIEW OF ELECTRICITY DEMAND MODELS

The models used in econometric studies of electricity demand can be divided broadly into two categories: static models and dynamic models. In the following, we discuss the basic assumptions and methodologies involved in these two approaches to the electricity demand problem.

#### 3.1 Static Models

The basic assumption underlying static demand models is that a formal representation of demand for a particular commodity need not include the time variable. The static approach may be used in either the year-by-year analysis or the cross-section analysis of market data. For time-series data, this approach carries the implicit assumption of a basic homogeneity among the different time periods. Such an assumption clearly is not plausible in the case of electricity demand, where technological change is bringing about many nonrecurring changes in market conditions. For this reason, the static-model approach to electricity demand is restricted to an analysis of cross-section data.

##### 3.1.1 Single-equation Models

Perhaps the simplest static-model approach to residential demand for electricity is to assume that, in each geographical market segment, aggregate residential demand depends upon average residential price,  $Q = Q(P)$ . Assuming the functional relation given in Eq. 1, we can write the following stochastic-demand relation:

$$\log Q_i = \log A + \alpha \log P_i + u_i, \quad (2)$$

where  $Q_i$  is the annual rate of consumption (kWh/yr),  $P_i$  is the average price (cents/kWh) in the  $i$ th market segment, and  $u_i$  is an unobserved random variable that expresses the effect of all factors other than price on electricity consumption (income, urbanization, climate, etc.). If the mean (expected value) of  $u_i$  is zero, then least-squares regression of  $\log Q$  on  $\log P$  gives "unbiased" estimates of  $A$  and  $\alpha$ . An actual least-squares regression of 1970 price-quantity data for the 48 coterminous states gives

$$\log Q_i = 0.57 \log 10 - 1.15 \log P_i + u_i, \quad (3)$$

where  $u_i$  has a mean value of zero and a standard deviation of 0.14. The coefficient of the price variable represents the price elasticity of demand.

Equation 2 can easily be extended to include other explanatory variables. For example, if natural-gas price  $G$  and personal income  $Y$  are included, the demand relation takes the form

$$\log Q_i = \log A + \alpha \log P_i + \beta \log G_i + \gamma \log Y_i + u_i. \quad (4)$$

This general type of multivariate demand relationship was used by Wilson and by Anderson to compute demand elasticities for a number of explanatory variables.\* Anderson performed regression analyses using state data; Wilson analyzed Standard Metropolitan Statistical Area (SMSA) data.<sup>2,5</sup>

\*In addition to fitting log-linear demand relations of the form given in Eq. 4, Wilson also estimated linear relations of the form  $Q = A + a \cdot P + b \cdot G + \dots$ ; Anderson's analyses included use of the price-ratio variable  $P/G$ .

It is tempting to interpret the fitted curve in Fig. 1 (Eq. 3) as a good approximation of the demand law for residential consumption of electricity. Such an interpretation, however, rests upon rather stringent assumptions about the role of supply as it affects the distribution of points in the price-quantity diagram. Rather than representing the intersections of a relatively stable demand curve with a supply curve that varies from state to state, the distribution of points in Fig. 1 might represent the intersections of a relatively stable supply curve with a shifting demand curve. The ambiguity is particularly troublesome because both the supply and the demand curves for electricity are downward sloping.

### 3.1.2 Supply-Demand Models

In principle, the supply-demand identification problem can be resolved by simultaneously analyzing the effects of both the supply and demand relations. The following represent a simple two-equation model of the electricity market:

$$q_i = a + \alpha p_i + u_i \quad (\text{demand})$$

and

$$p_i = b + \beta q_i + v_i \quad (\text{supply}).$$

Here, we have let  $q_i = \log Q_i$  and  $p_i = \log P_i$ . The terms  $u_i$  and  $v_i$  represent the effects of unobserved random variables. In this model the price and the quantity of electricity consumed are determined simultaneously by the supply and demand relations. It should be clear, however, that under this model, the supply and demand equations are not identified. That is, the values of the coefficients  $a$ ,  $\alpha$ ,  $b$ , and  $\beta$  cannot be determined uniquely from an observed sample of price-quantity data.

If we were to assume, however, that demand as a function of price is fairly stable across the United States while supply shifts back and forth according to interstate variations in the cost of producing electric energy, then the demand curve may be identifiable. If we let  $F_i$  denote an observed variable representing the collective effects of factors other than quantity consumed on supply price,\* then the market model can be written as

$$q_i = a + \alpha p_i + u_i \quad (5)$$

and

$$p_i = b + \beta q_i + \gamma f_i + v_i. \quad (6)$$

Figure 4 illustrates how such a model might account for the gross features of the distribution of data points in Fig. 1. The supply curve shifts back and forth because of systematic state-to-state variations in  $F_i$ . The slopes of the curves are greater than the slope of the demand curve to satisfy a necessary condition for stable market equilibrium; that is, the demand price must exceed the supply price over inframarginal units of output.

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\*Factors other than per-capita consumption that will affect price are: the availability of hydroresources, public versus private ownership, fuel costs, labor costs, etc.



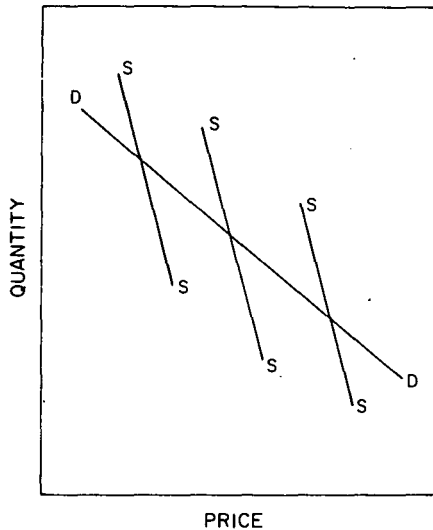


Fig. 4

Stable Demand: Shifting Supply Curves.

The slope of the supply curve is not directly related to the dynamic trend of the cost of supply, which cost trend is now rising. Under the static-model approach, the downward-sloping supply curve indicates that, in a particular state at a given time, the average unit cost of supply will be lower the greater the per-capita quantity of electricity consumed. This simply reflects scale efficiencies in the generation, transmission, and distribution of electric power.

