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USE OF ADDITIONAL FISSION SOURCES
OR SCATTERING SOURCES
TO MODEL INWARD AXIAL LEAKAGES
IN FAST-REACTOR ANALYSIS

by

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ABSTRACT

When calculations of flux are done in less than three dimensions, bucklings are normally used to model leakages (flows) in the dimensions for which the flux is not calculated. If the net leakage for a given energy group is outward (positive), the buckling is positive, and buckling methods work well. However, if the net leakage for a given energy group is inward (negative), the buckling is negative and can lead to numerical instabilities (oscillations in the iterative flux calculation). This report discusses two equivalent non-buckling methods to model inward leakages. One method (the χ_g method) models these incoming neutrons by additional fission sources. The other method [the $\Sigma_g(1+g)$ method] models them by increased downscatter sources. The derivation of the two methods is shown, and the flux spectra obtained by their use are compared with those obtained from two-dimensional (RZ) calculations.

I. INTRODUCTION

Experimental Breeder Reactor No. II (EBR-II) contains a heterogeneous core region, upper and lower steel axial reflectors, a steel radial reflector, and a fertile radial blanket. Run-to-run neutronic calculations are currently done in XY geometry using the discrete-ordinates transport code DOT-II (Ref. 1) in the S_4 approximation and assuming constant buckling in gross regions to model the axial leakage. The gross regions for which the bucklings are assumed constant in space and energy are the core, radial reflector, and radial blanket. The current XY calculations assume an energy-independent buckling value of 0.0023 cm^{-2} in the core, 0.00085 cm^{-2} in the radial reflector, and 0.0023 cm^{-2} in the radial blanket. Four mesh points of equal area are used in XY rectangular geometry to delineate each subassembly in the 16 rows of the reactor. The rectangular area associated with each subassembly is equal to the area of the actual subassembly hexagonal cell. The cell area includes the subassembly, the hexagonal can, the sodium area between the subassembly and the hexagonal can, and half the sodium area between the hexagonal cans of adjacent subassemblies.

Figure 1 shows a typical run configuration (proposed run 105A). The neutron cross sections used in the XY calculation are a 30-energy-group, transport-approximation P_0 set generated by the MC² code² from ENDFB/Version III data. Table I shows the 30-energy-group structure. To model the axial (Z) dimension, an axial-leakage-removal cross section $D_g B^2$ is used, where D_g is the region diffusion coefficient for energy group g , and B^2 is the region-dependent buckling. This leakage cross section is added to the transport and absorption cross sections for each region and energy group.

A- STEEL ROD P- HALF DRIVER,
B- STEEL DROP ROD HALF REFLECTOR
C- CONTROL ROD S- SAFETY ROD
D- DRIVER X- EXPERIMENT

Fig. 1. Core Loading Configuration for
Proposed EBR-II Run 105A

TABLE I. Thirty-group-set Energy Structure
Used in Neutronic Calculations for EBR-II Runs

Group No.	Upper Energy, eV	Lethargy Width
1	0.10000+8 ^a	0.5
2	0.60653+7	0.5
3	0.36788+7	0.5
4	0.22311+7	0.5
5	0.13534+7	0.5
6	0.82085+6	0.5
7	0.49787+6	0.5
8	0.30197+6	0.5
9	0.18316+6	0.5
10	0.11109+6	0.5
11	0.67379+5	0.5
12	0.40868+5	0.5
13	0.24788+5	0.5
14	0.15034+5	0.5
15	0.91188+4	0.5
16	0.55308+4	0.5
17	0.33546+4	0.5
18	0.20347+4	0.5
19	0.12341+4	0.5
20	0.74852+3	0.5
21	0.45409+3	0.5
22	0.27536+3	0.5
23	0.16702+3	0.5
24	0.10130+3	0.5
25	0.61442+2	0.5
26	0.37267+2	1.0
27	0.13710+2	1.0
28	0.50435+1	1.0
29	0.18554+1	1.0
30	0.68256+0 ^b	∞

^aRead 0.10000+8 as 0.10000×10^8 .

^bLower energy of group 30 = 0.0 eV.

Use of buckling values in this manner gives satisfactory eigenvalues and high-energy reaction rates. However, large errors occur in neutron-flux spectra at low energies, where the flux levels are small. Figures 2 and 3 show the full-height, volume-integrated fluxes in EBR-II core rows 1 and 5 for proposed run 105A when the system is described by an RZ model (reference) model and by a one-dimensional (1D) model with constant buckling. The details of the RZ and 1D models, which will be discussed later, are unimportant for the current discussion. What is important are the big differences in flux that are seen in the lower energy groups. Figure 4 shows the percent flux error (relative to the RZ calculation) in reflector row 9 and blanket row 12 for proposed run 105A when the 1D model is used. Although the differences are not as striking as for the core, they still are large.

