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CONCEPTUAL DESIGN STUDY OF
SPHEROMAK REACTORS

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CONCEPTUAL DESIGN STUDY OF SPHEROMAK REACTORS

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ABSTRACT

Preliminary design studies are carried out for a spheromak fusion reactor. Simplified circuit theory is applied to obtain characteristic relations among various parameters of the spheromak configuration for an aspect ratio $A \gtrsim 1.6$. These relations are used to calculate the parameters for the conceptual designs of three types of fusion reactor: 1) DT two-component, 2) DT ignited, and, 3) catalyzed DD ignited reactors. With a total wall loading of $\sim 4 \text{ MWm}^{-2}$, it is found that edge magnetic fields of only $\sim 4\text{T}$ (DT) and $\sim 9\text{T}$ (cat. DD) are required for ignited reactors of one-meter plasma (minor) radius with output powers in the gigawatt range. Assessment of various methods of generating reactor-grade spheromak plasmas is discussed briefly.

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1. Introduction

In the spheromak reactor, a toroidal fusion plasma is confined by its own internal (self-generated) magnetic field and the vertical-field which is produced by coils not linking the plasma. It has recently attracted considerable attention as a possible candidate for ideal fusion reactor due to the engineering simplicity associated with the elimination of toroidal magnetic field coils and center blanket systems.

The equilibrium and stability of the spheromak configuration have been investigated theoretically and an oblate-shaped spheromak plasma configuration with a center hole has been shown to possess better MHD stability and higher attainable beta value than the classical 'spherical' spheromak [1,2]. This oblate spheromak (Oblimak) configuration of aspect ratio 2, ellipticity 0.5 having a peaked current profile is considered to be stable to ideal MHD modes up to $\beta_{\text{edge}} = 15\%$. Global modes are expected to be stabilized by a loose-fitting shell. These encouraging predictions motivated us towards making a conceptual design study of spheromak reactors.

In this paper we present a study of spheromak reactors containing a toroidal plasma, and derive approximate functional relations between various spheromak plasma parameters based on a simplified energy principle of circuitry theory. In the derivation we employ an empirical current distribution based on the earlier, more exact numerical calculation by Okabayashi and Todd [2], which represents approximately force-free conditions ($\beta_0 < 0.1$). Furthermore, a large-aspect-ratio approximation ($a/R \ll 1$) was used for simplicity even for the present low-aspect-ratio toroid. Although the toroidal effects are not negligible in assessing the MHD stability of the spheromak configuration, one can justify the use of a large-aspect ratio approximation for the basic parameters of the present reactor such as average toroidal, poloidal magnetic fields and plasma currents, etc. [1,3].

For obtaining and sustaining the ignition temperature of the spheromak reactors, various supplementary heating schemes have been [3] considered such as (1) adiabatic compression, (2) injection of energetic particles (neutral beams or relativistic electron beams), and (3) radio frequency heating. In this paper special considerations are made for scaling laws for an adiabatic compression of spheromak plasmas.

Finally, using the derived relationship among various parameters of the spheromak reactor, we carry out a preliminary search for conceptual spheromak reactors with a stationary plasma as well as with a moving plasma produced by a formation scheme employed in the S-1 experiment at PPPL, Princeton University [3].

2. Study of the Spheromak Configuration

2-1. Approximate Analytical Treatment of the Spheromak Configuration

In the framework of the electric circuit theory, the spheromak confinement system is regarded as a kind of inductance circuit, where the spheromak plasma corresponds to a set of current loops immersed in the magnetic field produced by the external coil, as shown in Fig. 1.

The energy function of this system is given by

$$W = \frac{1}{2} L_0 I_0^2 - M I_0 i_t + \frac{1}{2} L_t i_t^2 + \frac{1}{2} L_p i_p^2 + V_p p, \quad (1)$$

where L_0 and I_0 are the self-inductance and the current of the external coil, M is the mutual inductance between the external coil and the plasma ring, L_t (L_p) and i_t (i_p) are the toroidal (poloidal) inductance and current of the plasma ring, and, V_p and p are the volume and the pressure of the plasma, respectively. The shape of the plasma is approximated by a circular torus with a major radius R and a minor radius a .

Spatial profiles of the toroidal current $i_t(r)$ and the toroidal magnetic field $B_t(r)$ shown in Fig. 2 in the cross-section of the plasma are assumed as follows with respect to the minor radius r :

$$i_t(r) = i_{t0} [1 - (r/a)^2]^\gamma \quad (2-a)$$

$$B_t(r) = B_{t0} [1 - (r/a)^2]^\gamma \quad (2-b)$$

where γ denotes a degree of current peaking. Here we have neglected toroidal effects. Because of the approximate force-free condition requiring $\vec{i} \propto \vec{B}$, where $\vec{i}$ is the plasma current density vector and $\vec{B}$ is the magnetic field vector, $i_t(r)$ and $B_t(r)$ are assumed to have the same spatial distribution, resulting in the same exponential factor of γ in Eqs. (2-a) and (2-b). This approximation can be justified for assessing a performance of low β ($\beta \ll 1$) spheromak reactors because of the resemblance of Eqs. (2-a) and (2-b) to the more exact numerical results (Ref. 2). Then, the distribution of poloidal magnetic field produced by the toroidal current of Eq. (2-a) is written as

$$B_p(r) = B_{pe} \frac{a}{r} \left\{ 1 - [1 - (r/a)^2]^\gamma \right\}^{\gamma+1} \quad (3)$$

$$B_{pe} \equiv \frac{\mu_0 a i_{t0}}{2(\gamma+1)} \quad (4)$$

For the total toroidal current and the toroidal magnetic field at center, the following relations are obtained.

