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MASTER

QUENCH DETECTION BY FLUID DYNAMIC MEANS IN CABLE-IN-CONDUIT SUPERCONDUCTORS*

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ABSTRACT

The tight confinement of the helium in cable-in-conduit superconductors creates protection problems because of the substantial pressure rise that can occur during a quench. But the same pressure rise offers the useful possibility of a non-electrical means of detecting incipient quenches by monitoring the outflow from the various hydraulic paths of the magnet. If the method is to work, (1) the signal must be large enough to be detected unambiguously at an early enough time, and (2) the signal must not depend too strongly on the length, Joule power density, or rate of growth of the initial normal zone (because these things are not entirely within our control). This paper explores by calculation the degree to which these conditions can be met. The Westinghouse coil for the Large Coil Task (LCT) is used as the basis for illustrative examples.

INTRODUCTION

The Westinghouse (WH) coil of the International Fusion Superconducting Magnet Test Facility (IFSMTF) at Oak Ridge National Laboratory is one of six large magnets of different design that operate in a torus. If one of its neighbors is dumped, the rapidly changing magnetic flux induces large voltages in the WH coil. These voltages can only be compensated for imperfectly, and the WH protection system responds to them as to an incipient quench. Such false positives may require a dump of the WH coil if we do not wish to risk a period of rapidly changing flux during which the protection system is temporarily blinded.

For this reason, a non-electrical means of detecting incipient quenches is desirable. Monitoring the outflow from the various hydraulic paths may provide such a means, but if it is to work, the following conditions must be met:

- (1) The signal must be large enough to be detected unambiguously.
- (2) The signal must not depend too strongly on the length, Joule power density, or rate of growth of the initial normal zone (since these are beyond our control).

The purpose of this paper is to explore by calculation the degree to which these conditions are met.

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BASIC EQUATIONS*

The key to making the calculations is the very large length-to-diameter (L/D) ratio of the helium volume inside the conductor. The length of the hydraulic paths in the Westinghouse IFSMTF magnet is 120 m, whereas the hydraulic diameter of the helium-filled interstices is only 0.4 mm; the L/D ratio is thus 3×10^5 . During a quench, the Joule heating raises the pressure of the helium. The helium tries to relieve this pressure by expanding, but its expansion is opposed by friction with the wires and the walls and by the inertia of the fluid. Because of the very large L/D ratio, the pressure gradient in the helium is almost entirely expended in overcoming friction, and accordingly we neglect the inertia of the fluid. This simplification enables us to obtain formulas that show explicitly the dependence of the expulsion velocity on the various parameters of the conductor.

The flow equations (continuity, momentum, energy) for a heated pipe are

$$\frac{d\rho}{dt} + \rho \frac{\partial v}{\partial x} = 0 \quad (1a)$$

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial x} - \rho F \quad (1b)$$

$$\rho \frac{d}{dt} \left(e + \frac{v^2}{2} \right) = -\frac{\partial}{\partial x} (pv) + Q \quad (1c)$$

where F , the frictional force per unit mass, is given by $F = 2fv^2/D$. (Symbols are defined at the end of the paper.) The frictional force appears in the momentum equation (1b) just as any external force would, but not in the energy equation (1c) because the work done by the fluid against the frictional force is not removed from the fluid (as it would be if the work were against an external force) but is returned to it as heat.

If we multiply (1b) by v and subtract it from (1c), we find, after using (1a) and the second law of thermodynamics, $T ds = de + p d\tau$, that

$$T \frac{ds}{dt} = \frac{Q}{\rho} + Fv \quad (2)$$

The term Fv on the right-hand side of (2) represents entropy production due to irreversible conversion by friction of kinetic energy to internal energy. Had the term $-\rho Fv$ been present on the right-hand side of (1c), as it would have been had F been an external force, then the term Fv would not appear in (2).