Fig. 2. Volume-integrated Flux in FBR-II Core Row 1 for RZ Reference Calculation and for One-dimensional (1D) Calculation with Full-height Constant Buckling

Fig. 3. Volume-integrated Flux in EBR-II Core Row 5 for RZ Reference Calculation and for 1D Calculation with Full-height Constant Buckling

Fig. 4

Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Full-core-height Constant Buckling: Reflector Row 9 and Blanket Row 12

The large flux errors at the lower energies are mainly due to the assumption of energy-independent, regionally constant buckling. A regionally constant positive buckling implies a net outward current. In the core region, the neutron leakage is outward for the higher energy groups, but because of the axial reflectors, the leakage is inward for the lower energy groups. In this buckling methodology, an outward net leakage implies a positive buckling, whereas an inward net leakage implies a negative buckling.

In our analysis, the bucklings, which can be positive or negative, were then calculated for each row, using boundary currents. Sometimes, however, the use of these negative bucklings resulted in calculational instabilities.

To circumvent the difficulties associated with a negative buckling (a negative outward-leaking source), these leaking-in neutrons were modeled by either an additional fissioning source (the χ_g method) or an increased down-scatter source $[\Sigma_g(1+g)]$. This report describes the two methods and compares the results of their use to the results from a reference RZ calculation.

II. BUCKLING CALCULATION

In general, a buckling for energy group g and region R is calculated as if it is an additional removal by equating the integrated Dg^2 leakage over the region volume (V_R) with the integrated normal axial currents ($\vec{J} \cdot \vec{n}$) at the axial surfaces of the region, viz.,

$$\int_{V_R} D_g B_g^2 \phi_g \, dV = \int_S \vec{J}_g \cdot \vec{n} \, dA,$$

where

D_g is the diffusion coefficient for group g ,

B_g^2 is the groupwise buckling,

ϕ_g is the groupwise scalar flux,

$\vec{J}_g$ is the groupwise current,

and

$\vec{n}$ is the unit vector normal to the regional axial surface S .

Hence, for a specific region and energy group, if we know the diffusion coefficient, and can obtain the scalar flux and the boundary angular fluxes from a reference calculation, the bucklings can be calculated. Since codes are not yet available for calculating the axial-leakage buckling for each subassembly

by using three-dimensional analysis, azimuthally symmetric row-wise bucklings were calculated from an RZ reference calculation. The reference fluxes and boundary angular fluxes were calculated with the DOT-IV computer code.³

The RZ configuration used throughout this study as the reference was based on EBR-II proposed run 105A. Figure 5 shows the RZ geometrical configuration for that run. The XY configuration for which the row-wise atom-density smearing was done was shown in Fig. 1. The reference calculation was normalized so that the total integrated fission-neutron source was 1 neutron/s. To show the usefulness of these bucklings in equivalent 1D cylindrical transport calculations with space- (region-) and energy-dependent bucklings, these fluxes are compared in Sec. IV to the fluxes from the reference 2D RZ calculation.

Fig. 5
Geometrical configuration for
RZ Reference Calculation

The 1D transport code that was used was ANISN,⁴ modified to accept space- and energy-dependent bucklings. Any axial reactor portion (slice) for the reference RZ calculation can be modeled in this way. The output flux from the 1D calculation is, however, the axial average of the flux over the original RZ slice. In the geometrical configuration used in the ANISN calculation (shown in Fig. 6), and also in the reference RZ calculation, row 7 was split

into an inner-half core region and outer-half radial-reflector region, and rows 13-16 were combined as one region. The portion of the reference RZ calculation chosen for initial testing was a full-core-height 34.49-cm (13.5-in.) slice of the core, radial reflector, and radial blanket. Table II shows representative bucklings for core rows 1 and 5, reflector row 9, and blanket row 12 for this one-slice problem.

Fig. 6
Mesh Description for One-dimensional Calculations

TABLE II. Regional Bucklings for 34.49-cm
(Core-height) Axial Slice

Energy Group	Core Row 1	Core Row 5	Reflector Row 9	Blanket Row 12
1	4.17-3 ^a	4.48-3	4.34-3	2.45-3
2	4.21-3	4.52-3	4.15-3	2.20-3
3	3.92-3	4.21-3	3.46-3	2.25-3
4	3.38-3	3.62-3	2.82-3	2.24-3
5	3.03-3	3.27-3	2.31-3	1.90-3
6	2.69-3	2.87-3	1.89-3	1.57-3
7	2.13-3	2.20-3	1.53-3	1.39-3
8	1.77-3	1.87-3	1.38-3	1.26-3
9	1.36-3	1.23-3	1.13-3	1.16-3
10	6.61-4	5.33-4	1.00-3	1.04-3
11	-3.70-4	-5.09-4	8.84-4	9.53-4
12	-1.52-3	-1.89-3	7.54-4	9.12-4
13	-3.43-3	-3.51-3	5.19-4	8.33-4
14	-6.85-3	-7.54-3	3.89-4	7.93-4
15	-1.35-2	-1.38-2	3.60-4	7.69-4
16	-2.07-2	-2.46-2	4.73-5	7.52-4
17	-3.30-2	-3.68-2	3.20-5	8.01-4
18	-3.38-2	-3.48-2	-2.13-6	6.05-4
19	-6.82-2	-6.74-2	-2.24-4	5.10-4
20	-1.06-1	-1.09-1	-5.29-4	3.40-4
21	-1.27-1	-1.27-1	-8.71-4	1.96-4
22	-2.03-1	-2.05-1	-1.24-3	1.47-5
23	-2.12-1	-2.21-1	-1.66-3	-2.06-4
24	-1.87-1	-1.85-1	-2.15-3	-5.78-4
25	-4.62-1	-4.75-1	-2.72-3	-1.70-4
26	-4.06-1	-4.09-1	-3.85-3	-8.44-4
27	-4.59-1	-4.45-1	-5.37-3	-3.16-3
28	-1.43-1	-1.51-1	-7.28-3	-1.71-3
29	-2.71-1	-2.71-1	-9.25-3	-2.47-3
30	-1.05+0	-9.85-1	-1.23-2	-7.11-3