$$I_t = \int_0^a i_t(r) 2\pi r dr = \frac{\pi a^2}{\gamma+1} i_{t0} \quad (5)$$

$$B_{t0} = \frac{\mu_0 I_p}{2\pi R} \quad (6)$$

The toroidal magnetic energy W_t corresponding to the fourth term of Eq. (1) is then given by

$$\begin{aligned} W_t &= \frac{1}{2} L_p I_p^2 \\ &= 2\pi R \int_0^a \frac{B_t(r)^2}{2\mu_0} 2\pi r dr \\ &= \frac{1}{2\gamma+1} \frac{\pi^2}{\mu_0} R a^2 B_{t0}^2 \end{aligned} \quad (7)$$

which leads to

$$L_p = \mu_0 \frac{a^2}{2R} \frac{1}{2\gamma+1} \quad (8)$$

The poloidal magnetic energy associated with the toroidal current, W_p , is given by the sum of the external and internal energies:

$$W_p = W_{p,ex} + W_{p,in} \quad (9)$$

where, each term is expressed by

$$\begin{aligned} W_p &= \frac{1}{2} L_t I_t^2 \\ W_{p,ex} &= \frac{1}{2} L_{t,ex} I_t^2 \\ W_{p,in} &= \frac{1}{2} L_{t,in} I_t^2 \end{aligned} \quad (10)$$

$$= 2\pi R \int_0^a \frac{B_p^2(r)}{2\mu_0} 2\pi r dr.$$

The inductances appearing in the above equations are given by

$$L_t = L_{t,ex} + L_{t,in}$$

$$L_{t,ex} = \mu_0 R \left(\ln \frac{BR}{a} - 2 \right) \quad (11)$$

$$L_{t,in} = \mu_0 R \frac{\ell(\gamma)}{2}$$

$$\ell(\gamma) = \gamma_E + 2\psi(\gamma+2) - \psi(2\gamma+1), \quad (12)$$

where $\gamma_E = 0.5772$ ---- is the Euler number and $\psi(x) \equiv \Gamma'(x)/\Gamma(x)$ is the Digamma function. The value of $\ell(\gamma)$ is presented in Fig. 3.

The magnetic field produced by the external coil is assumed spatially uniform in the vicinity of the plasma, which is denoted by $B_\perp$; therefore, the magnetic flux of the vertical field inside the plasma ring $\Phi_\perp$ is given by

$$\Phi_\perp = -MI_0 = -\pi R^2 B_\perp, \text{ therefore,}$$

$$M = \pi R^2 C_{bI} \quad (13)$$

where $C_{bI} \equiv B_\perp / I_0$ is a constant which is determined only by the geometrical design of the external coil. The toroidal magnetic flux and total poloidal magnetic flux inside the plasma are given by

$$\Phi_t = \int_0^a B_t(r) 2\pi r dr, \quad \Phi_{p,in} = \int_0^a 2\pi R B_p(r) dr. \quad (14)$$

On the other hand, the total toroidal and poloidal interlinkage fluxes associated with given current distributions are written as

$$\langle \Phi_t \rangle = L_p I_p, \quad \langle \Phi_p \rangle = L_t I_t + \Phi_\perp. \quad (15)$$

The total poloidal magnetic energy inside the plasma, W_I is defined by

$$W_I = 2\pi R \int_0^a \frac{|\vec{B}_p(r) + \vec{B}_\perp(r)|^2}{2\mu_0} 2\pi r dr. \quad (16)$$

The spheromak configurations are characterized by the following two conditions:

(i) the balance of force in the "R" direction,

$$\left(\frac{\partial W}{\partial R} \right)_{I_0, I_t, I_p, p} = 0 \quad (17)$$

(ii) the balance of force in the "a" direction

$$\left(\frac{\partial W}{\partial a} \right)_{I_0, I_t, I_p, p} = 0 \quad (18)$$