The basic assumption of this work is that the frictional forces greatly dominate inertial forces in a long, narrow tube. This means that the left-hand side of (1b) is very much less than either term on the right. In other words, the pressure gradient expends itself in overcoming friction, not in accelerating the fluid. Hence, we set $dv/dt = 0$ in (1b). We can eliminate the derivative of ρ from (1a) using the thermodynamic identity $d\rho = dp/c^2 - (B\rho/c_p)T ds$ so that

$$\frac{1}{\rho} \frac{d\rho}{dt} = \frac{1}{\rho c^2} \frac{dp}{dt} - \frac{B}{c_p} \left(\frac{Q}{\rho} + Fv \right) \quad (3)$$

*This section is quoted nearly verbatim from Ref. 1.

Using (1a,b) and (3), we find

$$\frac{\partial v}{\partial x} + \frac{1}{\rho c^2} \frac{\partial p}{\partial t} = \frac{B}{c_p} \left[\frac{Q}{\rho} + Fv \left(1 + \frac{c_p}{Bc^2} \right) \right] \quad (4)$$

Finally, consulting NBS-621, we find that Bc^2/c_p is always close to 1, so the bracketed quantity on the right-hand side of (4) is always close to 2.

Further Reduction of the Equations

We use (1b), with its left-hand side set equal to zero, and (4) to calculate the expulsion velocity at the ends of a long, slender pipe caused by a thermal perturbation near its center. We simulate the effect of the thermal perturbation and the subsequent growth of a nonrecovering normal zone by a pressure p in the central plane $x = 0$ that rises proportionally to a power n of the time t : $p(0, t) = p_0 t^n$. So throughout the pipe, i.e., for $x > 0$, the Joule power density, Q , is zero. When $Q = 0$, the right-hand side of (4) is proportional to v^3 . Early enough, when the velocity v is still small, we expect v^3 to be small compared with $\partial v / \partial x$. Accordingly, we drop the entire right-hand side of (4). Later, when we have solved the resulting equations, we shall use the solution to evaluate the right-hand side of (4) and the term $\partial v / \partial x$. Comparing them will give a limit on the elapsed time t , below which the right-hand side of (4) is negligible compared with $\partial v / \partial x$.

With these approximations, (1b) and (4) become

$$\frac{\partial p}{\partial x} = -\frac{2f}{D} \rho v^2, \quad \frac{\partial v}{\partial x} + \frac{1}{\rho c^2} \frac{\partial p}{\partial t} = 0 \quad (5)$$

Early, when not much helium has been expelled from the pipe, the helium remains on the high-density side of the pseudo-critical curve and behaves like a liquid. We therefore take the physical properties ρ and c to be constants. This done, we henceforth interpret p as the pressure rise above ambient pressure. Finally, to simplify the appearance of the equations, we work in a special system of units in which $\rho = c = D/4f = 1$. Then (5) becomes

$$\frac{\partial p}{\partial x} + \frac{v^2}{2} = 0, \quad \frac{\partial v}{\partial x} + \frac{\partial p}{\partial t} = 0 \quad (6)$$

To these we add the boundary and initial conditions

$$p(x, 0) = v(x, 0) = 0; \quad p(\infty, t) = v(\infty, t) = 0; \quad p(0, t) = p_0 t^n \quad (7)$$

Strictly speaking, the second boundary condition refers to a semi-infinite pipe. But we can use the solution corresponding to it to estimate the expulsion velocity from a pipe of finite length L for small enough times: if the pressure rise $p(L, t)$ is small compared with the driving pressure rise $p_0 t^n$, the expulsion velocity can be approximated by $v(L, t)$. This puts another condition on the elapsed time.

SIMILARITY SOLUTION*

Equation (6) is invariant to the one-parameter family of one-parameter groups of affine transformations

$$p' = \lambda^\alpha p, \quad v' = \lambda^\gamma v, \quad t' = \lambda^\beta t, \quad x' = \lambda x, \quad 0 < \lambda < \infty \quad (8)$$

* A general reference to the material in this section can be found in Ref. 2.

where the constants α , β , and γ are related by the two linear equations $\alpha - 2\beta = -3$, $\gamma - \beta = -2$. The number λ is the group parameter that labels the transformations of a particular group; any one of the numbers α , β , and γ can be used to label the various groups in the family.

