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**TITLE ANALYTICAL TREATMENT OF ELECTRON TRAJECTORY STRAIGHTENER ISSUES
IN FREE-ELECTRON LASERS**

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Analytical Treatment of Electron Trajectory Straightener Issues in Free-Electron Lasers

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Abstract

The poicare orbit of the electron's journey must be straight and narrow through a many period FEL. To utilize an available technology level, the straightest wiggler orbit operating point (SWOOP) is chosen by adjusting the spacing parameter of an orbit correction scheme. This eases the requirements for achieving the full potential of FEL wigglers. One quality index, one cost index, two quality factors, two measures of field noise, an improvement index, and a straightener classification scheme are concepts that facilitate the analysis. Only when the field noise is large, is this analysis of direct use. A reader-friendly description culminates in the comparison of two schemes to illustrate the methods. Two coupled first order langevin equations have solutions that are proven to be gaussian variables. The solutions are used to give the required cross correlations. These results are coupled with a simple graphical technique and give an expression for the angular variance in the correction corridor. The two components of this variance are compared to a fiducial scheme and yield the two quality factors from which the quality, improvement and cost indices are computed.

I. Introduction

The need for electron trajectory control in the wiggler was recognized in the beginning of FEL operation. Originally[1] a solenoidal guide field was present to prevent the electron beam from hitting the walls. The first room temperature, RF LINAC FEL[2] employing a permanent magnet halbach wiggler[3-6], had trajectory steering control in two planes, but it was not required. Departures of the electron beam from axis were measured and the need for trajectory correction cited[7]. The first report of tapered wiggler FEL oscillation utilized a multi-component wiggler with 14 fluorescent screens and corresponding vertical and horizontal steering coils[8]. Even though the ACO optical klystron had only 7 magnetic periods in the buncher and 7 in the radiator, the electron beam was aligned to $100\mu m$ all along the 1.3m straight section[9].

Steering control is required for many period wigglers having large field errors. Short-optical-wavelength FEL wigglers[10-35] including the THUNDER[13] and NISUS[14] devices that in addition are long length or low energy devices, are particularly susceptible to trajectory errors. Means of producing good orbits include judicious magnet selection[36] and shim tuning[14]. The approach we describe accepts a residual perversity of the wiggler trajectories and applies an external means of correction with electromagnetic coils that are diagnosed by beam position monitors(BPMs) as the active straighteners. A related approach is the pulsed wire technique[37,38].

The major issue is how well steering errors can be indemnified. The second issue is whether or not a particular correction scheme is performing at its best. A third issue relates to characterization of the faithfulness or quality of a correction technique. The fourth issue relates to the number of correction coils and BPMs needed to make the scheme work. To answer the first issue, all possible effects in all possible correction schemes need to be examined. Here we look at only a few. We deal with the second issue by adopting the straightest wiggler orbit operating point (SWOOP). SWOOP minimizes the phase associated

with the longitudinal coherence of the electron beam. The third issue is addressed by defining a SWOOP quality index relative to a particularly simple tandem correction scheme. Other schemes are measured relative to this scheme by BPM and kick error quality factors, a combination of which gives the SWOOP quality index. This index permits comparison of techniques on the basis of longitudinal phase coherence, or equivalently on the straightest trajectories as described below. It is quite clear that this index of straightness is only one of many considerations[36] in determining the overall efficiency of a laser operating at a fixed wavelength, for instance. The fourth issue is addressed by the introduction of a measure of the relative density of correction coils and BPMs with respect to the fiducial scheme. This cost factor is expressed as a power of the ratio of the two quality factors. The improvement factor contrasts the phase and cost.

The technology we examine for active multiple trajectory corrections utilizes BPMs to measure the trajectory position and determine relevant angles. Coils are engaged as straighteners to supply corrective dipole kicks. To obtain a simple description, we model these dipole kicks as being instantaneous impulses located at the center of mass of the coils; we take the length between correction coils and BPMs to be small compared to the betatron[39] wavelength; we assume the motion is relativistic ($\gamma \gg 1$); we take the field kick errors and the BPM diagnostic errors to be random, independent and in one plane; and we assume that the properties of the trajectory of an electron initially aligned and located at the centroid of the electron beam, the guide electron, are an adequate measure for the entire beam.

Three correction schemes are examined in detail and they illustrate the features of the computational methods. The placement of the BPMs and correction coils is periodic on a lattice of period L_c . Each type of lattice examined falls into one of two general classes. The T class has the correction coils and the BPMs located *together* at the same axial location, and the N class has them interlaminated. A scheme in either of these classes is called a TN scheme. A correction cycle is the irreducible unit of the pattern of placement (corresponding to a unit cell in crystallography); thus, to add another TN correction cycle of length L_c , requires both a BPM and a coil. To distinguish various schemes or subclasses of schemes within a class, one or more numerical subscripts are appended to the class designation. The subscripts designate the lattice increments of the BPMs supplying information used to correct a given coil. Thus, a T_1 scheme uses the BPM information one correction cycle downstream from the coil to correct the coil. The corresponding interlaminated scheme is $N_{1/2}$ whose subscript indicates that the BPM whose information is used to correct this coil is located downstream, midway through the correction cycle. Because all the subscripts of the N class are half-integers, we adopt the shorthand of using only the numerator in the designation, i.e., N_1 signifies $N_{1/2}$. Schemes utilizing information from multiple BPMs are designated by multiple subscripts. Single digit subscripts (signed or unsigned) are not separated by commas; whenever any multi-digit number is used, it is delimited by commas fore and aft; however, an initial comma is not used. The three schemes examined in detail are T_1 in Section III, N_1 in Section IV, and N_{13} in Section V. We also give results[40] for a scheme in the subclass of schemes, N_{-1135} . The N_1 scheme is particularly nasty in that it accumulates upstream errors and never unburdens itself of these. The N_{13} scheme averages the two BPM readings and treats that interpolated value as the measured position of the trajectory at the mean location.

The schemes described here have *set points* one correction cycle downstream from the adjusted coil. The set point is where the orbit is positioned to cross the axis. The remaining angular error can be completely corrected only if the set point is located at a coil that removes the residual angle by corrective orbit rotation.

The reduction of the steering error contribution to the longitudinal phase is not necessary when that phase is much less than unity. The phase can be estimated approximately by computing the product of the number of wiggler periods with field noise level. A rough measure of the ensemble averaged phase introduced to the guide electron is the product of the field kick error, σ_k , measured in percent and defined more precisely below, with the number of periods, N , of the wiggler. This we call the rough kick noise (RKN). For many FELs the RKN is approximately equal to a more precise determination we call the gross kick noise (GKN), the GKN is $\sqrt{4\xi}100\sigma_k N$ where ξ is a parameter that varies between 0 and 1/2, often appears as the argument of Bessel functions, and depends only on the (peak) scaled vector field amplitude a_w as $a_w^2/(4 + 2a_w^2)$. At a value of $a_w = \sqrt{2}$, the $\sqrt{4\xi} = 1$. An example is helpful. A 50 period wiggler operating at $a_w = \sqrt{2}$ and 1/2% kick errors has a GKN of 25. When the GKN is 25, the guide electron experiences a phase shift of $\pi/2$, at a GKN value of 35, it is π , and at 50, 2π .

If the GKN is not much less than 20, then corrective means need be considered. To minimize the phase using coils and BPMs, the length of the correction cycle is set equal the SWOOP correction length L_s , and

this procedure gives a new scaling that makes short optical wavelengths easier to correct than long, at fixed number of wiggler periods.