Eqs. (17) and (18) are transformed into

$$g_1 (\mu_0 I_t)^2 - 4 \pi R B_z (\mu_0 I_t) = 2 \pi^2 a^2 \left(\frac{1}{2\gamma+1} - \langle \beta_0 \rangle \right) B_{to}^2 \quad (19)$$

$$(\mu_0 I_t)^2 = 4 \pi^2 a^2 \left(\frac{1}{2\gamma+1} + \langle \beta_0 \rangle \right) B_{to}^2 \quad (20)$$

where

$$g_1 = \ell n \frac{BR}{a} - 1 + \frac{\ell(\gamma)}{2} \quad (21)$$

$$\langle \beta_0 \rangle = - \frac{p}{(B_{to}^2 / 2 \mu_0)} \quad (22)$$

Another characteristic feature of the spheromak configuration is pointed out that the stored toroidal magnetic energy W_t is equal to the total poloidal magnetic energy inside the plasma W_I [5], i.e.,

$$W_t = W_I \quad (23)$$

Eq. (23) is written as

$$\ell(\gamma) (\mu_0 I_t)^2 + 4 \pi^2 a^2 B_z^2 = 4 \pi^2 a^2 \frac{1}{2\gamma+1} B_{to}^2 \quad (24)$$

It is easily seen from Eqs. (19) and (20) that, in the low- β limit when $\langle \beta_0 \rangle \ll (\gamma+1)^{-1}$ is assumed, the effect of the plasma pressure is neglected.

By the Eqs. (19) and (20) combined with the Eq. (24), the following relation concerning $\ell(\gamma)$ is derived

$$\ell(\gamma) = 1 - \frac{1}{4} (g_1 - \frac{1}{2})^2 (\frac{a}{R})^2 . \quad (25)$$

Therefore, when the aspect ratio $A = R/a \gg 1$, this equation leads to $\ell(\gamma) \sim 1$ or $\gamma \sim 1.3$. When A is reduced as small as ~ 2 , $\ell(\gamma)$ is ~ 0.8 , and the accuracy of the model where $\ell(\gamma) \sim 1$ is assumed is somewhat less, but still remains in the tolerable range in view of the approximate treatment of this paper. The spatial profile of $i_t(r)$ and $B_t(r)$ given by Eqs. (2-a) and (2-b), is shown in Fig. 4 for $\gamma = 1.3$. The dotted line in the figure is the average of profile calculated by the numerical analysis in Ref. [2]. In the low- β limit, where effects of plasma pressure is neglected, the following relationships are derived.

From Eqs. (19) and (20), the vertical field is determined to be

$$B_z = \frac{\mu_0}{4\pi} \frac{I_t}{R} \left(\ln \frac{8R}{a} - \frac{3}{2} + \frac{\ell(\gamma)}{2} \right) . \quad (26)$$

The equilibrium condition for the tokamak configuration also leads to the similar relation [4].

The relationship between the poloidal and toroidal currents can also be obtained as,

$$I_p = \frac{R}{a} \sqrt{(2\gamma+1)} I_t . \quad (27)$$

The total magnetic energy stored inside the plasma is expressed as

$$W_M = 2W_t = \frac{\mu_0 R}{2} I_t^2 \quad (28)$$

The total toroidal flux and the poloidal flux inside the plasma are written as

$$\Phi_t = \frac{\pi a^2}{\gamma+1} B_{t0} \quad (29)$$

$$\Phi_{p,in} = \frac{\pi a R}{\sqrt{2\gamma+1}} [\gamma_E + \psi(\gamma+2)] B_{t0} \quad (30)$$

A comparison of the present formulae with the numerically calculated results (Ref. 2) which includes plasma pressure β ($\beta \ll 1$) effect shows that the present approximation holds with error scale of less than 10% even for $A \sim 2$.

2-2. Summary of Formulae

The approximate relationships between plasma and field parameters are summarized as follows,

$$\begin{aligned} (\mu_0 I_t) &= \frac{1}{G_1} (\pi R B_\perp) \\ &= \frac{2}{\sqrt{2\gamma+1}} (\pi a B_{t0}) \end{aligned} \quad (31)$$

$$B_\perp = \frac{2}{\sqrt{2\gamma+1}} G_1 \frac{a}{R} B_{t0} \quad (32)$$

$$\langle \Phi_t \rangle = \frac{\pi a^2}{2\gamma+1} B_{t0} \quad (33)$$

$$\langle \Phi_p \rangle = 2G_2 \sqrt{2\gamma+1} \left(\frac{R}{a}\right) \langle \Phi_t \rangle \quad (34)$$

where

$$G_1 = \frac{1}{4} \left(\ln \frac{BR}{a} - 1 \right) \quad (35)$$

$$(35)$$

$$G_2 = \frac{1}{4} (3 \ln \frac{8R}{a} - 5) \quad (36)$$

Various kinds of energies shown in Fig. 5 are expressed by the following formulae,

$$\begin{aligned} W_t = W_I &= \frac{2\gamma+1}{\mu_0} \frac{R}{a^2} <\psi_t>^2 \\ &= \frac{\pi}{4\mu_0 G_1^2} R^3 B_z^2 \end{aligned} \quad (37)$$

$$W_p = (2 \ln \frac{8R}{a} - 3) W_t \quad (38)$$

$$W_{op} = M I_0 I_t = (\ln \frac{8R}{a} - 1) W_t \quad (39)$$

$$W = \frac{1}{2} I_0 I_0^2 + (\ln \frac{8R}{a} - 1) W_t \quad (40)$$

The distance, $2d$, between two null points on the separatrix surface, which are located on the major center axis, is calculated by