^aRead 4.17-3 as 4.17×10^{-3} .

Severe calculational instabilities resulted when these bucklings were used in a 1D ANISN calculation. Although the effective transport cross section never became negative, the effective removal cross section (i.e., the effective transport cross section less the in-group scattering) became negative in some regions and energy groups. To obtain an indication of the result if negative removal cross sections were neglected, the ANISN code was provisionally modified so that the effective transport cross section was set equal to the in-group scattering cross section whenever a negative removal cross section was encountered. This modification ensured a non-negative removal cross section and resulted in a stable iteration history. The converged low-energy flux in the core region, however, still exhibited considerable error. To reduce that error, positive bucklings were handled in the normal way, whereas negative bucklings were handled in one of the two methods described in the next section.

III. THE χ_g METHOD AND THE $\Sigma_g(1+g)$ METHOD

The neutron-transport equation is just a statement of neutron conservation (losses = sources). The normal transport source term contains a fission source term and a scattering source term. When a negative-buckling (negative-loss) term is transposed from the loss side to the source side of the transport equation, algebraically it may be disguised as either a fission source or a scattering source. With the χ_g method, negative bucklings are modeled by additional fission sources; with the $\Sigma_g(1+g)$ method, they are modeled by additional scattering sources. The two methods are discussed below.

A. The χ_g Method

For a specific region R, the total fission-neutron source for energy group g is

$$\frac{\chi_g}{k} \sum_{i \in R} \sum_{h=1} \nu_h^i \Sigma_{fh}^i \phi_h^i V_i,$$

where

χ_g is the fission spectrum for group g,

k is the system eigenvalue,

ν_h^i is the number of neutrons per fission for energy group h at mesh point i,

Σ_{fh}^i is the macroscopic fission cross section for energy group h at mesh point i,

ϕ_h^i is the scalar flux in energy group h at mesh point i,

and

V_i is the volume associated with mesh point i.

The meshwise source due to group-h fissions ($v_h^i \Sigma_{fh}^i \phi_h^i v_i$) has been summed over all N energy groups and over all mesh points (i) in region R.

In an incremental fission spectrum ($\Delta\chi_g$) for energy group g is defined as

$$\Delta\chi_g \equiv \frac{\sum_{i \in R} D_g^i |B_{g,i}^2| \phi_g^i v_i}{\frac{1}{k} \sum_{i \in R} \sum_{h=1}^N v_h^i \Sigma_{fh}^i \phi_h^i v_i}$$

for energy groups in which there are negative buckling values, then the modified fission spectrum ($\chi_g + \Delta\chi_g$), when multiplied by the flux as was done above, will model both the fission neutrons and the neutrons axially leaking in. Although the incremental fission spectrum is dependent on the reference calculation, it should be relatively insensitive to run when the run-to-run modifications are small. If the run-to-run modifications are large, new incremental fission spectra have to be calculated.

As seen by the definition, the incremental fission spectrum is region dependent, so our version of the 1D transport code ANISN had to be modified to accept a region-dependent fission spectrum. The major effect of this modification was to slightly increase the code container size needed to run a problem.

Since the denominator must not be zero if $\Delta\chi_g$ is to be defined, further adjustments have to be made to model leakage of neutrons into nonfissioning regions. To accomplish this, in the reference calculation a small amount (10^{14} atoms/cm³) of ²³⁵U was inserted into what were originally nonfissioning regions. The choice of ²³⁵U was arbitrary; any fissionable material will suffice. The amount of fissionable material used in the nonfissioning regions, although arbitrary, should be kept small so that it does not alter the flux values or the system eigenvalue. The same number density of fissionable material that appears in regions of the reference calculation has to appear in the equivalent regions of the 1D model.