The coil and BPM correction technology results in a new expression for the guide electron's longitudinal phase that is the sum of two terms. The first term is proportional to the square of the GKN and inversely proportional to the product of the number of SWOOP configured correctors and the square of a kick error quality factor that is dependent on the details of a particular corrector scheme. The second term of the phase is proportional to the square of the BPM induced angle and the number of optical periods in the wiggler. This induced angle is measured by the ratio of the standard deviation of the BPM position error, σ_x , to the product of the correction cycle length and the BPM quality factor; again, this quality factor is scheme dependent. At the SWOOP the correction cycle length is optimized and this sum is proportional to the 4/3 power of the ratio of the field kick errors to its corresponding quality factor and the 2/3 power of the ratio of the BPM errors to the BPM quality factor as will be shown in Section VI. The reduction factor of the longitudinal phase by the correction at the SWOOP can be written in a number of ways. Remembering that at the SWOOP, the number of correctors N_{cc} depends on both quality factors, both standard errors, the electron energy, the wiggler length and its wavelength, it follows that the inverse phase reduction factor, the overall improvement factor of the correction system for the phase, is $2N_{cc}Q_k^2$. Fixing the method of correction and the values of σ_k , σ_x and a_u , implies that N_{cc} varies inversely as the 2/3 power of the optical wavelength alone. Short wavelengths then require more TN BPMs at optimum, but because they do, at the same number of wiggler periods, they have the potential to end up with better quality correction systems than at longer wavelengths.

Therefore, many period wigglers, especially long wavelength many period devices put stringent requirements on the kick and diagnostic errors; ignoring the dependence of the required number of wiggler periods on wavelength, short wavelength corrector systems, on the other hand, may cost more. At fixed correction scheme type and wavelength, a decade increase in wiggler periods from 100 to 1000 can be achieved by a factor of 6 improvement in kick errors or a factor of 30 improvement in the BPM precision. In combination, the same factor of ten can be achieved by factor of three improvement in *both* the BPM error and the kick error. Because the wiggler periods tend to increase where best mirror reflection coefficients are small, low reflectivity also stresses the technology.

One question that can be answered is: Is there a way to beat the cost and maintain the quality? The answer hinges on whether the Q_k in the improvement factor $2N_{cc}Q_k^2$ can be made arbitrarily large. In Appendix A, we prove Q_k cannot be larger than unity in any TN scheme. If we abandon the TN SWOOP, and assume the Q_x factor is infinite (see Appendix A), the best improvement factor is $3N_{cc}Q_k^2$. When quality is at stake, the better quality has to be paid for.

The T_1 SWOOP is taken as the fiducial against which other schemes are measured. Any other scheme can be transformed into a similar T_1 scheme having the same SWOOP phase value, $\langle \phi_{lc} \rangle_S$, by the evaluation of two quality factors. The T_1 scheme expression for $\langle \phi_{lc} \rangle_S$ is the sum of two quadratic terms, one in σ_k and the other in σ_x . A general scheme uses this same expression but with $\sigma_k \rightarrow \sigma_k/Q_k$ and $\sigma_x \rightarrow \sigma_x/Q_x$ where Q_k and Q_x are the quality factors for the kick error and the diagnostic error respectively. Consequently the SWOOP- T_1 quality index Q_{1S} for the reduction of $\langle \phi_{lc} \rangle_S$ is $Q_{1S} = Q_k^{4/3}Q_x^{2/3}$, i.e. $\langle \phi_{lc} \rangle_S = \langle \phi_{lc} \rangle_{ST1} / Q_{1S}$ where $\langle \phi_{lc} \rangle_{ST1}$ indicates the SWOOP T_1 value. Although the quality factors are the object of the computations, Sections III-V are concerned with the intermediate goal of expressing the average angular variance for the T_1 and N_{1S} schemes in as simple a way as possible. The latter scheme is sufficiently complicated that it may serve as a model for similar computations. Those not interested in these details may skip these sections and the background material in Section II. The results are summarized in Section VI.

Experimental and theoretical progress[41-62] has been made with a variety of strategies to control the electron trajectories, it is a major focus of these proceedings. Simple arguments also show that we are not fettered to the 4/3 2/3 rule described above which has a coefficient that is correction scheme dependent. Early work suggested that the longitudinal phase coherence, ϕ_{lc} , requirements could be reduced by a factor of three with the case of perfect BPMs. For that case, in the T_1 scheme, the phase is proportional to $1/2N_{cc}$, where N_{cc} is the number of correction cycles. However, after the initial decrease in phase with this T_1 scheme our model suggests that further addition of correctors no longer reduce the angular spread of the trajectories. Then the angular error introduced by the BPMs is of order of magnitude of that of the

Fig.(1) The interface between the macroscopic and microscopic variables. The trajectory enters with displacement x_j and angle θ_j and leaves with x_{j+1} and θ_{j+1} . The bracketed superscript indicates the number of the microscopic interval. The displacement at the beginning of the k 'th microscopic interval is $x_j^{(k)}$ and the angle is $\theta_j^{(k)}$. The random walk gives an increment to the angle in the microscopic interval an amount $\Delta\theta_j^{(k)}$.

sector mid-way corrected trajectory. The angular error of the T_1 BPMs is measured by its variance, where a single stochastic angular BPM error is $(\delta x_{j+1} - \delta x_j)/L_c$; here δx_j is the value of the diagnostic position error of the j 'th BPM. Adding more correctors per unit length makes L_c smaller and the BPM angular error even larger. The minimum in phase corresponds to a minimum in the composite angular variance. Because, loosely speaking, small angular variance means straight trajectories, we characterize the minimum phase operating point as the straightest wiggler orbit operating point (SWOOP).

II. Simplified Bookkeeping for the Microscopic Scale

A typical microscopic trajectory is described in Fig (1). The electron enters at angle θ_j and displacement x_j . After undergoing many random changes in angle, it emerges a unit distance later with angle $\theta_j + \Delta\theta_j$ and displacement $x_j + \Delta x_j$. In transit, the instantaneous position and angle of the electron are denoted by $x_j + \Delta x_j(s)$ and $\theta_j + \Delta\theta_j(s)$ respectively. It is the object of this section to compute the statistical properties of $\Delta x_j(s)$ and $\Delta\theta_j(s)$. These quantities are found to be gaussian variables whose cross correlations and variances are governed by Eqs (11)-(13). Those interested in knowing the calculation technique but not its justification may skip to Section III, and refer back to the results cited in Eqs (11)-(13) when needed.

Although the grid upon which this random motion is impressed was originally continuous and was able to describe the wiggler period sinusoidal motion of the particle itself, the grid we use has been altered. Conceptually, the following two step process has occurred. First the continuous interval is replaced by a finite mesh with separation of $\lambda_w/2$, half of the wiggler magnetic field wavelength. The departures from the true trajectory are then described discretely on these poincaré planes. The excursions are modeled as a random walk in the angle space of the departure trajectory or as viscous-free brownian motion in the displacement space of the departure trajectory.

A correspondence is established with the Kincaid model. That model describes field errors over a single half period as following the profile of the ideal field, but increased or decreased a random amount. With the

field on a half-period modeled as

$$B_y(z) = B_0(1 + \sigma_k) \sin(k_w z), \quad (1)$$

a corresponding change, $\delta\theta$, accrues to the angle that is the integral over the deviated field $B_0\sigma_k \sin(k_w z)$, given in MKS units as

$$\delta\theta = \frac{e}{m\gamma} \int_0^{\lambda_w/2} B_0\sigma_k \sin(k_w z) dz,$$

or

$$\delta\theta = \frac{2a_w}{\gamma} \sigma_k; \quad (2)$$

this is the product of the half-period kick with the fractional error of the field. We employ standard notation: the electron has mass m , charge magnitude e and has achieved a speed component that is a fraction β of the speed of light. The departure of β from unity is measured by the lorentz factor γ , where $\gamma^{-2} = 1 - \beta^2$. The scaled field, a_w , is that for the field amplitude: $a_w = eB_0/mk_w$. Although Eq.(2) corresponds to the Kincaid model, it may be re-interpreted as the experimental definition of σ_k . From henceforth we regard σ_k as the fractional error of the half-period kick. In the model then, we retain the stochastic half-period kick of the Kincaid model, but throw out the high spatial-frequency field effects[63].