$$d = R \sqrt{\left(\frac{2\pi}{\ln(8R/a)-1} \right)^{2/3} - 1} \quad (41)$$

for $A = 2$ and 3 , $d = 1.2 R$ and $1.0 R$ respectively,

The edge field B_e is the sum of the poloidal magnetic field at $r = a$, $B_p(a)$, and the vertical magnetic field B_z , whereas, a local value of $B_p(a)$ calculated by Eq. (4) based upon the cylindrical plasma model is insufficient in accuracy due to the toroidal effect. Here, the toroidal

current is approximated by the ring current of I_t with the radius R .

The edge field is then given by

$$B_e = B_i + \frac{\mu_0 I_t}{2\pi a} f(A) \quad (42)$$

$$\frac{B_e}{B_{t0}} \approx \frac{1}{\sqrt{2\gamma+1}} \left(f(A) + \frac{2G_2}{A} \right) \quad (43)$$

where

$$f(A) \sim 0.3 A^{0.56} \quad (2 \lesssim A \lesssim 4) \quad (44)$$

$f(A)$ is the ratio of the poloidal magnetic field at a point located a distance "a" away from the ring current of I_t to that at the point located the same distance away from the center of a straight conductor carrying the same amount of current I_t . For the aspect ratio A of 2 to 4, B_e/B_{t0} is changed 0.46 to 0.50, which roughly agrees with the results of numerical calculation ($B_e/B_{t0} \sim 0.42$ for $A = 2$ in [2]).

2-3. Adiabatic Compression

In the present scheme of forming reactor grade spheromak plasma, an adiabatic compression scheme is a key factor in the heating of the plasma. During the compression, the toroidal and poloidal fluxes inside the plasma are assumed to be conserved,

$$\Phi_t \approx \text{const.} \quad \Phi_{p,in} = \text{const.} \quad (45)$$

For simplicity, we make the assumption that the current peaking factor γ does not change by the compression. From the flux conservation requirement we obtain concerning the average q value inside the plasma,

$$\langle q_p \rangle_{IN} = \frac{\Phi_t}{\Phi_{p,in}} = \frac{a}{R} \frac{\sqrt{2\gamma+1}}{\gamma+1} \frac{1}{\gamma_E + \psi(\gamma+2)} = \text{const.} \quad (46)$$

It is then concluded that during the compression of a spheromak plasma the minor radius must remain proportional to the major radius (constant aspect ratio):

$$\frac{a}{R} = \text{const.} \quad (47)$$

Also, the following equation of adiabaticity is applicable

$$T_p V_p^{2/3} = \text{const.} \quad (48)$$

where T_p and V_p are the plasma temperature and volume. On the basis of Eqs. (47) and (48), the following scalings can be obtained

$$n_e \propto R^{-3} \quad (49a)$$

$$T_p \propto R^{-2} \quad (49b)$$

$$B_{t0} \propto B_{pe} \propto B_\perp \propto R^{-2} \quad (49c)$$

$$\langle \beta \rangle \propto R^{-1} \quad (49d)$$

$$W_M \propto \overline{B_1} \propto R^{-1} \quad (49e)$$

$$I_p \propto I_t \propto R^{-1} \quad (49f)$$

$$P_M \propto R^{-3} \quad (49g)$$

where, P_M is the required input power for creation of the magnetic flux defined by

$$P_M = W_M / \tau_M$$

τ_M is the magnetic diffusion time.

3. Reactor Studies

3-1. Steady State Reactor Condition

In this section, studies on the spheromak reactor are presented, based upon the above approximate analytical solutions. A basic configuration of the reactor with a stationary plasma is shown in Fig. 6, where magnetic fields denoted by B_{t0} , B_z , and B_e are the center toroidal field, the vertical field, and the edge poloidal field, respectively. As for the spatial distribution of the toroidal magnetic field and current density given by Eqs. (2-a) and (2-b), $\gamma = 1.3$ is assumed. When the aspect ratio $A = R/a$ is in the range of $1.7 < A < 4$, the following relation is obtained from Eqs. (32) and (43)*

$$B_{t0} \sim 2.1 B_e \quad (51)$$

The location of the spherical first wall is specified by a_w in Fig. 6 and $a_w = 1.5a$ is assumed.

Three types of reactors are investigated:

- 1) DT reactor with ICT[7] (two-component type) operation with a fusion gain Q_F of 1.7;
- 2) ignited DT reactor;
- 3) ignited catalyzed-type DD (Cat.D) reactor [8,9].

Spatially uniform temperature and density are assumed for simplicity, and operating conditions for temperature are listed in Table 1, together with the corresponding Lawson parameters $(n_e \tau_E)_0$. The Lawson parameters $(n_e \tau_E)_0$ in the table are calculated on the simple model where only the Bremsstrahlung loss is taken into account as the radiation loss out of the reacting plasmas.