B. The $\Sigma_s(l \rightarrow g)$ Method

For a specific region R, the total neutron scattering source for energy group g, assuming isotropic scattering, is

$$\sum_{i \in R} \sum_{J=1}^N \Sigma_{J \rightarrow g}^i \phi_J^i v_i,$$

where $\Sigma_{j \rightarrow g}^i$ is the macroscopic scattering cross section for scattering between group j and group g at meshpoint i . A set of pseudoscattering cross sections ($\Sigma_{j \rightarrow g}^R$, $j = 1, N$) that model the axial in-leakage for region R and energy group g can be defined as follows:

$$\sum_{i \in R} \sum_{j=1}^N \Sigma_{j \rightarrow g}^R \phi_j^i V_i = \sum_{i \in R} D_g^i |B_{g,i}^2| \phi_g^i V_i.$$

For convenience, all the pseudoscattering was put into the transfer from group 1 to group g ; hence,

$$\Sigma_{1 \rightarrow g}^R = \frac{\sum_{i \in R} D_g^i |B_{g,i}^2| \phi_g^i V_i}{\sum_{i \in R} \phi_1^i V_i}.$$

When multiplied by the flux, as was done above, these pseudoscattering cross sections model the axial in-leakage of neutrons. As with the incremental fission spectra in the χ_g method, these cross sections are region dependent even for a region-independent negative buckling. Once these cross sections are calculated, they are put into the 1D transport calculations by normal cross-section input/mixing methods. The $\Sigma_S(1 \rightarrow g)$ method involves more numbers (cross-section sets) than the χ_g method does, but there is no code modification involved.

IV. JLTS

The quantitative comparisons between the reference RZ flux spectra and the 1D flux spectra calculated with bucklings and either the χ_g method or $\Sigma_s(1+g)$ method are presented here in four different sections. The first section (Sec. IV.A) details the differences between the row-wise flux spectra calculated with the χ_g method and the $\Sigma_s(1+g)$ method when the full-height core region is used as the calculational slice for the buckling. The 1D fluxes in this case are axially averaged over the full core height because the bucklings were calculated for the full height.

When a finer axial spectral definition is required, several axial slices may have to be used for the calculations. Sections IV.B and IV.C consider the cases where the full-height core is described by three and 13 axial slices, respectively. The 13-axial-slice case corresponds to one calculational slice per axial mesh interval in the reference calculation. The fourth section (Sec. IV.D) considers two axial noncore-slice calculations: one in the lower axial reflector, and one in the upper axial reflector.

Since the differences between the χ_g and the $\Sigma_s(1+g)$ methods are shown in Sec. IV.A, the negative bucklings are modeled only by the χ_g method in Sec. IV.B-IV.D.

A. One Slice Consisting of Full Core Height

This section compares the RZ reference flux spectra with the 1D flux spectra calculated using bucklings and either the χ_g or the $\Sigma_s(1+g)$ method. The full-height core was the calculational slice. When each region is a full-core-height row, the representative directly calculated bucklings are as was shown in Table II. Tables III, IV, and V show, respectively, representative positive bucklings, the additional fission spectra ($\Delta\chi_g$), and the pseudoscattering cross sections [$\Sigma_s(1+g)$].

Figures 7-10 show the percent errors in group flux (relative to the RZ reference flux) for core rows 1 and 5, reflector row 9, and blanket row 12 when using the χ_g method and the $\Sigma_s(1+g)$ method. These figures show that the difference between the results from the two methods are small. The results are for the axially averaged flux over the full height of the slice. The figures show that at least the axially averaged core flux from the calculation can be approximated with a 1D calculation. However, what happens when we use a finer axial distribution than just the axial mean value? This is looked at in the following two sections.

B. Three Slices across Full Core Height

In this case, the core is divided axially into three slices as shown in Fig. 11. In the reference RZ calculation, 37.4% of the fissions occurred in the top slice, 24.0% in the middle slice, and 36.1% in the bottom slice. The other 2.5% of the fissions occurred in the radial blanket regions both above the top slice and below the bottom slice.

TABLE III. Positive Regional Bucklings for 34.49-cm
(Core-height) Axial Slice

Energy Group	Core Row 1	Core Row 5	Reflector Row 9	Blanket Row 12
1	4.17-3 ^a	4.48-3	4.34-3	2.45-3
2	4.21-3	4.52-3	4.15-3	2.20-3
3	3.92-3	4.21-3	3.46-3	2.25-3
4	3.38-3	3.62-3	2.82-3	2.24-3
5	3.03-3	3.27-3	2.31-3	1.90-3
6	2.69-3	2.87-3	1.89-3	1.57-3
7	2.13-3	2.20-3	1.53-3	1.39-3
8	1.77-3	1.87-3	1.38-3	1.26-3
9	1.36-3	1.23-3	1.13-3	1.16-3
10	6.61-4	5.33-4	1.00-3	1.04-3
11	0.0	0.0	8.84-4	9.53-4
12	0.0	0.0	7.54-4	9.12-4
13	0.0	0.0	5.19-4	8.33-4
14	0.0	0.0	3.89-4	7.93-4
15	0.0	0.0	3.60-4	7.69-4
16	0.0	0.0	4.73-5	7.52-4
17	0.0	0.0	3.20-5	8.01-4
18	0.0	0.0	0.0	6.05-4
19	0.0	0.0	0.0	5.10-4
20	0.0	0.0	0.0	3.40-4
21	0.0	0.0	0.0	1.96-4
22	0.0	0.0	0.0	1.47-5
23	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0

^aRead 4.17-3 as 4.17×10^{-3} .