If the variable  $f_i = \log F_i$  in the above model is observed, then the coefficients  $a$  and  $\alpha$  in the demand relation are uniquely determined. The coefficients  $b$ ,  $\beta$ , and  $\gamma$  of the supply relation, however, are not identified.\*

Plausible a priori arguments can be advanced to support the general form of the above market model. First, it seems more reasonable to assume that the distribution of points in Fig. 1 is explained by a shifting supply curve and a relatively stable demand curve, rather than by the converse, that is, by shifting demand curves and a relatively stable supply curve. For example, the difference between the price-quantity points for the states of Washington (1.01¢/kWh, 4750 kWh/yr) and Massachusetts (2.75¢/kWh, 1570 kWh/yr) are more likely due to the differences between the supply than between the demand characteristics of these states. Factors tending to hold down the supply price in Washington relative to Massachusetts include the larger hydroelectric generating component (95% of capacity versus 2%) and the greater participation (subsidization) by government through the ownership of generating and transmission plant (94% of capacity versus 2%).<sup>11</sup> On the other hand, the factors (other than electricity price) likely to affect demand appear rather comparable--for example, per-capita income: \$4,000 versus \$4,290; degree of

\*To see this, first write the reduced-form equations by solving the supply and demand relations (Eqs. 5 and 6), for  $p_i$  and  $q_i$

$$p_i = c + df_i + w_i$$

and

$$q_i = g + hf_i + x_i,$$

where  $c = (\beta a + b)/(1 - \alpha\beta)$ ,  $d = \gamma/(1 - \alpha\beta)$ ,  $g = (a + \alpha b)/(1 - \alpha\beta)$ , and  $h = \alpha\gamma/(1 - \alpha\beta)$ . If  $f_i$  is observed, then regressing  $p$  on  $f$  and  $q$  on  $f$  gives  $c$ ,  $d$ ,  $g$ , and  $h$ . The coefficients  $a$  and  $\alpha$  can be computed from  $\alpha = h/d$  and  $a = g - \alpha c$ . The coefficients  $b$ ,  $\beta$ , and  $\gamma$ , however, are not determined. Were the demand relation to include another exogenous variable (a variable not appearing in the supply relation), then the coefficients of both the supply and the demand equations can be determined.

urbanization: 73 versus 85%; heating degree days: 5500 versus 6100; and competing-fuel prices: \$1.32 versus \$1.46 per million Btu. (Competing fuel price equals weighted average residential price of natural gas and No. 2 heating oil. Weighting factor equals sales in Btu.)

Another reason for rejecting the shifting-demand, stable-supply hypothesis is that, were the supply price to fall off roughly as  $Q^{-1}$  (curve DD in Fig. 1), then total revenues,  $P \cdot Q$ , could not keep up with total costs if demand were increased.\* Clearly, the average unit price of supply must fall off more slowly as a function of quantity supplied. In Figs. 1 and 4, this corresponds to supply curves that are much steeper than curve DD.

The foregoing argues strongly in favor of the stable-demand, shifting-supply hypothesis. However, having adopted this hypothesis, we cannot conclude that the regression of  $Q$  on  $P$  will yield unbiased estimates of the demand relation. In general, unbiased estimates for the demand relation can be obtained only through regressions of the reduced forms of the supply-demand equations. (See footnote, page 13.) The regression of quantity on price will yield unbiased estimates of the demand relation only for the special case that the supply curve for each state is perfectly price-elastic (is a vertical line in Fig. 4). To the extent that this assumption is not valid, the fit obtained will be biased in the direction of overestimating the slope, or elasticity, of the demand relation. (For a discussion of this point, see pp. 506-511 of Ref. 15.)

Finally, a third assumption is necessary before the fit of quantity to price in Fig. 1 can be interpreted as a demand law. It must be assumed that there are no other important exogenous variables that are collinear with price. Omission of such variables from the demand relation could produce misleading results. Specifically, changes in consumption caused by variation in the omitted variables might be mistakenly attributed to variations in price.

The plausibility of the above assumptions is discussed in greater detail in Sec. 4 below.

### 3.2 Dynamic Models

As discussed above, technological advancement and changing consumer preferences make the static-model approach unsuitable for the analysis of time-series market data. One way to handle these time-dependent variables is to add a linear time trend to the constant elasticity (log-linear) market model,

$$q_{it} = a + \alpha p_{it} + \dots + r_t + u_{it} \quad (7)$$

and

$$P_{it} = b + \beta q_{it} + \dots + r'_t + v_{it}, \quad (8)$$

\*A similar criticism does not hold with regard to a stable-demand curve that falls off as  $P^{-1}$ . If a supply curve in Fig. 4 were to shift to the left (for example, a result of technological improvement or increased government subsidy), then revenues, which would remain approximately constant, would still cover total costs, because of the reduction in unit costs of supply.

where  $i$  identifies the state and  $t$  the period (or year) of observation.† The coefficients  $r$  and  $r'$  in the demand and supply equations represent the rates of change of the respective time trends. Such a model can be directly applied to cross-section data that have been pooled for different years. Halvorsen used a variation of this model to analyze residential market data for 48 states for 1961-1969.<sup>4</sup>

There are objections to the above method of including a time trend as a surrogate variable for technological change. First, because the explanatory variables used in the analysis are already time-trended, the introduction of an explicit time trend can lead to high intercorrelation among the explanatory variables, resulting in statistical bias and loss of significance of the coefficient estimates. Second, the values obtained for  $r$  and  $r'$  do not provide direct information concerning the source and the mode of technological change.

More sophisticated than the above model is the one used by Baxter and Rees in their analysis of industrial demand for electricity in the United Kingdom.<sup>7</sup> This model assumes that variations in demand are directly related to changes in the explanatory variables, but that the equilibrium level of consumption will not be attained until some time after the change in the independent variables has taken place. Baxter and Rees considered three time patterns of adjustment, preferring the geometrically distributed form

$$\frac{Q_t}{Q_{t-1}} = \left( \frac{Q_t^*}{Q_{t-1}} \right)^\lambda, \quad (9)$$

where  $Q_t$  = actual demand and  $Q_t^*$  = equilibrium demand (as estimated by the static model) during period  $t$ . Equation 9 states that the ratio of demand during period  $t$  to demand during the previous period is some power of the ratio of equilibrium to actual demand during the previous period. With the substitution of a static model demand function for  $Q^*$ , Eq. 9 takes the form

$$q_t = (1 - \lambda) q_{t-1} + a + \lambda\alpha \cdot p_t + \lambda\beta \cdot g_t + \dots,$$

where  $\lambda\alpha$ ,  $\lambda\beta$ , etc., represent the short-run (first-year response) elasticities. The short-run elasticities, the lag parameter  $\lambda$ , and, therefore, the long-run elasticities are estimated through least-squares regression analysis.