Solutions of (6) that are themselves invariant to all the transformations of a particular group must have the form

$$p = t^{\alpha/\beta} P(x/t^{1/\beta}), \quad v = t^{\gamma/\beta} V(x/t^{1/\beta}) \quad (9)$$

where P and V are as yet undetermined functions of the single argument $y = x/t^{1/\beta}$. The numbers α , β , and γ are determined by the boundary and initial conditions: substituting (9) into (7) gives

$$\alpha = 3n/(2-n), \quad \beta = 3/(2-n), \quad \gamma = (2n-1)/(2-n) \quad (10a)$$

$$P(0) = p_0, \quad P(\infty) = 0, \quad V(\infty) = 0 \quad (10b)$$

If we substitute (9) into the partial differential equations (6), we obtain the following two *ordinary* differential equations (ODEs) for P and V :

$$\dot{P} + V^2/2 = 0, \quad \beta \dot{V} + \alpha P - y \dot{P} = 0 \quad (11)$$

Here $\dot{P}$ is an abbreviation for dP/dy . Since we are mainly interested in V , we eliminate P from (11) and obtain the following second-order ODE for V :

$$\beta \ddot{V} - \frac{1}{2}(\alpha-1)V^2 + yV\dot{V} = 0 \quad (12)$$

The ODEs (11) and (12) are themselves invariant to the associated group of affine transformations

$$P' = \mu^{-3}P, \quad V' = \mu^{-2}V, \quad y' = \mu y, \quad 0 < \mu < \infty \quad (13)$$

The quantities $u = y^2V$ and $w = y^3\dot{V}$ are both invariant to (13) [$\dot{V}' = \mu^{-3}\dot{V}$ follows immediately from (13)]; they are called an invariant and a first differential invariant, respectively. According to a theorem of Lie's,³ using an invariant and a first differential invariant as new independent and dependent variables reduces (12) to a first-order differential equation:

$$\frac{dw}{du} = \frac{(3\beta - u)w + \frac{1}{2}(\alpha-1)u^2}{\beta(2u + w)} \quad (14)$$

Figure (1a) shows schematically the direction field of (14) for $0 < n < 1/2$ and Fig. (1b) shows the direction field of (14) for $1/2 < n < 2$. (For $n > 2$, the problem posed by (5) and (7) is not solved by a similarity solution.) Some typical integral curves are sketched in each figure. Only the fourth quadrant is shown because, y and V being positive and $\dot{V}$ being negative, $u > 0$ and $w < 0$. There are two singular points, the origin $(0,0)$ and the point $R: (6, -12)$. Note carefully that the coordinates of the point R are independent of α , β , and γ .

The integral curve of (14) that we seek passes through the origin, for when $y = 0$, $u = y^2V$ and $w = y^3\dot{V}$ are both zero. The integral curves emanating from the origin are of three types: those that intersect the locus of zero slope, those that intersect the locus of infinite slope, and the separatrix that divides them. The separatrix passes through the singular point R . As we approach R along the separatrix, $y \rightarrow \infty$. Since $u_R = 6$, $V = u/y^2 \approx 6/y^2$ near R . So the separatrix is the integral curve we want, for the V it determines obeys the boundary condition $V(\infty) = 0$.

(a) $0 < n < 1/2$

 (b) $1/2 < n < 2$

Fig. 1. The direction field of Eq. (14).

The separatrix is a relation of the form $w = F(u)$ or equivalently $y^3 \dot{V} = F(y^2 V)$, i.e., a first-order ordinary differential for V in terms of y . The general solution of a first-order ODE is a one-parameter family of curves. Since the differential equation is invariant to the group (13), so must be the entire family of solution curves. This means that each solution curve is carried into another by the transformations (13). The common asymptote of all the solution curves, $V = 6/y^2$, is carried into itself since $y^2 V$ is an invariant of the group (13).

