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ON MODAL PHOTON DENSITIES

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ABSTRACT

The short wavelength laser code XRASER uses ine radiation fields whose dimensions are photons/mode. In this document, we discuss modal photon densities and provide formulas relating these units to units more familiar to the LLNL community.

I. DEFINITION OF A MODAL PHOTON DENSITY

We begin by examining the cgs energy density of a blackbody field, which is (Landau and Lifshitz)

$$U d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT}-1} d\nu \ \frac{\text{ergs}}{\text{cm}^3} \tag{I.1}$$

where $U d\nu$ is an energy density, h is Planck's constant, ν is frequency, c is the speed of light, kT is the product of the Boltzmann constant and a temperature describing the blackbody. The blackbody field is isotropic, and may be described by

$$n_p = \frac{1}{e^{h\nu/kT}-1} \quad \text{photons/mode} \ . \tag{I.2}$$

For isotropic radiation fields, we have

$$n_p = \frac{c^3}{8\pi h\nu^3} U \quad \text{photons/mode} \tag{I.3}$$

For non-isotropic radiation fields, we have the averaged modal photon density is given by (I.3); however, in any given direction, the value may be different. For example, in cgs units and a blackbody field, the intensity is

$$I d\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT}-1} d\nu \quad \frac{\text{ergs}}{\text{cm}^2 \text{ sec ster}} \tag{I.4}$$

where $I d\nu$ is the intensity and the other quantities are defined above. In this case, the modal photon density is (I.2), and we may write

$$n_p = \frac{c^2}{2h\nu^3} I \quad \text{photons/mode} \ . \tag{I.5}$$

This formula may be used to compute the modal photon density for a non-isotropic source. The modal photon density will be angle-dependent (and probably time-, space-, and energy-dependent in the general case).

For additional information on uses of modal photon densities elsewhere, see Marcuse or Louisell.

II. DISCUSSION OF MODAL PHOTON DENSITIES

What is the motivation for using modal photon densities? Well, if one is interested in kinetics processes, a photoexcitation rate between two states may be computed as

$$\gamma_{ex} = \frac{g_u}{g_L} A \int_0^{\infty} \phi(x) \; n_p(x) dx \tag{II.1}$$

where γ_{ex} is the photoexcitation rate, g_L and g_u are the statistical weights of the lower and upper laser states, A is the radiative decay rate, and $\phi(x)$ is the normalized absorption cross section satisfying

$$\int_0^{\infty} \phi(x) dx = 1 \; , \tag{II.2}$$

where x is normalized frequency.

Similarly, radiative decay rates are

$$\gamma_{dx} = [\, 1 + \int_0^{\infty} \phi(x) \, n_p(x) dx] \, A \tag{II.3}$$

where the first term is due to spontaneous decay, and the second term is due to stimulated emission.

Another feature of the unit which is of interest is that quantum electromagnetic noise (virtual photons) has a mean value of

$$n_p = \frac{1}{2} \text{ photons/mode} \tag{II.4}$$

for all angles and energies. One might think of spontaneous decay as being caused by a virtual photon field causing stimulated emission where the cross section is twice as large as for real photons (see for example, Feynman).

The radiative transfer equation in modal photon densities is

$$- \mu \frac{\partial}{\partial s} I = \kappa (I - S) \tag{II.5}$$

where μ is the direction cosine, κ is the opacity (cm^{-1}), I is the intensity (photons/mode), and S is the source function

$$S = \frac{n_u}{\frac{g_u}{g_L} n_L - n_u} \tag{II.6}$$

where n_L and n_u are populations (cm^{-3}) and complete redistribution has been assumed.

Finally, for an optically thick problem in equilibrium,

$$n_p = \frac{1}{e^{h\nu/kT} - 1} \tag{II.7}$$

where T is the equilibrium temperature.

III. VARIOUS FORMULAE

Energy density:

$$U = \frac{8\pi h\nu^3}{c^3} \; n_p \; \frac{\text{ergs}}{\text{cm}^3\text{hz}} \qquad \text{(III.1)}$$

$$= 6.181 \times 10^{-57} \; \nu^3 \; n_p \; \frac{\text{ergs}}{\text{cm}^3\text{hz}} \qquad \text{(III.2)}$$

$$= 2.112 \times 10^{1} \; \varepsilon_{eV}^3 \; n_p \; \frac{\text{ergs}}{\text{cm}^3\text{eV}} \qquad \text{(III.3)}$$

$$= 2.112 \times 10^{13} \; \varepsilon_{keV}^3 \; n_p \; \frac{\text{ergs}}{\text{cm}^3\text{keV}} \qquad \text{(III.4)}$$

$$= 2.112 \times 10^{-3} \; \varepsilon_{keV}^3 \; n_p \; \frac{\text{Jerks}}{\text{cm}^3\text{keV}} \qquad \text{(III.5)}$$

$$= 1.318 \times 10^{13} \; \varepsilon_{eV}^2 \; n_p \; \frac{\text{photons}}{\text{cm}^3\text{eV}} \qquad \text{(III.6)}$$

Intensity:

$$I = \frac{2h\nu^3}{c^2} \; n_p \; \frac{\text{ergs}}{\text{cm}^2\text{ sec ster hz}} \qquad \text{(III.7)}$$

$$= 1.474 \times 10^{-47} \; \nu^3 \; n_p \; \frac{\text{ergs}}{\text{cm}^2\text{ sec ster hz}} \qquad \text{(III.8)}$$

$$= 5.037 \times 10^{10} \; \varepsilon_{eV}^3 \; n_p \; \frac{\text{ergs}}{\text{cm}^2\text{ sec ster eV}} \qquad \text{(III.9)}$$

$$= 3.109 \times 10^{22} \; \varepsilon_{eV}^3 \; n_p \; \frac{\text{keV}}{\text{keV cm}^2\text{ sec ster}} \qquad \text{(III.10)}$$

$$= 5.037 \times 10^{3} \, \varepsilon_{eV}^{3} \, n_p \, \frac{\text{Watts}}{\text{cm}^2 \text{ ster eV}} \tag{III.11}$$

$$= 3.144 \times 10^{22} \, \varepsilon_{eV}^{2} \, n_p \, \frac{\textit{photons}}{\text{cm}^2 \text{ sec ster eV}} \tag{III.12}$$

Emission from an optically thick half space

$$\frac{c}{4} U = \frac{2\pi h \nu^3}{c^2} n_p \frac{\text{ergs}}{\text{cm}^2 \text{ sec hz}} \tag{III.13}$$

$= \pi \times$ results III.7 – III.12

Computing keV/keV sphere from a disk target

$$\left(\frac{\text{keV}}{\textit{keV sphere}}\right) = 3.907 \times 10^{23} \, \varepsilon_{eV}^{3} \, n_p \, \Delta t \, A \, \frac{\text{keV}}{\textit{keV sphere}} \tag{III.14}$$

where Δt is pulse duration in seconds and A is area.

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