The Baxter-Rees model was generalized by Mount et al. (MCT) in their study of U.S. demand for electricity.<sup>1</sup> The generalized version of the model allowed each elasticity to vary with the level of the corresponding exogenous variable. The MCT variable-elasticity model was of the form

$$q_{it} = a + (1 - \lambda) q_{it-1} + \alpha\lambda \log p_{it} + \alpha_1\lambda/p_{it} + \beta\lambda \log g_{it} + \beta_1\lambda/g_{it} + \dots,$$

where the coefficients  $\alpha_1$ ,  $\beta_1$ , etc., are additional unknown parameters. The interpretation of  $\lambda$  is the same as before, but the short-run elasticities are now given by  $\lambda(\alpha - \alpha_1/p_{it})$ ,  $\lambda(\beta - \beta_1/g_{it})$ , ..., and the long-run elasticities,

†An early precedent for incorporating a time trend in this fashion is provided by Schultz.<sup>16</sup> However, in his analysis of U.S. demand for beef, pork, and mutton for 1922-1933, Schultz found that the effect of time variable was insignificant. (See Ref. 16, pp. 633-638.)

by  $(\alpha - \alpha_1/p_{it})$ ,  $(\beta - \beta_1/g_{it})$ , ... . The elasticities thus vary with the level of the corresponding exogenous variable, and the rate and direction of variation depends upon the fitted values of  $\alpha_1$ ,  $\beta_1$ , ... . In MCT's analysis, the coefficient  $a$  was not constant over the entire data sample (pooled interstate cross-section data for 1947-1970); rather, a value for this coefficient was determined for each of nine different power regions.

Using the above demand model, MCT analyzed residential, commercial, and industrial demand for electricity in the United States. The variable-elasticity form was used for the population, income, and electricity-price variables; constant-elasticity forms were used for natural-gas price and electric appliance price variables. The analysis included separate regressions involving ordinary least-squares and instrumental variable techniques. The results for the two procedures were generally consistent, ordinary least squares yielding better predictions as determined by comparisons with actual 1971 consumption figures.

Results of MCT's analysis of the residential market include a lag parameter  $\lambda$  equal to about 0.89 and a long-run price elasticity equal to about -1.3. The results indicate that the first-year response to a 1% increase in electricity price is a 0.14% decrease in consumption and that about 7 years are required before 50% of the total response occurs.

### 3.3 Dynamic Models as Predictive Tools

Demand relations statistically inferred from dynamic models should provide a useful basis for predicting future electricity-consumption trends. However, the models so far developed suffer shortcomings that may seriously impair their reliability when used to predict future consumption. Although the analysis of time-series data, like the analysis of cross-section data, is subject to the supply-demand identification problems, neither the Baxter-Rees nor the MCT model includes a supply equation.

Several limitations of the single-equation demand model as used in the analysis of time-series data are illustrated in Fig. 5. In each part of the figure, the curves SS and DD represent "real" relations describing long-run supply-and-demand conditions. The problem for market analysis is to "find" these curves, given the timewise dependence of price-quantity and other market data. To simplify the following discussion we will assume for the four situations depicted in Fig. 5 that the market at time  $t = 0$  is in a condition of stable long-run equilibrium (exogenous variables constant over a long period of time) at point  $(Q_0, P_0)$ .

Figure 5a shows the market response to an abrupt change in supply price at the beginning of the first time period. If all other exogenous variables remain constant (to first order we can ignore such effects as price-induced changes in real consumer income), the price-quantity point moves to Point 1 after the first time period. Thereafter, consumption asymptotically approaches the new long-run equilibrium point  $Q, P$ . A regression analysis on this sequence of data points (if it uses a realistic time pattern of adjustment) can be expected to produce an accurate demand relation. The analysis will associate the entire increase in consumption with the initial decrease in supply price, and it should yield a long-run price elasticity that coincides with the slope of the actual long-run demand curve. Furthermore, when used to

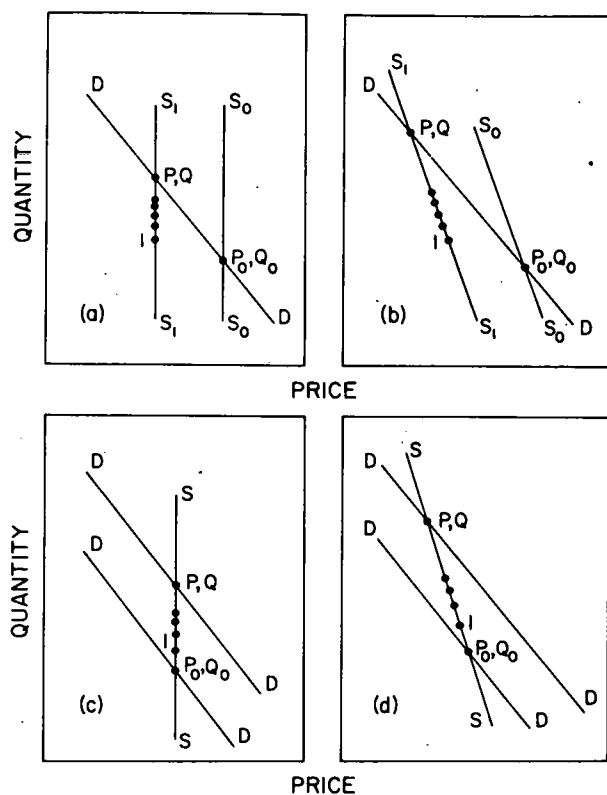


Fig. 5

Dynamic Model under Different  
Supply-Demand Conditions.

consumption. For example, if an expected price increase (due, say, to a tax on output or an increase in fossil-fuel costs) is input to demand relation,\* the demand model ignores supply-induced changes and thus underestimates the actual change in consumption.

Figures 5c and 5d show another aspect of the supply-demand simultaneity problem. In this case, an exogenous variable affecting demand (personal income, say) is assumed to undergo an abrupt change at the beginning of the first time period. No special difficulties are encountered in Fig. 5c because, as in Fig. 5a, the supply curve is perfectly elastic with respect to price. However, in Fig. 5d, the increase in income produces not only an increase in consumption due to the increase in demand, but also an additional increase in consumption due to supply-induced price changes. A regression analysis performed on such a data set will "pick up" the induced as well as the direct effects; thus it is unlikely that significant bias will be introduced into the calculated price and income elasticities of demand. However, serious error can arise if the estimated demand relation is used to predict changes in consumption caused by changes in income. In the predictive mode, the demand model is not capable of estimating the induced price changes (these are a function of the supply curve), so that the model will tend to underestimate the actual change in consumption.

predict future consumption, the demand relation should yield reliable estimates as long as supply is perfectly elastic with respect to price.

Figure 5b shows a situation in which a single-equation demand model may not perform well. Here the supply curve is not perfectly elastic with respect to price, so that the initial decrease in supply price induces further price changes through the action of the supply curve. A regression fit on this sequence of data points probably will not lead to a seriously biased demand relation; however, in an important sense, the estimated demand relation will not be useful as a method for predicting future consumption rates. The reason the estimated demand relation may not be seriously biased is that the regression-fit data sample will include not only the supply-induced quantity increases, but also the supply-induced price decreases. Moreover, the ratio of the total quantity change to total price change is fixed by the slope of the actual demand curve. The problem arises when the estimated demand relation is used to predict future

\*Mount et al. have used their model in this way. (See Ref. 17.)