Any value of y can correspond to any point on the separatrix except 0 and R . Suppose we have a set of three values $y, V, \dot{V}$ that map onto a point (u, w) on the separatrix, i.e., for which $u = y^2 V$ and $w = y^3 \dot{V}$. Any image $\mu y, \mu^{-2} V, \mu^{-3} \dot{V}$ of $y, V, \dot{V}$ will also map onto the same point (u, w) , but with a different value of y . We can find points (u, w) on the separatrix by using $u_s = 6 - \epsilon, w_s = -12 - \epsilon \dot{w}_R$ as starting values for a numerical integration in the (stable) direction $P \rightarrow O$. To the values u, w we append a guess for y and thereby obtain starting values $y, V, \dot{V}$ for a (stable) backward integration of (12). If the value obtained for $P(0) = -\beta \dot{V}(0)/\alpha$ [follows from (11)] does not equal p_0 (which will generally be the case), we scale the calculated curve $V(y)$ according to (13) to obtain the curve we want. Shown in Fig. 2 are curves of $V(y)$ normalized to $V(0) = 1$ obtained in this way for $n = 0, 1/4, 3/8, 1$, and $3/2$. Shown in the insert is the quantity $A = -\dot{V}(0) = \alpha P(0)/\beta$ as a function of n when $V(0) = 1$. A useful approximation to the curves in Fig. 2 is

$$V = 6/(6 + 6Ay + y^2), \quad A = 0.529n^{0.703} \quad (15)$$

Fig. 2. Curves of $V(y)$ normalized to $V(0) = 1$ for various n . The insert shows the quantity $A = -\dot{V}(0) = \alpha P(0)/\beta$ when $V(0) = 1$ as a function of n .

Henceforth, we shall denote the curve $V(y)$ normalized to 1 at $y = 0$ as $V_*(y)$. Then $V_*(0) = 1$ and $\dot{V}_*(0) = -A$.

RESTRICTIONS ON THE ELAPSED TIME

There are two restrictions on the elapsed time. The first arises from the condition $v^3 \ll |\partial v / \partial x|$, which can be written $t^n \ll -\dot{V}/V^3$. We evaluate the right-hand side for $y = 0$. Since $\dot{V}(0) = -(\alpha/\beta)P(0) = -np_0$, the value of μ that transforms $V_*(y)$ into $V(y)$ is $\mu = (A/np_0)^{1/3}$. Then $V(0) = \mu^{-2}V_*(0) = (np_0/A)^{2/3}$ so that $t^n \ll -\dot{V}(0)/V^3(0) = A^2/np_0$. It appears from the numerical results that the quantity $-\dot{V}/V^3$ is monotone increasing, so t^n is always $\ll -\dot{V}/V^3$.

This last result is correct in special units. To write it in any units, e.g., mks units, we must make the various terms dimensionless by introducing powers of ρ , c , and $D/4f$, all of which are numerically equal to 1 in special units. Since $p_0 t^n$ is a pressure, we can write in ordinary units $t^n \ll A^2 \rho c^2 / np_0$.

*To see how this comes about, differentiate the equation $u = y^2 V$ to get $y(du/dy) = 2u + w$. Near R , this becomes $dy/y = du/(2 + \dot{w}_R)(u - \bar{u})$, where $\dot{w}_R$ is the slope of the separatrix at R . Thus near R , y varies as $(\bar{u} - u)^{2 + \dot{w}_R}$. We can find $\dot{w}_R$ from (14) by using l'Hospital's rule: $\dot{w}_R = [\beta - \bar{u} - \sqrt{(\beta - \bar{u})^2 + 24\beta(\alpha + 1)}]/2\beta$. From this formula we see that $\dot{w}_R < -2$, which shows that $y \rightarrow \infty$ as $u \rightarrow \bar{u}$ from below.