TABLE IV. Regional Additional Fission Spectra for
34.49-cm (Core-height) Axial Slice

Group	Original χ	Additional Fission Spectrum ($\Delta\chi_B$)			
		Core Row 1	Core Row 5	Reflector Row 9	Blanket Row 12
1	2.18-2 ^a	0.0	0.0	0.0	0.0
2	9.99-2	0.0	0.0	0.0	0.0
3	1.97-1	0.0	0.0	0.0	0.0
4	2.27-1	0.0	0.0	0.0	0.0
5	1.85-1	0.0	0.0	0.0	0.0
6	1.22-1	0.0	0.0	0.0	0.0
7	7.08-2	0.0	0.0	0.0	0.0
8	3.79-2	0.0	0.0	0.0	0.0
9	1.93-2	0.0	0.0	0.0	0.0
10	9.55-3	0.0	0.0	0.0	0.0
11	4.64-3	8.98-4	1.32-3	0.0	0.0
12	2.23-3	1.81-3	2.38-3	0.0	0.0
13	1.06-3	4.04-3	4.65-3	0.0	0.0
14	5.05-4	2.96-3	3.71-3	0.0	0.0
15	2.40-4	1.35-3	1.84-3	0.0	0.0
16	1.13-4	9.06-4	1.13-3	0.0	0.0
17	5.36-5	3.61-4	4.30-4	0.0	0.0
18	2.54-5	2.08-3	2.51-3	2.97+1	0.0
19	1.20-5	2.06-3	2.45-3	2.18+3	0.0
20	5.66-6	1.63-3	1.91-3	3.83+3	0.0
21	2.68-6	1.24-3	1.43-3	4.80+3	0.0
22	1.26-6	1.01-3	1.16-3	5.34+3	0.0
23	5.97-7	8.09-4	9.28-4	5.60+3	1.01-4
24	2.82-7	6.02-4	6.97-4	5.59+3	4.57-4
25	1.33-7	4.60-4	5.39-4	5.29+3	4.00-4
26	9.27-8	6.24-4	7.36-4	8.58+3	7.75-5
27	2.07-8	3.03-4	3.62-4	5.21+3	2.17-4
28	4.62-9	6.65-5	8.64-5	2.25+3	3.98-5
29	1.03-9	2.13-5	2.74-5	5.63+2	2.18-5
30	1.03-9	6.97-6	8.94-6	2.09+2	6.14-6

^aRead 2.18-2 as 2.18×10^{-2} .

TABLE V. Regional Pseudoscattering Cross Sections for
34.49-cm (Core-height) Axial Slice

Group	Pseudoscattering Cross Section $\{\Sigma_g(1+g)\}$, cm ⁻¹			
	Core Row 1	Core Row 5	Reflector Row 9	Blanket Row 12
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0
11	4.51-3 ^a	5.48-3	0.0	0.0
12	9.09-3	9.88-3	0.0	0.0
13	2.03-2	1.93-2	0.0	0.0
14	1.48-2	1.54-2	0.0	0.0
15	6.78-3	6.64-3	0.0	0.0
16	4.55-3	4.70-3	0.0	0.0
17	1.81-3	1.79-3	0.0	0.0
18	1.04-2	1.04-2	4.79-5	0.0
19	1.03-2	1.02-2	3.50-3	0.0
20	8.19-3	7.95-3	6.17-3	0.0
21	6.21-3	5.96-3	7.72-3	0.0
22	5.06-3	4.83-3	8.60-3	0.0
23	4.06-3	3.86-3	9.01-3	4.82-4
24	3.02-3	2.90-3	9.00-3	2.19-3
25	2.31-3	2.24-3	8.52-3	1.92-3
26	3.13-3	3.06-3	1.38-2	3.71-4
27	1.52-3	1.51-3	8.39-3	1.04-3
28	3.34-4	3.59-4	3.61-3	1.91-4
29	1.07-4	1.14-4	9.07-4	1.05-4
30	3.50-5	3.72-5	3.36-4	2.95-5

^aRead 4.51-3 as 4.51×10^{-3} .