*In the low aspect ratio, B_e can be defined as $B_e = \frac{1}{2} (B_{eI} + B_{eo})$, where B_{eI} and B_{eo} are the edge poloidal field inside and outside of the plasma toroid.

Therefore, the effect of other kinds of radiation losses such as the impurity radiation loss and the cyclotron radiation loss, which is specially large in a Cat.D reactor operation, should be taken into account in the energy confinement time.

The cyclotron radiation loss is estimated to be less than 40% of the Bremsstrahlung loss in the cat. D plasmas if the formula used for the tokamak reactor is employed based on the average magnetic field values [9].

Concerning the beta value, the center beta value is defined by Eq. (24), that is

$$\langle \beta_0 \rangle = \frac{p}{(B_{t0}^2 / 2\mu_0)} \quad (52)$$

where p is the total plasma pressure given by

$$p = n_e kT + n_i kT + P_h \quad (53)$$

P_h is sum of the pressures of high energy beam particles and fusion produced ions. k is the Boltzmann constant.

The magnetic field which is referred to from the engineering point of view, is the maximum magnetic field B_m applied to the external windings. The maximum value depends on the configuration and positions of the external coils, however, the relation

$$B_i \leq B_m \leq B_e \quad (54)$$

is generally preserved. The condition $B_m = B_e$ takes place particularly when the conductor is placed just outside the plasma. Usually, the blanket and the radiation shield is installed between the plasma and the

conductor, whose total thickness is about 1.5 m in the DT reactor and 1m in the Cat.D reactor, and when the conductor is placed outside the blanket, B_m becomes less than B_e . For simplicity, the outer edge magnetic field B_e is referred as a measure of field strength which external coil should sustain. The engineering beta value is defined using this B_e as

$$\beta_e = \frac{p}{(B_e^2/2\mu_0)} \quad (55)$$

The attainable beta value in a spheromak plasma has been discussed on the basis of MHD instability theory [1,2], and the center beta value $\langle\beta_0\rangle$, is expected to reach $0.06 \sim 0.07$. In this study, a rather conservative value of the center beta value is assumed, namely 0.45 . For $\langle\beta_0\rangle = 0.045$, $\beta_e = 0.2$, by the relation of Eq. (51) and (55). Then, the electron density is given by

$$n_e = \frac{2.5B_e^2}{T(\text{keV})\gamma_1} \times 10^{20} [\text{m}^{-3}], \quad (56)$$

where γ_1 is defined by

$$\gamma_1 = \frac{n_e kT + n_i kT + p_h}{2n_e kT} \quad (57)$$

The conceivable values of γ_1 are presented in Table 1.

The required energy confinement time for the operation is given by use of the Lawson parameters as

$$\tau_E^* = \frac{(\eta_e \tau_L)_c}{n_e}$$

$$= \begin{cases} 0.56 & B_e^{-2} & [\text{sec}] & (\text{DT-TCT}) \\ 18 & B_e^{-2} & [\text{sec}] & (\text{DT-IGN}) \\ 490 & B_e^{-2} & [\text{sec}] & (\text{cat. D-IGN}) \end{cases} \quad (58)$$

When the total fusion power density is denoted by p_t , the total output power P_t and the total wall loading P_w are given by

$$P_t = 2\pi^2 A a^3 p_t, \quad (59)$$

$$P_w = \frac{a}{3} P_t. \quad (60)$$

The total output power, P_t , is approximately calculated for the DT TCT reactor by an energy relation given by

$$p_b + Q_F p_b \eta_i = \frac{3n_e k T}{\tau_E} \quad (61)$$

where p_b is the injected beam power density, and η_i is the ratio of fusion energy carried by the charged particles to the total fusion energy, which is about 0.2 for the DT reaction. The total output power density is then given by

$$p_t = (1 + Q_F) p_b \sim \frac{6 n_e k T}{\tau_E}. \quad (62)$$

For ignited reactors

$$p_t = \frac{n_e^2}{\gamma_2} <\sigma v>_{DT,DD} (kE_t). \quad (63)$$

Values of γ_2 and E_t are also presented in Table 1. Using Eqs. (59), (60), (62), and (63), the total output power P_t and the total wall loading P_w are given by (the aspect ratio A is assumed $1.7 \lesssim A \lesssim 4$, a in [m], B_e in [T]),

$$P_t = \begin{cases} 4.2 A a^3 B_e^4 \times 10^6 & [W] & (DT-TCT) \\ 1.0 A a^3 B_e^4 \times 10^6 & [W] & (DT-IGN) \\ 1.9 A a^3 B_e^4 \times 10^4 & [W] & (cat. D -IGN) \end{cases} \quad (64)$$

$$P_w = \begin{cases} 7.1 a B_e^4 \times 10^4 & [W/m^2] & (DT-TCT) \\ 1.7 a B_e^4 \times 10^4 & [W/m^2] & (DT-IGN) \\ 3.2 a B_e^4 \times 10^2 & [W/m^2] & (cat. D -IGN) \end{cases} \quad (65)$$

The total magnetic energy, Eq. (28), inside the plasma is given by

$$W_m = 1.9 a^3 B_e^2 \times 10^7 [J] \quad (66)$$

The total toroidal current and the total poloidal current are

$$I_t = 5.5 a B_e \times 10^6 [A] \quad (67)$$

$$I_p = 1.1 A a B_e \times 10^7 [A] \quad (68)$$

The magnetic diffusion time is approximately

$$\tau_M \sim 10 a^2 T(\text{keV})^{3/2} [\text{sec}] \quad (69)$$

where $Z_{\text{eff}} = 2$ and $\ln \Lambda = 20$ are assumed.

