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Supersymmetric Statistical Models on the Lattice

Yonathan Shapir

Physics Department, Brookhaven National Laboratory
Upton, New York 11973* U.S.A.

and

Department of Physics and Materials Research Laboratory
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801 U.S.A.

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Yonathan Snapir

Physics Department, Brookhaven National Laboratory
Upton, New York 11973* U.S.A.

and

Department of Physics and Materials Research Laboratory
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801 U.S.A.

Abstract

SUSY models in statistical mechanics involve spinless fermions and form an ideal laboratory for latticization. Extension of SUSY relations to the lattice may clarify, as well, the question of their validity beyond perturbation theory. A non-linear approach to lattice SUSY is introduced by the assignment of coupled commuting and anticommuting variables to each lattice site. As a typical example we demonstrate a SUSY dimer Hamiltonian which generates lattice branched-polymers configurations.

We suggest a new SUSY model whose configurations are generated by the Ising super-Hamiltonian. These configurations cannot be simply related to the high temperature expansion of the random-field Ising model. The critical behavior of the random-field model is probably in the non-perturbative regime with thermal fluctuations squeezed into $\bar{d}$ -dimensional manifolds in the boundaries of frozen domains. Self-consistent dimensional reduction yields $\bar{d} = (d + 1/\nu)/2$ if $\alpha < 0$ and $\bar{d} = d - 1/\nu$ if $\alpha > 0$. This may be explained in terms of anomalous "anticommuting dimensions."

I. Introduction

Supersymmetric concepts, originally introduced in the attempt to explain high-energy phenomena, have recently infiltrated to the statistical mechanics literature. This process has been initiated by G. Parisi and N. Sourlas [1] and is extensively reviewed by the second author in his presentation before this conference. Non-perturbative aspects are discussed by J. Cardy in his talk. Our contribution focuses on the attempt to locate the SUSY statistical theories on a discrete lattice and to discuss the potential failure of the naive SUSY to describe some of the related models which are investigated experimentally as well.

The SUSY models encountered in statistical mechanics may be divided into two classes. In the first class [2] the SUSY relations involve only transformations in the order-parameter space. Linear polymers [3], quantum localization [4] and classical diffusion [5] are the typical examples in this class. In these models, the anticommuting fields are introduced simply to replace the $n \rightarrow 0$ replica trick. There are no major difficulties to latticize the models of this class and we shall discuss it elsewhere [5].

The second class is less trivial. In this class the SUSY transformations are superrotations in superspace which induce mixing between the bosonic and the "fermionic" components. The fermionic fields, however, have spin-zero; they are Grassmann ghost fields with kinetic terms quadratic in the momenta. This way the troubles associated with the species-doubling of spin-1/2 fermions are avoided. We still face, however, the problem of the SUSY character manifested by

the supergeometry. The challenge arises because on one hand the Leibnitz chain-rule for differentiation does not hold for lattice derivation and, on the other hand, no natural discretization exists for the "anticommuting directions" (see, however, J. Rabin's talk on his joint work with A. Kosteletzky). We shall try to overcome these difficulties which are purely due to the SUSY character. The understanding of such theories may provide some insight to the latticization of others based on spinor representations of the Poincaré group. We shall discuss two typical representatives: the lattice branched-polymer (LBP) model [6] and the random-field (RF) systems [1]. The underlying SUSY arises from a formal summation of a family of diagrams (tree-like) in the form of a stochastic equation. Those diagrams are the most divergent and have been argued to control the critical properties in the vicinity of the upper critical dimensionality (below which the infrared divergences change the value of the exponents). This SUSY leads to the dimensional-reduction: the critical exponents are the same as these of the non-SUSY versions of the same models but in $\bar{d} = d - 2$ dimensions [7]. The persistence of SUSY to lower dimensions is still questionable. Finding lattice relations between the models, independent of the dimensionality, may provide strong support for the validity of SUSY in all dimensions. Of course, irrelevant terms which break continuous superrotations are necessary to construct the lattice theories. We do not expect such terms to prevent the restoration of continuous SUSY at the critical point of the lattice model. This program has been carried on successfully for the LBP model [8,9] and is presented in Section II.

