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**General Formulation of an
HCDA Bubble Rising in a Sodium
Pool and the Effect of
Nonequilibrium on Fuel Transport**

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TABLE OF CONTENTS

	Page
Abstract	i
List of Figures	ii
List of Tables	iii
1. Introduction	1
2. Analytical Model	4
3. Bubble Mixture Field Formulation	6
3.1 Mass Balance Equations	6
3.2 Interfacial Mass Transfer	9
3.3 Energy Balance Equations	11
3.4 Constitutive Equations	15
4. Liquid Sodium Field Formulation	18
4.1 Dynamic Field Formulation	18
4.2 Energetic Field Formulation	19
5. Cover Gas Field Formulation	21
6. Bubble Rise Velocity and Displacement	24
7. Solution Method	25
8. Results and Discussions	26
9. Appendices	
A. Derivation of the Extended Rayleigh Equation	34
B. Derivation of the Integral Energy Equation	38
10. Nomenclature	42
11. References	45
12. Distribution List	48

ABSTRACT

This report which improved the formulation of the previous reports is designed to investigate the effect of the interfacial nonequilibrium mass transfer and the radiative heat transfer on the amount of the fuel vapor condensed before the bubble reaches to the cover-gas region.

Consideration is given to a fuel dominated bubble which is assumed to have just penetrated into the sodium pool in a spherical form subsequent to an Hypothetical Core Disruptive Accident (HCDA). The two-phase bubble mixture as it rises through the sodium pool to the cover-gas region is formulated. The formulation takes into account the effects of the nonequilibrium mass transfer at the interfaces and of the radiative cooling of the bubble as well as the kinematic, dynamic and thermal effects of the surrounding fields.

The results of calculation for the amount of the fuel vapor condensed before the bubble reaches the cover-gas region are presented over a wide possible range of the evaporation coefficient as well as the liquid sodium-bubble interface absorbtivity. It is shown that the effects of nonequilibrium mass transfer become more meaningful at the later stage of the bubble rise where the temperature difference between the liquid fuel and the gaseous mixture has been increased. The thermal radiative cooling is found to be very effective in attenuating the fuel content of the bubble; depending on the value of the liquid sodium-bubble absorbtivity, a great reduction of fuel vapor is found to be possible. As a result, if the condensed fuel falls out of the bubble, the thermal radiation - which condenses out most of the fuel vapor - can effectively prevent and eliminate most of the fuel leaking out of the reactor vessel.

LIST OF FIGURES

1. Development of an HCDA Bubble
2. Buoying Bubble in Liquid Sodium
3. Energy Balance at the Bubble-Liquid Sodium Interface
4. Cover-Gas Leakage From Reactor Vessel
5. Fuel Vapor Condensation vs. Time (C_f is parameter)
6. Fuel Vapor Condensation vs. Time (α_j is parameter)
7. Bubble Expansion During Bubble Buoyancy
8. Heat Transfer on Sodium Side

LIST OF TABLES

1. Initial Bubble and Cover-Gas Conditions for the Example Calculations
2. Material Properties Used in the Calculations

1. INTRODUCTION

In problems associated with nuclear safety, it is of interest to know the amount of fuel that will be released to the environment in the event of mechanical disassembly of a core disruptive accident in liquid Metal Fast Breeder Reactors, (LMFBR). According to some Hypothetical Core Disruptive Accident (HCDA) concepts, Ref. [1], an HCDA bubble emerging from the high-pressure and high-temperature fuel-coolant interaction zone would rise in the form of a bubble through the sodium pool into the cover-gas region (see Fig. 1). Since this is the basic mechanism for radionuclide transfer through the sodium pool, evaluation of the bubble motion and cooldown processes is important in determining leakage of radionuclides to the environment.

During the bubble rise, the heat transfer from the hot two-phase mixture inside the bubble takes place both by convection and radiation to a variety of heat sinks, such as the surfaces of the structural material on the path of the bubble, and the liquid sodium surrounding the bubble. This thermal interaction of the bubble with its environment will result in fuel condensation on the fuel droplets inside the bubble and sodium evaporation and/or condensation through the bubble exterior surface. Furthermore, during this period, the bubble would expand or contract, and therefore, there would be a dynamic interaction between the bubble and the surrounding sodium as well as the cover gas. In order to predict what fraction of the fuel vapor inside the bubble would condense during the bubble rise time, it is necessary to have a realistic formulation that takes into account the simultaneous coupling effects of the kinematic, dynamic and thermal processes which characterize the

phenomenon.

Significant research has been done on the analysis of core disruptive accidents and fuel-coolant interaction, which gives results on the condition of the fuel immediately after an accident. Much work has also been done on the vapor and fuel transport, after the accident, to the cover-gas plenum. It should be noted that in all these analyses, Refs. [2-6], a complete formulation that takes into account the simultaneous effects of the kinematic, dynamic and thermal processes within a rising HCDA bubble as well as the effects of interactions between the bubble and its surrounding is still lacking. In particular, equilibrium thermodynamics has generally been assumed in the transport process. However, the basic theory of nonequilibrium evaporation and condensation used for the analysis of early expansion in a whole-core accident indicated significant differences over the use of equilibrium theory for the same set of conditions, Ref. [7].

It appears from the present state of the art that for an HCDA bubble rising through a sodium pool there is a need for a formulation which takes into account: 1) the thermal interaction within the rising bubble, 2) the kinematic, dynamic and thermal interactions of the buoying bubble with its surrounding field, 3) the dynamic interaction of the bubble with the cover-gas, 4) the radiative cooling process of the bubble, 5) the nonequilibrium mass transfer at the interfaces, and 6) the bubble rising velocity.