Fig. 7

Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling and the χ_g and $\Sigma_g(1+g)$ Methods: Full Core Height, Core Row 1

Fig. 8. Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling with the χ_g and $\Sigma_s(1 \rightarrow g)$ Methods: Full Core Height, Core Row 5

Fig. 9. Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling with the χ_g and $\Sigma_s(1 \rightarrow g)$ Methods: Full Core Height, Reflector Row 9

Fig. 10. Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling with the χ_g and $\Sigma_s(1 \rightarrow g)$ Methods: Full Core Height, Blanket Row 12

Fig. 11. Geometrical Description for Case of Three Axial Slices across Core Height

Table VI lists representative axially dependent bucklings for a core region (row 1) and a reflector region (row 9). The bucklings and the flux errors for the top slice (not listed) are similar to those for the bottom slice, because the RZ reference problem is almost symmetrical. The row-1 negative bucklings for the two end slices start in a higher energy group because those slices are closer to the in-leakage source for the axial reflectors. The row-9 negative bucklings for the middle slice start in a higher energy group. The reason is that incoming source neutrons from both the top and bottom slices are transported to the middle slice, but the neutrons entering either the top or bottom slice are primarily transported from the middle slice.

TABLE VI. Regional Bucklings for Bottom and Middle Slice of Three-slice Problem

Energy Group	Buckling, cm^{-2}			
	Row 1, Bottom	Row 1, Middle	Row 9, Bottom	Row 9, Middle
1	5.06-3 ^a	2.11-3	4.81-3	3.52-3
2	5.07-3	2.20-3	5.29-3	1.61-3
3	4.60-3	2.31-3	3.98-3	2.43-3
4	3.79-3	2.33-3	3.00-3	2.68-3
5	3.25-3	2.37-3	2.35-3	2.48-3
6	2.72-3	2.41-3	1.86-3	2.22-3
7	1.99-3	2.17-3	1.50-3	1.84-3
8	1.57-3	2.01-3	1.36-3	1.66-3
9	1.10-3	1.70-3	1.11-3	1.35-3
10	2.98-4	1.28-3	1.00-3	1.19-3
11	-8.67-4	6.45-4	8.93-4	1.02-3
12	-2.12-3	1.02-4	7.64-4	8.71-4
13	-4.38-3	-1.33-3	5.21-4	5.77-4
14	-7.58-3	-3.36-3	4.06-4	4.20-4
15	-1.48-2	-4.28-3	3.75-4	3.83-4
16	-2.51-2	-5.76-3	-1.55-5	3.12-4
17	-3.89-2	-6.19-3	-1.95-5	2.56-4
18	-3.50-2	-1.43-2	-1.18-6	4.17-5
19	-6.83-2	-3.20-2	-2.45-4	-1.35-4
20	-1.07-1	-7.02-2	-5.93-4	-3.37-4
21	-1.26-1	-1.16-1	-9.93-4	-5.40-4
22	-2.01-1	-2.03-1	-1.43-3	-7.44-4
23	-2.10-1	-2.49-1	-1.94-3	-1.00-3
24	-1.85-1	-2.44-1	-2.54-3	-1.22-3
25	-4.65-1	-2.55-1	-3.20-3	-1.62-3
26	-4.04-1	-4.46-1	-4.63-3	-2.13-3
27	-4.58-1	-4.60-1	-6.56-3	-2.73-3
28	-1.47-1	-1.95-1	-9.12-3	-3.11-3
29	-2.73-1	-2.56-1	-1.16-2	-3.56-3
30	-1.05+0	-2.98-1	-1.57-2	-4.00-3

^aRead 5.06-3 as 5.06×10^{-3} .

Figures 12 and 13 show the errors in group flux (relative to the RZ reference flux) for the middle and bottom slices of core rows 1 and 5, reflector row 9, and blanket row 12 when using the χ_g method. The flux errors for rows 1 and 5 are within a few percent until energy group 20. After group 20, the error becomes oscillatory and relatively large. This large error is due to an unsatisfactory mesh spacing for the lower-energy groups and the relative hardness of the lower-energy source in the middle slice. The low-energy source for the middle slice is mainly a combination of transport (leaking in) and downscatter, not fission. These low-energy neutrons [as represented by the $\Delta\chi_g$ or the $\Sigma_s(1+g)$ method] see a relatively large removal cross section (about half the transport cross section). In the low-energy groups, these neutrons are removed mostly by absorption. Because of the large removal cross section, a fine mesh is needed to follow the rapidly varying low-energy spatial source and flux.

Fig. 12. Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling and the χ_g Method: Middle Slice of Three-slice Problem

Fig. 13. Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling and the χ_g Method: Bottom Slice of Three-slice Problem

Figure 14 plots the interaction mean free path against energy for core rows 1 and 5, reflector row 9, and blanket row 12. The core axial mesh spacing used was 2.65 cm. Radial meshing was done using two mesh intervals per row. The radial mesh spacing used in row 1 was 1.56 cm. Because of the ^{10}B distribution used in the row-5 control rod, the radial mesh spacing in row 5 was 2.36 cm whereas the outer mesh interval of row 5 was 3.07 cm. The radial mesh spacing of rows 9 and 12 was 2.70 cm. In most instances, the mean free path of the lower-energy groups, was less than the axial and radial mesh spacing.

