MASTER

STREAK CAMERA TECHNIQUES

Rex Avara

QUALITY DIVISION

June 1977

Mason & Hanger-Silas Mason Co., Inc.
Pantex Plant

P. O. BOX 30020
AMARILLO, TEXAS 79177
806 335-1581

operated for the
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
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Printed in the United States of America
Available from

National Technical Information Service
U. S. Department of Commerce

5285 Port Royal Road
Springfield, VA 22161

Price: Printed Copy \$6.00; Microfiche \$3.00

ABSTRACT

An introduction to streak camera geometry, experimental techniques, and limitations are presented. Equations, graphs and charts are included to provide useful data for optimizing the associated optics to suit each experiment. A simulated analysis is performed on simultaneity and velocity measurements. An error analysis is also performed for these measurements, utilizing the Monte Carlo method to simulate the distribution of uncertainties associated with simultaneity-time measurements.

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ACKNOWLEDGEMENT

The author wishes to express appreciation to Cathie Harris for suggesting and implementing the Monte Carlo technique of analysis. He also wishes to thank T. W. Warren, J. K. Boettner, C. E. Canada, and W. C. Davis for their instructive discussion and encouragement. Correspondence of technical information to practical application would have been impossible without the effort given by the firing site technicians and supervisors. The author is also grateful to Martha Head for her guidance and instruction during the preparation of this report. Finally, he wishes to thank Mary Helen Lowry, Lou Garrett, and Gilbert Perkins for the formal preparation of the paper.

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INTRODUCTION

Various high-speed cameras are used to study detonation processes of high explosives (HE). Rotating-mirror cameras are among the most reputable instruments in obtaining time-related information about these events. A rotating-mirror framing camera records a series of photographs, called frames, which are taken at intermittent time intervals. A time interval, in this case, is called the interframe time and is typically on the order of microseconds (10^{-6} seconds). The streak (or smear) camera is another rotating mirror type. This camera produces a continuous time record instead of the discrete record obtained with a framing camera. A rotating mirror streak camera typically resolves time to a few nanoseconds (10^{-9} seconds). Streak cameras have been in use for some time at the firing site and some have previously been designed and fabricated at Pantex.

This report is an effort to compile some useful information for optimizing the capabilities of a streak camera. The basic operating principles and some camera geometry are discussed. In most cases equations, graphs, and charts are presented in the text, while derivations are explained in Appendix A unless an appropriate reference is adequate. Some experimental measuring techniques are discussed so that the use of the text information can be demonstrated. A simulated error analysis is performed to illustrate a technique used to estimate the uncertainty of shot statistics.

STREAK CAMERA GEOMETRY

The use of a streak camera can be understood with reference to Fig. 1. The detonator is initiated by a high voltage pulse which has been time-synchronized to the rotating mirror. The detonation shock wave progresses through the explosive material and emerges at the air gap. Light is emitted continuously (with increasing intensity) as the shock moves across the air gap. A higher intensity flash occurs as the shock impacts the quench plate and is followed (in time) by a quench plate "afterglow." This is a simplified version of the light emission, but it should adequately describe the time sequence for this case. A more detailed study has been presented by Worley(1). A short duration light pulse is imaged onto the slit plate by the objective lens and is re-imaged onto the film plane by the camera internal relay lenses. The slit plate will mask the first image so that only a narrow slit portion of the explosive surface is seen at the film plane. The rotating mirror sweeps the final image down the film plane and a streak trace (Fig. 2) is produced. The time direction is denoted as the direction of image motion produced by the rotating mirror. If two detonators are initiated, then the resulting traces can be measured (with reference to a trace edge) and a time interval can be calculated provided that the sweep velocity (or writing rate) of the image is known.

It should be useful at this point to consider the geometry of a streak camera with a cylindrical film plane and its effect on writing rates. Fig. 3 shows the descriptive parameters of a cylindrical film track with the film track axis displaced from the rotation axis to obtain optimum focus. If the rotation axis were located at the reflecting surface of the mirror, then constant focus could be obtained with the cylindrical film plane centered at the rotation axis. The image velocity could be determined from a measurement of the rotation frequency and writing arm. However, in high-speed cameras, centrifugal forces require that the mirrors be fabricated with a finite thickness symmetrically in balance to the mirror rotation axis. This consequently requires a non-circular (transcendental) film plane to obtain constant focus as an image is swept along the film. For practical purposes, many camera designs employ a circular film plane positioned for best focus

(along the track length) as determined by a least squares calculation. Useful design calculations have been presented by Davis and the following equation defines the writing rate as a function of mirror rotation angle(2):

$$\frac{ds}{d\phi} = 2(R + X_0 \cos 2\phi + Y_0 \sin 2\phi - a \cos \phi), \quad (1)$$

where X_0 and Y_0 are the displacements of the film track axis with respect to the rotation axis. Note that for every degree rotation of the mirror, the image rotates two degrees; a consequence of the reflecting properties of light. Since the mirror period is usually monitored, Equation 1 can be expressed as a function of the rotation period(τ).

$$\frac{ds}{dt} = (4\pi/\tau) (R + X_0 \cos 2\phi + Y_0 \sin 2\phi - a \cos \phi). \quad (2)$$

One could design a camera to optimize a constant writing rate instead of constant focus, but both cannot be optimized for the same design(3). If the linear portion of Equation 2 is acceptably accurate for computing time intervals, then the following equation can be used.

$$\frac{ds}{dt} \approx 4\pi R/\tau. \quad (3)$$

For critical analyses, one can determine the mirror angle (corresponding to measured values of film track distance (S) by solving numerically for ϕ in the following equation(2).

$$S = 2 R\phi + X_0 \sin 2\phi - 2a \sin \phi + 2 Y_0 \sin^2 \phi. \quad (4)$$

With known values of ϕ and the mirror period the time interval can be calculated from Equation 2. Since the writing rate changes slowly for small-time intervals, an average value of ϕ should be adequate for most measurements.

Equation 3 is used for writing rate calculations at the test site, and the value ($R = R'$ in this case) is the writing arm length given in the

handbook data for a respective camera. A list of the geometric parameters for three cameras used extensively at the test site is given below. These parameters were determined with reference to design sketches of the respective cameras. To the author's knowledge, there have been no modifications of these cameras that would change the values given. They are in millimeters, and (R) is defined in Fig. 3 while R' is the value ordinarily used with Equation 3 for computing writing rates.

	<u>R'</u>	<u>R</u>	<u>X₀</u>	<u>Y₀</u>	<u>a</u>
LASL (Los Alamos Scientific Laboratory) Mark II	146.05	146.05	2.63	0.498	3.96
Cordin Model 132	318.31	318.97	3.25	0.965	3.96
Beckman & Whitley Model 770	265.99	267.87	0.86	1.47	3.96

The relative difference in writing rates from Equation 3 (with R') and Equation 2 (with R) can be determined as:

$$\text{Rel. Diff.} = [R'/(R + \alpha)] - 1, \quad (5)$$

where $\alpha = X_0 \cos 2\phi + Y_0 \sin 2\phi - a \cos \phi$.

These differences have been plotted in Fig. 4 for the three cameras mentioned above. A new writing arm (R') could be selected for the Mark II camera to plot a curve similar to the other two cameras (see Fig. 4). Two assumptions were made for Equation 2.

1. The rotation speed is held constant during the cycle of the recorded event and monitored mirror period.
2. The optical axis passes through the film track axis.

The Model 770 curve (Fig. 4) is approximate since assumption 2 is not precisely met (see Fig. B-3 in Appendix B). Suppose a mirror was

accelerating so that its period changed one microsecond for every second of laboratory time. The resulting error in assumption 1 would be one part per million and could be neglected. The three cameras discussed above were designed so that the optical axis passes through (or very near) the film track center as shown in Fig. 3. See Appendix B for geometry schematics of the cameras discussed above.

Direct measurements of writing rates have been obtained at the test site by pulsing two independent light sources (10 nanosecond rise time) into the single ends of dual-branched fiber optic cables (2 each). One branch of each cable was attached to the slit plate to yield a streak trace for each light source. Similarly, the other branch of each cable was coupled to a photodiode (less than 1 ns rise time) and monitored on a raster oscilloscope (see Fig. 5). The light sources were triggered at different times and the writing rate was calculated from the distance measurement (on the streak film) and the time interval (from the raster photograph). These values agreed to less than 1% with writing rates obtained from the mirror period and Equation 3 (with R'). The uncertainty of the raster measurements and film measurements were nearly as large as the difference in the values and, therefore, the more correct value is still undetermined.