The second conceptual step is to solve the random walk on the discrete grid. This is done in the limit of an infinite number of half periods over a unit length. The rather remarkable paper[64] of P. Langevin on brownian motion was an early realization of the great mathematical simplicity that a differential equation with random driving terms provides to describe a process that up until that time had been modeled as an intrinsically discrete process involving nearly infinitesimal steps. The same simplicity is found in this problem in computing the correlations of Δx and $\Delta\theta$. The sums that appear there may be evaluated in the limit of infinitely many kicks. In that limit, the sums become integrals that are given directly by the differential equation approach. The limiting process replaces the discrete grid by a continuous one, over which the angular variance of the trajectory is matched at the discrete interval to that required by the random kick error. (A discrete approach is used in Appendix B.) The unencumbered random motion is, thus, governed by

$$\begin{aligned} \frac{d\Delta\theta}{ds} &= \sqrt{(\sigma_\theta^2)'} \dot{u}(s), \\ \Delta\theta(0) &= 0, \\ \frac{d\Delta x}{ds} &= \Delta\theta, \\ \Delta x(0) &= 0. \end{aligned} \quad (3)$$

In the above equations, $\dot{u}$ contains the random (white noise) variables and $\langle \dot{u}^2 \rangle$ is independent of s (its exact value is chosen later for convenience), and $(\sigma_\theta^2)'$ is a constant chosen such that at $s = \lambda_w/2$,

$$\langle (\Delta\theta)^2 \rangle^{1/2} = \langle (\delta\theta)^2 \rangle^{1/2} = \frac{2a_w}{\gamma} \sigma_k, \quad (4)$$

i.e. the variance of the random walk matches the expected variance from a half-period, and where the brackets, $\langle \rangle$, indicate an ensemble average over the random walk variables of $u(s)$, as it were, averaging over the ensemble of wigglers, where $\sigma_k^2 = \langle \sigma_k^2 \rangle$. In order for Eq (4) to have meaning, it must have a definite limit as the problem is modeled on a finer and finer mesh of separation Δs . Therefore, the expression

$$\Delta\theta(s) = \sqrt{(\sigma_\theta^2)'} \int_0^s u(s_1) ds_1 = \lim_{\Delta s \rightarrow 0} \sum_{n=1}^{s/\Delta s} u(n\Delta s) \Delta s, \quad (5)$$

must have invariant statistical properties[65]. Now, the mean of $\Delta\theta(s)$ is zero and the variance is given by

$$\langle [\Delta\theta(s)]^2 \rangle = (\sigma_\theta^2)' (\Delta s)^2 \sum_{n=1}^{s/\Delta s} \langle u^2 \rangle.$$

Because $\langle \dot{u}^2 \rangle$ is independent of s , the sum is $s/\Delta s$ times the single instance value, and therefore

$$\langle [\Delta\theta(s)]^2 \rangle = (\sigma_\theta^2)' s \Delta s \langle \dot{u}^2 \rangle.$$

Furthermore, we choose $\langle \dot{u}^2 \rangle = 1/(\Delta s)$, which when combined with the preceding equation gives

$$\langle [\Delta\theta(s)]^2 \rangle = (\sigma_\theta^2)' s, \quad (6)$$

where

$$(\sigma_\theta^2)' = \left(\frac{2a_w \sigma_k}{\gamma} \right)^2 \frac{2}{\lambda_w}, \quad (7)$$

and $(\sigma_\theta^2)'$ is the spatial rate of increase of the angular variance from kick errors. The limit process is, therefore, valid, and the resulting computations simplified.

Combining Eqs.(5) and (6) we arrive at

$$\langle \int_0^s \dot{u}(s_1) ds_1 \int_0^s \dot{u}(s_2) ds_2 \rangle = s.$$

This may be economized by writing

$$\langle \dot{u}(s_1) \dot{u}(s_2) \rangle = \delta(s_1 - s_2), \quad (8)$$

where δ is the dirac delta function, and this expresses the independence of $\dot{u}(s_1)$ and $\dot{u}(s_2)$. This is the continuous form of the discrete condition $\langle \dot{u}_n \dot{u}_m \rangle = \delta_{mn}$ where δ_{mn} is the kronecker delta. The displacement can, thus, be written

$$\Delta x(s) = \int_0^s \Delta\theta(s_1) ds_1.$$

Using this integral expression for $\Delta\theta(s_2)$, switching the order of integration, and integrating once displays the leverage of the angular kicks upon the displacement,

$$\Delta x(s) = \sqrt{(\sigma_\theta^2)'} \int_0^s (s - s_1) \dot{u}(s_1) ds_1. \quad (9)$$

In Appendix C, we prove that for positive A and B

$$\left\langle \int_0^A f(x) \dot{u}(x) dx \int_0^B g(y) \dot{u}(y) dy \right\rangle = \int_0^{\min(A,B)} f(x) g(x) dx. \quad (10)$$

Denoting $\min(s_3, s_4)$ by $s_<$ we then have

$$\langle \Delta x(s_3) \Delta x(s_4) \rangle = (\sigma_\theta^2)' \int_0^{s_<} (s_3 - s_1)(s_4 - s_1) ds_1 = (\sigma_\theta^2)' [s_3 s_4 s_< - (s_3 + s_4) s_<^2 / 2 + s_<^3 / 3], \quad (11)$$

which can be written

$$\langle \Delta x(s_1) \Delta x(s_2) \rangle = \frac{(\sigma_\theta^2)'}{12} [|s_1 - s_2|^3 + (s_1 + s_2)^3 - 2s_1^3 - 2s_2^3].$$

Likewise,

$$\langle \Delta x(s_3) \Delta\theta(s_4) \rangle = (\sigma_\theta^2)' \int_0^{s_<} (s_3 - s_1) ds_1 = (\sigma_\theta^2)' [s_3 s_< - s_<^2 / 2], \quad (12)$$

or alternately,

$$\langle \Delta x(s_1) \Delta\theta(s_2) \rangle = \frac{(\sigma_\theta^2)'}{4} [2s_1^2 - (s_1 - s_2)^2 + |s_1 - s_2|(s_2 - s_1)],$$

and

$$\langle \Delta\theta(s_3)\Delta\theta(s_4) \rangle = (\sigma_\theta^2)' \int_0^{s_4} ds_1 = (\sigma_\theta^2)' s_4, \quad (13)$$

rewritten as

$$\langle \Delta\theta(s_1)\Delta\theta(s_2) \rangle = \frac{(\sigma_\theta^2)'}{2} [s_1 + s_2 - |s_1 - s_2|].$$