This report which improves the formulation of the previous report, Ref. [6], has two purposes:

1. to present a formulation which takes into account the various

effects enumerated above, and

2. to investigate the effect of interfacial nonequilibrium mass transfer on estimating the transport of fuel through the sodium pool to the cover-gas region.

Fig. 1 Development of an HCDA Bubble Mixture

2. ANALYTICAL MODEL

Following a hypothetical core disruptive accident, a high-pressure and high-temperature two-phase mixture is generated in the core, which may consist primarily of fuel vapor and liquid fuel droplets as well as other minor constituents, such as fission gases and steel vapor. The mixture will sail through the upper core structure, and emerge as a jet or bubble into the liquid sodium pool above the core (see Fig. 1). The bubble at this point provides the starting point for our analysis, and the two-phase bubble mixture as it rises through the sodium pool to the cover-gas region is the subject of the present study.

The main assumptions utilized in our formulation are summarized as follows:

1. Without loss of generality, we shall assume, as many others do, the two-phase mixture forms a spherical bubble shape as it buoys to the cover-gas region.
2. It is assumed that the bubble would grow or contract spherically in a pool of sodium for which the pressure far from the bubble is the variable cover-gas pressure.
3. The liquid fuel within the bubble is assumed to exist in spherical droplets of specified initial diameter.
4. As fuel vapor condenses on the droplet surfaces, the particles are allowed to increase in size; however, the number of particles remains constant throughout the calculation.
5. As indicated above, the heat removal from the hot bubble mixture takes place both by convection and radiation to a variety of heat sinks. However, the thermal radiation is assumed to be a significant

mode of heat transfer as evidenced by recent studies which examined the effect of thermal radiation in safety analyses, Refs. [8-12].

6. The nonequilibrium mass transfer occurs at the interface.

The analytical formulation based on the physical model described above is, for convenience, divided into the following four parts, each of which is described in turn:

- Bubble Mixture Field Formulation,
- Liquid Sodium Field Formulation,
- Cover-Gas Field Formulation,
- Bubble Rise Velocity and Displacement.

3. BUBBLE MIXTURE FIELD FORMULATION

In this section we illustrate the governing differential equations which designate the mass and energy balances within the rising HCDA bubble as well as the equations that characterize the interfacial mass and energy transport processes.

3.1 Mass Balance Equations

Let us consider an HCDA bubble resulting from a mechanical disassembly of a voided core that emerges from the sodium pool to the cover gas region (see Fig. 2). Since the two-phase mixture consists primarily of fuel vapor and liquid fuel as well as other minor constituents, such as steel vapor and fission gases, integrated mass balance equation is written for each constituent. They are as follows:

- Fuel Vapor Mass Balance

A mass balance over the fuel vapor within the HCDA bubble yields

$$\frac{dm_{fg}}{dt} = A_{if} \dot{m}_{fg} \quad (3.1)$$

where m_{fg} is the fuel vapor mass present in the bubble and $\dot{m}_{fg}$ is the interfacial net fuel evaporation rate per unit area while A_{if} is defined as the effective interfacial area for fuel droplets. The effective interfacial area for fuel droplets of radius R_d is given by

$$A_{if} = C_{Af} \frac{3 m_{f1}}{R_d \rho_{f1}}$$

where C_{Af} is defined as the interfacial area coefficient. In fact, the interfacial area coefficient is defined as the ratio of effective interfacial area to the geometric interfacial area.

-Sodium Vapor Mass Balance

A heat exchange between the bubble and the surrounding sodium occurs. A radiative flux is incident on the bubble interface. Part of the energy is conducted and convected into the sodium bulk while the rest is either reflected back or used to evaporate sodium into the bubble. Integrated sodium vapor mass balance within the bubble is given by

$$\frac{dm_{jg}}{dt} = A_{ij} \dot{m}_{jg} \quad (3.3)$$

where m_{jg} is the sodium vapor mass present in the bubble, $\dot{m}_{jg}$ is the interfacial net sodium evaporation rate per unit area, and A_{ij} is the interfacial area which is the bubble surface, i.e.,

$$A_{ij} = 4\pi R_b^2 \quad (3.4)$$

- Fission Gasses Mass Balance

It is expected that an HCDA bubble contains a quantity of noncondensable fission gases. The mass balance for the inert gas in the bubble yields

$$\frac{dm_{ig}}{dt} = 0 \quad (3.5)$$

where m_{ig} is the mass of the noncondensable fission gases present in the bubble.

- Steel Vapor Mass Balance

Steel vapor mass balance is given by

$$\frac{dm_{sg}}{dt} = A_{is} \dot{m}_{sg} \quad (3.6)$$

where m_{sg} is the steel vapor mass within the rising bubble, A_{is} is the interfacial area, and $\dot{m}_{sg}$ is the net evaporation rate of steel.

As shown in Ref. [6], steel is unlikely to evaporate for the condition of interest. Thus,

$$\dot{m}_{sg} = 0 \quad (3.7)$$

is assumed throughout this study.