The importance of considering streak camera geometry when estimating the time measurement error associated with non-linearity of the writing rate has been shown. It was noted that one could design a camera for an optimized constant writing rate, however, the resulting focus errors could not be corrected. The importance of correct focus will be discussed later for both static and dynamic photographs obtained with a streak camera. The technique of determining a time interval from a combination of static and dynamic photographs will be presented next.

TIME MEASUREMENTS

Time intervals between detonation breakouts at different locations on a test part are of primary importance in obtaining shot statistics. As a simple case consider the test part in Fig. 1, but with multiple detonators placed at known positions on the HE piece. A slit plate can be fabricated so that each slit allows a narrow image portion of the HE surface to be reimaged at the film plane. With precise alignment, HE surfaces exactly opposite the detonators can be observed. The time intervals for simultaneity can be measured from the resulting streak by determining the distances between streak traces and subtracting the corresponding slit image to slit image distances. The resulting differences are each divided by the writing rate to compute the times. A streak record will usually contain a static photograph of the slit pattern followed by the various traces from each detonation point.

Once the times are determined, statistics (such as means and standard deviations) can be calculated that describe the particular information sought from a test fire. Conclusions can then be made concerning the results of the test fire as compared to anticipated results predicted from the test part design. Often this is how a test part is perfected, long before it is included in a final design. The importance of measurement accuracy can be seen as new designs produce time data approaching simultaneity, or other desired conditions. Often one must know the uncertainty of the data to justify a conclusion concerning design parameters of the test part. If the uncertainty is too large, then knowing what equipment and shot parameters determine the time resolution of a streak record can be helpful in obtaining better data. It is equally important that an uncertainty is estimated on data obtained for shot statistics in quality control. Some causes and effects of data uncertainty will be presented in this section.

EFFECTS OF OBJECT DEFOCUS

If the image of the test fire is defocussed at a slit plate, and consequently at the film plane, then the resulting static photographs will be of poor quality. This, and the induced magnification error can cause difficulty aligning the shot image with the slits. Slit alignment errors can cause time measurement errors if, for example, a shot contains an explosive track system such as that shown in Fig. 6. If the slits are displaced to the right, then a trace might be recorded from the left slit first even though both pellet surfaces could have detonated simultaneously. Besides alignment errors, defocusing can cause small exposure variations for different traces on a record. The better focused portions will produce more concentrated light as compared to badly defocused portions. For optimum records all observed surfaces should be focused for a well-defined image. This can be difficult if the surface is curved instead of plane. When photographing curved surfaces, one must sacrifice some resolution for depth of field. The depth of field is defined as the variation of the object plane for an acceptable resolution.

The image resolution is thus limited by the quality of the objective lens and the required depth of field. Objective lenses used at the firing site are achromatic doublets which combine a negative flint glass element bonded (or air spaced) to a positive crown glass element. These lenses are designed for an object at infinity, and some image degrading results when the lenses are used at object distances less than 10 times the image distance. Various focal length lenses were tested for image resolution at different magnifications. The results are shown in Fig. 7. This data should be adequate for estimating resolution of the objective lenses used at the test site. Note that the positive (convex) lens element should always face the longest conjugate, otherwise, the image will have poor contrast. The curves are given for magnifications from 1/10 to 1. These lenses approach their respective diffraction limits as the magnification nears 1/10.

The diffraction limit represents the theoretical resolution limit of a perfect lens. This limit is a consequence of the fact that light behaves

as a wave and bends around corners to a small degree. Light diffraction concepts are explained in most introductory textbooks on physical optics. The following equation is given as the limiting observable separation between two image points(4).

$$B = 1.22 \lambda (f/\#), \quad (6)$$

with λ being the light wavelength and $(f/\#)$ the ratio of the lens focal length to the diameter. Equation 6 is valid if the object is at infinity so that the image cone is defined by the lens $f/\#$. For shorter object conjugates, the image cone is obtained by multiplying the $f/\#$ by $(1 + m)$ where m is the object magnification. Taking the reciprocal of Equation 6 gives an estimate of the image diffraction limited resolution in line pairs per millimeter (with λ given in millimeters).

$$N_d' = [1.22 \lambda (f/\#) (1 + m)]^{-1}. \quad (7)$$

Equation 7 is plotted for various values of $(f/\#)(m + 1)$ and the results are given in Fig. 8. These values are useful for comparing the actual resolution of a lens to its theoretical perfect limits. Equation 7 can also be applied to a streak camera where the image cone is defined by the ratio of the writing arm to the effective mirror width $2b$.

$$N_d' = 2b \cos \phi / 1.22 \lambda R, \quad (8)$$

where (R) represents the writing arm length, $2b$ the mirror width, and ϕ the mirror angle as previously defined. This is an approximation since the mirror is actually rectangular-shaped instead of round. The diffraction limit is also useful as a prediction of image resolution for points on the lens focal plane at object magnifications of 1/10 or less. Points off the focal plane require consideration of the depth of field.

Image resolution for a given depth of field, $f/\#$, and object magnification can be derived from the geometry in Fig. 9. It can be seen that:

$$\frac{A}{f(1 + m) + \delta'} = \frac{B}{\delta'}, \quad (9)$$

where (m) is the image magnification. Since the image depth δ' is approximately equal to the object depth δ times the square of the magnification, the image blur (B) can be determined as a function of the object depth, f/#, and object magnification. The reciprocal of the image blur yields an estimate of resolution and the following equation results, where the negligible term δ' has been dropped from the left side of Equation 9.

$$\delta = \frac{(f/\#)(1 + m)}{N^2 m^2} . \quad (10)$$

Curves for Equation 10 have been plotted (Fig. 10) for various values of image resolution (N'). This data can be used in predicting a depth of field for a given lens and magnification. Note, however, that the resulting depth of field, as seen at the film plane, is dependent on the limiting aperture of the optical system. Fig. 11 shows a typical optical system where the camera contains two lenses arranged so that the slit plate and the film plane are located at the focal points of lens 2 and 3, respectively. Useful expressions of the depth of field have been obtained in Appendix A-4 for three cases where it is assumed that lens 2 and 3 have equal diameters. The conditions given are to be checked to see which case applies to the optical system.

CASE I: The objective lens is the limiting aperture.

CONDITIONS:

$$F_1 (1 + m_1) \geq F_2.$$

$$F_1 (1 + m_1) \geq \left(\frac{f_2}{f_3} \right) \left(\frac{R}{2b \cos \phi} \right).$$

DEPTH OF FIELD:

$$\delta = \left(\frac{f_2}{f_3} \right) \left(\frac{1 + m_1}{m_1^2} \right) \left(\frac{F_1}{N'^2} \right) \quad (11)$$

where (f_i) represents the focal length and (F_i) the (f/#) of the ith lens; (m_i) is its magnification and N' is the image resolution at the

film plane. In many cameras $f_2 = f_3$ and the relay lens system operates at unit magnification. Equation 11 then simplifies to Equation 10 and the curves in Fig. 10 can be used.

CASE II: The camera relay lens system is the limiting aperture.

CONDITIONS:

$$F_1 (1 + m_1) \leq F_2.$$

$$F_3 \geq \frac{R}{2b \cos \phi}.$$

DEPTH OF FIELD:

$$\delta = \left(\frac{f_2}{f_3} \right)^2 \left(\frac{1}{m_1} \right)^2 \left(\frac{F_3}{N'} \right). \quad (12)$$

CASE III: The rotating mirror is the limiting aperture.

CONDITIONS:

$$\frac{R}{2b \cos \phi} \geq F_3.$$

$$\left(\frac{f_2}{f_3} \right) \left(\frac{R}{2b \cos \phi} \right) \geq F_1 (1 + m_1).$$

DEPTH OF FIELD:

$$\delta = \left(\frac{f_2}{f_3} \right)^2 \left(\frac{1}{m_1} \right)^2 \left(\frac{R}{2b \cos \phi} \right) \left(\frac{1}{N'} \right). \quad (13)$$

If the relay lens system is a single lens, then the following conversions apply:

$$F_2 \longrightarrow F (1 + 1/m).$$

$$F_3 \longrightarrow F (1 + m).$$

$$f_2/f_3 \longrightarrow 1/m.$$

Defocus of the slit plate (as seen at the film plane) also has an effect on the time resolution of test shot data. The causes and effects of slit defocus will be presented next.