Section III is devoted to the construction of a SUSY generalization to the classical Ising model. We present the obstacles encountered in trying to relate this lattice SUSY model to the Ising model with random-fields. In Section IV other arguments are advanced in favor of the breaking of the direct link between the RF system and the SUSY Ising model. We present another physical approach which is equivalent to giving anomalous dimensions to the "Grassmann parameters." The conclusions are summarized in Section V.

II. Branched Polymers as SUSY Dimer Model

1. The monomer-dimer (MD) model.

This model [8] addresses one of the simplest problems in statistical mechanics: the number of configurations, n_k , of k hard rods such that each of them is placed on a link of a d -dimensional lattice and no more than one rod reach any vertex. To study the problem we associate a generating (or partition) function as a polynomial:

$$G(z) = \sum_k n_k z^k, \quad (1)$$

where z is the fugacity per rod (dimer).

In the analytic continuation to $z < 0$, there is a critical point at which the free-energy has a cusp-like singularity:

$$F(z) = \ln G(z) - (|z| - |z_c|)^{2-\alpha} \text{ for } |z| > |z_c|. \quad (2)$$

In this model we denote $2 - \alpha = 1 + \sigma$, where σ is the exponent of the "magnetization":

$$M = \partial F / \partial z - (|z| - |z_c|)^\sigma. \quad (3)$$

This notation is borrowed from the Yang-Lee model which, according to a conjecture made a few years ago [10], share the same critical properties. To demonstrate that indeed both models belong to the same universality class we construct the Hamiltonian for the monomer-dimer configurations.

To each lattice site we associate a variable a_i such that:

$$\text{Tr } a_i = \text{Tr } 1 = 1, \quad (4)$$

and

$$a_i^2 = 0. \quad (5)$$

This constraint kills any diagram in which two rods meet at a lattice site. A simple expansion then yields:

$$G(-|z|) = \text{Tr } \exp(-H), \quad (6)$$

with

$$H = |z| \sum_{\langle ij \rangle} a_i a_j \quad (7)$$

and the sum is over nearest neighbors sites on the lattice. From this Hamiltonian the field-theory may be derived by the standard transformation:

$$\begin{aligned} & \exp\left\{-\frac{|z|}{2} \sum_{(ij)} a_i A_{ij} a_j\right\} \\ &= (\det A_{ij})^{-1/2} z^{-N/2} \int \prod_i d\phi_i \exp\left\{-\frac{1}{2|z|} \sum_{(ij)} \phi_i \phi_j\right. \\ & \quad \left.+ i \sum_i \phi_i a_i\right\}. \end{aligned} \quad (8)$$

where $A_{ij} = 1$ if i and j are nearest neighbors and vanishes otherwise and N is the total number of sites. The trace over the a 's may be performed, giving

$$G(-|z|) = (\det A_{ij})^{-1/2} z^{-N/2} \int \prod_i d\phi_i \exp\left\{-\frac{1}{2|z|} \sum_{(ij)} \phi_i A_{ij}^{-1} \phi_j + \sum_i \ln(1 + i\phi_i)\right\} \quad (9)$$

Neglecting fluctuations, $G(-|z|)$ is proportional to $\exp[-N\Gamma_0(\bar{\phi})]$. The zero-loop approximation of the potential in the thermodynamic limit is:

$$\Gamma_0(\bar{\phi}) = - (1/2) c |z| \bar{\phi}^2 + \ln(1 + i\bar{\phi}) \quad (10)$$

up to a constant, where

$$\bar{\phi} = \frac{1}{2} [1 - (1 - 4c|z|)^{1/2}] \quad (11)$$

is the solution to the saddle-point equation:

$$\left. \frac{\partial \Gamma}{\partial \phi} \right|_{\phi=\bar{\phi}} = - \frac{\phi}{|z|c} + \frac{1}{(1 + i\bar{\phi})} = 0 \quad (12)$$

By substituting $\phi = \bar{\phi} + \xi$ and expanding A_{ij}^{-1} in terms of gradients one obtains the effective Lagrangian for the fluctuations:

$$\mathcal{L}(\xi) = \frac{1}{2} r \xi^2 + \frac{1}{2} (\bar{\nabla} \xi)^2 + \frac{g}{3!} \xi^3 + \dots \quad (13)$$

The ξ^3 interaction has an imaginary coupling-constant:

$$g = (\bar{\phi}/|z|c)^3, \quad (14)$$

reproducing the Yang-Lee field theory [11].