- Liquid Fuel Mass Balance

Total liquid fuel mass balance present in the HCDA bubble is given by

$$\frac{dm_{f1}}{dt} = A_{if} \dot{m}_{f1} \quad (3.8)$$

where m_{f1} is the total mass of liquid fuel droplets, $\dot{m}_{f1}$ is the interfacial condensation rate per unit area. It should be noted that the interfacial mass balance requires that

$$\dot{m}_{fg} + \dot{m}_{f1} = 0 \quad (3.9)$$

- Fuel Droplet Mass Balance

As fuel condenses on the droplet surfaces, the particles are allowed to increase in size. Hence, mass balance for a single droplet should be considered by a separate equation. Assuming particles are spherical in shape, the differential equation for a fuel droplet with radius R_d can be expressed by

$$4\pi \rho_{f1} R_d^2 \frac{dR_d}{dt} = A_{id} \dot{m}_{f1} \quad (3.10)$$

where A_{id} is the effective single particle interfacial area which is defined by

$$A_{id} = C_{Af} 4\pi R_d^2 \quad (3.11)$$

where C_{Af} has its usual meaning as defined above.

3.2 Interfacial Mass Transfer

-For Fuel

The evaporation and condensation processes must be considered separately because the liquid is evaporating at the liquid fuel temperature T_{f1} while the condensation occurs at the fuel vapor temperature T_g . From kinetic theory, Ref. [13], one may write, for the surface evaporation rate per unit area

$$\dot{m}_{fe} = \frac{C_{fe}}{\sqrt{2\pi R_f}} \frac{P_{f1}^+}{\sqrt{T_{f1}}} \quad (3.12)$$

and for the condensation rate per unit area

$$\dot{m}_{fc} = \frac{C_{fc}}{\sqrt{2\pi R_f}} \frac{P_{fg}^*}{\sqrt{T_g}}$$

Here, P_{f1}^+ stands for the fuel saturation pressure for the liquid temperature T_{f1} , and P_{fg}^* is the fuel vapor partial pressure in the bubble. Finally, C_{fe} and C_{fc} are evaporation and condensation coefficients. In fact, these coefficients account for the variation in experimental data from purely theoretical predictions for which these coefficients are unity. The expression for $\dot{m}_{fc}$ with $C_{fc}=1$ describes the rate at which molecules strike the liquid surface. The coefficient then describes the fraction of these molecules which stay in the liquid.

The relationship for the net rate of fuel evaporation is the difference between two absolute rates with the sign taken as positive when net evaporation is occurring. Thus,

$$\dot{m}_{fg} = -\dot{m}_{f1} = \dot{m}_{fe} - \dot{m}_{fc}$$

where the interfacial mass balance expressed by Eq. (3.9) is used.

In view of Eqs. (3.2,12,13 and 14), the fuel vapor mass balance,

Eq. (3.1) can be expressed by

$$\frac{dm_{fg}}{dt} = C_{Af} \left(\frac{3m_{f1}}{R_d \rho_{f1}} \right) \frac{1}{\sqrt{2\pi R_f}} \left(C_{fe} \frac{P_{f1}^+}{\sqrt{T_{f1}}} - C_{fc} \frac{P_g^*}{\sqrt{T_g}} \right) . \quad (3.15)$$

Previous analyses have assumed that the evaporation and condensation coefficients, C_{fe} and C_{fc} , were equal. Without loss of generality, we shall assume that

$$C_{fe} = C_{fc} \quad (3.16)$$

With this assumption the fuel vapor mass balance, Eq. (3.15), becomes

$$\frac{dm_{fg}}{dt} = \left(\frac{3m_{f1}}{R_d \rho_{f1}} \right) \frac{C_f}{\sqrt{2\pi R_f}} \left(\frac{P_{f1}^+}{\sqrt{T_{f1}}} - \frac{P_g^*}{\sqrt{T_g}} \right) \quad (3.17)$$

where C_f is called as the effective condensation or evaporation coefficient and defined by

$$C_f = C_{Af} \cdot C_{fc} \quad (3.18)$$

Similarly, in view of Eqs. (3.2,9,12,13,14 and 18), the liquid fuel mass balance, Eq. (3.8) can be expressed by

$$\frac{dm_{f1}}{dt} = - \frac{dm_{fg}}{dt} = \left(\frac{3m_{f1}}{R_d \rho_{f1}} \right) \frac{C_f}{\sqrt{2\pi R_f}} \left(\frac{P_g^*}{\sqrt{T_g}} - \frac{P_{f1}^+}{\sqrt{T_{f1}}} \right) \quad (3.19)$$

Finally, substituting Eqs. (3.11) into Eq. (3.10) and making use of Eqs. (3.9,14 and 18), the fuel droplet mass balance may be expressed by the following equation:

$$\frac{dR_d}{dt} = \frac{1}{\rho_{f1}} \frac{C_f}{\sqrt{2\pi R_f}} \left(\frac{P_g^*}{\sqrt{T_g}} - \frac{P_{f1}^+}{\sqrt{T_{f1}}} \right) \quad (3.20)$$

-For Sodium

Nonequilibrium interfacial mass transfer relations between the sodium vapor and the liquid sodium can be developed similar to Eqs. (3.12-14).

Surface evaporation and condensation rates are given by

$$\dot{m}_{je} = \frac{C_{je}}{\sqrt{2\pi R_j}} \frac{P_{j1}^+}{\sqrt{T_{j1}}} \quad (3.21)$$

and

$$\dot{m}_{jc} = \frac{C_{jc}}{\sqrt{2\pi R_j}} \frac{P_g^*}{\sqrt{T_g}} \quad (3.22)$$

Definitions of various terms appearing in Eqs. (3.21 and 22) are similar to those appearing in Eqs. (3.12 and 13) with indices f and g denoting fuel and sodium, respectively.