DYNAMIC TIME RESOLUTION

The time resolution of a streak camera is dependent upon the sharpness of the referenced edge of a streak trace. Holland and Davis have shown the relation of time resolution to the effective slit width(5). A similar approach will be taken here except that a linear approximating function will be assumed for the light intensity history. This approximation will allow us to see how the slit, the rise and fall times (of the light source), and writing rate relates to trace sharpness. Fig. 12 shows the time history for a light source. The broken lines represent an approximation so that the history becomes linear and:

$$I_r = I_m \frac{t}{t_r} \quad (14)$$

$$I_f = I_m \left(1 - \frac{t - t_r}{t_f} \right) \quad (15)$$

The intensity (I_r) is the intensity at any time (t) during the light source rise time (t_r). Similarly, (I_f) is the intensity at any time (t) during the fall time (t_f). The exposure of a point on the film is the summed product of the light intensity and the time interval of the point's exposure to light. Consider Fig. 13, where a slit image (filled with uniform light intensity at any time) at width (S') is swept along the film at a writing rate $\tau/4\pi R$. A given point on the film will receive a maximum exposure during some time interval $S'\tau/4\pi R$. The blur of a trace edge is defined by points on the film representing the minimum and maximum exposures. The minimum exposure point would receive light for an infinitely small time interval and would be located at the left side of the slit (opposite side from time direction) at the time $t = 0$. The maximum exposure point can be determined with reference to Fig. 12.

The exposure for a given point during a time interval $\Delta t = S\tau/4\pi R$ is the area under the curve for that time period. Mathematically, this is the integral sum:

$$\epsilon = \int_{t'}^{t' + \Delta t} I dt \quad (16)$$

Intuitively, the maximum exposure can be seen to include portions of the curve for I_r and I_f , and Equation 16 becomes:

$$\epsilon = \int_{t'}^{t_r} I_r dt + \int_{t_r}^{t' + \Delta t} I_f dt \quad (17)$$

Substituting Equations 14 and 15 into Equation 17:

$$\epsilon = \int_{t'}^{t_r} I_m \frac{t}{t_r} dt + \int_{t_r}^{t' + \Delta t} I_m \left(1 - \frac{t - t_r}{t_f}\right) dt \quad (18)$$

The time (t_m) for maximum exposure is found by differentiating Equation 18 with respect to time and setting the result equal to zero.

The solution for t_m is then:

$$t_m = t_r \left(1 - \frac{\Delta t}{t_r + t_f}\right) \quad (19)$$

Note, however, that this is the time the maximum exposure point begins to receive light from the slit image. This point receives its total amount of light at time $t_m + \Delta t$ and the time interval (δt) from the minimum (zero) exposure point to the maximum exposure point becomes:

$$\delta t = \Delta t + t_r \left(1 - \frac{\Delta t}{t_r + t_f} \right), \quad (20)$$

Equation 20 corresponds to referencing to a streak trace at its first (in time) edge. If the trace is read at its last edge, then the time interval becomes $T - (t_m + \Delta t)$ so that:

$$\delta t = \Delta t \left(\frac{t_r}{t_r + t_f} - 1 \right) + t_f. \quad (21)$$

Note that if the rise and fall times are equal, then Equation 21 yields a smaller value for δt than does Equation 20.

Equations 20 and 21 define the time intervals for film points receiving minimum and maximum exposure. It is possible to detect a change in density much less than normally found for the minimum to maximum density of a streak trace. The intensity history is reproduced on the film by the logarithmic relation $\Delta D = \gamma \log \Delta E$, where $\gamma \approx 1$ and ΔD is the difference in density for two points on the film corresponding to an exposure difference ΔE (see Ref. 6). If two light sources have identical intensity histories, then their corresponding exposures produce identical density gradients, and hence, identical streak traces.

Equations 20 and 21 show the importance of reducing a slit width and increasing a writing rate to obtain a sharp trace edge. A slit width reduction is limited by the image resolution obtainable on the film. This is dependent upon the camera optics and film. There is also a practical limit for fabricating an acceptably precise slit plate. Slits less than 0.05 mm approach fabrication and resolution limits. Writing rate increase is limited by the intensity of the light, speed of fast films, and the material strength of rotating mirrors. These relationships are presented in Ref. 2 where the time resolution is shown to be inversely proportional to the cube root of the source brightness.

Maximum peripheral velocities for rotating mirrors are between 600 m/s and 700 m/s, but practical speeds are around 400 m/s(2). Rotating

mirror distortion is an important factor in time resolution because the slit image is defocused by the optical power induced by the curvature of a distorted mirror. Beryllium and steel mirrors have been studied for distortion effects(5). Since beryllium distorts only about 1/80 that of steel, its distortion effects are negligible. Steel mirrors, however, require correction lenses for the astigmatism induced by distortion. The astigmatic focal length (for time direction) of a distorted mirror is given by(5):

$$1/f = 1.98 \left(\frac{\sigma \rho \omega^2 a}{E} \right) + 340 b^2 a \left(\frac{\sigma \rho \omega^2}{E} \right)^2 \quad (22)$$

for, $0.25 \leq a/b \leq 0.425$; $0.95 \leq b \leq 2.5$ cm,

where (ω) is the angular velocity, (ρ) the material density, (E) the elastic modulus, (σ) the Poisson ratio of the mirror material, and dimensions (a) and (b) are defined in Fig. 3. The focus shift causes the defocused slit width image to become(5):

$$S' = mS + 2bR/f, \quad (23)$$

(S) being the slit width, $2b$ the mirror width in the time direction, (R) the writing arm length, and (m) the magnification of the relay optics (including the optical power effects of mirror distortion). The dynamic magnification can be different than the static magnification for an uncorrected optical system. The difference can be measured by illuminating a pin hole pattern (instead of a slit plate) and comparing static and streak photographs for hole spacing(7). Slit image can also be effected by turbulence within the camera housing. Some camera mirrors are driven with air while the camera is internally purged with helium so that the rotating mirror is surrounded with a nearly pure helium atmosphere. The atmosphere should be 95% helium for good image resolution(8). Otherwise, the air-helium mixtures create a random lens effect from the variations of optical refractive index and the image becomes blurred, or defocused. Turbulence effects can be determined

by imaging a resolution chart onto a slit (resolution bars perpendicular to the slit) and shadow graphing the chart during a streak(9). Comparisons of the dynamic and static resolution can be made to detect air-helium turbulence.

The following discussion will present some measuring techniques followed by a simulated error analysis. This should demonstrate some practical uses for the information that has been presented.

SOME FIRING SITE TECHNIQUES

Two classical measurements will be discussed; simultaneity and velocity. Simultaneity measurements yield time comparisons of shock arrivals at various geometrical locations on a test part. For simplicity, it will be assumed that a proper technique is employed to limit the source light pulses to an acceptable duration for streak photography. Depth of field and exposure will be considered for the optical setup. Some electronics will be discussed and an error analysis will be performed in the section on error analysis. The concepts of velocity measurements will be presented and sources of errors will be considered in the error analysis section.

SIMULTANEITY MEASUREMENTS

Consider as an example a test shot composed of spherical shell segment of explosive material. Suppose multiple detonators are placed at various locations on the convex surface and points opposite the detonators (i.e., points on the concave surface) are to be observed for simultaneity. A slit plate is fabricated so that the test shot (~ 15 cm diameter) is imaged at 1/10 magnification. The slit pattern will be about 1.5 cm in size. Referring to Appendix A-2, we calculate S and S' from Equations A-4 and A-5. A 24-inch focal length lens operating at 1/10 magnification will require an object distance of 22 feet (6.7 m) and an image distance of 26 inches (67 cm). If the shot data from a similar light source (i.e., the same type of explosive material) shows satisfactory exposure, we can estimate new exposure parameters by considering what variables effect exposure. Since the image intensity varies inversely as the square of the effective f/# (see Ref. 4) it can be seen that exposure (E) varies approximately as:

$$E = \left(\frac{A^2}{f^2(1 + m_1)^2} \right) \left(\frac{m_2 S}{v} \right) \times \text{const.}, \quad (24)$$

where (A) is the lens aperture, (f) the focal length, (m_1) the objective magnification, (m_2) the camera relay magnification, S the slit width,

and (v) the writing rate. Equation 24 can be used to find how parameters can be changed without changing (E). The ratio of two expressions for $E^{1/2}$ yields:

$$A^* = KA \left(\frac{f^*}{f} \right) \left(\frac{1 + m_1^*}{1 + m_1} \right) \left(\frac{m_2 S v^*}{m_2^* S^* v} \right)^{1/2} \quad (25)$$

The asterisked symbols (*) define the new optical parameters and the non-asterisked symbols define the optical data of the auxiliary or reference test shot. The K factor has been added so that Equation 25 can be corrected for different film speeds or for additional optical elements that were not a part of the reference setup. Suppose the reference data is:

$$\begin{aligned} v &= 4 \text{ mm}/\mu\text{s} \\ S &= 0.1 \text{ mm} \\ m_1 &= 1.0 \\ m_2 &= 1.5 \\ A &= 38 \text{ mm} \\ f &= 42'' = 106.7 \text{ cm} \end{aligned}$$