Until now we have simply inquired into the mean, variance and cross correlations of Δx and $\Delta\theta$. Here we ask the question: what is the joint distribution of Δx and $\Delta\theta$. Eqs.(5),(9), and (11)-(13) give the required expressions, and in the Appendix B, a generalization of the central limit theorem[66] is used to prove that $\Delta x(s)$ and $\Delta\theta(\bar{s})$ follow a bivariate normal distribution[67]; for $s = \bar{s}$,

$$Pr[\Delta x(s), \Delta\theta(s)] = \frac{\exp \left\{ -\frac{1}{2(1-\rho^2)} \left[\frac{(\Delta x)^2}{\sigma_{\Delta x}^2} - 2\rho \frac{\Delta x}{\sigma_{\Delta x}} \frac{\Delta\theta}{\sigma_{\Delta\theta}} + \frac{(\Delta\theta)^2}{\sigma_{\Delta\theta}^2} \right] \right\}}{2\pi\sqrt{1-\rho^2}\sigma_{\Delta\theta}\sigma_{\Delta x}}. \quad (14)$$

The corresponding cross correlation coefficient ρ is

$$\rho = \frac{\langle \Delta x \Delta\theta \rangle}{\sqrt{\langle \Delta x^2 \rangle \langle \Delta\theta^2 \rangle}} = \frac{\sqrt{3}}{2},$$

and the variances are

$$\sigma_{\Delta\theta} = \langle \Delta\theta^2 \rangle^{1/2} = \sqrt{(\sigma_\theta^2)' s},$$

$$\sigma_{\Delta x} = \langle \Delta x(s)\Delta x(s) \rangle^{1/2} = s\sigma_{\Delta\theta}/\sqrt{3}.$$

Such a joint distribution is a general property of any distinct pair of gaussian variables, and the characteristic function given in the Appendix B is the generator for calculation of the bivariate moments.

Simply speaking, the standard deviation of x is that of θ multiplied by a lever arm that is $\sqrt{3}/3$ times the distance of travel. The cross correlation coefficient of x and θ is $\sqrt{3}/2$, the positive sign of which means that any displacement Δx in a particular direction is likely to be accompanied by an angular misalignment $\Delta\theta$ in that same direction. Knowing the probability distribution and the initial conditions, the whole microscopic scale can be crossed by the trajectory in one step.

III. The T_1 Scheme

Although the name may suggest retribution by the great Norse God who brings fire to the earth, THUNDER was the first corrector scheme with intentionally-contained errors. This T_1 scheme[13] may be thought of as single correctors of the variety described by Kincaid[54] in tandem. Each coil is used to adjust the adjacent downstream BPM to have a zero reading. The corrector at the wiggler entrance positions the electron beam on axis at the end of the correction cycle. That removes position errors that have accumulated from steering errors up to that point. With a single corrector, the scheme does not remove the accompanying angular errors although it reduces their ensemble average variance by a factor of 3. In a multiple-corrector scheme, the angular error at the end of the first cycle traverses an uncorrected second cycle and is leveraged into a displacement. When the second cycle is corrected, that displacement goes to naught, and therefore, the angular error is completely removed from the device. Thus, the results of steering errors originating in the first cycle are contained in the first two cycles only.

As a part of the macroscopic/microscopic approach, we model the steering errors to accumulate during a cycle but they have no effect until just before the end of the cycle. Such a model is aimed at a correct description of the electron trajectory at the correctors and BPM stations. Superimposed upon this trajectory is the microscopic angle and displacement information contained in $\Delta\theta(s)$ and $\Delta x(s)$.

Examine the errors. First consider the consequences of a displacement Δx resulting from steering errors in the first interval. It appears as the chained line of Fig(2a). The deviation is discovered at the end of the second cycle and nulled as shown by the solid arrow. No alteration of the trajectory in the first cycle has occurred, nor will any effect of the error be felt at the third and subsequent cycles. Next look at the corresponding angular error in Fig (2b). at first it is uncorrected and chained, then corrected as an arrow

Fig.(2) The adjusted T_1 trajectories for four errors: (a) The accumulated position error from interval 1 is the dashed line when uncorrected and the solid arrow when corrected; (b) The dashed angular error is completely removed; (c) The arrow pointing to midnight is the diagnostic error. The other two arrows are the induced trajectory. The solid slanted line owes to the diagnostic error of the BPM at the end of cycle 2. The three coils are labeled C_1 , C_2 , and C_3 and the 3 BPMs B_1 , B_2 , and B_3 .

The BPMs errors may not be as important in single corrector systems, but they give an ultimate limitation on how well multiple correction schemes work. The error arises from the inability to achieve perfect placement to a spatial reference, the difficulty of establishing a good reference line or keeping it up to standards, and fluctuations in the currents of the correction coils. The BPM model functions as follows: All BPM errors are independent. Any BPM error adds to the trajectory position and results in an apparent position of the electron beam. This apparent position is erroneously nulled by adjusting the upstream correction coil. The additive errors of the second BPM and the first part of the error of the third BPM are shown in Fig.(2c). The offset of the trajectory at the BPM is required to make its apparent position lie true. In an actual system, if the BPM induced angular error were larger than the angular errors from steering, the BPM could be ignored[63]. This suggests that correction systems with more coils than BPMs could be beneficial, but we will not investigate those here. Rather, SWOOP is promoted in Section VI, and it equalizes the BPM induced angular error with the scaled mid-stream angular error, thereby decreasing the detune or increasing the Q_{15} of the system. This may be accomplished in the design stage by adjusting the separation distance between coil-BPM pairs, but to do so effectively, requires knowledge of the state of technology for magnetic field quality, BPM precision, power conditioning, and position standards. To the authors' knowledge, no technological review of these areas, appropriate to wigglers, lies in the public domain.

All possible errors of unity amplitude are assembled. The resulting expression is the sum of the error variables weighted by their corresponding slope shown in cycle 2 of Fig.(2a-c) plus the accumulated angle $\Delta\theta_j(p)$, thus

$$\theta_j(p) = (\delta\dot{x}_{j-1} - \delta\dot{x}_j)/L_c + \Delta\theta_j(p) - \Delta x_j/L_c. \quad (15)$$

An alternate method to derive this equation is to solve algebraic equations written for position and angle in the two cycles, in conjunction with the expression for the correction angle. We now see in Eq.(15) that the trajectory displacement Δx_j that arises from leveraged accumulated steering errors throughout the interval is not independent of $\Delta\theta_j(p)$ that also depends on the steerings in common up to position p . The cross correlation and auto correlation of $\Delta\theta_j$ and Δx_j that appear below are given in Eqs.(11)-(15), and

$$\langle \theta_j^2(p) \rangle = \langle [(\delta\dot{x}_{j-1} - \delta\dot{x}_j)/L_c]^2 \rangle + \langle (\Delta\theta(p) - \Delta x(1)/L_c)^2 \rangle, \quad (16)$$

evaluates to the ensemble averaged profile trajectory

$$\langle \theta_j^2(p) \rangle = \frac{2}{L_c^2} \sigma_x^2 + (\sigma_\theta^2)' L_c \left[\frac{1}{12} + \left(p - \frac{1}{2} \right)^2 \right]. \quad (17)$$

Fig.(3) Correction of an initial displacement and angle error for two values of f in an N_1 system. The $f = 1$ way is uncorrected. The $f = 0$ way has correction to make BPMs read null at the three BPMs assuming perfect BPMs. The 4 coils are labeled C_1 , C_2 , C_3 , and C_4 and the 3 BPMs are B_1 , B_2 and B_3 .

If we set σ_x^2 to zero we obtain the result of Kincaid. It is the interference, here as well as there, of Δx and $\Delta\theta(p)$ that reduces the variance of θ_j . With Δx , δx , and δx_{j-1} nulled, the uncorrected wiggler angle, θ_j , reduces to $\Delta\theta_j(p)$ and results in the value of $\langle \theta_j^2(p) \rangle$ of $(\sigma_\theta^2)'p$ for comparison purposes. Averaging Eq.(17) over the profile gives by integration from nil to unity with respect to p yields

$$\overline{\langle \theta_j^2(p) \rangle} = 2 \frac{\sigma_x^2}{L_c^2} + \frac{1}{6} \frac{(\sigma_\theta^2)'}{L_c}. \quad (18)$$

The corresponding optimized quality factor Q_{15} (and its cost C_{15}) is discussed in Section VI.