Net sodium evaporation rate per unit area is the difference between the two absolute rates with sign taken positive when evaporation is occurring. Thus,

$$\dot{m}_{jg} = \dot{m}_{je} - \dot{m}_{jc} \quad (3.23)$$

In view of Eqs. (3.4 and 23), the sodium vapor mass balance, Eq. (3.3) can be expressed as

$$\frac{dm_{jg}}{dt} = 4\pi R_b^2 \frac{C_j}{\sqrt{2\pi R_j}} \left(\frac{P_{f1}^+}{\sqrt{T_{f1}}} - \frac{P_g^*}{\sqrt{T_g}} \right) \quad (3.24)$$

where $C_{fe} = C_{fc} \equiv C_j$ is used

3.3 Energy Balance Equations

Since thermal nonequilibrium exist between the liquid fuel droplets and the gaseous phase, two energy balance equations should be considered as one for each phase.

- Gaseous Mixture Energy Balance

Assuming the uniform properties throughout the HCDA bubble, the enthalpy energy balance on the gaseous mixture of the bubble yields

$$\begin{aligned} \frac{d}{dt} \left[\sum_k m_{kg} h_{kg}(T_g) \right] &= A_{if} [\dot{m}_{fe} h_{fg}(T_{f1}) - \dot{m}_{fc} h_{f1}(T_g)] + A_{ig} [\dot{m}_{je} h_{jg}(T_{j1}) - \\ &\dot{m}_{jc} h_{j1}(T_g)] - (1-\alpha_d) V_b \frac{dP_g}{dt} - A_{if} \dot{q}_{gj} - A_{is} \dot{q}_{gs} - \\ &\left(\frac{m_{fg} C_{pfg}}{m_{fg} C_{pfg} + m_{f1} C_{p1}} \right) A_{if} [\dot{m}_{fe} \Delta h_f(T_{f1}) - \dot{m}_{fc} \Delta h_f(T_g)] \end{aligned} \quad (3.25)$$

where the summation on k covers the bubble gaseous mixture constituents, i.e., fuel vapor, sodium vapor, fission gases and steel vapor. Expanding the left hand side of Eq. (3.25) with

$$\frac{dh}{dt} = C_p \frac{dT}{dt}, \quad (3.26)$$

and using the mass balance equations given by Eqs. (3.1, 3.5 and 7), the energy balance can be expressed by

$$\begin{aligned} \left(\sum_k m_{kg} C_{pkg} \right) \frac{dT_g}{dt} &= A_{if} \dot{m}_{fc} C_{pfg} (T_{f1} - T_g) + A_{ig} \dot{m}_{je} C_{pjg} (T_{j1} - T_g) - \\ &(1-\alpha_d) V_b \frac{dP_g}{dt} - A_{if} \dot{q}_{gj} - A_{is} \dot{q}_{gs} - \left(\frac{m_{fg} C_{pfg}}{m_{fg} C_{pfg} + m_{f1} C_{p1}} \right) \cdot \\ &A_{if} [\dot{m}_{fe} \Delta h_f(T_{f1}) - \dot{m}_{fc} \Delta h_f(T_g)] \end{aligned}$$

In the above equations, $h_{fg}(T)$ and $h_{f1}(T)$ are the specific enthalpies for the fuel vapor and liquid fuel at temperatures T , and

$h_{jg}(T)$ and $h_{jl}(T)$ are the specific enthalpies for the sodium vapor and liquid sodium, respectively. $\Delta h_f(T)$ is the latent heat of evaporation at temperature T , α_d is the droplet void fraction in the two-phase bubble mixture, and V_b is the bubble volume given by

$$V_b = \frac{4}{3} \pi R_b^3 \quad (3.28)$$

The net rate of radiative heat exchange between the bubble gaseous mixture and the bounding sodium, $\dot{q}_{gj}$ is given by

$$\dot{q}_{gj} = \left[\frac{\alpha_j \epsilon_m}{\alpha_j + (1 - \alpha_j) \epsilon_m} \right] \left(\frac{\kappa_g}{\kappa_g + \kappa_l} \right) \sigma T_g^4 \quad (3.29a)$$

and between the bubble gaseous phase and the steel structure is the bubble $\dot{q}_{ls}$ is given by

$$\dot{q}_{gs} = \left[\frac{\alpha_s \epsilon_m}{\alpha_s + (1 - \alpha_s) \epsilon_m} \right] \left(\frac{\kappa_l}{\kappa_g + \kappa_l} \right) \sigma T_g^4 \quad (3.29b)$$

where α is the surface absorptivity of sodium or steel, ϵ_m is the equivalent emissivity from the bubble mixture, κ is the extinction coefficient of the gaseous mixture or the liquid fuel, and σ is the Stefan-Boltzmann constant.

In the derivation of the energy balance equation, it is assumed that the heat of vaporization came out of and deposited into each phase in proportion to the relative total heat capacity of each phase. The heat drawn from each phase should be related to the thermal diffusivities of the two media. Uniform temperature distributions in the liquid and in the gaseous mixture which were assumed in the model do not permit the second case in mathematical sense. However, in either case, most of the

latent heat is still transferred to and from the liquid phase. The last term in Eq. (3.25) or (3.27) accounts for dividing the source or deposition of latent heat in that way.