We will assume that the same film is used so that no correction is necessary for film speed. If components are added then K must be used as follows.

$$\left. \begin{aligned} K &= 1 \text{ (no corrections needed)} \\ K &= 2^{n/2} \text{ (need additional light)} \\ K &= 2^{-n/2} \text{ (need less light)} \end{aligned} \right\} \quad (26)$$

(n) is the number of stops to be added or subtracted from the optics. The rotator transmits approximately 50% of the light (i.e., one stop). Suppose the new setup is:

$$\begin{aligned} v^* &= 10 \text{ mm}/\mu\text{s} \\ S^* &= 0.05 \text{ mm} \\ m_1^* &= 1/10 \\ m_2^* &= 1.0 \end{aligned}$$

$$f^* = 24'' = 61 \text{ cm}$$

$$K = \sqrt{2} \text{ (an image rotater is added)}$$

Equation 25 becomes:

$$A^* = \sqrt{2} \quad (38) \quad \left(\frac{610}{1067} \right) \left(\frac{1.1}{2.5} \right) \left[\left(\frac{1.5}{1.0} \right) \left(\frac{0.1}{0.05} \right) \left(\frac{10}{4} \right) \right]^{\frac{1}{2}} = 37 \text{ mm}$$

$$f/\# = f^*/A^* = 16.5$$

$$f/\# (1 + m_1) = 18$$

From Fig. 7, we see that the objective lens resolution approaches its diffraction limit when used at 1/10 magnification. Fig. 8 shows the diffraction limit (for an aperture of 37 mm) to be approximately 85 line pairs per millimeter resolution at the image.

Suppose the Cordin Model 132 streak camera is used and the streak will be photographed at $\phi = 35^\circ$. Referring to Appendix B-1 we see that:

$$R = 318 \text{ mm}$$

$$2b = 25.4 \text{ mm}$$

The limiting aperture test conditions on the depth of field (see conditions for Equations 11, 12 and 13) show that:

$$F_2 = F_3 = 5$$

$$\frac{R}{2b \cos \phi} = 15$$

$$F_1 (1 + m_1) = 18$$

Hence, the objective lens is the limiting aperture (or aperture stop) and Fig. 10 can be used for estimating the depth of field for Equation 11.

$$\delta = 18 \text{ mm for } N'' = 100 \text{ lines/mm.}$$

This implies that the optics should resolve the diffraction limit (85 lines/mm) for a depth of field of $\pm 18 \text{ mm}$. If the test part curvature

is such that this depth is not needed, then we can enlarge the aperture to obtain a better diffraction limit resolution and insert a neutral density filter to remove the extra light.

If the objective lens is not the limiting aperture stop, then Equation 25 can be used to determine an exposure difference by expressing K in terms of f stops.

$$n = \log \left[\left(\frac{\alpha}{\alpha^*} \right)^2 \left(\frac{\beta^*}{\beta} \right) \right] / \log (2),$$

$$\text{where: } \alpha = \begin{cases} F_1 (1 + m_1), & \text{if the objective lens is the aperture stop} \\ \frac{R}{2b \cos} \left(\frac{f_2}{f_3} \right), & \text{if the mirror is the aperture stop} \\ F_2, & \text{if the relay lens No. 2 is the aperture stop} \end{cases}$$

$$\beta = \frac{m_2 S}{v}$$

The asterisk symbols are the same as previously defined and (n) is the number of stops.

Preparing an optical setup requires considerations such as those just presented.

Often, the setup will include fiber optics systems with auxiliary light sources such as those previously described for measuring a writing rate. That technique also allows one to measure breakout times relative to some selected time. Fig. 14 shows (for reference in the analysis which follows) a typical electro-optic system that measures shock emergence times relative to the time that a firing voltage pulse arrives at the detonators. Proper cable length matching is necessary so that corrections need not be computed to relate streak times to corresponding electronic times as monitored on the raster oscilloscopes. In this system, cables are matched so the high voltage pulse arrives at the detonator the same time its reduced signal arrives at the pulse shaper. Also, the transmission time from the shaper to the raster is equal to

the transmission time from the photodiode to the raster. This provides a common reference for electronic and photographic times relative to the pulsed auxiliary light.

Routine data reduction at the test site often involves analyzing a number of test shots to obtain shot statistics of a particular design. A typical streak record (see Fig. 15) will contain a static photograph of the slit pattern followed by a streak trace pattern. The following equation can be used to compute a time interval between two streak traces:

$$T_1 = \left(\frac{\tau}{4\pi RM} \right) \left(\frac{W_n}{W_a} C_a - C_n \right), \quad (27)$$

where (M) is the film reader magnification. The subscript (a) corresponds to "actual" measurements on the projected image of each record and the subscript (n) represents "nominal" slit spacing measurements from some previous film record. Multiplying C_a by the factor W_n/W_a corrects C_a (for any given record) to the same magnification used during the C_n measurements. This technique corrects all streak record data (for various test shots of a common design) to a common (nominal) magnification. This eliminates the necessity of measuring (M) for each record analysis (due to focus repeatability). Similarly, the auxiliary light source times (L_a) are measured relative to the selected streak trace (C_{a1} in Fig. 15) in Equation 27.

$$T_2 = \left(\frac{\tau}{4\pi RM} \right) \left(\frac{W_n}{W_a} L_a - L_n \right). \quad (28)$$

A raster scope record (see Fig. 4) will have three pulses; two auxiliary light pulses (from the photodiodes) and a pulse from the capacitor discharge unit (CDU). This time difference is denoted T_{LS} where only one light pulse needs to be measured. The function time (including the detonator function time) then becomes:

$$T = T_{LS} + T_1 - T_2, \quad (29)$$

or:

$$T = T_{LS} + \left(\frac{\tau}{4\pi RM} \right) \left[\left(\frac{W_n}{W_a} \right) (C_a - L_a) - C_n + L_n \right] . \quad (30)$$

The time (T) is computed for each trace by a computer program containing memory of the nominal measurements C_n , W_n , L_n , and M . These equations will be used in calculating uncertainty data in the section on error analysis. First, velocity measurements will be presented briefly and the error analysis section will include velocity error data.

VELOCITY MEASUREMENTS

Velocity measurements can be obtained for flying objects, expanding cylindrical walls, or any other experiment where sufficient light is provided for adequate film exposure. Consider a simple case where the object is self-luminous as in Fig. 16. A cylindrical HE stick is detonated from the bottom with a plane shock wave and the resulting masked image is swept perpendicular to, and along with, the slit image. The resulting streak record can be measured for incremental values of $\Delta X'$ and $\Delta Y'$ where the primed values represent measurements from the streak record. The instantaneous value of the detonation velocity is:

$$V = \frac{dy}{dt} = \left(\frac{1}{m_1 m_2} \right) \frac{dy'}{dt} , \quad (31)$$

where the factor $m_1 m_2$ is the product of the magnification for the objective lens and camera relay optics respectively. Now

$$v = dx'/dt , \quad (32)$$

where (v) is the writing rate. Combining Equations 31 and 32 and replacing the differentials with incremental values:

$$V = \left(\frac{v}{m_1 m_2} \right) \frac{\Delta y'}{\Delta x'} . \quad (33)$$

This represents an incremental velocity measurement and the increments can be made as small as necessary for computing velocity or acceleration values.

ERROR ANALYSIS

Accuracy of velocity measurements have been studied for application to streak cameras(6,10). The accuracy of (V) in Equation 33 will depend upon errors from (v), $(m_1 m_2)$, $\Delta Y'$, and $\Delta X'$. Since $\Delta Y'/\Delta X' = \tan \theta$, then the fractional error in (V) from measuring θ is(6):

$$\epsilon_{\theta} = \frac{dV}{V} = 2d\theta/\sin 2\theta . \quad (34)$$

Three possible errors (time, angular, and reading uncertainty) sum to give $d\theta$ (10):

$$d\theta \simeq \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 , \quad (35)$$

where $(\Delta\theta_1)$ is the uncertainty in determining the time direction. This direction can be obtained by streaking an auxiliary (long duration) light source on the film as a reference for time direction. Time direction can also be established by referencing to the film edge provided that some mechanical-optical technique has measured the alignment error of the writing direction to the film track edge retaining the film. This error is preferably $\leq \pm 6'$ angular error. The angular error $\Delta\theta_2$ is a measure of the film reader errors (including optical distortion errors and mechanical alignment errors) and is approximately 0.3% for the firing site reader systems. The error $\Delta\theta_3$ represents the uncertainty of reading coordinate points on the blurred trace edges. The blur is a consequence of the exposure variables of slit scanning such as that discussed previously. Since we now have the detonation front as another variable in the Y' direction, the minimum to maximum exposure time is calculated differently. The resulting error is(10):

$$\Delta\theta_3 = \frac{S'}{vt} \sin \theta \cos \theta , \quad (36)$$

where (t) is the total recording time for the streak, Equation 34 now becomes(10):

$$\epsilon_{\theta} = \frac{S'}{vt} + \frac{2(\Delta\theta_1 + \Delta\theta_2)}{\sin 2\theta} . \quad (37)$$

Note that Equation 36 is based on minimum to maximum exposure points and that a density change less than these points can be detected. Also, object defocus has been neglected. If the detonation front's image is blurred, the trace edge will be affected. Equation 37 is useful to analyze error sources, but for estimating errors the term S'/vt can be replaced by a statistical angular measurement of repeatability for errors from $\Delta X'$ and $\Delta Y'$ on an actual streak record.