IV. The N_1 Method

The N_1 way is a method that adjusts the preceding coil to make a BPM reading f times its uncorrected value. N_1 is a sub-class of schemes when all values of f are considered. Although this method is uninteresting from an actual applications point of view, it is interesting pedagogically and for completeness. It is a simple example of a correction technique that has a free parameter. It also illustrates a feature of systems that needs to be avoided. The bookkeeping for the N_1 way is simplified by the consideration of special values of f , and then the general case is constructed by linear interpolation.

In Fig.(3) the principal of the scheme is illustrated. The four coils C_1 to C_4 are interspersed with three BPMs B_1 to B_3 making up the six sectors. A displacement and angle error exists at C_1 and the uncorrected trajectory is labeled by $f = 1$. The correction scheme takes the value of the trajectory as measured by BPM B_1 and adjusts the reading on the BPM to be a factor f times the original value. The analogue of the T_1 scheme is to take $f = 0$. For $f \neq 1$, the preceding coil bends the trajectory. When $f = 0$, the trajectory crosses the axis at the BPM location. For other values of f , the trajectory is the weighted average of the two trajectories shown.

This scheme or its derivatives has two serious disadvantages that makes its further consideration unprofitable. For $f = 1$ the BPMs and coils are not used. For $f \neq 0, 1$ trajectory walk off from the optical mode is a problem as is accumulation of upstream trajectory errors. For $f = 0$, BPM and field angular errors also accumulate from upstream and are undamped. The $f = 0$ case also may have an optical mode problem for tapered wigglers[68]. These disadvantages amount to unacceptably large errors. Inclusion of BPM errors does not change this conclusion.

Fig.(4) The N_{13} way for an arbitrary displacement and angular error at C_1 . The original trajectory as shown by extending the first dashed line is correctly measured by the B_1, B_2 pair and brought to its axis at C_2 . Subsequent to C_2 , the remaining angular error is indicated by the second dashed line; this is then fully corrected from the B_2, B_3 readings by adjusting C_2 to make the trajectory horizontal.

V. The N_{13} Method

The N_{13} method uses the average of the next two downstream BPM readings as a measure of the trajectory at their average position. This location is the set point and coincides with the position of the next downstream coil, C_2 of Fig.(4). The N_{13} method, in contrast to the N_1 technique, keeps the trajectory errors from accumulating. In Fig.(4), the correction of an arbitrary trajectory angle and position displacement at a coil is shown to occur within the two following sectors.

Just as the trajectory of the N_1 method is linear in the adjustment factor f , the individual trajectory errors are additive and the trajectory may be computed from the knowledge of the amplitude of each error. The trajectories induced for 18 possible errors are shown in Fig.(5) for a two sector cycle of length 2. In accord with the previous microscopic treatment, the trajectory field errors Δx_i and $\Delta \theta_i$ are modeled to occur at the end of the interval over which they originate. All errors that originate prior to C_1 have been fully corrected within the region $C_3 - C_4$, the correction corridor. Thus, the 18 random variables are sufficient to describe the trajectory there completely. Except for the $\Delta x_i, \Delta \theta_i$ pairs, the random trajectory variables are entirely independent and the statistical analysis, therefore, simplifies greatly. From Fig.(5) we can see that $\Delta x_1, \Delta \theta_1, \Delta x_2, \Delta \theta_2, \Delta x_3, \Delta \theta_3, \delta_1$ and δ_7 do not contribute to the correction corridor. This leaves 10 random variables that do. The variable δ_3 has a position error but zero slope in the correction corridor, and, thus, we have but 9 random trajectory variables that enter in the computation of $\langle (\Delta \theta)^2 \rangle$.

The graphical trajectory presentation offers many conceptual advantages. The even and odd subscripted Δx_i and $\Delta \theta_i$ sets form four subgroups whose elements are related by translation over distances that are multiples of two sectors. Likewise δ_2, δ_3 and δ_4 are similarly related. Since δ_1 does not enter the correction corridor, there are only five basic types of errors to consider. The 4 other errors of the 9 are simply translations. We show and explain these 5 types in more detail in Fig.(6). After familiarization is achieved with use of Fig.(5) to write down Eqs (19)-(21), the next level of proficiency is to use the 5 prototype error trajectories shown in Fig.(6) to do so. Gurus need only 2 trajectories.

Although the trajectory displacements are depicted graphically, the trajectory angle information is also available. Except in the region of accumulation of the trajectory field errors, the slope of the trajectory gives the angle of the trajectory. In the region of accumulation, the trajectories are drawn as if the accumulated errors $\Delta x_j(p)$ and $\Delta \theta_j(p)$ occur immediately at the end of the trajectory. Although rare, such errors are possible as a statistical quirk. As far as correcting the trajectory is concerned, it doesn't matter what the distribution of stochastic field errors are within the interval. The Δx_j and $\Delta \theta_j$ variables are, therefore, sufficient statistics for trajectory properties. For the more general question of computing the average rms angle over the accumulation interval j , the angle within the interval is obtained by adding $\Delta \theta_j(p)$ where the pointer p varies from 0 to just less than unity. At the beginning $\Delta \theta_j(p)$ is zero, and at the end, $\Delta \theta_j(1)$

Fig.(5) The complete assembly of the corrected trajectory responses to 18 unit amplitude errors using the N_{13} way. These errors that are in the $C_3 - B_3 - C_4$ region, the correction corridor, are taken as typical of the wiggler. The Δ_j curves are responses to a unit displacement error at axial location $jL_c/2$, and $\Delta\theta_j$ are the corresponding responses to an angular error. The responses to a unit error of a BPM at location B_j are shown in the lower four curves. The height can be calibrated by noting the area under the δ_2 curve is -2.

is nothing other than the trajectory variable $\Delta\theta_j$. Therefore, a plot of the sum would show a continuous, albeit tortuous, trajectory in angle space.

We may now write down by inspection the angle $\theta_c(p)$ in the sector $C_3 - B_3$ and $\theta_b(p)$ for sector $B_3 - C_4$. These two sectors will have the same statistical expectations as all subsequent sector pairs, and therefore, we use these two sectors to represent the general quality of errors throughout the wiggler. Examination of Fig.(6) shows that for the 9 trajectory variables slopes of $\pm 1/4$ and $\pm 1/2$ are possible. The only variables showing slope changes or discontinuities between these two sectors are $\Delta\theta_6$ and Δx_5 . The sum of the other 8 angles common to both sectors is denoted by θ_{ic} , independent of p , and is given by the slope-weighted sum of those trajectories in the correction corridor as shown in Figs.(5)-(6)

$$\theta_{ic} = \frac{1}{4} [-\Delta x_4 - \Delta x_6 - \Delta x_7 + \Delta\theta_4 - \Delta\theta_6 + \delta_2 - \delta_4] \quad (19)$$

Assembling the rest we obtain

$$\theta_c(p) = \theta_{ic} - \frac{1}{4} (\Delta x_5 + 2\Delta\theta_5) + \Delta\theta_5(p) \quad (20)$$

and

$$\theta_b(p) = \theta_{ic} - \frac{1}{4} (\Delta x_5 - 2\Delta\theta_5) + \Delta\theta_5(p) \quad (21)$$

For $\theta_c(p)$, we note that no field error variables with subscript 5 appear in its θ_{ic} term. It is also convenient to separate out variables with subscript 6 from θ_{ic} and include those with subscript 5 when computing $\theta_b(p)$ because it has the p variation over interval 5. This variable θ_{ib} is said to have 6 exclusion and 5 inclusion