S. Samuel suggested [12] a Hamiltonian for this model in terms of Grassmann variables, η_i^* and η_i such that:

$$(\eta_i^*)^2 = (\eta_i)^2 = \eta_i^* \eta_i + \eta_i \eta_i^* = 0. \quad (15)$$

His representation is equivalent to the one described above if we relate:

$$a_i = \eta_i^* \eta_i, \quad (16)$$

with the measure

$$\text{Tr}(\cdot) = \int \left[\prod_i d\eta_i^* d\eta_i \exp\{\eta_i^* \eta_i\} \right] (\cdot). \quad (17)$$

2. Lattice Branched-Polymers (LBP).

This model may also be simply described [13]. It concerns the enumeration of the configuration, N_m , of one connected cluster formed by m bonds without loops (Fig. 1). Again we define a generating function over the space of polynomials by:

$$\tilde{G}(Z) = \sum_m N_m Z^m. \quad (18)$$

We now proceed to supersymmetrize the monomer-dimer Hamiltonian [8]. Namely, we generalize H in Eq. (7) to:

$$\tilde{H} = -z \sum_{\langle ij \rangle \bar{\theta}, \theta} \tilde{a}_i(\bar{\theta}, \theta) \tilde{a}_j(\bar{\theta}, \theta) \quad (19)$$

and all we need is to define the supervariable $\tilde{a}_i(\bar{\theta}, \theta)$ as to obtain a "SUSY" model.

Naturally we are looking for a non-linear discrete realization. Three candidates for the components of $\tilde{a}_i$ are: $a_i = \eta_i^* \eta_i$, η_i^* and η_i and we complete the multiplet by a commuting auxiliary variable P_i , to be defined later:

$$\tilde{a}_i(\bar{\theta}, \theta) = a_i + \eta_i^* \theta + \bar{\theta} \eta_i + \theta \bar{\theta} P_i. \quad (20)$$

We now use the requirements of a SUSY theory to define P_i :

a. Since $\langle P_i \rangle = 0$ we are led to impose

$$\text{Tr } P_i = 0 \quad . \quad (21)$$

b. The relation $\langle P_i a_j \rangle = \langle \eta_i^* \eta_j \rangle$ we choose to implement by the constraint:

$$\text{Tr } P_i a_i = \text{Tr } \eta_i^* \eta_i = \text{Tr } a_i \quad , \quad (22)$$

and, therefore:

$$P_i^2 = P_i \quad . \quad (23)$$

c. Odd elements have vanishing expectation values, hence:

$$\text{Tr } P_i \eta_i = \text{Tr } P_i \eta_i^* = 0 \quad . \quad (24)$$

One may incorporate the effects of P_i into a change in the local measure, for all $k > 1$:

$$\text{Tr } P_i^k (\cdot) = \int \left[\prod_{j \neq i} d\eta_j^* d\eta_j \exp \eta_j^* \eta_j \right] d\eta_i^* d\eta_i (\cdot) \quad . \quad (25)$$

We now proceed to compute

$$\tilde{Z}(-z) = \text{Tr } \exp \{-\tilde{H}\} \quad . \quad (26)$$

It may be expanded

$$\tilde{Z}(-z) = \text{Tr } \prod_{\langle ij \rangle} \exp \{ P_i a_j + P_j a_i + \eta_i^* \eta_j + \eta_j^* \eta_i \} \quad , \quad (27)$$

and expressed as a sum of diagrams with bosonic and fermionic lines.

The fermionic lines are self-avoiding, while bosonic bonds may be self-assembled or attached to the fermionic vertices by terms of the form $P_i^k a_i$. The most simple, nontrivial graph consists of one loop (or a degenerate loop in the form of two lines on the same link). Each loop may be traced in two directions (or one in the degenerate case) by each type of variables. The fermionic loops carry a factor (-1) ,

leading to exact cancellation. The most general graph (Fig. 2) may have disconnected clusters. Each one of them is built from bosonic dangling arms rooted to one basic loop. The basic loop itself, however, may be traced by either bosonic or fermionic lines. No more than one loop can be formed on each cluster and lines of two different clusters cannot meet in a common vertex. The basic loop, therefore, assigns the parity to the whole cluster (+1 if it is bosonic and -1 for fermionic). Since this parity is independent for each cluster in the diagram there is exact cancellation. The only graph which survive is the trivial one without any line and we conclude:

$$\tilde{Z}(-z) = 1. \quad (28)$$

We may, however, use this Hamiltonian [9] to compute $\tilde{G}(z)$ (Eq. 18), the LBP generating function:

$$\tilde{G}(z) = \langle a_0 \rangle = \frac{1}{N} \frac{\delta}{\delta j} \left[\text{Tr} \exp \left\{ -\tilde{H} + \frac{j}{4} \sum_{1, \bar{\theta}, \theta} \tilde{a}_1^2 \right\} \right]_{j=0}. \quad (29)$$