The failure of single-equation dynamic models to properly resolve the price-quantity simultaneity problem seriously impairs the models' usefulness as tools for predicting electricity consumption. Only if the electricity supply curve were perfectly elastic could the models be expected to yield accurate results. As should be intuitively clear and as we demonstrate in Sec. 5.2, such is not the case.

### 3.4 The Electricity Price Variable

#### 3.4.1 General Discussion

Because of the importance of the price-quantity relation and because of the complications arising from electric-utility pricing practices, the problem of selecting the appropriate price variable deserves special attention. The problem has been discussed by several previous investigators.

Halvorsen, in his discussion of the question of marginal versus average price, argues in favor of average price.<sup>4</sup> His argument rests mainly on the belief that the individual consumer faced with the complication of declining block rates will not make the effort necessary to implement the marginal conditions for welfare maximization. Wilson points to declining block rates as a source of bias, because buyers in separate market areas, even if confronted with identical rate schedules, will have different average prices if they consume different quantities of energy.<sup>5</sup> For this reason, Wilson (although he obtained higher statistical correlation using average price) preferred the FPC's typical electrical bill for 500 kWh/month as the price variable.\* The thrust of MCT's argument in defense of the use of average price is not entirely clear; however, they note that Wilson obtained similar elasticity values using typical billing and average price.<sup>1</sup>

We believe that most previous discussions of the question of the appropriate price variable have not fully described the problem, especially its relationship with the supply-demand simultaneity problem. A principal source of confusion has been the failure to draw the appropriate distinction between rate schedules and actual cost of supply. Wilson's attempt to resolve the supply-demand simultaneity problem within his single-equation demand model can be rigorously defended only if it is assumed that rate schedules accurately represent actual cost of supply.\*\* Given this assumption, the price variable (= the typical bill for 500 kWh) will not vary from region to region because of interregional differences in the quantity supplied.†

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\*The Federal Power Commission annually compiles a list of "typical bills" for specified quantities of service for urban market areas. The bills are computed directly from rate schedules submitted by electric utilities. The bills are not based upon actual consumer purchase data. As discussed in the text below, the FPC's method of computing these bills may underestimate actual rates for certain quantity-of-service categories. However, for a particular quantity-of-service category, the bills probably represent a reasonably consistent measure of interregional price variation.

\*\*Whether rate schedules actually represent cost of supply is currently a controversial issue. Under a given rate schedule in a particular service area, large consumers may be subsidizing small ones, or vice versa. As we show in Sec. 5.2, there is some empirical evidence that rate schedules in some aggregate sense do reflect actual cost of supply.

†For each region, quantity supplied plotted against the typical bill for 500 kWh will be a vertical line. Thus, any bias due to supply-demand simultaneity is effectively removed. (See Sec. 3.1.)

On the demand side, Wilson's model further requires that the typical-bill value uniquely characterize the whole of the rate schedule. Rate schedules having different structures, but yielding equal typical bills for a specified quantity of service, are assumed not to occur. This apparently stringent assumption is required before the demand relation can be interpreted as assigning a unique quantity value to each price value (all other variables remaining fixed). As we show below, fortunately for Wilson's model there is some empirical evidence to support the plausibility of his assumptions on both the demand and the supply side.

In his analysis of the relative merits of average versus marginal price, Halvorsen discusses both the analytical and the behavioral aspects of the problem. He analyzes whether a difference might exist between elasticities computed for a demand equation incorporating marginal price and the same equation incorporating average price. Retracing Halvorsen's discussion of the elasticity difference, we can assume (as Halvorsen implicitly assumes) that the rate schedule closely parallels the supply curve. We then note his later empirical result that the relationship between average supply price and quantity is reasonably approximated by  $P_a \approx Q^{-\phi}$ , where  $\phi$  is the constant elasticity of price with respect to quantity supplied. From the equation relating marginal price,  $P_m$ , and average price

$$P_m = \frac{d}{dQ}(QP_a),$$

it then follows that

$$P_m = (1 - \phi) P_a.$$

Because  $P_m$  and  $P_a$  differ only by a constant factor, it follows that the fitted elasticity value for a log-linear demand relation (see Eq. 4) is independent of whether the equation is expressed in terms of marginal or average price. This result is especially comforting because price-quantity data distributed as in Fig. 1 (whether the price variable equals marginal or average price) are likely to be best fit using demand relations of the log-linear form.

Halvorsen's arguments on behavioral grounds, that consumer purchase decisions are made on the basis of average price rather than marginal price, are not especially convincing. We believe that the consumer's decision to purchase an energy-intensive appliance is influenced mostly by information or impressions about the expected increase in his total electric bill. In other words, even in the absence of direct information about the actual rate structure, the consumer is deciding on the basis of marginal price. (Such energy-intensive appliances as water heaters and space-heating equipment often qualify for special rates. In this case, the special rate is the marginal price.) The same principle would apply with regard to decisions about the intensity of appliance use.

Decisions regarding appliances that are not energy-intensive will be influenced mostly by factors other than electricity price. However, to the extent electricity cost is a consideration, the consumer will base his decision on the expected increment in his total bill, and thus upon marginal price.

### 3.4.2 Typical Bills

One might expect that a cursory examination of typical-bill values as compiled by the FPC would shed some light on the question of average versus

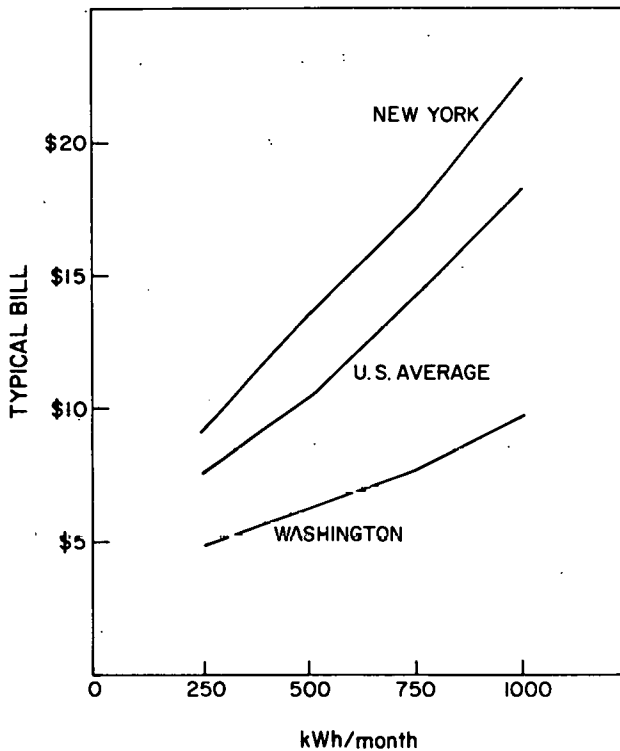


Fig. 6

FPC Typical Residential Bills:  
January 1, 1970.

marginal versus typical-bill price. Unfortunately, this is not the case. The most disconcerting aspect of the FPC's typical-bill values is the failure of the total bill to "bend over" as the quantity purchased increases.