The second restriction on the time comes from the requirement that the pressure rise at the end of a hydraulic path be small compared with the driving pressure rise. This is the same as the requirement that we be able to use the asymptotic forms $6/y^2$ and $6/y^3$ for V and P , respectively ($P = 1/2 \int_y^\infty V^2 dy = 6/y^3$ if $V = 6/y^2$). Figure 2 shows the curves $V_*(y)$, which are normalized to $V_*(0) = 1$, on a log-log plot. If we transform $V_*(y)$ by (13) we can get $V(y)$ having any value at the origin, depending on the value of μ we use. Such a transformation on a log-log plot is simply translation of the curves $V_*(y)$ parallel to lines of slope -2 , i.e., parallel to the line $6/y^2$, which maps into itself. The requirement on t can then be written

$$\left(\frac{np_0}{A}\right)^{2/3} = V(0) \gg \frac{6}{y^2} = \frac{6t^{2/\beta}}{L^2} \quad \text{or} \quad \frac{np_0}{A} \gg \frac{6^{3/2}t^{2-\alpha}}{L^3} \quad (16)$$

Equation (16) is correct in special units. In ordinary units, it becomes

$$t \ll \left(\frac{4}{6\sqrt{6}} \frac{n f L^3 p_0}{A D c^2 \rho c^2} \right)^{1/(2-\alpha)} \quad (17)$$

NUMERICAL EXAMPLE

The following numerical data are appropriate to the WH coil: $\rho = 150 \text{ kg} \cdot \text{m}^{-3}$, $c = 290 \text{ m s}^{-1}$ (10 atm, 4.0 K), $D = 4 \times 10^{-4} \text{ m}$, $L = 60 \text{ m}$ (half-length of hydraulic path), $f = 0.01$. According to Ref. 1, the pressure in an initially small, nonrecovering normal zone grows proportionally to the time so that $\alpha = 1$. In Ref. 1, the proportionality constant p_0 is given as 35 atm/s for the WH coil. With these data, the limits on the time turn out to be 1.0 s and 3.0 s, respectively. The time it takes for a sonic signal to travel from the center to the end of a hydraulic path is $60/290 = 0.21 \text{ s}$; before this much time elapses no disturbance at the center of a path can be detected at the ends. The expulsion velocity is given by $v = t^{7/\beta} / 6t^{2/\beta} / L^2 = 6t/L^2$ in special units, or

$$v = \frac{3 c^2 D t}{2 f L^2} \quad (18)$$

in ordinary units. At $t = 0.5 \text{ s}$, $v = 70 \text{ cm/s}$, corresponding to an outward mass flow of 9.3 g/s. This is nearly five times the ambient flow of 2 g/s in a hydraulic path and so should certainly be large enough to detect unambiguously, if we monitor each flow path individually. Even if we monitor only the total (48 parallel paths), it will increase by nearly 10% after 0.5 s and 20% after 1 s, which should be enough to be clearly noticeable.

CONCLUDING REMARKS

Whether the increased outflow is large enough to be detected unambiguously (condition 1 of the introduction) can be determined for a particular design using (18) and the restrictions on the time mentioned earlier. The independence of the asymptotic formula (18) of the parameters p_0 and α means that the outflow velocity does not depend strongly on the details of the nonrecovering normal zone. In this way, condition 2 of the introduction is fulfilled. It should not be forgotten that p_0 and α enter the formulas restricting the time interval during which (18) is valid.

List of Symbols

A	$-\dot{V}(0) = \alpha P(0)/\beta$
B	volume coefficient of thermal expansion, $\rho(\partial\tau/\partial T)_p$
c	sonic speed
c_p	specific heat at constant pressure
D	hydraulic diameter
e	specific internal energy
f	Fanning friction factor
F	frictional force per unit mass, $2fv^2/D$
L	half-length of pipe
n, p_0	constants in power law for driving pressure; see Eq. (7)
$P(y), V(y)$	functions defined in Eq. (9)
Q	Joule power density
s	specific entropy
t	time
T	temperature
u	$y^2\dot{V}$; $w = y^3\dot{V}$
$V_*(y)$	the curve $V(y)$ normalized to $V(0) = 1$
x	distance down the tube from the center
y	$z/t^{1/\beta}$
$\alpha, \beta, \gamma, \lambda, \mu$	constants in the transformation formulas (8) and (13)
ρ	density
τ	specific volume, ρ^{-1}

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