Since in this case any cluster with m bosonic bonds emanating from the origin will make a contribution z^m . Other clusters, not connected to the origin, must again have a basic loop and any disconnected graph will drop as before.

This new formalism allows us to express all higher correlation functions as well. For example, the total number of LBP passing through the points x and y is given by:

$$C_{xy}(z) = \langle a_x p_y \rangle = \langle \eta_x^* \eta_y \rangle, \text{ etc.} \quad (30)$$

To check the SUSY properties of this Hamiltonian we first go to the continuum limit. We introduce a conjugate superfield:

$$\tilde{\phi}_i(\bar{\theta}, \theta) = \phi_i + \bar{\psi}_i \theta + \bar{\theta} \psi_i + \omega_i \theta \bar{\theta} \quad (31)$$

and use again the identity (8), this time in superspace:

$$\begin{aligned} \text{Tr} \exp\left\{ \frac{-z}{2} \sum_{i,j,\bar{\theta},\theta} \bar{a}_i A_{ij} \bar{a}_j \right\} \\ = \int d\bar{\phi} \text{Tr} \exp\left\{ \frac{1}{2z} \sum_{i,j,\bar{\theta},\theta} \tilde{\phi}_i A_{ij}^{-1} \tilde{\phi}_j + \sum_{i,\theta,\bar{\theta}} \bar{a}_i \tilde{\phi}_i \right\} = 1 \end{aligned} \quad (32)$$

The supermeasure is

$$\int d\tilde{\phi}_i = \int d\phi_i d\omega_i d\bar{\psi}_i d\psi_i \quad (33)$$

We may perform the trace over the components of the non-linear supervariable $\tilde{a}$. After performing the integration with respect to the anticommuting directions we find:

$$1 = \tilde{Z}(-z) = \int d\phi d\omega d\bar{\psi} d\psi \exp \int d^d x \mathcal{L}[\phi, \omega, \bar{\psi}, \psi] \quad (34)$$

where

$$\mathcal{L}[\phi, \omega, \bar{\psi}, \psi] = \frac{1}{2z} \sum_{(ij)} (\phi_i A_{ij}^{-1} \omega_j + \bar{\psi}_i A_{ij}^{-1} \psi_j) + \sum_i W(\phi_i, \omega_i, \bar{\psi}_i, \psi_i) \quad (35)$$

and

$$W(\phi, \omega, \bar{\psi}, \psi) = \ln[1 + e^{\phi(\omega + \bar{\psi}\psi)}] \quad (36)$$

To recover the SUSY Yang-Lee model of Parisi and Sourlas [6] the following steps have to be performed:

- To rotate the contour of integration in the ϕ and ω planes by $\pi/2$.
- To shift ϕ in order to eliminate the linear term in ω .
- To replace the lattice by a continuum and to expand A_{ij}^{-1} in term of gradients (or Fourier components).
- To neglect, near $d = 8$, irrelevant terms such as $\omega^2 \phi$, $\omega \bar{\psi} \psi$, $\omega \phi^3$, $\bar{\psi} \psi \phi^2$, etc. These terms are breaking the SUSY explicitly.

To prove the restoration of SUSY at criticality in all dimensions, one has to check explicitly the irrelevance of these terms at lower dimensions. The very existence, however, of the lattice relations, valid in all dimensions and for all values of the fugacity (in the SUSY phase), is a major argument supporting a positive conclusion [9].