-Liquid Fuel Energy Balance

Integrated form of the enthalpy energy equation over the liquid fuel present in the bubble can be expressed by

$$\begin{aligned} \frac{d}{dt} [m_{f1} h_{f1}(T_{f1})] = & - A_{if} \dot{m}_{fc} h_{f1}(T_{f1}) + A_{if} \dot{m}_{fc} h_{f1}(T_g) + \alpha_d V_b \frac{dP}{dt} - \\ & A_{if} \dot{q}_{lj} - A_{is} \dot{q}_{ls} - \left(\frac{m_{f1} C_{f1}}{m_{fg} C_{pfg} + m_{f1} C_{f1}} \right) A_{if} [\dot{m}_{fc} \Delta h_f(T_{f1}) - \dot{m}_{fc} \Delta h_f(T_g)] \end{aligned} \quad (3.30)$$

which can be further simplified by the substitution of the liquid fuel mass balance equation given by Eq. (3.8) into Eq. (3.30) and making use of Eq. (3.26) to obtain

$$\begin{aligned} m_{f1} C_{f1} \frac{dT_{f1}}{dt} = & A_{if} \dot{m}_f C_{f1} (T_g - T_{f1}) - \alpha_d V_b \frac{dP}{dt} - A_{ij} \dot{q}_{lj} - A_{is} \dot{q}_{ls} - \\ & \left(\frac{m_{f1} C_{f1}}{m_{fg} C_{pfg} + m_{f1} C_{f1}} \right) A_{if} [\dot{m}_{fc} \Delta h_f(T_{f1}) - \dot{m}_{fc} \Delta h_f(T_g)] \end{aligned} \quad (3.31)$$

where $\dot{q}_{lj}$ is the net radiative heat transfer rate per unit area between the liquid fuel droplets and the bounding sodium, and $\dot{q}_{ls}$ is the net radiative heat transfer rate per unit area between the liquid fuel droplets and the steel structure within the bubble. They are expressed by the following equations:

$$\dot{q}_{lj} = \left(\frac{\alpha_j \epsilon_m}{\alpha_j + (1 - \alpha_j) \epsilon_m} \right) \left(\frac{\kappa_l}{\kappa_g + \kappa_l} \right) \sigma T_{f1}^4 \quad (3.32a)$$

and

$$\dot{q}_{1s} = \left(\frac{\alpha_s \epsilon_m}{\alpha_s + (1-\alpha_s) \epsilon_m} \right) \left(\frac{\kappa_l}{\kappa_g + \kappa_l} \right) \sigma T_{fl}^4 \quad (3.32b)$$

3.4. Constitutive Equations

The bubble mixture formulation cannot be completed without the appropriate constitutive relations between the state variables. The equations of state and Clausius-Clapeyron equations are to be introduced for the additional relationships needed to complete the bubble mixture field formulation. They are as follows:

-Fuel Vapor Equation of State

The fuel vapor is assumed to behave like a perfect gas. That is,

$$(1-\alpha_d) V_b P_{fg}^* = m_{fg} \bar{R}_f T_g \quad (3.33)$$

which upon differentiation with respect to time, and after some rearrangement can be expressed by

$$\frac{1}{P_{fg}^*} \frac{dP_{fg}^*}{dt} = \frac{1}{T_g} \frac{dT_g}{dt} + \frac{1}{m_{fg}} \frac{dm_{fg}}{dt} + \frac{1}{1-\alpha_d} \frac{d\alpha_d}{dt} - \frac{1}{V_b} \frac{dV_b}{dt} \quad (3.34)$$

-Sodium Vapor Equation of State

The sodium vapor is again assumed to follow the perfect gas law.

That is

$$\frac{1}{P_{jg}^*} \frac{dP_{jg}^*}{dt} = \frac{1}{T_g} \frac{dT_g}{dt} + \frac{1}{m_{jg}} \frac{dm_{jg}}{dt} + \frac{1}{1-\alpha_d} \frac{d\alpha_d}{dt} - \frac{1}{V_b} \frac{dV_b}{dt} \quad (3.35)$$

-Mixture Equation of State

Since the gaseous mixture consists of the ideal gases, then the mixture itself obeys the ideal gas law. That is,

$$(1-\alpha_d) V_g P_g = \left(\sum_k m_{kg} \bar{R}_k \right) T_g \quad (3.36)$$

where the summation on k is taken over the gaseous mixture constituents, i.e., fuel vapor, sodium vapor, fission gases and steel vapor. Differentiation of Eq. (3.36) with respect to time yields,

$$\frac{1}{P_g} \frac{dP_g}{dt} = \frac{1}{T_g} \frac{dT_g}{dt} + \frac{1}{(\sum_k m_k \bar{R}_k)} \left(\sum_k \bar{R}_k \frac{dm_k}{dt} \right) + \frac{1}{1-\alpha_d} \frac{d\alpha_d}{dt} - \frac{1}{V_b} \frac{dV_b}{dt} \quad (3.37)$$

-Fuel Vapor Saturation Pressure

The Clausius-Clapeyron equation,

$$P_{f1}^+ = 0.1 \exp \left(69.979 - \frac{76800}{T_{f1}} - 4.34 \ln T_{f1} \right) \quad (3.38)$$

is an explicit relationship between the saturation pressure and the temperature for the fuel, Ref. [6]. In this equation P_{f1} is measured in Pa and T_{f1} in K. From Eq. (3.38) by differentiation with respect to time, we have

$$\frac{1}{P_{f1}^+} \frac{dP_{f1}^+}{dt} = \left(\frac{76800}{T_{f1}^2} - 4.34 \right) \frac{1}{T_{f1}} \frac{dT_{f1}}{dt} \quad (3.39)$$

-Sodium Saturation Pressure

A correlation between the sodium saturation pressure and the temperature based on Ref. [14] is found as

$$\frac{1}{P_{j1}^+} \frac{dP_{j1}^+}{dt} = \frac{5180}{T_{j1}^2} \frac{dT_{j1}}{dt} \quad (3.40)$$

where P_{j1}^+ is in Pa and T_{j1} in K.