Fig (6) The 5 prototype errors of the N_{13} way. The subscripts on Δx and $\Delta \theta$ indicate the sector interval, whereas those on δ give the ordinal number of the corresponding BPM. The unit displacement error at B_2 where $s = 3S_c$ and $S_c = L_c/2$, accumulated from the 3rd sector, is Δx_3 , and its correction alters the trajectory beginning with C_1 whose setting depends on both the B_1 and B_2 readings; because of averaging, only half the error is corrected at C_2 ; C_2 makes no correction because the trajectory is already heading to zero at C_3 . The B_3, B_4 readings cause the left over angular error at C_3 to be removed. The displacement error, Δx_2 , at coil C_2 first registers on B_2 . The algorithm corrects the trajectory as if it had half this displacement error at C_2 . The B_2, B_3 pair correctly measure the remaining trajectory, and adjustment of C_2 nulls the displacement at C_3 . The angular error $\Delta \theta_3$ taken at B_2 does not initially change the B_2 reading nor the preceding B_1 reading; its correction is based on the B_2, B_3 pair and the C_2 setting. The angular error $\Delta \theta_2$ taken at C_2 first appears on the B_2 reading; the averaging process causes half this error to be corrected at C_2 ; the B_2, B_3 pair correctly detects the post C_2 trajectory and completely nulls the trajectory at C_3 . The error δ_2 occurs at B_2 as an erroneous reading of unity. Half this reading is corrected by C_1 ; the B_2, B_3 pair still contain the δ_2 error and so the coil C_2 cannot but correct half the remaining error. Not until the B_3, B_4 pair measure the trajectory can it be fully nulled at the corresponding coil C_4 . The text takes $L_c = 2$ and the slopes are $0, \pm 1/4$ for all but $\Delta \theta_3$, and there the slopes are $\pm 1/2$.

The variables in common to θ_{1r} and θ_{1b} are then those of θ_{1r} with 6 exclusion and they are labeled θ_1 given by

$$\theta_1 = \frac{1}{4}[-\Delta x_4 - \Delta x_7 + \Delta \theta_4 + \delta_2 - \delta_4] \quad (22)$$

In summary, these definitions give

$$\theta_r(p) = \theta_1 + \theta_{rb}(p) + \theta_{rn}, \quad (23)$$

$$\theta_b(p) = \theta_1 + \theta_{rb}(p) + \theta_{bn}(p), \quad (24)$$

$$\theta_{rb}(p) = \Delta \theta_b(p) - (1/4)(\Delta x_b + 2\Delta \theta_b), \quad (25)$$

$$\theta_{rn} = -\frac{1}{4}(\Delta x_n + \Delta \theta_n), \quad (26)$$

$$\theta_{b5} = -\frac{1}{4}(\Delta x_5 - 2\Delta\theta_5), \quad (27)$$

and

$$\theta_{b6}(p) = \Delta\theta_6(p) - \frac{1}{4}(\Delta x_6 + \Delta\theta_6). \quad (28)$$

To obtain $\overline{\langle \theta^2 \rangle}$, several simplifications are made. First we note that the variables in θ_i are independent of those on sectors 5 and 6; therefore, it is useful to compute

$$\langle \theta_i^2 \rangle = \frac{1}{16}[\langle (\Delta\theta_4 - \Delta x_4)^2 \rangle + \langle (\Delta x_7)^2 \rangle + \langle \delta_2^2 \rangle + \langle \delta_4^2 \rangle]. \quad (29)$$

Using Eqs.(11)-(13) we readily find

$$\langle \theta_i^2 \rangle = \frac{1}{16} \left(\frac{2}{3}(\sigma_\theta^2)' + 2\sigma_x^2 \right). \quad (30)$$

We also require $\langle \theta_{c6}^2 \rangle$ and $\langle \theta_{b5}^2 \rangle$ that are computed in similar fashion giving

$$\langle \theta_{c6}^2 \rangle = \langle \theta_{b5}^2 \rangle = \frac{7}{48}(\sigma_\theta^2)'. \quad (31)$$

Using Eqs.(11)-(13) again, the variances of the $\theta_{c5}(p)$ and the $\theta_{b6}(p)$ terms are

$$\langle \theta_{c5}^2(p) \rangle = (\sigma_\theta^2)' \left(\frac{7}{48} + \frac{(1-p)^2}{4} \right), \quad (32)$$

and

$$\langle \theta_{b6}^2(p) \rangle = (\sigma_\theta^2)' \left(\frac{7}{48} + \frac{p^2}{4} \right), \quad (33)$$

giving

$$\overline{\langle \theta_{c5}^2(p) \rangle} = \overline{\langle \theta_{b6}^2(p) \rangle} = \frac{11}{48}(\sigma_\theta^2)'. \quad (34)$$

Adding together all the contributions over $L_c = 2$ and dividing by two to get the average over a unit length gives:

$$\overline{\langle \theta^2 \rangle} = \frac{5}{12}(\sigma_\theta^2)' + \frac{1}{8}\sigma_x^2. \quad (35)$$

By the dimensional argument that δx_j combines as $\delta x_j/L_c$ and $(\sigma_\theta^2)'$ combines as $(\sigma_\theta^2)'L_c$, the general expression is

$$\overline{\langle \theta^2 \rangle} = \frac{5}{24}(\sigma_\theta^2)'L_c + \frac{1}{2} \left(\frac{\sigma_x}{L_c} \right)^2. \quad (36)$$

VI. The SWOOP

For any local correction scheme, the error quantities $(\sigma_\theta^2)'$ and σ_x^2 will appear in $\overline{\langle \theta^2 \rangle}$. For those readers joining us at this point, the quantities $(\sigma_\theta^2)'$ and σ_x^2 are the spatial growth rate of the angular variance and the variance of the BPM diagnostic, respectively. The growth rate of the angular variance is given by Eq (7) and is proportional the σ_k^2 the square half period kick error as described further in Section II. They always combine to have the form

$$\overline{\langle \theta^2 \rangle} = 2 \left(\frac{\sigma_x}{Q_r L_c} \right)^2 + \frac{(\sigma_\theta^2)' L_c}{Q_k^2 6} \quad (37)$$

The values of Q_r and Q_k for the N_{13} wiggler are 2 and $\sqrt{4/5} = 0.89$, respectively, and both are unity for the T_1 scheme. An ensemble average of wigglers of length L , divided up into L/L_c equal length sectors and operating at optical wave number k then has a longitudinal coherence phase $\langle \phi_{lc} \rangle$ equal that of the

trajectory component of a single electron $\phi_{lc} = (k/2) \int_0^L \theta^2 ds$ [51,41]; invoking the periodic nature of the correction scheme and an ensemble average over wigglers,

$$\langle \phi_{lc} \rangle = \frac{L}{L_c} \left\langle \frac{k}{2} \int_0^{L_c} \langle \theta \rangle^2 ds \right\rangle = \frac{kL}{2} \overline{\langle \theta^2 \rangle}. \quad (38)$$

In minimizing $\overline{\langle \theta^2 \rangle}$, we reduce this phase.