Table 2 lists the main required parameters for different conceptual designs of a spheromak reactor with a stationary plasma. A total wall loading of 4 MW/m^2 is assumed in Table 2. The edge magnetic field is about 4T for DT ignited reactors, which is about half the value for tokamaks. The scaling law for the energy confinement time actually attained in spheromak plasmas is not known experimentally. If the value of τ_E^* listed in Table 2 are longer than the actual attainable value, the edge magnetic field should be increased in order to reduce the values of τ_E^* according to Eq. (58). For example, if the edge magnetic field is raised from 4T to 8T, the required energy confinement time is reduced by a factor of ~ 4 because of Eq. (58); however, the wall loading is increased by a intolerable factor of 16. (Eq. 65) In these stationary spheromaks, the wall loading is found to be a major constraint against increasing the fusion power density and decreasing the required energy confinement time, in contrast with the tokamak reactor where the maximum magnetic field at the conductor is the major limiting factor.

3-2. Structural Problems of the Spheromak Reactor

Various methods have been proposed to produce the *spheromak plasma in the reaction chamber*. However, they usually require installations of various plasma forming components such as coils, conductors, and sometimes electrodes in the space between the plasma and the first wall in the start-up phase. Installations of such plasma-forming components invoke numerous difficult technological problems associated with their radiation shielding and cooling, and also result in the poor neutron economy of blanket systems.

Therefore, it is highly recommended that those components are isolated from the reactor chamber so that their radiation damages caused by the direct bombardments of neutrons are avoided. This requirement can be fulfilled by applying either of the following two types of reactor configurations.

Type 1. Configuration with stationary plasma and removable plasma formation components.

Type 2. Configuration with moving plasma and stationary plasma formation components.

In type 1, the spheromak plasma is produced and heated up in the same place, whereas, various components specially used for the production of the initial spheromak plasma are quickly removed out of the reactor chamber before the onset of reaction, so that they are not irradiated by neutrons. This concept is realized by a toroidal ZETA Pinch type production scheme combined with mechanically removable center conductor for toroidal field generation in the startup phase [10]. Though this scheme is simple and easy in the operation principle, the removing of components might bring in various difficult mechanical problems. In place of the removable center conductors, the application of high current discharge or intense charged particle beams is proposed in order to produce the initial toroidal field [10,11]. However, considering that the required value of the current is $\sim 10^8$ A, their application to the reactor grade machine is somewhat in speculative state.

In type 2, the production chamber and the reaction chamber are separated spatially, and the spheromak initially produced by a non-reacting

plasma in the former is magnetically moved into the latter followed by the fuel supply and further heating. Examples of schematic structure are presented in Fig. 7, where spheromak plasma is injected or carried into the reaction chamber by specially arranged pinch gun[12,13] (a), or

S-1 type excitors(s)[3](b). In addition, if the concept of merging of spheromaks[13] are proved practical, its application is very attractive because it enables relatively small plasma-generators with small aperture to be used to feed a larger spheromak in the reaction chamber. Successive injection of plural of spheromaks may be able to produce one large spheromak in the chamber by this method and to hold it in the steady state condition.

3-3. Moving Spheromak Reactor

In the reactor configuration of the type 2, the spheromak plasma, fed specially in the cylindrical reaction chamber, can be moved along the center axis during the fusion reaction process, which enables the effective wall loading to be reduced compared with the stationary case. Therefore, the required energy confinement time given by Eq. (58) can be reduced by increasing the edge magnetic field allowed by the relief of the wall loading restriction. An example of the configuration is shown in Fig. 8 where the plasma moves inwards according to the numbers (1) to (4), in a continuously repeating cycle. An example of the operation, whose plasma parameters for (1) to (4) are listed in Table 3, is as follows. Column (2) shows the starting phase of reaction, with the magnetic fields chosen to be ~ 1.6 times larger than in the second column of Table 2,