Fig. 2 Buoying Bubble in Liquid Sodium

4. LIQUID SODIUM FIELD FORMULATION

As the bubble moves upward the cover-gas region, it expands or contracts depending on the complicated phase change at the boundaries and on the cover-gas pressure. The kinematic dynamic as well as the energetic field of the liquid sodium surrounding the bubble will be affected by the bubble motion and by the interfacial mass, momentum and energy exchange processes. Here we shall develop the dynamic and energetic field equations coupled with the interfacial conditions at the bubble surface.

4.1 Dynamic Field Formulation

During its buoyancy, it is assumed that the HCDA bubble would grow or contract spherically in the sodium for which the pressure far from the bubble surface is the variable cover-gas pressure. In addition to the bubble motion, condensation or evaporation of sodium may occur at the bubble exterior surface, therefore; the existing Rayleigh equation to describe the bubble dynamics in an infinite medium is not applicable here. The Rayleigh equation is extended (see Appendix A) to account for the bubble motion and for the possible interfacial mass transfer as

$$\begin{aligned}
 & \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{j1}} \right) \left[2R_b^2 \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{j1}} \right) \left(1 - \frac{R_b}{R_e} \right) \frac{dR_b}{dt} + R_b^3 \left(\frac{d^2 R_b}{dt^2} - \frac{1}{\rho_{j1}} \frac{d\dot{m}_{jg}}{dt} \right) \right. \\
 & \left. \left(1 - \frac{R_b}{R_e} \right) - \frac{1}{2} R_b^2 \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{j1}} \right) \left(\frac{dR_b}{dt} \right) \right] + \frac{1}{2} \left(1 - \frac{R_b}{R_e} \right) \left(v_b \frac{dR_b}{dt} + \frac{1}{3} R_b \frac{dv_b}{dt} \right) \cdot \\
 & R_b^2 v_b - \frac{1}{4} R_b^2 v_b^2 \frac{dR_b}{dt} = \frac{1}{2} \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{j1}} \right) \frac{R_b^2 \dot{m}_{jg}}{\rho_{j1}} - \frac{1}{4} \frac{\dot{m}_{jg} R_b^2 v_b^2}{\rho_{j1}} + \\
 & \cdot \frac{R_b^2}{\rho_{j1}} \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{j1}} \right) (P_g - P_c)
 \end{aligned} \tag{4.1}$$

where R_b is the radius of the HCDA bubble, R_e is the equivalent radius of the reactor container, P_c is the pressure of cover-gas region, P_g is the pressure inside the bubble, ρ_{j1} is the density of the surrounding liquid sodium, and v_b is the instantaneous bubble rising velocity. Note that Eq. (4.1) reduces to the classical Rayleigh equation as

$$\begin{aligned} R_e &\rightarrow \infty \\ v_b &= 0 \\ \dot{m}_{jg} &= 0, \end{aligned}$$

and furthermore reduces to the Taylor's equation, Ref. [15], as

$$\begin{aligned} R_e &\rightarrow \infty \\ \dot{m}_{jg} &= 0. \end{aligned}$$

With the initial conditions:

$$\begin{aligned} R_b(0) &= 0 \\ \left. \frac{dR_b}{dt} \right|_{t=0} &= 0 \end{aligned}$$

Eq. (4.1) governs the liquid sodium dynamic field.

4.2 Energetic Field Formulation

Assuming a finite thermal boundary layer thickness in the liquid sodium, δ , integral energy balance method is used to obtain the energetic field equation in Appendix B. That is,

$$\begin{aligned} \delta(\delta + 10 R_b) (T_{j1} - T_{j\infty}) \frac{dR_b}{dt} + R_b(5 R_b + 2\delta)(T_{j1} - T_{j\infty}) \frac{d\delta}{dt} + R_b\delta(5 R_b + \delta) \frac{dT_{j1}}{dt} = \\ \frac{20 R_b^2}{\rho_{j1} C_{j1}} [\dot{q}_{ij} - \dot{m}_{je} C_{j1} (T_{j1} - T_{j\infty}) + \dot{m}_{jc} C_{j1} (T_g - T_{j\infty})] \end{aligned} \quad (4.3)$$

where T_{j1} is the liquid sodium temperature at the bubble surface, $T_{j\infty}$ is the temperature of the liquid sodium at a great distance from the interface, C_{j1} is the specific heat of the liquid sodium, and $\dot{q}_{ij}$ is the heat flux at the bubble surface which is given by Eq. (B.11).

$$\dot{q}_{ij} = \left(3 + \frac{\delta}{R_b}\right) \left(\frac{k_{j1}}{\delta}\right) (T_{j1} - T_{j\infty}) \quad (4.4)$$

where k_{jl} is the thermal conductivity of the liquid sodium.

An energy balance at the bubble-liquid sodium is needed in order to complete the liquid sodium energetic field formulation. Consistent with the thermal nonequilibrium phase change, an energy exchange model is shown in Fig. 3. From this figure, the boundary condition involving the radiative heat exchange and the net mass transfer may be written as

$$\dot{q}_{gj} + \dot{q}_{lj} + \dot{m}_{jc} h_{jg}(T_g) + \dot{m}_{je} h_{jl}(T_{jl}) = \dot{q}_{ij} + \dot{m}_{je} h_{jg}(T_{jl}) + \dot{m}_{je} h_{jl}(T_g) \quad (4.5)$$

where $\dot{q}_{ij}$ is the conductive heat flux in the liquid sodium. After rearranging, Eq. (4.5) can be rewritten as

$$\dot{q}_{gj} + \dot{q}_{lj} - \dot{q}_{ij} = \dot{m}_{je} \Delta h_j(T_{jl}) - \dot{m}_{je} \Delta h_j(T_g) \quad (4.6)$$

where the latent heat of evaporation $\Delta h_j = h_{jg} - h_{jl}$ is used.