The writing rate error $\Delta v/v$ represents the corresponding error ϵ_v . The writing rate is given by Equation 3, so the relative error is:

$$\epsilon_v = \frac{\Delta R}{R} + \frac{\Delta \tau}{\tau} . \quad (38)$$

Errors for $\Delta R/R$ can be determined from curves such as that in Fig. 4. Mirror period errors are generally ± 1 microsecond. The magnification factor for the overall object to film plane magnification ($m_1 m_2$) can be measured from a static photograph of a rule or grid placed at the test part. The magnification error can be determined from the repeatability tests on the static photograph. If the rule, or grid, itself contains negligible error, then the magnification error is:

$$\epsilon_m = \frac{\Delta L'}{L'} , \quad (39)$$

(L') being the image length measured for reference. The total maximum error is the sum of $\epsilon_{\theta} + \epsilon_v + \epsilon_m$ and:

$$\frac{\Delta V}{V} = \frac{S'}{vt} + \frac{2(\Delta\theta_1 + \Delta\theta_2)}{\sin 2\theta} + \frac{\Delta R}{R} + \frac{\Delta \tau}{\tau} + \frac{\Delta L'}{L'} . \quad (40)$$

The minimum value of the second term occurs when $\theta = 45^\circ$, which can be obtained by selecting an appropriate writing rate and magnification.

The uncertainty of simultaneity data is important when computing shot statistics. The error sources have already been discussed and it is evident that each parameter of Equation 30 contributes in some way to the measurement error in (T). The Monte Carlo method of analysis can be used to estimate the combined effects of component errors for Equations 27, 28 or 30(11). With this method, a computer program simulates the individual component distributions then computes empirically the characteristics of the system probability distribution. Each component distribution is assumed to be normal, uniform (rectangular), triangular, or point. The program output consists of descriptive statistics such as average, standard deviation, range, median and mode for the system equation. The program also provides statistics that can be used to test for normality of the resulting probability distribution (or histogram data).

All errors (i.e., τ , R, M, W_n , W_a , C_n , C_a , L_n , and L_a) are considered independent in the Monte Carlo model. Table 1 lists these error components along with their respective assigned distributions. These values were determined to represent errors for the example discussed in the section on simultaneity measurements. The writing rate error ΔR was selected from Fig. 4 and reader distortion was determined with reference to a precision grid pattern. Static and streak trace reading repeatabilities were expressed as standard deviations. The standard deviation values must be determined by a technique that realistically represents routine reading operations. If a streak record contains a sufficiently large number of traces, the standard deviation for C_a can be determined by reading the record twice and computing differences (between the two readings) for various independent measurements. A sample of 30 differences adequately represents the repeatability for reading a time interval. Similarly, a repeatability for reading static slit lines can be obtained, but more than two readings will usually be necessary to obtain 30 independent differences. The standard deviation for T_{LS} must be determined with consideration given to both the image quality and the

linearity of the time mark intervals produced by the oscilloscope (see Fig. 5). These factors have been studied by T. W. Warren (Pantex) and a report (for curve fitting the data) is in progress(12).

The Monte Carlo results which characterize the error distribution for T_1 (Equation 27) appear in Table 2 and the corresponding histogram is shown in Fig. 17. Suppose that a test shot produces 20 streak traces then the range, average, and standard deviation of the corresponding trace times are computed. Using the Monte Carlo results, values can be computed which represent 95% prediction limits for these shot statistics. In the following calculations σ_1 denotes the expected standard deviation of T_1 which was computed in the Monte Carlo analysis.

I. SAMPLE RANGE (R)

The sample range is the difference in extreme times from each sample of 20 points. Percentage points for the relative range $W = R/\sigma$, have been tabulated for various sample sizes(13). The upper prediction limit on R is obtained from:

$$\text{Prob } [R/\sigma_1 \leq W_{.95} (20)] \simeq 0.95$$

where $W_{.95} (20) = 5.0117$

Hence, the 95% prediction limit is

$$W_{.95} (20) \sigma_1 = 0.014 \mu s$$

The range error will increase for larger sample sizes since the probability of obtaining a maximum error increases with sample size.

Table I. Error Components for Equation 30 and
Their Assumed Distributions

<u>Error Component</u>	<u>Error Source</u>	<u>Error</u>	<u>Distribution</u>
ΔT	Period Counter	$\pm 1\mu s$
ΔR	Writing Arm & Writing Rate	$\pm 0.25\% = \pm 0.7 \text{ mm}$
ΔM	Film Reader Distortion	$\pm 0.3\% = \pm 0.06$
ΔW_n	Reading Static Slit Lines	$\sigma = 0.006 \text{ mm}$
ΔW_a	Reading Static Slit Lines	$\sigma = 0.006 \text{ mm}$
ΔC_n	Reading Static Slit Lines	$\sigma = 0.006 \text{ mm}$
ΔC_a	Reading Streak Traces	$\sigma = 0.01 \text{ mm}$
ΔL_n	Reading Static Slit Lines	$\sigma = 0.006 \text{ mm}$
ΔL_a	Reading Streak Traces	$\sigma = 0.01 \text{ mm}$
ΔT_{LS}	Reading Raster Photograph	$\sigma = 0.004 \mu s$

PARAMETER VALUES

$T = 799 \mu s$ (for a writing rate of $5 \text{ mm}/\mu s$)
 $R = 318 \text{ mm}$
 $M = 20$
 $W_n = 15 \text{ mm}$
 $W_a = 15 \text{ mm}$
 $C_n = 15 \text{ mm}$ (largest value)
 $C_a = 15 \text{ mm}$ (largest value)
 $L_n = 7 \text{ mm}$
 $L_a = 7 \text{ mm}$
 $T_{LS} = 7.8 \mu s$

Table 2. Statistics Generated by the Monte Carlo
Program for T_1 (Equation 27)

T1
Sample Size = 2000

Mean	0.00005
Std. Dev.	0.00277
Range	0.01837
Variance	0.00001
Maximum X	0.00890
Maximum Possible X	0.00890
Upper 3 - Sigma Limit	0.00835
Minimum X	-0.00946
Minimum Possible X	-0.00946
Lower 3 - Sigma Limit	-0.00825
Most Likely Value	0.0
Mode	-0.00025
Median	0.00025
Summary X	0.10202797D 00
Summary X Square	0.15293012D-01
Correction Term	0.52048528D-05
Summary Small X Square	0.15287808D-01
Standard Deviation % Mean	0.61837398D-04
Variance % Mean	0.38238638D-08
Normality Test	
Third Moment	0.25382238D-09
Fourth Moment	0.17091693D-09
Sqrt B Sub 1	0.12010390D-01
B Sub 2	0.29251939D 01

II. SAMPLE AVERAGE ($\overline{\delta T}$)

The average time deviation of the sample of 20 points is designated as ($\overline{\delta T}$). Since the variance for one point is given by σ_1^2 , the variance of the average is $\sigma_1^2/20$. Hence, approximately 95% prediction limits are computed from:

$$\text{Prob } (-2\sigma_1/\sqrt{20} < \overline{\delta T} < 2\sigma_1/\sqrt{20}).$$

The system prediction limits are:

$$\pm 2 \sigma_1 / \sqrt{20} = \pm 0.001 \text{ } \mu\text{s}$$

III. SAMPLE STANDARD DEVIATION (S)

The sample standard deviation of 20 point times, designated by S, is distributed approximately as $\sigma_1^2 \chi^2/19$ where χ^2 has 19 degrees of freedom. We obtain a 95% upper prediction limit from:

$$\text{Prob } (S^2 < \sigma_1^2 \chi_{.95}^2/19) \simeq 0.95$$

or:

$$\text{Prob } (S < \sqrt{\sigma_1^2 \chi_{.95}^2/19}) \simeq 0.95$$

Substituting $\chi_{.95}^2 (19) = 30.14$ into the above equation, the upper prediction limit becomes:

$$\sqrt{\frac{\sigma_1^2 (30.14)}{19}} = 0.004 \text{ } \mu\text{s}$$

The three calculations (R, $\overline{\delta T}$, and S) were based on difference measurements and consequently only σ_1 was needed for computing prediction limits. The error associated with an individual time measurement (including the detonator function time) must be determined from a Monte Carlo standard deviation of the time (T) (Equation 30). Table 3

presents Monte Carlo results for (T), and the corresponding histogram appears in Fig. 18. Approximate 95% prediction limits on (T) are:

$$\pm 2 \sigma = \pm 0.011 \mu s$$

where σ is taken from Table 3. If an average measurement is obtained for (T), then the associated error can be computed using Monte Carlo simulation which reflects the fact that some error components (i.e., T_{LS}) do not affect each point time independently while other error components (i.e., C_a) do affect each point time independently. Considerations such as this must be made when applying Monte Carlo results to shot statistics.