Figure 6 shows bill values for the highest and lowest ranking states and for the U.S. as a whole. The failure of the curves to reflect decreasing marginal costs is due to the assumptions used by the FPC in computing bill values. The bill values are not based upon actual customer purchase data, but are computed directly from electric-utility rate schedules. The bill value for 500 kWh and, to a lesser extent, the one for 750 kWh, involve unrealistically high estimates of consumption for water heating.\* Because electric rate schedules usually provide special rates for water-heating use, the tendency is to underestimate these bill values.

Despite the failure of the typical bills to reflect declining marginal prices, they should provide a reasonably consistent measure of interstate variations in the electricity rates. An examination of the typical-bill curves for a number of states reveals that the curves are roughly parallel. It is in this sense that Wilson's use of the typical bill for 500 kWh can be justified.

Our use (in subsequent sections of this report) of average price as the price variable can be justified only in terms of the a posteriori finding that rate schedules apparently do follow the supply curve and that the supply curve can be roughly approximated by  $P_a \approx Q^{-\phi}$ .

\*Unless the utility rate schedule provides its own formula, the FPC assumes water-heating use equals 250 kWh at 500 kWh, 350 kWh at 750 kWh, and 350 kWh at 1000 kWh. Whether the FPC or the utility formula is used, all customers using 500 kWh or more are assumed to own electric water heaters.



## 4. RESULTS OF PREVIOUS STUDIES

Table II presents selected results from several previous studies of electricity demand. The table lists estimated demand elasticities for three explanatory variables: electricity price, income, and natural-gas price.

TABLE II. Elasticity Estimates from Previous Studies of Electricity Demand

Study	Model	Market	Elasticities		
			Electricity Price	Income	Gas Price
Mount et al. <sup>1,3</sup>	Dynamic, single equation	Residential	-1.3 <sup>a</sup>	+0.3 <sup>a</sup>	+0.15 <sup>a</sup>
		Commercial	-1.5 <sup>a</sup>	+0.9 <sup>a</sup>	+0.15 <sup>a</sup>
		Industrial	-1.7 <sup>a</sup>	+0.5 <sup>a</sup>	+0.15 <sup>a</sup>
Anderson <sup>2</sup>	Static, single equation	Residential	-0.85	+0.94	+0.21
Halvorsen <sup>4</sup>	Static (+ time trend), supply-demand	Residential	-1.2	+0.61	+0.036
Wilson <sup>5</sup>	Static, single equation	Residential	-1.33	-0.46	+0.31
Stull and MacAvoy <sup>b</sup> (Ref. 6)	Static, single equation	Combined	-1.24	+0.86	-

<sup>a</sup>Elasticities estimates for constant-elasticity model.<sup>3</sup> See Ref. 1 for variable-elasticity estimates.

<sup>b</sup>Capacity demand model.

Despite differences in underlying assumptions, methodology, and data base, the studies all indicate that electricity price is the most important determinant of electricity demand. The results of Chapman et al.,<sup>3</sup> Halvorsen,<sup>4</sup> and Wilson<sup>5</sup> for the residential market indicate a price elasticity of demand of about -1.25. Stull and MacAvoy<sup>6</sup> find a similar elasticity value for the entire U.S. electricity market, and Anderson's result<sup>2</sup> for the residential market is not in too serious disagreement.

MCT's finding<sup>1</sup> that commercial and industrial demand is more price-elastic than residential demand is not too surprising. In one sense, however, their result for the industrial market may involve a certain amount of double counting. Energy-intensive industries, by choosing to locate in states with low power costs, cause an increase in consumption in these states while effectively reducing demand in states with high power costs. Unlike the analysis of the residential market in which the redistributive effects of population shifts are normalized out via the population variable, in the analysis of the industrial market the corresponding effects will largely be attributed to the variations in price. Thus, although the computed price elasticity of demand may describe the effects of a local price increase on local consumption, it will tend to overestimate the corresponding effect for the nation as a whole.

The interstudy agreement among the income elasticities is less satisfying than among the price elasticities; however, the results generally indicate that a given increase in income will cause a less than proportionate increase in consumption. Halvorsen suggests that Wilson's negative correlation between

electricity sales and income may be due to Wilson's failure to include an urbanization variable in his demand relation. However, as Wilson's data refer to SMSA's, such an explanation is questionable. Wilson attributes the effect to the concentration of federal power projects in low-income areas. Presumably, the increased consumption in these areas is not entirely explained by lower typical-bill values.

The estimated cross elasticities with respect to gas price are generally lower than might be expected. To the extent that natural gas is intended as a surrogate for all competing fuels, the elasticity estimates probably underestimate the true effect. The total effect of competing-fuel prices is masked by variations in consumption of heating oil.

## 5. ANALYSIS OF THE RESIDENTIAL MARKET

To further explore the causal factors underlying demand for electricity, we have undertaken, in certain areas, to reproduce and extend the analyses discussed in the previous sections. Although the analysis presented here is restricted to the residential market, the methodology and many of the inferences are applicable to the commercial and industrial markets as well.

### 5.1 Single-equation Model

#### 5.1.1 General Discussion

To reproduce and check the results of earlier studies, we performed a demand analysis using a single-equation demand model of the type described in Sec. 3.1. The study method consisted of regressing electricity consumption against explanatory variables for 1959, 1965, and 1970, and then comparing the fitted elasticity values for these years. Variables included in the demand relation were: electricity price P, competing fuel price F, per-capita personal income Y, heating degree days H, average July temperature T, and a population-density variable D.

Electricity price (cents/kWh) was calculated by dividing total electric-utility revenues for each state by total sales for that state (average price variable). Revenue and sales data were obtained from Ref. 9.

Two different techniques were used to compute competing-fuel prices. One method consisted of dividing the sum of revenues from No. 2 heating oil and natural gas by the sum of sales of the two fuels as expressed in Btu's. The other method was to use natural-gas price as the competing-fuel price. Natural-gas prices were determined from revenue and sales data presented in Ref. 12, and heating-oil revenues were computed from sales and price data in Ref. 13. Because the long-run average retail price of fuel oil is nearly uniform across the United States, retail fuel-oil prices for each state were set equal to the national average value. Thus, the principal effect of including fuel-oil price in the composite-price variable was to smooth the interstate variations that would occur if only natural-gas price were used.

Per-capita personal income and the data required to compute the population-density variable were obtained from Ref. 11. The population-density variable was actually a distance measure computed from the formula  $D = (A/N_R)^{1/2} \cdot R$ , where A equals the area of a particular state,  $N_R$  equals the rural population, and R is the percentage of the state's population living in rural areas. Use of the variable D consistently yielded better fits than use of either R or  $1 - R$  (= percentage of population in urban areas).

Values for the climatological variables were developed from 30-year-average city data presented in Ref. 18. State-average values for July temperature and annual heating degree days were developed by averaging the population-weighted city data.