Eq. (4.1) together with Eqs. (4.3), (4.4) and (4.6) complete the liquid sodium field formulation, and should be coupled with the equations derived in Sections 3 and 5.

Fig. 3 Energy Balance at the Bubble-Liquid Sodium Interface

5. COVER-GAS FIELD FORMULATION

Since the sodium plug above the core would have hit the cover during the early bubble expansion process in the core, flow passages through the cover may be available during this time. As a result, some cover-gas will leak through these flow passages while the bubble is rising. As the pressure in the cover-gas plenum is decreased, the bubble would tend to expand. These phenomena would change the two-phase thermodynamic conditions in the bubble interior. Changes in the bubble in turn affect the cover-gas conditions and the cover-gas leakage rate.

This phenomenon is taken into consideration with the assumption of isentropic flow of a perfect gas through ideal channels during the bubble rise. The flow passage is assumed to be converging-diverging nozzle, with the further idealization that the cross-sectional area at the throat approaches that of the ruptured hold through the cover. Such an idealized nozzle is illustrated schematically in Fig. 4. With these assumptions, standard gas dynamic equations Ref. [16] can be used to formulate the cover-gas field.

The isentropic flow through the nozzle, the Mach number is unity at the throat when the ratio of the downstream pressure, P_e , to the upstream pressure, P_c , is less than the critical pressure ratio,

$$(P_e/P_c)_{cr}, \text{ where } \left(\frac{P_e}{P_c}\right)_{cr} = \left(\frac{2}{\gamma_c + 1}\right)^{\frac{\gamma_c}{\gamma_c - 1}} \quad (5.1)$$

where γ_c is the isentropic exponent for the cover-gas, which is the ratio of specific heats at constant pressure and volume.

The downstream pressure is the pressure in the secondary containment, which is 1 atm, and the upstream pressure is the high

pressure in the cover gas region. In our analysis,

$$\frac{P_e}{P_c} < \left(\frac{P_e}{P_c}\right)_{cr} = \left(\frac{2}{\gamma_c + 1}\right)^{\frac{\gamma_c}{\gamma_c - 1}} \quad (5.2)$$

so that choked flow is assumed.

The rate of change of mass in the cover-gas region due to the leakage through the nozzle is

$$\frac{dm_c}{dt} = -A_e v_e \rho_e \quad (5.3)$$

where the density, ρ_e , the speed of sound, v_e , and the area, A_e are calculated at the throat. The speed of sound and the density can be related to the upstream conditions by

$$v_e = \left(\frac{2\gamma_c}{\gamma_c + 1} \bar{R}_c T_c\right)^{1/2} \quad (5.4)$$

and

$$\rho_e = \left(\frac{2}{\gamma_c + 1}\right)^{\frac{1}{\gamma_c - 1}} \rho_c \quad (5.5)$$

Substituting Eqs. (5.4 and 5) into Eq. (5.3), we obtain

$$\frac{dm_c}{dt} = -A_e \left(\frac{2}{\gamma_c + 1}\right)^{\frac{1}{\gamma_c - 1}} \left(\frac{2\gamma_c}{\gamma_c + 1}\right)^{1/2} (\bar{R}_c T_c)^{1/2} \rho_c \quad (5.6)$$

and furthermore, making use of the definition of the density

$$\rho_c \equiv \frac{m_c}{V_c} \quad (5.7)$$

in Eq. (5.6), the rate of change of mass in the cover-gas plenum can be expressed by

$$\frac{1}{m_c} \frac{dm_c}{dt} = -A_e \left(\frac{2}{\gamma_c + 1}\right)^{\frac{1}{\gamma_c - 1}} \left(\frac{2}{\gamma_c + 1}\right)^{1/2} \frac{(\bar{R}_c T_c)^{1/2}}{V_c} \quad (5.8)$$

where V_c is the volume of the cover-gas region. Since the liquid sodium is assumed to be incompressible, then the cover-gas volume plus the bubble volume stays constant during the rise period. That is,

$$V_c + (1 - \alpha_d) \frac{4}{3} \pi R_b^3 = \text{Constant} \quad (5.9)$$

or in the differential form

$$\frac{dV_c}{dt} = -4 \pi (1-\alpha_d) R_b^2 \frac{dR_b}{dt} . \quad (5.10)$$

The cover-gas pressure and the temperature can be calculated by making use of the perfect-gas equation

$$P_c V_c = m_c \bar{R}_c T_c \quad (5.11)$$

and of the isentropic process relation

$$\frac{P_c}{T_c^{\frac{\gamma_c}{\gamma_c-1}}} = \text{Constant} . \quad (5.12)$$

From Eqs. (5.11) and (5.12), it can be shown that

$$\frac{1}{P_c} \frac{dP_c}{dt} = \gamma_c \left(\frac{1}{m_c} \frac{dm_c}{dt} - \frac{1}{V_c} \frac{dV_c}{dt} \right) , \quad (5.13)$$

and

$$\frac{1}{T_c} \frac{dT_c}{dt} = \left(\frac{\gamma_c-1}{\gamma_c} \right) \left(\frac{1}{P_c} \frac{dP_c}{dt} \right) . \quad (5.14)$$

Equations (5.8, 10, 13 and 14) constitute the governing equations for the cover-gas field during the bubble rise period.