Although the test-fire just discussed is fictional (i.e., it would be impractical to assemble 20 detonators within the given boundaries) a general approach to error analysis is analogous to the example. If HE track systems are employed, consideration must be given to the effects of slit displacement. Also, it has been assumed that the slit is uniformly illuminated at any given time during the light duration. In many cases a detonation front arriving at the surface will be sufficiently plane so that shock arrival time differences (within the slit aperture) are much less than the slit width exposure time $S \tau / 4 \pi R$. The arrival time differences can often be approximated by calculations involving detonation initiation points, HE material geometry, and the material detonation velocity.

Table 3. Statistics Generated by the Monte Carlo
Program for T (Equation 30)

T
Sample Size = 2000

Mean	7.80022
Standard Deviation	0.00538
Range	0.03708
Variance	0.00003
Maximum X	7.81916
Maximum Possible X	7.81916
Upper 3 - Sigma Limit	7.81635
Minimum X	7.78207
Minimum Possible X	7.78207
Lower 3 - Sigma Limit	7.78409
Most Likely Value	7.80000
Mode	7.79900
Median	7.79900
Summary X	0.15600443D 05
Summary X Square	0.12168696D 06
Correction Term	0.12168691D 06
Summary Small X Square	0.57811392D-01
Standard Deviation % Mean	0.12025006D-03
Variance % Mean	0.14460078D-07
Normality Test	
Third Moment	0.19018898D-07
Fourth Moment	0.20154403D-08
Sqrt B Sub 1	0.12237993D 00
B Sub 2	0.24121431D 01

SUMMARY

The concepts of time measurements obtained with streak cameras have been presented in the text. It was necessary to study the relations of optical parameters to streak record data so that appropriate techniques could be derived to improve time resolution and to estimate uncertainties. Tables and graphs presented should aid in optimizing a streak camera optical system for a given test shot measurement. In addition, it has been shown that uncertainties in shot statistics can be estimated using Monte Carlo simulation data.

The Appendices present additional information that can be applied in general for an optical system geometry. Appendix A presents some concepts of image resolution, lens equations, and calculations for depth of field and field of view for a streak camera optical system. Geometrical information is presented in Appendix B for some cameras at the firing site. This data should be useful for designing an optical setup for new test shots. Photographs of some cameras and bunker electronics are also presented in Appendix B. It is hoped that the material presented in this report will aid technical personnel involved with high speed photography at the test sites.

Fig. 1. Streak Camera Optical Setup for Measuring Simultaneity

Fig. 2. Resulting Streak Trace from Light Pulse

Fig. 3. Geometry of Rotation Axis Displacement for Optimum Focus
(The film track axis can be inside or outside the mirror
thickness $2a$) (Reference 2)

Fig. 4. Relative Writing Rate Difference for Various Mirror Rotation Angles
 (Plus and minus signs denote positive and negative errors from
 using Equation 3 and R' .)

Fig. 5. Sketch of a Record from a Raster Oscilloscope
[Dual raster scopes (with master-slave systems)
are usually employed to record during sweep fly-
back cycles.]

Fig. 6. Two HE Pellets Initiated by an Explosive Track

Fig. 7. Image Resolution v/s Magnification for Various Focal Lengths Achromatic Objectives (Contrast was observed to decrease considerably at magnifications from 3/4 to 1.)

Fig. 8. Diffraction Resolution v/s Image Cone ($\lambda \sim 0.51 \mu\text{m}$)

Fig. 9. Geometry for Calculating Depth of Field

Fig. 10. Depth of Field (v/s, f/#)

Fig. 11. Typical Streak Camera Optical System for Determining the Limiting Aperture for Depth of Field

Fig. 12. Light Source Intensity v/s Time,
Simplified to a Linear Function

Fig. 13. A Point on the Film Receives Light for a Slit Width Time Interval $S'\tau/4\pi R$.

Notes

1. C_1 & C_2 have equal transmission lines
2. $C_3 + C_4$, $C_5 + C_6$, and C_7 transmission times are equal.
3. Safety interlock system is not shown.

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Fig. 14. Block Diagram for a System Designed to Measure Function Times of a Test Part and Detonators

Fig. 15. Typical Streak Record for Data Reduction (This trace represents 12 detonations, 3 aligned to each slit.)

Fig. 16. Streak Record for Velocity Measurement

INTERVAL		FREQ	HISTOGRAM
%	0.0005		
-0.0100 -	-0.0095	0	
-0.0095 -	-0.0090	1 LO	
-0.0090 -	-0.0085	1	
-0.0085 -	-0.0080	1 LL	
-0.0080 -	-0.0075	7	XXXX
-0.0075 -	-0.0070	2	X
-0.0070 -	-0.0065	6	XXXX
-0.0065 -	-0.0060	10	XXXXXXXX
-0.0060 -	-0.0055	9	XXXXXX
-0.0055 -	-0.0050	17	XXXXXXXXXXXX
-0.0050 -	-0.0045	43	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0045 -	-0.0040	46	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0040 -	-0.0035	57	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0035 -	-0.0030	61	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0030 -	-0.0025	90	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0025 -	-0.0020	119	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0020 -	-0.0015	111	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0015 -	-0.0010	139	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0010 -	-0.0005	136	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
-0.0005 -	0.0	142MPL	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0	0.0005	139	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0005 -	0.0010	128	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0010 -	0.0015	128	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0015 -	0.0020	125	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0020 -	0.0025	111	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0025 -	0.0030	76	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0030 -	0.0035	70	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0035 -	0.0040	71	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0040 -	0.0045	42	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0045 -	0.0050	39	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.0050 -	0.0055	26	XXXXXXXXXXXXXXXXXXXX
0.0055 -	0.0060	12	XXXXXXXXXX
0.0060 -	0.0065	13	XXXXXXXXXX
0.0065 -	0.0070	12	XXXXXXXXXX
0.0070 -	0.0075	6	XXXX
0.0075 -	0.0080	1	
0.0080 -	0.0085	1 UL	
0.0085 -	0.0090	2 HIX	
0.0090 -	0.0095	0	
0.0095 -	0.0100	0	
0.0100 -	0.0105	0	
0.0105 -	0.0110	0	
0.0110 -	0.0115	0	
0.0115 -	0.0120	0	
0.0120 -	0.0125	0	
0.0125 -	0.0130	0	
0.0130 -	0.0135	0	
0.0135 -	0.0140	0	
0.0140 -	0.0145	0	
0.0145 -	0.0150	0	
0.0150 -	0.0155	0	
0.0155 -	0.0160	0	
0.0160 -	0.0165	0	
0.0165 -	0.0170	0	

Fig. 17. Monte Carlo Frequency Histogram of Time Measurement Errors for T_1 (Equation 27)

Fig. 18. Monte Carlo Frequency Histogram of Time Measurement Errors for T (Equation 30)

APPENDIX A-1

RESOLUTION AND MODULATION TRANSFER FUNCTIONS {1}

APPENDIX A-1

RESOLUTION AND MODULATION TRANSFER FUNCTIONS(4)

A resolution bar chart consists of a series of alternating light and dark bars of equal width (see Fig. A-1). Several pattern sets test the optical system for spatial resolution N . This type of measurement yields only qualitative information concerning contrast since only visual judgements can be made about the contrast. The modulation transfer function represents a measurement of resolution at various contrast function values. The contrast function is obtained as follows:

$$\text{Modulation} = \frac{\text{Maximum} - \text{Minimum}}{\text{Maximum} + \text{Minimum}} \quad (\text{A-1})$$

where maximum and minimum are the respective maximum and minimum values of image brightness (see Fig. A-2). The modulation transfer function is the ratio of image modulation to object modulation.

$$\text{MTF} = \frac{M_i}{M_o} \quad (\text{A-2})$$

A plot of Equation A-2 for various values of frequency N (line pairs/mm) is shown in Fig. A-3. This represents the resolution corresponding to contrast and provides quantitative information that would otherwise be qualitative. MTF can be measured for lenses, films, phosphors, image tubes, the eye, and even complete optical systems. An important advantage is that one can multiply the MTF's of several components to obtain the MTF of the combination. Readers interested in the theory and application of MTF can begin with Reference (4). Also, note that strict environmental conditions are necessary to obtain an MTF measurement(14).