III. Lattice SUSY Ising Model.

Following, in the reverse direction, the steps undertaken in Section II, we shall try to find one (others may differ by irrelevant terms) non-linear lattice SUSY Ising model [1,5]. We are looking for a Lagrangian of the form:

$$\mathcal{L}(\phi, \omega, \bar{\psi}, \psi) = - \frac{1}{2z} \sum_{(i,j)} \phi_i A_{ij}^{-1} \phi_j + \bar{\psi}_i A_{ij}^{-1} \psi_j + W_I(\phi, \omega, \bar{\psi}, \psi) , \quad (37)$$

with

$$\exp(W_I) = 1 + \omega v'(\phi) + \left(\frac{\omega^2}{2} + \bar{\psi}\psi\right) v''(\phi) . \quad (38)$$

In order to reproduce a representative of the SUSY Ising model which has the same continuum limit and differs only by irrelevant operators we need

$$V(\phi) = v(\phi) + \frac{1}{2cz} \phi^2 \quad (39)$$

to be in the $:\phi^4:$ universality class, namely:

$$v(\phi) = - a\phi^2 + b\phi^4 + \dots \quad (40)$$

We expect exponential functions to be adequate and we thus choose:

$$v(\phi) = - \cos\phi . \quad (41)$$

Then:

$$W_I(\phi, \omega, \bar{\psi}, \psi) = \ln\left\{1 + \omega \sin\phi + \left(\frac{\omega^2}{2} + \bar{\psi}\psi\right) \cos\phi\right\} . \quad (42)$$

From which we derive variables b_i , η_i , $\bar{\eta}_i$ and D_i , such that

$$\exp\{W_I(\phi_i, \omega_i, \bar{\psi}_i, \psi_i)\} = \text{Tr} \exp\{b_i \omega_i + \eta_i^* \psi_i - \bar{\eta}_i \psi_i^* + D_i \phi_i\} \quad (43)$$

expanding the right hand side and comparing to (42) will produce the Trace rules requested:

$$b_i^2 = \eta_i^* \eta_i \quad (44)$$

$$\text{Tr} D_i^{2k} b_i^2 = \text{Tr} D_i^{2k+1} b_i = (-1)^k, \quad (45)$$

$$\text{Tr} D_i^{2k} b_i = \text{Tr} D_i^{2k+1} b_i^2 = 0 \quad (46)$$

and Traces of odd-elements always vanish. The commuting components b_i and D_i may be considered as the square roots of a_i and P_i of

Section II.

The Super-Ising Hamiltonian is:

$$\tilde{H} = z \sum_{\langle ij \rangle \bar{\theta}, \theta} \tilde{b}_i(\bar{\theta}, \theta) \tilde{b}_j(\bar{\theta}, \theta), \quad (47)$$

where

$$\tilde{b}_i = b_i + \eta_i^* \theta + \bar{\theta} \eta_i + \bar{\theta} \theta D_i. \quad (48)$$

One may check that the corresponding Hamiltonian:

$$H = z \sum_{\langle ij \rangle} b_i b_j, \quad (49)$$

generates the configurations of a loop-gas (in presence to dimers) which is in the Ising universality class.

So we have achieved our goal and we would like now to comment on the relation of the configurations generated by the super-Ising Hamiltonian and those of the High-Temperature expansion of the Ising model with RF [14].

The first problem arises from the fact that the RF model has two coupling constants the nearest-neighbors interactions (K) and the RF strength $\lambda = H^2$ (both are scaled by the temperature). We need, in order to make a sensible comparison, to look for a limit where a unique coupling (function of λ and K) controls the theory. The mathematically possible limit $\lambda \rightarrow \infty$, $K \rightarrow 0$ is not acceptable physically. Indeed, this limit drives us to the completely disordered regime and we are unable to approach, this way, the critical line.

This still does not rule out a potential connection between the critical behavior of these two models. Yet, we are prevented from finding a relatively simple, dimension-independent, relation to confirm the SUSY of the RF model and the dimensional-reduction. Other question marks have been raised concerning the SUSY of this model. We relate to them in the next section where we also suggest an alternative approach.