and according to Eq. (64), the resultant wall loading is 6.3 times higher (25 MW/m^2). This spheromak plasma is moved quasi-statically with a controlled velocity, by shaping the "vertical" magnetic field along the center axis, so that the plasma dimension and the local wall loading are kept unchanged. For the purpose of attaining spatially uniform wall loading along the whole length of the reactor wall, the plasma is periodically moved back and forth between both ends during the reaction phase of (2) to (3). While the reacting spheromak plasma moves, the magnetic field inside the plasma is expected to decrease according to $\exp(-t/\tau_M)$ due to a resistive flux diffusion. Therefore, the vertical magnetic field should be properly shaped and altered along the center axis. Column (3) represents the final condition. During the reaction phase between (2) and (3), the Lawson parameter $n_e \tau_E$ should be maintained at the required value so that the steady state ignition condition is sustained. This requirement brings in the necessary adjustment of the actual energy confinement time to the required energy confinement time given by Eq. (58). Investigation of possible means for such kind of adjustment finds a number of common aspects to the studies of the burning control of Tokamak ignition experiments [14]. In fact, it is not necessarily required to keep the constant plasma size during the move, and the operation of moving should be discussed in connection with the control problems mentioned above. In this preliminary study, the detailed investigation has not been made.

Because of the fourth power dependence of the fusion output power on the magnetic field in Eq. (64) for a constant β values, the average output is calculated by

$$\begin{aligned}\overline{P_t} &= \frac{1}{T_r} \int_0^{T_r} P_{t0} (e^{-t/\tau_M})^4 dt \\ &= \frac{\tau_M}{4T_r} (1 - e^{-4T_r/\tau_M}) P_{t0} \sim 850 \text{ MW}\end{aligned}\quad (70)$$

where $P_{t0} = 3000 \text{ MW}$ is the initial output power, $\tau_M = 700 \text{ sec}$ is the magnetic diffusion time, and $T_r = 600 \text{ sec}$ is the specified reaction time. The average value of the total wall loading can be kept as low as 2 MW/m^2 when the length of the cylindrical reactor chamber is chosen to be $\sim 16 \text{ m}$.

Adiabatic compression (with flux conservation) discussed in section 2-3 is applied to increase the inside magnetic energy during the startup phase. Plasma parameters before and after the compression are shown in columns (1) and (2) of Table 3 where a compression ratio 2 is considered. The inside magnetic energy for column (1) is 750 MJ, which is half the value for column (2).

The expansion scheme is used to recover the remaining magnetic energy inside the plasma during the finishing phase. As shown in column (4) of Table 3, the decrease of the vertical magnetic field by a factor of 16 reduces the inside energy by a factor of 4, resulting in the recovery of most of the remaining magnetic energy.

3-4. The S-1 Device

The initial spheromak plasma of column (1) in Table 3 is assumed to be formed by use of the flux core and the pinching coils with the same manner as that used in the S-1 machine [3]. A specification of the core for the excitation of the initial plasma of column (1) is also presented in Table 3. The transfer efficiencies of the poloidal and toroidal fluxes from the core to the plasma are assumed to be 30% and 55% respectively. If the required peak current in the pinching coil is assumed to be about the same as the plasma toroidal current, the Joule loss of this plasma forming system,

when nonsuperconducting windings are applied both to the core and to the pinching coils, is estimated to be 3 to 5 GJ. Meanwhile, the total fusion output energy of the present hypothetical reactor is 510 GJ; therefore, even if the total magnetic energy initially stored in the flux core (2.4 GJ) can not be recovered, the net fusion gain reaches as much as 70 to 100.

As is noted previously, in order to minimize the radiation damage of various components in the core chamber, the spheromak plasma should be formed by hydrogen or helium gas in the start up phase and, after it is *pulled off into the reaction section and the shutter for the neutron shield* is closed, it should be supplied D-T fuels.

Conclusion

The spheromak reactor is very attractive from the view point that the magnet systems are quite simple and the field strength of the external coil is relatively low compared with low-beta tokamak machines, which permits immediate use of superconducting magnets for producing the externally applied vertical magnetic field. The proposed spheromak formation scheme has been successfully demonstrated in a proto-type S-1 device (Proto S-1A)[15,16].

Engineering uncertainties come mainly from the physics problems associated with the transport properties of its steady state plasma and the dynamics of start-up. These problems are expected to be investigated in future experiments[3].

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REFERENCES

- [1] BUSSAC, M.N., et al., Plasma Physics and Controlled Nuclear Fusion Research (Proc. 7th Int. Conf., Innsbruck, 1978) IAEA.
- [2] OKABAYASHI, M. and TODD, A.M.M., Nuclear Fusion .
- [3] YAMADA, M., et al., Bulletin of APS, 24, 1022, 1979, 555. Proceedings of the US-JAPAN Joint Symposium Compact (orus. P. 171, December 12-14, 1979, OKABAYASHI, M., et al., *ibid* P. 167.
- [4] MUKHOVATIV, V.S. and SHAFRANOV, V.D., Nuclear Fusion 11, 1971, P. 605.
- [5] CHANDRASEKHAR, S., Proc. National Academy of Science, 42, 1, 1956.
- [6] FURTH, H.P. and YOSHIKAWA, S., Phys. Fluids 13, 1970, P. 2593.
- [7] JASSBY, D.L., Nuclear Fusion, 17, 2, 1977, P. 309.
- [8] MILLS, R.G., IEEE, Trans. Nuclear Science, NS-18, 1971, P. 205.
- [9] SOUTHWORTH, F.H., EPRI Report, ER-536-SR, 1977, P. 355.
- [10] YAMADA, M. and OKABAYASHI, M., and FURTH, H.P., Preliminary communications to DoE from Princeton Plasma Physics Laboratory, December 15, 1978.
- [11] GOLDENBAUM, G.C., et al., Phys. Rev Lett. 44, 1980, P. 393.
- [12] ALFVEN, H., et al., J. Nucl. Energy, Pt.C Plasma Physics, 1, 1960, P. 116.
- [13] WELLS, D.E., et al., Phys. Rev. Lett. 33, 20, 1974, P. 1203.
- [14] BROMBERGY, L., et al., M.I.T. PFC/RR-79/17, 1979.
- [15] YAMADA, M., et al., (to be published).
- [16] JARDIN, S., et al., (to be published).