Fig. A-1. Resolution Bar Chart

Fig. A-2. Variation of Image Brightness for a Bar Chart

Fig. A-3. Modulation Transfer Function Curve (Reference 4 and 14)

APPENDIX A-2

SOME BASIC LENS EQUATIONS

APPENDIX A-2

SOME BASIC LENS EQUATIONS

A list of equations will be presented which are useful in geometrical optical calculations(4,15).

$$1/f = 1/S' + 1/S \quad (A-3)$$

$$m = S'/S \quad (A-4)$$

$$S' = f (1 + m) \quad (A-5)$$

$$S = f (1 + 1/m) \quad (A-6)$$

$$S + S' = f (m + 1/m + 2) \quad (A-7)$$

$$f = m (S + S')/(m + 1)^2 \quad (A-8)$$

$$m = \frac{S + S' - 2f \pm \sqrt{(S + S')^2 - 4f (S + S')}}{2f} \quad (A-9)$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{f_1 f_2 m_2}{m f_1 + f_2} \quad (A-10)$$

where:

$$m_2 = h''/h'$$

$$m = h''/h$$

$$S_2' = \frac{f_2 (f_1 - d)}{f_1 + f_2 - d} \quad (A-11)$$

The optics are shown in Figs. A-4 and A-5, where m is the magnification and f is the focal length. Equations A-5 through A-11 can be deduced from Equations A3 and A4.

Fig. A-4. Optics for Equations A-3 through A-9

Fig. A-5. Optics for Equations A-10 and A-11

APPENDIX A-3
FIELD STOPS AND VIGNETTING

APPENDIX A-3

FIELD STOPS AND VIGNETTING

Sometimes it is necessary to determine the field of view for a camera optical system to assure that light loss within the camera does not degrade the final image. The field stop is the component aperture that limits the field of view of the complete optical system. The field stop can cause vignetting of the image, where outer boundary points of the image are weak or lost. Consider the system shown in Fig. 11 of the text. The two relay lenses have equal diameters and the slit plate and film plane are located at the respective focal lengths f_2 and f_3 . Some cameras contain rotating mirrors sufficiently small so that only a portion of the light cone reflects to the film plane. For the following discussion, we define the mirror size as:

$$\text{Small Mirror:} \quad \frac{2b \cos \phi}{R} \leq \frac{A_3}{f_3}$$

$$\text{Large Mirror:} \quad \frac{2b \cos \phi}{R} \geq \frac{A_3}{f_3}$$

First, we will examine the slit plate and relay optics of the camera. Small mirrors reflect only the light cone portion that is intercepted by the effective mirror aperture $2b \cos \phi$. As the slit position h' (see Fig. A-6) increases, the image radiance will not decrease provided that the full mirror aperture $2b \cos \phi$ remains within the light cone. Fig. A-6 shows the geometry that determines the maximum slit position for this condition. The following equations can be obtained from the geometry:

$$h''/f_3 = a/d \quad (A-12)$$

$$h' = h'' f_2/f_3 \quad (A-13)$$

$$1/2 A_2 = a + c, A = \text{lens aperture} \quad (\text{A-14})$$

$$\frac{C + h''}{f_3} \approx \frac{b + h''}{R} \quad (\text{A-15})$$

Combining these equations yields:

$$h' \leq \frac{(1/2 A_2 R - f_3 b \cos \phi) f_2}{f_3 (f_3 - R) + R d} \quad (\text{A-16})$$

Equation A-16 defines the maximum slit position for equal exposures from various slits that are imaged by a small mirror. If the exposure varied for different slits, then the static photograph of the slit pattern would present varying slit widths, and consequently, possible errors for static slit line measurements.

We now examine the slit projection from a large mirror. Since a large mirror reflects the complete light cone, the maximum slit position is determined by the maximum image position h'' (see Fig. A-7) that receives the full light cone. Fig. A-7 shows the geometry for this condition and it can be seen that:

$$\frac{h'' - b \cos \phi}{R} \approx \frac{h'' - 1/2 A_3}{f_3}, A_3 = A_2 \quad (\text{A-17})$$

From this and Equation A-13, we get:

$$h' \leq \left(\frac{f_2}{f_3} \right) \left(\frac{f_3 b \cos \phi - 1/2 A_2 R}{f_3 - R} \right). \quad (\text{A-18})$$

This equation defines the maximum slit position for zero vignetting at a large mirror and, hence, equal exposures for various slit positions.

We now consider vignetting of the relay lenses. It can be seen from Fig. A-7 that vignetting of the relay lenses will decrease the diameter (C) of the collimated light beam between the two relay lenses. The resulting cross-section beam area is determined by the relay lens apertures and is shown in Fig. A-8. As h' gets larger, this area

decreases and the final image irradiance is proportional to the area ΔA . This area can be calculated by integral calculus provided the diameter (C) is known. From the geometry of Fig. A-7:

$$\frac{h'}{f_2} = \frac{A_2 - C}{d}.$$

Calculation of the area ΔA yields:

$$\Delta A = \pi r^2 - 2 \left[X \sqrt{r^2 - X^2} + r^2 \sin^{-1} \left(\frac{X}{r} \right) \right], \quad (A-19)$$

where

$$X = \frac{h' d}{2 f_2} \quad \text{and} \quad r = 1/2 A_2$$

Since $h' = 0$ for a slit positioned at the optical axis, zero vignetting yields the area, $A = \pi r^2$. Now the ratio $\Delta A/A$ is proportional to the ratio of the image irradiance, H/H_0 , where H is the irradiance (power per unit area incident on the film) for a slit at h' and H_0 is the irradiance for $h' = 0$. The ratio becomes:

$$\frac{H}{H_0} = \frac{\Delta A}{A} = 1 - \frac{2}{\pi r^2} \left[X \sqrt{r^2 - X^2} + r^2 \sin^{-1} \left(\frac{X}{r} \right) \right], \quad (A-20)$$

Note that for determining the arc sine, X/r is expressed in radians. Equation A-20 can be expressed in terms of f stops by the definition:

$$\frac{H}{H_0} = 2^{-n}, \quad (A-21)$$

where (n) is the number of stops. Solving for n yields:

$$n = \frac{\log \left(\frac{\Delta A}{A} \right)}{\log (2)}. \quad (A-22)$$

For maximum values of h' (as determined previously) the relative difference is usually less than 10% (or 0.15 stops) and represents a negligible variation of exposure from different slits.

The discussions just presented describe vignetting effects on the static photograph of the slit pattern. We will now consider vignetting effects on the streak traces. First, we note that if light from the object is properly coupled into the first relay lens aperture A_2 without vignetting, then the final image irradiance variation for different streak trace positions is as previously described for the static slit images. Fig. A-9 shows the geometry for calculating the maximum h' for zero vignetting at the lens aperture A_2 . From the geometry in Fig. A-9:

$$\frac{1/2 (A_1 + A_2)}{f_1 (1 + m_1) + f_2} = \frac{h' + 1/2 A_1}{f_1 (1 + m_1)} \quad (A-23)$$

and solving for h' ;

$$h' \leq \frac{1/2 f_1 (A_1 + A_2) (1 + m_1)}{f_2 + f_1 (1 + m_1)} - 1/2 A_1. \quad (A-24)$$

Often, a large diameter objective lens will be used at a small magnification ($\sim 1/10$) and the calculated h' (from Equation A-24) will be smaller than h' (from Equation A-16 or A-18). This value (or a smaller value) is chosen for the slit pattern, however, values of h' greater than that from Equations A-16, A-18 or A-24 should not be used since an observable variation of irradiance might result on the streak traces. A negative solution for Equation A-24 implies that zero vignetting cannot be accomplished for the given data and a smaller aperture A_1 must be selected to couple h' into the relay lens. The aperture for a given h' can be determined by solving for A_1 in Equation A-24:

$$A_1 \leq (A_2 - 2 h') \left[\frac{f_1 (1 + m_1)}{f_2} \right] - 2 h'. \quad (A-25)$$

A negative solution for Equation A-25 implies that zero vignetting cannot be obtained for the stipulated h' , regardless of reduction in the aperture size (A_1). One must then select a smaller value for h' and/or employ a field lens.