IV. Self-consistent Dimensional Reduction and Anomalous "Anticommuting Dimensions".

Stability considerations of the ground-state in the random-field Ising model, initiated by Imry and Ma [15] and pushed recently one step close to mathematical rigor [16] yield $d_{l.c.d.} = 2$ for the lower critical dimension (below which there is no long-range order). This cast doubts on the validity of the SUSY formulation [1] which predicts dimensional-reduction by two [17] and, therefore, $d_{l.c.d.} = 3$. To understand what may go wrong with the perturbation expansion which yield the SUSY theory we recall the argument given by Aharony, Imry and Ma (AIM) [7] which we understand as follows: if the magnetic free-energy

density scale as $f(\xi) \sim \lambda \xi^{-(d-d_\lambda)}$ and if the critical behavior is associated with some reduced dimension $\bar{d}$, then: $\bar{d} = d - d_\lambda$ [5]. It results from the fact that for any larger reduced dimension the probability for configurations with diverging correlation length ξ will decay exponentially. This is better understood by a physical picture where d_λ dimensions are frozen or "compactified" due to the local fields. Approaching T_c from above, local ordered domains of size ξ are pinned by the excess of the RF in any finite volume. Thermal fluctuations are squeezed into $\bar{d}$ -dimensional manifolds located in the boundaries between domains. The AIM relation then follows from the requirement to have the same magnetic free-energy in the domains as the thermal one ($kT \xi^{-\bar{d}}$) in the boundaries. This picture gives a physical content to $\bar{d}$ and should provide an improved description as T approaches T_c . One should bare in mind the existence of an irreversibility cut-off L_D . Since the experimental measured value of L_D is very large ($>700 \text{ \AA}$), we expect the present treatment for $\xi < L_D$ to describe adequately the observed critical behavior.

The SUSY perturbative approach gives $\bar{d} = d - 2$. To compute d_λ we look at the operator to which it is coupled. Using the replicated Hamiltonian for this model:

$$H_{\text{rep}} = \int d^d x \sum_{\alpha=1}^n \left\{ \left(\frac{1}{2} r |\phi^\alpha(x)|^2 + \frac{1}{2} |\nabla \phi^\alpha(x)|^2 + \frac{g}{4!} (\phi^\alpha(x))^4 \right) \right. \\ \left. - \lambda \sum_{\alpha, \beta=1}^n \phi^\alpha(x) \phi^\beta(x) \right\}, \quad (50)$$

we find:

$$-\frac{\partial \bar{f}}{\partial \lambda} = \lim_{n \rightarrow 0} n^{-1} \sum_{\alpha, \beta} \langle \phi^\alpha(x) \phi^\beta(x) \rangle = \overline{\langle \phi^2 \rangle} - \langle \phi \rangle^2; \quad (51)$$

(the bar denotes averages over the RF distribution assumed to be Gaussian). Or according to this same theory the critical behavior of this operator is given by that of ϕ^2 in $d - 2$ dimensions and therefore:

$$d_\lambda = d - 2 - d_{\phi^2} \{d - 2\} \neq 2 \text{ for any } d < 6, \quad (52)$$

which disagree with the AIM relation between $\bar{d}$ and d_λ . The SUSY theory is not consistent because the effects of the RF have to be considered in the fully renormalized averaged free-energy and not in the bare Hamiltonian. Since the critical behavior is dominated by the RF it is essential to correct d_λ according to its own renormalization. To look for a consistent solution to this non-perturbative regime we assume the thermal (but not the magnetic!) properties of the random model still to be described by those of the pure model in $\bar{d} = d - d_\lambda$ dimensions. After each order, both $\bar{d}$ and d_λ are corrected according to the dimension of the operator ϕ^2 in $\bar{d}$ of the previous order [5]. This recursive scheme has a unique self-consistent limit for the dimensional reduction:

$$\bar{d} = \begin{cases} d - 1/\nu\{\bar{d}\} & \text{if } \alpha > 0 \text{ or } d > 6 \\ (d + 1/\nu\{\bar{d}\})/2 & \text{if } \alpha < 0 \end{cases} \quad (53a)$$

$$(53b)$$

α and ν are the specific-heat and correlation length exponents respectively.

The straightforward consequences from these equations are:

- $\bar{d} = d - 2$ for all $d > d_{u.c.d.} = 6$.
- The l.c.d. at which $\alpha \rightarrow -\infty$ and $\nu \rightarrow \infty$ is given by $d_{l.c.d.} = 2\bar{d}_{l.c.d.}$ which yield $d_{l.c.d.} = 2(4)$ for Ising (Heisenberg)-like models respectively.

- c. The 3-d Ising model has the thermal exponents of the pure model in 2-d. This is in agreement with the logarithmic divergence of the specific-heat measured experimentally [18].