In general, log-linear demand relations provided better fits than did equations linear in the demand variables. Table III lists results of ordinary least-squares regression fits for different combinations of the explanatory

TABLE III. Computed Elasticities for Eq. 10

Year	Elasticities						R <sup>2</sup>
	$\alpha$	$\beta$	$\gamma$	$\delta$	$\epsilon$	$\eta$	
1959	-0.98						0.72
1965	-1.04						0.78
1970	-1.05						0.81
(Error) <sup>a</sup>	(0.08)						
1959	-0.98	0.15					0.76
1965	-1.03	0.14					0.80
1970	-1.14	0.15					0.82
(Error)	(0.08)	(0.09)					
1959	-1.02	0.08	0.32				0.77
1965	-1.07	0.12	0.09				0.80
1970	-1.11	0.21	-0.19				0.83
(Error)	(0.08)	(0.10)	(0.13)				
1959	-0.88	0.31	0.48	0.16			0.88
1965	-0.95	0.30	0.34	0.14			0.89
1970	-1.07	0.26	0.09	0.06			0.85
(Error)	(0.07)	(0.08)	(0.09)	(0.03)			
1959	-0.90	0.30	0.44	0.15	0.05		0.89
1965	-0.93	0.30	0.40	0.15	-0.04		0.89
1970	-0.99	0.31	0.19	0.11	-0.11		0.87
(Error)	(0.07)	(0.08)	(0.12)	(0.03)	(0.04)		
1959	-0.87	0.28	0.40	0.15	0.02	-0.05	0.89
1965	-0.92	0.30	0.38	0.15	-0.05	-0.02	0.89
1970	-1.03	0.35	0.20	0.11	-0.07	0.06	0.88
(Error)	(0.07)	(0.08)	(0.12)	(0.03)	(0.04)	(0.04)	
1959 <sup>b</sup>	-0.93	0.19	0.43	0.15	0.03		0.88
1965 <sup>b</sup>	0.97	0.14	0.40	0.13	-0.02		0.88
1970 <sup>b</sup>	-1.03	0.17	0.18	0.10	-0.10		0.86
(Error)	(0.07)	(0.05)	(0.13)	(0.03)	(0.04)		

<sup>a</sup>"Error" refers to standard errors for year 1965.

<sup>b</sup>Competing-fuel price equals natural-gas price.

variables in a demand relation of the form

$$\log Q = \omega \cdot \log I_0 + \alpha \cdot \log P + \beta \cdot \log F + \gamma \cdot \log Y + \delta \cdot \log D + \epsilon \cdot \log H + \eta \cdot \log T. \quad (10)$$

The most striking feature of the results is the stability of the electricity-price elasticity as other variables are introduced into the demand relation. For a given year, this elasticity is stable to within  $\pm 5\%$ . The cross elasticity with respect to competing fuel price is stable with respect to the introduction of the climatological variables. As expected, the estimated cross elasticities are lower when the demand equation incorporates natural-gas price as the competing-fuel variable than when it incorporates the composite oil-gas price variable.

Inclusion of the population-distance variable, D, increases the estimated value of the income elasticity. As Halvorsen pointed out, income and degree

of urbanization are positively correlated, but consumption and urbanization are negatively correlated. Thus, failure to include some variable representing population density leads to an underestimate of the income elasticity. The "break" between 1965 and 1970 in the values estimated for  $\gamma$  and  $\delta$  may represent an actual effect. On the other hand, it casts some doubt on the estimated income elasticities. All that can be said is that the results in Table III are consistent with an income elasticity of about 0.3.

The standard errors associated with the climatological parameters are too large to permit meaningful statistical inference. However, the time trend of these parameters is consistent with the observation that an increasing portion of electricity consumption is for air conditioning and space heating. These uses, of course, are proportionately greater in areas with high summer temperatures and mild winter temperatures.

### 5.1.2 Variable Elasticities

Considered as a whole, the data in Table III suggest that the price and income elasticities are changing with time. The decreasing income elasticity is consistent with the dependence exhibited by most goods.<sup>19</sup>

To explore the dependence of the price and income elasticities on the level of income, we first added a term  $\alpha_1 \cdot Y \cdot \log P$  to Eq. 10, corresponding to a variable price elasticity of the form  $\alpha + \alpha_1 \cdot (\log P) \cdot Y$ . In a second calculation, we added a term  $\gamma_1 \cdot Y$ , corresponding to an income elasticity of the form  $\gamma + \gamma_1 \cdot Y$ . The estimated dependence of price elasticity on income was statistically insignificant for 1959, 1965, and 1970, although the sign and magnitude of the dependence were consistent with the time trend shown in Table III. The value obtained for  $\gamma$  was statistically significant at the 0.03 confidence limit and tended to "explain" the income-elasticity time trend in Table III. However, its inclusion in the demand relation did not significantly improve the overall goodness of fit ( $\Delta R^2 = +0.01$ ), nor did it alter the values computed for the other elasticities.

### 5.2 Supply-Demand Analysis

As discussed in Sec. 3.1, the estimation of the coefficients in the single-equation demand model is likely to contain bias. The bias arises because of the nature of the price-quantity interrelation; not only does price affect quantity, but quantity, in turn, is influenced by price. Only in the special case that the supply curve is perfectly elastic with respect to price will the source of the bias disappear.

Fortunately, there exists a method that involves relatively light computation that often constitutes the best method of estimation in situations of this type. The method is two-stage least squares.

Essentially, the method consists of the following steps: (1) From the structural supply and demand relations, write the reduced-form equations that explicitly relate price and quantity to the predetermined variables (see footnote on page 13). (2) Apply the method of least squares to the reduced-form equation for price, and obtain best-fit values for the price variable. (3) Substitute these best-fit values into the original demand relation, and apply least squares to obtain estimates of the demand coefficients. (See pp. 225-232 of Ref. 20.)

To apply the method of two-stage least squares, we specified demand and supply equations of the form

$$\log Q = \omega \cdot \log 10 + \alpha \cdot \log P + \beta \cdot \log F + \gamma \cdot \log Y \\ + \delta \cdot \log D + \varepsilon \cdot \log H$$

and

$$\log P = \omega' \cdot \log 10 + \rho \cdot \log Q + \rho_1 \cdot \log D + \rho_2 \cdot \log Z_2 \\ + \rho_3 \cdot \log Z_3 + \rho_4 \cdot \log Z_4,$$

where the exogenous variables in the demand equation have the same meaning as in Sec. 5.1 and the  $Z_n$ 's are variables affecting supply price. For  $n = 2, 3, 4$ ,  $Z_n$ , respectively, is equal to: hydroelectric capacity as a percentage of total generating capacity, private ownership of generating capacity as a percentage of total ownership, and residential sales as a percentage of total sales. The population-distance variable,  $D$ , appears in both the supply and demand relations.