The form of Eq.(37) suggests that there is an optimum distance, L_{sc} , that depends on the technology or achievable levels of $(\sigma_\theta^2)'$ and σ_x^2 . Minimizing this expression with respect to L_c gives $L_c = L_{sc}$ where

$$2 \left(\frac{\sigma_x}{Q_x L_{sc}} \right)^2 = \frac{1}{2} \frac{(\sigma_\theta^2)' L_{sc}}{Q_k^2 6}, \quad (39)$$

or in terms of the BPM angle and the stream angle

$$\frac{\sigma_x}{L_{sc}} = \frac{Q_x}{Q_k} \sqrt{\frac{(\sigma_\theta^2)' L_{sc}}{24}}. \quad (40)$$

Writing

$$C_{1S} = \left(\frac{Q_x}{Q_k} \right)^{2/3}, \quad (41)$$

gives the SWOOP value

$$L_{sc} = \left[\frac{24 \sigma_x^2}{(\sigma_\theta^2)'} \right]^{1/3} / C_{1S}, \quad (42)$$

and using Eqs.(37) and (42) gives

$$\overline{\langle \theta^2 \rangle}_S = 1/2 (3 \sigma_x^2 \sigma_\theta^4)^{1/3} / Q_{1S}, \quad (43)$$

where

$$Q_{1S} = Q_x^{2/3} Q_k^{4/3}, \quad (44)$$

it follows

$$\langle \phi_{lc} \rangle_S = \frac{kL}{Q_{1S}} \left[\frac{3(\sigma_\theta^2)'^2 \sigma_x^2}{64} \right]^{1/3}, \quad (45)$$

where Q_{1S} is the method-dependent quality factor measuring the excellence of the reduction of $\langle \phi_{lc} \rangle$ the N_{13} scheme $Q_{1S} = 1.37$, in comparison to $Q_{1S} = 1$ in the T_1 scheme. The cost factor C_{1S} measures the number density of BPMs per unit length. For T_1 , $C_{1S} = 1$ and for N_{13} , $C_{1S} = 1.7$. Thus, in principle, a N_{13} system has a 37% better $\langle \phi_{lc} \rangle$ than a T_1 one but requires 1.7 times the number of BPMs to achieve that level. Use of an N_{13} algorithm is reported in [40].

Under the conditions reported in [40], where $\gamma = 265$, $L = 10m$, $\lambda_u = 3.89cm$, $B_0 = 4.42kG$, the T_1 and N_{13} designs are compared on the basis of 16 BPM screens each. Here σ_x varies from 0 to 0.20mm, and $\sigma_k = 0.005$. We make a comparison at $\sigma_x = 0.15mm$. With N_{13} we have $L_{sc} = 84cm$ compared to 62.5cm actually used, and for T_1 we have $L_{sc} = 142cm$ versus the 62.5cm actually used. The latter number suggests that about half the number of BPMs in T_1 might be advisable. This conclusion is born out by the higher BPM contribution to the T_1 method $\langle \phi_{lc} \rangle$ shown in Table 1. The values of $\langle \phi_{lc} \rangle$ for $L_c = 135cm$ are shown in parentheses. The N_{13} wiggler receives a 10% benefit to $\langle \phi_{lc} \rangle$ by making L_c optimal, it is, however, already optimal for $\sigma_x = 0.10mm$.

$\langle \phi_{lc} \rangle$	N_{13}	T_1
BPM	1.43	5.70(1.43)
FIELD	1.21	0.94(1.88)
TOTAL	2.64	6.64(3.31)
efficiency	1.43 $\pm$ 0.17%	1.20 $\pm$ 0.22%

Table 1. A tabulation of ϕ_{lc} and efficiencies[40] for the T_1 and N_{13} wigglers with $\sigma_x = 0.15mm$ and 1/2% field errors.

The tabulated values are consistent with expectations in most cases. The BPM and field error components to $\langle \phi_{lc} \rangle$ are separated out and totaled for comparison using Eq (37). The efficiency with no BPM error is 1.67% for both N_{13} and T_1 , and this value is consistent with the $\langle \phi_{lc} \rangle$ for the field components of 1.21 and 0.94, only slightly worse for N_{13} . Also shown in the table are the efficiencies averaged from 8 runs. The lower efficiency of the T_1 wiggler is consistent with its higher total $\langle \phi_{lc} \rangle$. Retuning, gain of 1.85, emittance, energy spread, and optical mode characteristics may account for why the drop in efficiency is less than a third of the no BPM error case when $\langle \phi_{lc} \rangle > 2\pi$.

VII. Conclusions

Analytical methods give a simple means of evaluating the phase shift of the guide electron, expected on average from an ensemble of wigglers having the same correction scheme. The most difficult parts of the description are obtaining the stochastic angle equations and computing the resultant angular variance. These operations have been reduced to simple trajectory tracing and the application of cross correlation formulas. The angular variance for the correction scheme is expressed relative to the T_1 fiducial scheme as two quality factors from which the quality, improvement and cost indices are calculated. Application of this method to choose the spacing between BPMs and coils in the NISUS wiggler shows that it gives useful guidelines and insight.

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Appendix A: The Greatest Q_k and Q_r

First we prove that the greatest value of Q_k is unity. To do so we show that $\overline{\langle \theta^2 \rangle} = (\sigma_\theta^2)'S/6$ is the minimum angular variance that can occur over an interval of length S , as it does for the T_1 scheme. The general expression for the angle in such an interval for a TN scheme always contains

$$\theta = \Delta\theta(s) + c_1\Delta\theta + c_2\Delta x.$$

Without loss of generality, we can take $S = 1$ and $(\sigma_\theta^2)' = 1$. We obtain the variance by forming θ^2 and using Eqs. (11)-(13) to obtain the ensemble average. The ensemble average is then averaged over distance with s going from 0 to 1,

$$\overline{\langle \theta^2 \rangle} = 1/2 + c_1 + (2/3)c_2 + c_1^2 + c_1c_2 + c_2^2/3,$$

which can be rewritten as

$$\overline{\langle \theta^2 \rangle} = 1/6 + (1/12)(c_2 + 1)^2 + (c_1 + c_2/2 + 1/2)^2.$$

Clearly the minimum of $1/6$ is demonstrated.

Next we produce an example where $Q_r = \infty$. The ground rules laid out in the introduction are that the BPMs may be placed at lattice sites, and their errors absolute-position are independent. No specifications were given as to how close the BPMs may be placed to each other. Here we imagine a lattice with infinitely many lattice sites. The lattice sites are chosen within the usual T_1 lattice. The first lattice site is chosen to coincide with the first T_1 site, and there a coil is placed. The rest of the lattice sites are chosen infinitesimally close to the second T_1 lattice site. On each of these infinitely many sites, another BPM is placed. The infinite number of BPMs near the second T_1 site have readings that are averaged to produce the reading in place of the usual T_1 reading. This model gives a variance of zero in the BPM error and therefore an infinite Q_r quality factor, while maintaining the Q_k factor at unity.

Appendix B: The Bivariate Normal Distribution

Here we examine the joint distribution function of two differently weighted sums of the same independent variables. For simplicity, we label the independent random variables, r_k and these correspond to the angular kicks $\delta\theta_j^{(k)}$. The total angular deflection is $\Delta\theta_j = \sum_{k=1}^N r_k$, and the displacement is $\Delta x_j = \sum_{k=1}^N s(N-k)r_k$. This is of the form

$$U = \sum_{k=1}^N W_k r_k,$$

$$V = \sum_{k=1}^n \tilde{W}_k r_k,$$

where W_k and $\tilde{W}_k$ are linearly independent. In our case $\langle r_k \rangle = 0$, and $\text{Var}(U) = \text{Var}(r) \sum W_k^2$, and $\text{Var}(V) = \text{Var}(r) \sum \tilde{W}_k^2$. This relationship suggests the introduction of the normalized variables U' and V' that have unity variance

$$U' = U / \left[\sum_{k=1}^N W_k^2 \right]^{1/2} = \sum_{k=1}^N w_k r_k,$$

$$V' = V / \left[\sum_{k=1}^N \tilde{W}_k^2 \right]^{1/2} = \sum_{k=1}^N \tilde{w}_k r_k,$$

where $\sum_{k=1}^N w_k^2 = \sum_{k=1}^N \tilde{w}_k^2 = 1$. Using these definitions it follows that

$$w_j = \frac{1}{\left[\sum_{k=1}^N 1 \right]^{1/2}} = \frac{1}{\sqrt{N}}$$

$$\tilde{w}_j = \frac{N-j}{\left[\sum_{k=0}^N (N-k)^2 \right]^{1/2}} = \frac{N-j}{\sum_{k=0}^{N-1} (j+1)^2} = \frac{N-j}{[(N-1)N(2N-1)/6]^{1/2}}$$

Both u_j and $\tilde{u}_j$ are of order $1/\sqrt{N}$ for large N .