TABLE CAPTIONS

Table 1. Operational conditions of fusion reactors.

Table 2. Examples of the spheromak reactor design.

Table 3. An example of design of the spheromak reactor with moving plasma.

	DT		CAT.DD
E_t (MeV)	17.6		21.6
Type of Operation	Two component type (TCT)	Ignited (IGN)	Ignited (IGN)
T (keV)	7	15	35
$(n_e \tau_E)$ ($m^{-3} sec$)	1×10^{19}	3×10^{20}	3.5×10^{21}
γ_1	2	1	1
γ_2		4	2.4

TABLE 1

Type of Reaction	DT			Cat, DD	
Type of Operation	TCT Q=1.7	IGN		IGN	
a Minor radius (m)	0.5	1	1.5	1.5	2
R Major radius (m)	1	2	3	3	4
P _t Total output power (GW)	0.12	0.47	1.1	1.1	1.9
P _w Total wall loading (MW/m ²)	4	4	4	4	4
B _z Vertical magnetic field (T)	1.6	1.9	1.7	4.6	4.4
B _{to} Toroidal magnetic field at center (T)	6.8	8.2	7.4	20	19
β _c Center beta	0.045	0.045	0.045	0.045	0.045
T Plasma temp. (keV)	7	15	15	35	35
n _e Electron density (10 ¹⁴ cm ⁻³)	1.9	2.6	2.1	6.5	5.6
τ _E * Required energy confinement time (sec)	0.053	1.2	1.4	5.4	6.2
(n _e τ _E) _o (cm ⁻³ sec 10 ¹⁴)	0.1	3	3	35	35
t _M Magnetic diffusion time (sec)	46	580	1,300	4,700	8,300
W _M Internal magnetic energy (MJ)	50	580	1.6 × 10 ³	1.2 × 10 ⁴	2.4 × 10 ⁴
I _t Toroidal current (MA)	8.9	22	29	79	98
I _p Poloidal current (MA)	34	82	110	300	370

Table 2

Table 3

Phase No. (Plasma)	(Flux core)	(1) (H' or H _e)	(2)	(3) (D-T)	(4)
Minor radius a(m)	(1.6)	2.0	1.0	1.0	4.0
Major radius R(m)	(10)	4.0	2.0	2.0	8.0
Vertical field B _v (T)		0.75	3.0	1.3	0.081
Center toroidal field B _{to} (T)	(3.5)	3.2	13.0	5.5	0.34
Total poloidal flux <Φ _p > (W _b)	(240)	72	72	31	31
Toroidal flux <Φ _t > (W _b)	(20)	11	11	4.8	4.8
Poloidal current I _p (MA)	(130)	65	130	55	14
Toroidal current I _t (MA)	(10)	17	34	14	3.5
Internal magnetic energy W _M (MJ)	(total magnetic energy) (2400)	750	1,500	260	65
Total wall load- ing, P _w (MW/m ²)			25	0.78	
Out-put power P _t (MW)			3,000	91	
(Average value)			$\frac{P_w}{P_t} = 2 \text{ MW/m}^2$ $P_t = 850 \text{ MW}$

(PPPL-792501)
Fig. 1. Circuit model for the spheromak configuration.

(PPPL-803579)
Fig. 2. Parameters for the spheromak configuration.

(PPPL-792504)
Fig. 3. Relationship between
Peaking parameters: $l(\gamma)$ vs γ .

(PPPL-792503)
Fig. 4. Spatial profile of toroidal
magnetic field and current (solid line).
A dotted line is the average value by
the numerical analysis in Ref. [2].

(PPPL-792524)

Fig. 5. Classification of various magnetic energies in the spheromak.

(PPPL-792502)

Fig. 6. Principle of structure for the spheromak reactor with stationary plasma.

(PPPL-792500)
Fig. 7. Examples of conceptual methods to feed spheromak plasma in the reactor chamber. Initial spheromak is produced by the pinch gun (a) and, the cormak type exciters (b).

(PPPL-793981)
Fig. 8. Conceptual design of the spheromak reactor with moving plasma and the cormak type excitor.