A field lens can be installed at the slit plate to couple light rays (which would otherwise be lost) into the relay lens A_2 . The field lens is placed very near (theoretically at) the slit plate and its focal length is chosen to image the objective lens onto the first relay lens A_2 (see reference 2). The required field lens focal length is determined by applying Equation A-3 to the geometry shown in Fig. A-10.

$$f = \frac{SS'}{S + S'} = \frac{f_1 f_2 (1 + m_1)}{f_1 (1 + m_1) + f_2} \quad (A-26)$$

An average value for (f) is usually selected for a range of values of f_1 and m_1 .

Referring to Fig. A-10, it can be seen that light rays from h' will pass above the aperture A_2 (without the field lens installed). We wish to install the field lens and determine the aperture A_1 which will couple light into aperture A_2 without vignetting. From the geometry of Fig. A-10, we see that:

$$A_1 = \frac{A_2 f_1 (1 + m_1)}{f_2} \quad (A-27)$$

When the aperture is set to this value, or less, the maximum h' (as determined in Equation A-16 or A-18) is coupled into the relay optics. Note that the aperture A_2 is now uniformly filled with light for any value of h' within the limits previously determined.

If sufficient light is available from the source so that the aperture A_1 can be significantly reduced (with considerations given to image resolution) then the final image light cone can be reduced. Small mirrors will couple the full image cone when A_1 is reduced as shown in Fig. A-11. The following relationships are obtained:

$$\frac{2b \cos \phi}{R} \simeq \frac{C}{f_3} \quad (A-28)$$

$$\frac{A_1}{f_1 (1 + m_1)} \simeq \frac{C}{f_2} \quad (A-29)$$

where the $\cos \alpha$ and $\cos \beta$ factors have canceled in the equations. Combining these equations yields:

$$A_1 \leq \frac{f_3 f_1 (1 + m_1) (2b \cos \phi)}{f_2 R} . \quad (A-30)$$

The equations presented are approximations since thin lens equations were applied and no account was taken for variation of R for ϕ . This correction term is $\pm b \sin \phi$ and can be applied if necessary. The equations should be sufficiently precise to reduce vignetting effects to a negligible amount as observed on the film.

Fig. A-6. Geometry for Maximum Slit Position h' for Full Mirror Aperture Within the Final Image Light Cone (Small Mirror)

Fig. A-7. Geometry for Maximum Slit Position h' for Zero Vignetting at a Large Mirror

Fig. A-8. Resulting Cross-Section Area of Collimated Light Beam Stemming from Vignetting Within the Relay Lens System

Fig. A-9. Geometry for Calculating Maximum Slit Position h' for Zero Vignetting at First Relay Lens A_2 .

Fig. A-11. Geometry for Zero Vignetting at Small Mirror

APPENDIX A-4

DEPTH OF FIELD FOR A CAMERA SYSTEM

APPENDIX A-4

DEPTH OF FIELD FOR A CAMERA SYSTEM

To calculate the depth of field, one must determine which aperture limits the exiting cone of light. Fig. A-12 presents the geometry for determining the aperture stop and defining the depth of field at the optical center line.

CASE I: The objective lens is the limiting aperture.

Referring to previous calculations in the text, the geometry of Fig. 9 (in the text) and Equation 9 (in the text) can be applied to Fig. A-12 and we get for the final image cone:

$$\frac{1}{\delta'''} = \frac{A_3}{f_3} \quad (A-31)$$

Noting that:

$$\delta''' = M^2 \delta', \quad (A-32)$$

where M is the magnification of the relay lens set,

$$M = f_3/f_2 \quad (A-33)$$

and noting that $\delta' = m_1^2 \delta$, we get the relation:

$$\delta''' = \left(\frac{f_3}{f_2} m_1 \right)^2 \delta \quad (A-34)$$

Since $A_3 = A_2$, we obtain the following for the light cones common to the slit.

$$\frac{A_2}{f_2} = \frac{A_3}{f_2} = \frac{A_1}{f_1 (1 + m_1)} \quad (A-35)$$

or

$$A_3 = \frac{f_2}{F_1 (1 + m_1)}, \quad (A-36)$$

where F_1 is the $f/\#$ of the objective lens and f_1 is the focal length. Combining Equations (A-31), (A-34) and (A-36) yields:

$$\delta = \left(\frac{f_2}{f_3} \right) \left(\frac{1 + m_1}{m_1^2} \right) \left(\frac{F_1}{N^2} \right). \quad (A-37)$$

For the objective lens to be the aperture stop, the objective lens image cone must be smaller than the first relay lens object cone.

$$F_1 (1 + m_1) \geq F_2. \quad (A-38)$$

Also, the objective lens image cone must be smaller than the first relay lens object cone which is accepted by the rotating mirror. The limiting cone at the last relay lens image is defined by:

$$\frac{A_3}{f_3} = \frac{2b \cos \phi}{R}. \quad (A-39)$$

Since $A_3 = A_2$, we can determine the first relay lens object cone. The objective lens image cone must be smaller than the relay lens object cone and

$$F_1 (1 + m_1) \geq \frac{f_2}{f_3} = \left(\frac{f_2}{f_3} \right) \frac{R}{2b \cos \phi}. \quad (A-40)$$

CASE II: The camera relay lens system is the limiting aperture.

This situation requires that the first relay lens object cone is smaller than the objective lens image cone.

$$F_1 (1 + m_1) \leq F_2, \quad (A-41)$$

and that the second relay lens image cone be smaller than a light cone accepted by the rotating mirror.

$$F_3 \geq R/2b \cos \phi . \quad (A-42)$$

From Equation (A-31) and (A-34) we obtain:

$$\delta \left(\frac{f_3}{f_2} m_1 \right)^2 = \frac{f_3}{A_3 N'''} = \frac{F_3}{N'''} . \quad (A-43)$$

Solving for δ yields:

$$\delta = \left(\frac{f_2}{f_3} \right)^2 \left(\frac{1}{m_1} \right)^2 \left(\frac{F_3}{N'''} \right) \quad (A-44)$$

CASE III: The rotating mirror is the limiting aperture.

This condition requires that the last relay lens image cone be greater than a light cone accepted by the mirror.

$$\frac{R}{2b \cos \phi} \geq F_3 . \quad (A-45)$$

Also, the objective lens image cone must be larger than the first relay lens object cone (as accepted by the rotating mirror). This is the opposite of a condition derived in Case I and we may state:

$$\left(\frac{f_2}{f_3} \right) \left(\frac{R}{2b \cos \phi} \right) \geq F_1 (1 + m_1) . \quad (A-46)$$

For the resulting depth of field, we note that:

$$\frac{1}{\delta'''} = \frac{2b \cos \phi}{R} . \quad (A-47)$$

Combining this with Equation (A-34) yields:

$$\delta = \left(\frac{f_2}{f_3} \right)^2 \left(\frac{1}{m_1} \right)^2 \left(\frac{R}{2b \cos \phi} \right) \left(\frac{1}{N} \right). \quad (\text{A-48})$$

For a single lens relay lens system, we convert:

$$f_2 \longrightarrow f (1 + 1/m)$$

$$f_3 \longrightarrow f (1 + m)$$

$$\text{and } f_2/f_3 = 1/m.$$

Fig. A-12. Geometry for Calculating Depth of Field for a Camera System

APPENDIX B-1

SOME CAMERA GEOMETRY SKETCHES

Fig. B-1. Model 132 Streak Camera Geometric Schematic

Fig. B-2. Model 770 Streak Camera Geometric Schematic

Fig. B-3. Enlargement of Model 770 Geometry
for X_0 , Y_0 Approximation

Fig. B-4. Model 132 Streak Camera Optical Schematic

Fig. B-5. Model 770 Streak Camera Optical Schematic

Fig. B-6. Model 168 Streak Camera Optical Schematic

Fig. B-7 Mark II Streak Camera Internal Optical Schematic

APPENDIX B-2

SOME FIRING SITE OPTICAL SYSTEMS

Fig. B-8. The Model 132 Streak Camera (Objective lens, image rotator, and laser alignment fixture are mounted on the optical bench rail.)

Fig. B-9. The Mark II Streak Camera [This camera was designed at LASL and utilizes a zoom lens system and (mechanical) translation - rotation for final alignments.]

Fig. 10. Firing Site Bunker Electronics Bay (Oscilloscopes are used to obtain electronic pin times from raster photographs.)

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