Fig. 14

Mean Free Path as a Function of Energy Group in Core Row 1, Reflector Row 9, and Blanker Row 12

Fig. 15. Polar Plot of Angular Flux for Energy Groups 1, 15, and 30 at Centerline of Core Row 1, 1.32 cm below Core Midplane

Figure 15 shows the energy dependence of the angular flux for energy groups 1, 15, and 20 in the central mesh interval of row 1, i.e., in the radial center of that row and 1.32 cm below the core midplane. (Figure 16 shows how to interpret the space and angular dependencies shown in Figs. 15 and 17-19.) The bucklings, and hence the 1D flux calculated from them, for the energy groups that exhibit the type of angular dependence as seen in the group-30 angular flux are suspect. In the middle slice of row 1, all the lower-energy groups (groups 20-30) exhibit the same type of angular dependence as seen in group 30. These bucklings lead to the large flux errors that were seen in Fig. 12.

Figure 17 shows the energy dependence of the angular flux for energy groups 1, 15, and 20 at the core bottom in the radial center of core row 1. (The angular fluxes at the core top are not shown, because they are essentially the same shape as the angular fluxes at the core bottom, but with a 180° phase shift.)

Fig. 16. Geometrical Definition Used for Figs. 15 and 17-19. In Fig. 15, for example, r_0 is the radial distance from core centerline to the outer boundary of the first mesh interval in row 1, and Z_0 is 1.32 cm below core midplane. The Group-1 angular flux in the cylindrical-quadrature angle defined by direction cosines $\eta = -0.33$ and $\mu = -0.94$ is 2.6 multiplied by the scale factor of $1.0 \times 10^{-6} \text{ n/cm}^2 \text{ s-unit weight}$.

Fig. 17. Polar Plot of Angular Flux for Energy Groups 1, 15, and 30 at Centerline of Core Row 1, at Core Bottom

Fig. 18. Polar Plot of Angular Flux for Energy Groups 1, 15, and 30 at Centerline of Reflector Row 9, 1.32 cm below Core Midplane

Fig. 19

Polar Plot of Angular Flux for Energy Groups 1, 15, and 30 at Centerline of Reflector Row 9 and at Elevation of Core Bottom

These bottom-boundary angular fluxes do not exhibit the same angular dependence of lower energies as seen in the middle slice (Fig. 16). This is due to two reasons: (1) the low-energy buckling for the whole core, the top slice, and the bottom slice are dominated by the leakage from the core top and/or core bottom, and (2) the regions immediately above and below the core are highly scattering (not absorbing) media and do not need as fine a mesh spacing to follow transported low-energy-source neutrons. When the angular-flux dependencies are satisfactory, the buckling values are acceptable.

The flux values for row 9 (radial reflector), calculated using 1D analysis, have the highest errors in the first few energy groups. The errors then moderate. This effect is seen irrespective of calculational slice. (Compare Figs. 12 and 13.) This error is caused by trying to model a highly anisotropic

axial out-leakage by an isotropic buckling. Figures 18 and 19 show the angular flux for energy groups 1, 15, and 30 at core center and core bottom, respectively, in row 1. These figures show how anisotropic the high-energy-group angular fluxes are relative to the intermediate- and low-energy-group angular fluxes. The error caused by the modeling method mentioned above is seen in both the core and blanket regions. (See Figs. 12 and 13.) This error, although occurring in all regions, is largest in regions where there is no high-energy source and, hence, the high-energy angular flux is quite directional. The error, however, is not limited to high energies. Whenever an isotropic effect (fissioning or scattering) is used to model an anisotropic in-leaking source, a flux error will result.

C. Thirteen Slices across Full Core Height

In this case, the core is divided axially into 13 slices. Each axial mesh interval of the curve in the RZ reference calculation was made into a slice. Table VII shows representative buckling values in row 1 for several axial slices. Also shown is the overall buckling values in row 1 when the whole core is modeled as one axial slice.

TABLE VII. Bucklings in Row 1 for Various Slices of 13-slice Problem

Energy Group	Buckling, cm ⁻²			
	Slice 1 (bottom)	Slice 7	Slice 13 (top)	Overall
1	1.43-2 ^a	1.72-3	1.33-2	4.17-3
2	1.41-2	1.84-3	1.32-2	4.21-3
3	1.14-2	2.02-3	1.08-2	3.92-3
4	7.64-3	2.15-3	7.48-3	3.38-3
5	5.46-3	2.26-3	5.64-3	3.03-3
6	2.45-3	2.34-3	3.84-3	2.69-3
7	1.66-3	2.15-3	2.41-3	2.13-3
8	9.19-4	2.02-3	1.51-3	1.77-3
9	7.78-5	1.73-3	8.42-4	1.36-3
10	-1.28-3	1.32-3	-4.70-4	6.61-4
11	-2.78-3	8.04-4	-2.31-3	-3.70-4
12	-5.32-3	1.97-4	-4.73-3	-1.52-3
13	-8.60-3	-1.23-3	-6.64-3	-3.43-3
14	-1.06-2	-2.63-3	-1.29-2	-6.85-3
15	-2.40-2	-3.66-3	-2.76-2	-1.35-2
16	-3.93-2	-4.96-3	-3.56-2	-2.07-2
17	-6.88-2	-4.99-3	-6.89-2	-3.30-2
18	-4.38-2	-1.00-2	-4.84-2	-3.38-2
19	-7.99-2	-1.83-2	-8.48-2	-6.82-2
20	-1.18-1	-1.28-2	-1.22-1	-1.08-1
21	-1.34-1	7.89-3	-1.38-1	-1.27-1
22	-2.18-1	1.65-1	-2.24-1	-2.03-1
23	-2.23-1	1.07-1	-2.30-1	-2.12-1
24	-1.94-1	3.37-2	-1.98-1	-1.87-1
25	-4.88-1	-3.43-1	-4.86-1	-4.62-1
26	-4.14-1	-3.31-1	-4.22-1	-4.06-1
27	-4.70-1	-4.31-1	-4.72-1	-4.59-1
28	-1.50-1	-1.98-1	-1.10-1	-1.43-1
29	-2.81-1	-8.85-3	-2.76-1	-2.71-1
30	-1.07+0	-4.26-1	-1.07+0	-1.05+0