Another important consequence is the breaking of the SUSY present only the tree-level [1]. We may try to preserve the SUSY character of the theory by introducing anomalous "anticommuting dimensions," to reflect the renormalization of the randomness parameter, λ . Instead of the dependence on $|\vec{r}|^2 + \bar{\theta}\theta$, all functions in superspace, $f(\vec{r}, \bar{\theta}, \theta)$, will depend on a variable of the form $|\vec{r}|^2 + |\vec{r}|^y \bar{\theta}\theta$ with $y = 2 - d_\lambda$. Then:

$$\int d^d r d\bar{\theta} d\theta f(r^2 + |\vec{r}|^y \bar{\theta}\theta) = \int d^{d+y-2} r f(r^2) \quad (54)$$

and the negative dimensions associated with the stochasticity are $-d_\lambda$ instead of -2. It is an example where, near a singular point, the Grassman parameters get anomalous dimension. This is certainly related to the fact that the underlying theory is not purely SUSY in this case, but the idea may still have some applications in other supersymmetric physical theories discussed in this workshop.

V. Conclusions

We have presented a new non-linear approach to lattice SUSY models. By constructing a super-Hamiltonian for the LBP model from the MD Hamiltonian, we have provided strong support for the relation (by dimensional reduction $d \rightarrow d-2$) between their critical exponents. We have proposed the SUSY generalization of the Ising model. The configurations generated by this super-Hamiltonian cannot be directly

related to the RF Ising model. For this latter model we have found that naive SUSY is broken by the non-trivial renormalization of the randomness parameter λ . This may, however, be understood as anomalous "Grassmann parameters".

The incorporation of anticommuting variables and SUSY approach to statistical physics has just started. Whatever future is reserved to SUSY in the high energy realm, it will certainly be a useful tool in analyzing some of the real problems of the condensed matter surrounding us.

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References

* Present Address

- [1] G. Parisi and N. Surlas, Phys. Rev. Lett. 43 (1979) 744.
- [2] A. McKane, Phys. Lett. 76A (1980) 22.
- [3] G. Parisi and N. Surlas, J., Physique Lett. 41 (1981) L-403.
- [4] K. B. Efetov, Zh. Eksp. Teor. Fiz. 82 (1983) 872 [Sov. Phys. JETP 55 (1982) 514].
- [5] Y. Shapir, to be published.
- [6] G. Parisi and N. Surlas, Phys. Rev. Lett. 46 (1981) 871.
- [7] A. Aharony, Y. Imry and S.-K. Ma, Phys. Rev. Lett. 37 (1976) 1364
and A. P. Young, J. Phys. C10 (1977) L-257.
- [8] Y. Shapir, J. Phys. A15 (1982) L433.
- [9] Y. Shapir, Phys. Rev. A28 (1983) 1893.
- [10] G. A. Baker and P. Moussa, J. Appl. Phys. 49 (1978) 1360 and
D. Kurze and M. E. Fisher, Phys. Rev. B20 (1979) 2785.
- [11] M. E. Fisher, Phys. Rev. Lett. 46 (1981) 871.
- [12] S. Samuel, J. Math. Phys. 21 (1980) 2806.
- [13] T. Lubensky and J. Isaacson, Phys. Rev. Lett. 41 (1978) 829.
- [14] Y. Shapir and A. Aharony, J. Phys. C15 (1982) 1361.
- [15] Y. Imry and S. K. Ma, Phys. Rev. Lett. 35 (1975) 1399;
G. Grinstein and S. K. Ma, Phys. Rev. Lett. 49 (1982) 685 and
J. Villain, J. Physique Lett. 43 (1982) L551.
- [16] J. Chalker, Heidelberg preprint (1983) and D. S. Fisher,
J. Fröhlich and T. Spencer, Courant Inst. preprint (1983).

- [17] H. S. Kogon and D. J. Wallace, J. Phys. A14 (1981) L257; A. Niemi, Phys. Rev. Lett. 49 (1982) 1808 and J. Cardy, Phys. Lett. 125B (1983) 470.
- [18] D. P. Belanger, A. R. King, V. Jaccarino and J. L. Cardy, Phys. Rev. B28 (1983) 2522.

Figure Captions

Fig. 1 Typical branched polymer configuration. The origin may be any one of the sites on the cluster.

Fig. 2 Typical graph of the superpartition function $\tilde{Z}(-z)$. It has four disconnected clusters each of them having exactly one closed loop.

Fig. 1