Fossil-fuel costs were not included in the supply relation because these costs do not vary greatly from state to state (as compared with the variations of the other variables) and because, for the period in question (1959-1970), fuel costs typically represented less than 20% of the overall cost of residential electricity.

Successive application of least squares, first to the reduced-form equation for price, and then to the structural-demand equation, yielded coefficient values that were little changed from those estimated under the single-equation demand model. Table IV lists the coefficient values for the two methods. As expected, two-stage least squares give a lower estimate of the own-price elasticity; however, the difference was not large. The other elasticities are generally consistent with those obtained under the single-equation demand model.

TABLE IV. Estimated Demand and Supply Elasticities: 1970

Method of Estimation	Demand Elasticities				
	Price ( $\alpha$ )	Gross Price ( $\beta$ )	Income ( $\gamma$ )	Distance ( $\delta$ )	Heating Degree Days ( $\varepsilon$ )
Ordinary least squares	-1.03	0.31	0.19	0.11	-0.11
Two-stage least squares	-0.89	0.34	0.18	0.12	-0.13
	Supply Elasticities				
	Quantity ( $\rho$ )	Distance ( $\rho_1$ )	% Hydro ( $\rho_2$ )	% Private ( $\rho_3$ )	% Residential ( $\rho_4$ )
Two-stage least squares	-0.66	0.024	-0.03	0.10	0.18

Having formulated simultaneous supply-demand relations, we can also apply the method for two-stage least squares to estimate the coefficients of

the supply relation. The procedure is essentially the same as before, except that now the best-fit values for  $Q$  from the reduced-form equation are inserted into the structural supply relation. Table IV gives elasticities computed for 1970 market data.

All the supply-equation elasticities have expected signs, with per-capita consumption identified as the most important determinant of electricity price. The value  $-0.66$  obtained for the elasticity of price with respect to quantity purchased is close to the value estimated by Halvorsen ( $-0.61$ ) for pooled data for 1960-1970.<sup>4</sup>

The rather strong dependence on the ratio of residential-to-total sales indicates that residential consumers benefit from the presence of electricity-intensive industry, either through cross-subsidization or through higher scale efficiencies in the generation and transmission of power. The dependence on the distance variable is less than might be expected, probably because of the offsetting effects of higher distribution costs and higher consumption rates in rural areas. As expected, greater public ownership and a larger hydroelectric component tend to reduce price.

It is instructive to display the supply and demand curves estimated according to two-stage least squares on a price-quantity diagram. Figure 7 shows these curves normalized to the U.S.-average price-quantity point for 1970. The figure also shows the corresponding points for 1959 and 1965. [All prices represent real (deflated) prices; 1957-1959 = 100.]

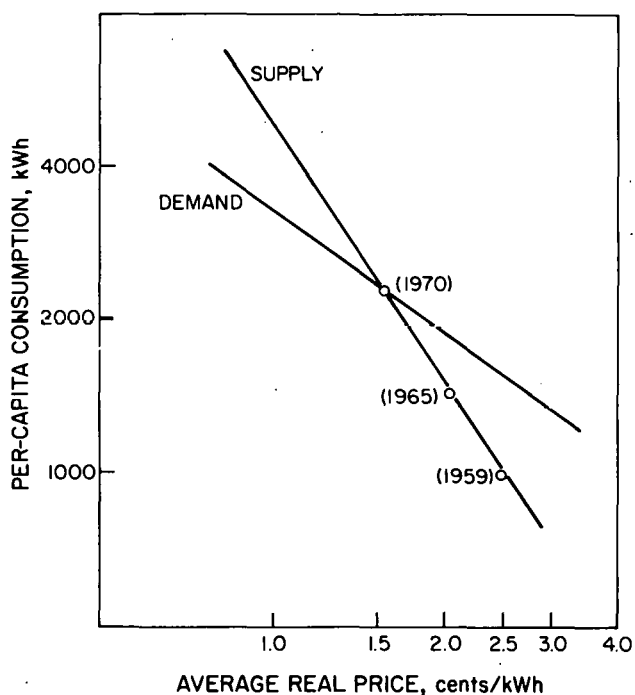


Fig. 7.

Electricity Supply and Demand Curves:  
Residential Market.

This suggests that, at least in some very aggregate sense, electric rate schedules were in close conformity with actual cost of supply.

Interestingly, the figure suggests that, during the time period in question, the price-quantity point has been moving along the supply curve. It is unlikely that the slopes of either the supply or the demand curve have changed significantly over this relatively short time. Accordingly, the loci of points can be explained as the intersections of an upward-shifting demand curve with a relatively stable supply curve. (This timewise behavior of the supply and demand curves should not be confused with the cross-section behavior illustrated in Fig. 4.)

The preceding interpretation of the supply-demand relationship brings to light another interesting feature of the supply curve. As Table V shows, the structure of electric rates has not changed significantly over recent years. In other words, electric utilities did not generally find it necessary to restructure their rate schedules as the price-quantity point moved along the supply

TABLE V. FPC Typical Residential Bills:  
U.S. Average (Nominal prices)<sup>9</sup>

Year	kWh/month				
	100	250	500	750	1000
1959	\$3.98	\$7.36	\$10.51	-	-
1965	4.02	7.38	10.41	\$14.34	\$18.59
1970	4.09	7.51	10.51	14.22	18.31

One effect of the approximate equality of the supply curve and the rate schedule is to "save" the interpretations previous investigators placed on their demand models (see Sec. 3.4). Furthermore, the goodness of the constant-elasticity fit of supply price to quantity ( $R^2 = 0.70$ ) indicates that no serious bias is introduced by using average rather than the marginal price as the price variable.

Probably the most important conclusion to be drawn from Fig. 7 is that future prices and consumption rates cannot be forecast on the basis of either a supply or a demand equation alone. Although the static-model curves in Fig. 7 cannot be used directly to forecast future consumption trends, they do illustrate how supply and demand are inextricably connected in the forecasting problem. Many electric-utility forecasts appear to be in conflict with even the most rudimentary elements of the supply-demand interaction. Many of these forecasts project a continuation of the historical rate of sales growth while at the same predicting an increase in the average real price of electricity. These forecasts imply truly massive shifts from the historical trends of electricity supply and demand.

### 5.3 Competing-fuel Analysis

The coefficient estimates obtained from both the single-equation and the simultaneous supply-demand models indicate that the elasticity of demand with respect to competing-fuel price is about 0.3. This estimate can be independently checked by developing a demand relation for heating oil plus natural gas. A comparison of the respective cross elasticities for electricity and competing-fuel demand then provides an independent test of the cross-elasticity estimates. Specifically, if electricity demand and competing fuel demand are represented by

$$\log Q_E = \omega \cdot \log I_0 + \alpha \cdot \log P + \beta \cdot \log F + \gamma \cdot \log Y + \delta \cdot \log D + \epsilon \cdot \log H \quad (11)$$

and

$$\log Q_F = \omega' \cdot \log I_0 + \alpha' \cdot \log P + \beta' \cdot \log F + \gamma' \cdot \log Y + \delta' \cdot \log D + \epsilon' \cdot \log H, \quad (12)$$

then, according to Slutsky's law of reversibility, the following relation must hold between the cross elasticities:\*

$$\frac{\alpha' Q_F}{P} + \frac{Q_E \gamma' Q_F}{Y} = \frac{\beta Q_E}{F} + \frac{Q_F \gamma Q_E}{Y}$$

\*See Ref. 21, Eq. 50.