^aRead 1.43-2 as 1.43×10^{-2} .

Figures 20 and 21 show the errors in group flux in the middle and bottom slices, respectively, of core rows 1 and 5, reflector row 9, and blanket row 12 when the χ_g method is used to model inward axial leakages. The curves follow the same basic shapes as in the three-slice calculations. The middle-slice calculations have large flux errors in the lower-energy groups because of inadequate meshing, and both the middle slice and bottom slice show the effect of modeling anisotropic leakage by isotropic buckling. Although there are inherent errors involved with the χ_g and $\Sigma_s(1+g)$ methods, it should be kept in mind that the errors are relatively small when compared with the errors when using a constant, positive buckling.

Fig. 20. Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling and the χ_g Method: Middle Slice of 13-slice Problem

Fig. 21. Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling and the χ_g Method: Bottom Slice of 13-slice Problem

D. Slice across Lower and Upper Axial Reflectors

This series of calculations modeled an axial slice one mesh interval wide in each axial reflector. The axial slice in the lower reflector went through 7-1/2 rows of lower reflector, 3-1/2 rows of radial reflector, and 4+ rows of radial blanket (blanket rows 15 and 16 are not full rows). Figure 22 shows the flux errors in core rows 1 and 5, reflector row 9, and blanket row 12 of the lower-reflector slice when the χ_g method was used to model axially incoming neutrons. (Recall that a small amount of ^{235}U was put into all nonfissioning regions so that the χ_g method could be used in these regions.) As in the other slices, the errors in the high-energy groups are due to modeling anisotropic leakage by isotropic buckling. The axial slice in the upper axial

reflector went through four rows of upper reflector followed by one row (row 5) composed of a radial smear of axial reflector and seven boron carbide control rods, 1-1/2 more rows of upper reflector, 3-1/2 rows of radial reflector, and 4+ rows of radial blanket. Because of cylindricization and self-shielding effects, the ^{10}B was put only into the outer zone of row 5.⁵ To conserve atoms, ^{11}B was substituted for ^{10}B in the inner zone.

Fig. 22

Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling and the X_g Method - Lower-reflector Slice

Figure 23 shows the flux errors in core rows 1 and 5, reflector row 9, and blanket row 12 of the upper-reflector slice when the X_g method was used to model axially incoming neutrons. Again, the errors in the high-energy groups are due to modeling anisotropic leakage by isotropic buckling. The relatively high errors in the lower-energy group in the outer zone of the row-5 control-rod section are due to mesh spacing problems in the highly absorbing ^{10}B -containing zone. Much of the error in the lower-energy groups of the inner (non- ^{10}B -containing) zone of row 5 is a result of the error in the outer zone of that row.

Fig. 23

Percent Error (Relative to RZ Reference Calculation) When Using a 1D Model with Variable Buckling and the X_g Method - Upper-reflector Slice

Figures 22 and 23 show that even noncore axial slices can be modeled if adequate care is taken in setting up the reference calculation.

V. CONCLUSIONS

The results show that good agreement can be obtained between the groupwise fluxes calculated with RZ geometry and those calculated with 1D geometry using bucklings calculated from boundary angular fluxes, with either the χ_g or the $\Sigma_g(1+g)$ method substituting for negative bucklings. However, adequate mesh spacing must be used in regions of relatively high absorption and low sources. Even if the reference calculation is adequate for the buckling calculation, a small error will be introduced because anisotropic leakage is modeled by an isotropic effect. This built-in error is usually largest in the high-energy groups of nonfissioning regions.

The bucklings used in the 1D analysis were generated from the reference fluxes, so exact values for boundary angular flux from the reference calculation were used to approximate the exact flux values by 1D analysis. However, fluxes from any RZ calculation of similar composition should give similar results. Just how dissimilar the reference calculation can be from the 1D model has not been analyzed. Before these methods can be used in the normal XY calculations with individually delineated subassemblies, further analysis has to be done to estimate individual subassembly bucklings from row-smeared values.

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