We then form the joint probability distribution

$$\text{Pr}(\tilde{U}, \tilde{V}) = \int \left[\prod_{j=1}^N \text{Pr}(r_j) \right] \delta \left(\tilde{U} - \sum_{k=1}^N u_k r_k \right) \delta \left(\tilde{V} - \sum_{k=1}^N \tilde{u}_k r_k \right) \prod_{j=1}^N dr_j.$$

Following a suggestion from Fraser[66], we fourier transform the distribution

$$F(s, t) = \int \exp(is\tilde{U} + it\tilde{V}) \text{Pr}(\tilde{U}, \tilde{V}) d\tilde{U} d\tilde{V}.$$

Substituting the definition of Pr, switching the order of integration, and evaluating the dirac delta functions gives

$$\begin{aligned} F(s, t) &= \prod_{j=1}^N \left[\int \text{Pr}(r_j) \exp[(is u_j + it \tilde{u}_j) r_j] dr_j \right], \\ &= \prod_{j=1}^N F_1(s u_j + t \tilde{u}_j), \end{aligned}$$

where F_1 is the fourier transform of $\text{Pr}(r)$, $F_1(\mathbf{k}) = \int_{-\infty}^{\infty} \exp(i\mathbf{r}\mathbf{k}) \text{Pr}(r) dr$.

The essence of the rest of the proof is that as $N \rightarrow \infty$, u_j and $\tilde{u}_j \rightarrow 0$. Thus, it is proper to expand F_1 in a taylor series. The linear term is zero as a requirement on our angle generation distribution. The quadratic term survives and the requirement of $\sum u_j^2 = \sum \tilde{u}_j^2 = 1$ guarantees it will survive in the limit $N \rightarrow \infty$ in F . The cubic terms involve $\sum \tilde{u}_j^3$, $\sum u_j^2 \tilde{u}_j$, $\sum u_j \tilde{u}_j^2$, $\sum \tilde{u}_j^3$ and these all vanish in the limit of $N \rightarrow \infty$ because their sums are $O(1/\sqrt{N})$. The first few terms in F_1 are

$$F_1(\mathbf{k}) = 1 - \frac{\mathbf{k}^2}{2!} (\sigma_\theta^2)' - i\mu_3 \frac{\mathbf{k}^3}{3!} + \dots$$

and this gives

$$F(s, t) = \prod_{j=1}^N \left[1 - \frac{(\sigma_\theta^2)'(s u_j + t \tilde{u}_j)^2}{2!} - i\mu_3 \frac{(s u_j + t \tilde{u}_j)^3}{3!} + \dots \right]$$

Because u_j and $\tilde{u}_j \rightarrow O\left(\frac{1}{\sqrt{N}}\right)$, we can neglect the μ_3 terms. Taking $\ln(F)$ we find

$$\ln F(s, t) = \sum_{j=1}^N \ln \left[1 - \frac{1}{2} (s u_j + t \tilde{u}_j)^2 \right] + O\left(\frac{1}{\sqrt{N}}\right),$$

again, because u_j and $\tilde{u}_j$ are small we may taylor expand $\ln$ and then invert to get

$$\begin{aligned} \ln F(s, t) &= \sum_{j=1}^N \left[-\frac{1}{2} (s u_j + t \tilde{u}_j)^2 + O\left(\frac{1}{\sqrt{N}}\right) \right], \\ &= -\left(\frac{s^2}{2} + \frac{t^2}{2}\right) - st \sum u_j \tilde{u}_j + O\left(\frac{1}{\sqrt{N}}\right), \\ F(s, t) &= \exp \left[-\left(\frac{s^2}{2} + \frac{t^2}{2}\right) - st \sum u_j \tilde{u}_j \right] + O\left(\frac{1}{\sqrt{N}}\right) = \exp \left[-\frac{1}{2} (s^2 + 2\rho st + t^2) \right] \end{aligned}$$

This we recognize as the fourier transform of the bivariate distribution[67]

$$\text{Pr}(U, V) = [2\pi\sqrt{1-\rho^2}]^{-1} \exp \left[-\frac{1}{2} \left(\frac{U^2 - 2\rho UV + V^2}{1-\rho^2} \right) \right]$$

Here the sum

$$\sum w_j \tilde{w}_j = \frac{\sum W_j \tilde{W}_j}{\left[\sum W_j^2 \sum \tilde{W}_j^2 \right]^{1/2}} \equiv \rho,$$

is the cross correlation coefficient of $\tilde{U}$ and $\tilde{V}$. Note that a trailing fraction c of the w_k or $\tilde{w}_k$ coefficients can be set to zero where $c = O(1)$ and the proof goes through unaltered. Thus, U and V may correspond to different spatial locations.

Appendix C: Sector Correlation Integrals

The manipulations are purely algebraic in nature and given narratively. They utilize the following notational convenience: $[exp]$ takes the value of unity or nil according to whether a logical expression exp is true or false respectively. For $a < b$ the following expression is useful; it is also true but trivial when $b < a$ or $b = a$,

$$[a < b] \int_a^b f(x) dx = \int_{-\infty}^{\infty} [a < x < b] f(x) dx. \quad (C1)$$

We also recite the orthogonality condition from Eq.(9),

$$\langle \dot{u}(x) \dot{u}(y) \rangle = \delta(x - y). \quad (C2)$$

The following rule applies when integrating a dirac delta function:

$$\int_{-\infty}^{\infty} \delta(y - x) f(x) dx = f(y). \quad (C3)$$

The manipulations also require

$$[a < x < b] = [a < x][x < b], \quad (C4)$$

$$[a < x][c < x] = [\max(a, c) < x], \quad (C5)$$

and

$$[x < b][x < d] = [x < \min(b, d)]. \quad (C6)$$

The form of the expressions we deal with are given on the left hand side of the following equation. The right hand side evolves by the seven typographical moves: (1) Use Eq (C1) to express the integrals with infinite limits. (2) Push $u(x)$ through the y integral and lodge it next to $\dot{u}(y)$. (3) Pull the left angular bracket (the bra of the bra-ket pair) next to the first u and translate the ket leftward to the right of the second u . (4) Substitute out the ensemble expectation with Eq (C2). (5) Remove the dirac delta function by Eq (C3), delete the dy and integration sign with its limits, and replace all remaining occurrences of y by x . (6) Change $[a < x < A][b < x < B]$ into $[\max(a, b) < x < \min(A, B)]$. The latter move is justified by using Eqs. (C4), (C5) and (C6). (7) Rewrite the definite integral by Eq (C1). Thus,

$$\left\langle \int_1^A f(x) u(x) dx \int_b^B g(y) u(y) dy \right\rangle = [\max(a, b) < \min(A, B)] \int_{\max(a, b)}^{\min(A, B)} f(x) g(x) dx \quad (7)$$

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