In the limit that electricity, natural gas, and fuel oil make up small portions of the consumer's budget, the above relation reduces to the so-called Slutsky-Hotelling relation,<sup>16,22</sup>

$$\alpha' \cdot R_F = \beta \cdot R_E,$$

where  $R_F = F \cdot Q_F$  and  $R_E = P \cdot Q_E$  represent total competing-fuel and electricity revenues, respectively. Although the Slutsky condition holds rigorously only for demand by a single individual, the simpler Slutsky-Hotelling condition is satisfied for the aggregated market demand of a large number of individuals.

Table VI presents coefficients obtained by fitting sales of No. 2 heating oil plus natural gas to Eq. 12. Although the estimated income elasticity for 1970 appears to be too low, the cross elasticity with respect to electricity price is consistent with the values obtained for 1959 and 1965. The strong dependence of sales on heating degree days follows the trend indicated in Fig. 3; the inverse dependence on the population-density variable is consistent with higher electricity sales in rural areas (see Table I).

TABLE VI. Computed Elasticities for Eq. 12

Year	Elasticities					R <sup>2</sup>
	$\alpha'$	$\beta'$	$\gamma'$	$\delta'$	$\epsilon'$	
1959	0.30	-1.06	0.50	-0.34	0.82	0.90
1965	0.44	-0.66	0.63	-0.18	0.75	0.91
1970	0.44	-0.47	0.10	-0.18	0.80	0.87
(Error) <sup>a</sup>	(0.10)	(0.12)	(0.19)	(0.04)	(0.06)	

<sup>a</sup>"Error" refers to standard errors for 1965.

Table VII compares the ratio of the fitted cross elasticities (from Eqs. 11 and 12; see Tables VI and VII) with the value of the ratio as predicted by the Slutsky-Hotelling condition. The rather good agreement lends credence to the estimated cross-price elasticities.

TABLE VII. Comparison of Estimated Cross Elasticities under Slutsky-Hotelling Condition

Year	Estimated Elasticities		Revenues, 10 <sup>9</sup> \$		$\beta/\alpha'$	$R_F/R_E$
	$\alpha'$	$\beta$	$R_F$	$R_E$		
1959	0.30	0.30	4.85	4.34	1.00	1.11
1965	0.44	0.30	6.60	6.33	0.68	1.04
1970	0.44	0.31	8.46	9.42	0.70	0.90

## 6. DISCUSSION AND RECOMMENDATIONS

A number of observations and conclusions can be drawn from the foregoing analyses of residential market for electricity.

### 6.1 Electricity Price

As indicated by several previous studies and as reaffirmed by this study, electricity price is the most important determinant of electricity demand. We find that the price elasticity of residential demand is approximately equal to -1.0. That is, everything else remaining constant, a 1% increase in price will, in the long run, cause a 1% reduction in demand.

On the basis of the static-model analysis, we can confidently predict that the recent upward trend in the cost of electricity supply will reduce the rate of growth of electricity consumption. The only occurrence that could upset this prediction is a massive shift to electricity in markets now served by competing fuels. For this to occur, the price of competing fuels would have to increase at several times the rate of increase of electricity price.

Just as electricity price is the most important determinant of electricity demand, so quantity purchased is the most important determinant of average supply price. The estimated elasticity of supply price with respect to quantity purchased is -0.66. We find that electric rate schedules have been designed to cover the aggregate cost of supply as the aggregate quantity purchased has increased. This finding by itself, however, is not sufficient to guarantee that existing rate structures are "efficient" in the sense of leading to an optimum allocation of resources. In fact, the importance of the price-quantity relationship suggests the need for a careful analysis of the welfare implications of present pricing practices.

### 6.2 Income Elasticity

Most recent studies of the demand for electricity indicate that the income elasticity of demand is greater than zero and less than one. The results of this study are consistent with an elasticity of 0.3. Therefore electricity is properly classified as a superior rather than an inferior good and as a necessity rather than a luxury. As for most goods, the income elasticity of demand appears to decrease with increasing income.

Because low-income groups spend proportionately more of their income on electricity than do high-income groups, it follows that across-the-board percentage increases in the real price of electricity have a regressive effect on the distribution of income. This problem is compounded by the quantity discounts offered under most electric-utility rate schedules.

### 6.3 Cross-price Elasticities

The cross-price elasticities estimated in this study are generally higher than those found by previous investigators. We find that cross elasticity of electricity demand with respect to competing-fuel prices is about 0.3, whereas the cross elasticity of competing-fuel demand with respect to electricity price is 0.4. The ratio of the estimated cross elasticities is in good agreement with the value predicted by the Slutsky-Hotelling condition.

The values obtained for the cross-price elasticities indicate that competition plays an important role in the allocation of energy resources. This suggests that consumer welfare may be better served when gas and electric utilities are under separate management and ownership than when they are combined.

#### 6.4 Other Variables

Residential demand for electricity is related to climatological and demographic variables as well as to price and income variables. Per-capita consumption is greater in rural than in urban areas, and demand is generally higher in the warmer parts of the country.

The price of residential electric service is generally lower in areas with large industrial users. Thus residential customers benefit either from cross subsidization or from spillover benefits due to scale efficiencies in the generation and bulk transmission of electric power.

Although electricity and electric appliances are close complementary goods, we did not investigate the dependence of electricity demand on appliance price. First, appliance prices are essentially uniform across the entire country, making an analysis on the basis of cross-sectional data virtually impossible. Second, the problems of multicollinearity encountered in time-series data would render the results of an analysis of such data highly suspect.

#### 6.5 Demand Forecast

Although the econometric approach to the analysis of the market for electrical energy has provided considerable insight into the causal factors underlying electricity supply and demand, none of the models developed to date can be expected to provide accurate estimates of future consumption. The existing dynamic models fail to incorporate the supply relation, while the static supply-demand models are ill suited to situations involving changes in long-run supply and demand trends.

We would recommend that a dynamic forecast model that includes both the supply and demand relations be developed. It is likely that the lag phenomenon, so prominent in the dynamic demand model will be equally important, if not more important, on the supply side of this model. Because of the importance of interfuel substitution effects, the development of the forecast model would entail the simultaneous analysis of the markets for competing fuels. For comprehensiveness, the forecast model would cover the industrial and commercial sectors as well as the residential-market sector.

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