Fig. 4 Cover-Gas Leakage From Reactor Vessel

6. BUBBLE RISE VELOCITY AND DISPLACEMENT

The velocity, v_b , at which an HCDA bubble rises through the sodium pool is evaluated from a force balance on the bubble in Ref. [17] (as derived by Herring). The expression for v_b for a bubble with variable radius is given by

$$v_b = \frac{2g}{R_b^3} \int_0^t R^3(\xi) d\xi \quad (6.1)$$

where g is the gravitational acceleration and ξ is a dummy variable. Eq. (6.1) accurately describes large bubble rise velocity in underwater explosions, Ref. [17]. When differentiated with respect to time, it forms the additional relationship needed in Eq. (4.1).

$$\frac{dv_b}{dt} = -\frac{3v_b}{R_b} \frac{dR_b}{dt} + 2g \quad (6.2)$$

The bubble displacement y_b , which is the distance that the center of the bubble has risen above its original position, can then be simply expressed by

$$\frac{dy_b}{dt} = v_b \quad (6.3)$$

The distance traveled by the bubble from its original position, which we assumed is the center of the core, determines the length of time scale because as soon as the bubble reaches the sodium free surface the calculations will be terminated (see Fig. 1). That is when

$$y_b = h - R_b \quad (6.4)$$

the bubble rise period will be terminated, and the contents of the bubble will mix with the cover-gas. Mixing and consecutive events are out of the scope of this report.

7. SOLUTION METHOD

In general, the set, Eqs. (3.17,19,20,24,27,31,34,35,37,39,40), Eqs. (4.1,3,6), Eqs. (5.8,10,13,14) and Eqs. (6.2,3) of twenty non-homogeneous non-linear, ordinary differential equations constitutes a complete system in the twenty time-dependent unknowns, m_{fg} , m_{jg} , m_{fl} , m_c , R_b , R_d , T_g , T_{fl} , T_{jl} , T_c , P_g , P_c , P_{fg}^* , P_{fl}^+ , P_{jg}^* , P_{jl}^+ , δ , V_c , V_b and Y_b .

Once their initial values are given, the evaluation of the system is uniquely determined.

The integration of the system was performed numerically by using a fourth-order Runge-Kutta procedure. Since Eq. (4.3) presents an infinite derivative at the initial point, Eq. (4.3), in the first few steps, were substituted with the closed form obtained with some simplifying assumptions. The choice of integration time step required special care. At the beginning, when the departure from equilibrium is very small, round-off errors could cause the reversing of the driving force.

8. RESULTS AND DISCUSSIONS

As an HCDA bubble emerges into the sodium pool above the core, it expands until slug impact occurs. This two-phase bubble, which contains mostly liquid and vaporous fuel with some minor fission gases and steel vapor, will rise to the cover-gas region. We are particularly interested in investigating the effect of interfacial non-equilibrium mass transfer and the radiative heat transfer on the amount of the fuel vapor condensed before the bubble reaches the cover-gas region.

For demonstration purposes, a calculation was performed on an HCDA bubble having the following conditions at the moment of slug impact: 4640 K at 21.4 atm (19.26 atm of fuel and 2.14 atm of noncondensable fission gases) total pressure, liquid fuel mass = 2445 kg, fuel vapor mass = 202 kg, and the bubble surface area = 29.4 m^2 .

In view of the bubble initial condition given above, initial conditions on the time-dependent variables, m_{fg} , m_{jg} , m_{fl} , m_c , R_b , R_d , T_g , T_{fp} , T_{jp} , T_c , P_g , P_c , P_{fg}^* , P_{fl}^* , P_{jg}^* , P_{jc}^+ , δ , V_c , v_b and y_b , are listed in Table I. Additional constants and properties used for the example calculations in this report are presented in Table 2.

As shown in Table 1, it is necessary to define, a priori, the characteristic size of the liquid fuel droplets. The droplet radius enters the calculation of the interfacial area for vaporization and condensation. However, the determination of an appropriate liquid droplet size to characterize initial dispersion of the liquid fuel requires judgement. As discussed in detail in Ref. [18], several different criteria can be considered for defining the appropriate liquid droplet size. Initially, the fluid dynamics is dominated by the flashing of the liquid fuel to form a two-phase expansion zone. Therefore,

a flashing criterion would be appropriate during this early phase of the transient to define the fuel droplet size. However, during the early phase of the transient dynamic stirring occurs in the active core region that may define liquid droplet sizes based on a critical Weber number considerations.

A flashing criterion based on that in Ref. [19] was employed in Ref. [18] to assess the droplet size range for flashing liquid fuel under the conditions expected in the core, and a droplet radius of 2×10^{-4} m was found. However, if the critical Weber number criterion is used in the slip flow situation expected in this problem, a particle radius in the range of 1×10^{-3} m is obtained. This is based on a relative velocity of 50 m/s. From these criteria for droplet size, it appears that droplet diameters in the range of 2×10^{-4} m to 2×10^{-3} m are reasonable. In our calculations, an average initial droplet radius of 0.5×10^{-3} m is assumed.

It appears from the literature that considerable disagreement exists in the literature regarding the actual numerical values that should be used for the fuel vapor condensation coefficient and for the absorptivity of the bubble-sodium interface. As pointed out by Ref. [5], a wide range of absorptivity values reported in the literature results from possible contaminations of the interface by steel or fuel condensate.

On the other hand, as emphasized in Ref. [13], numerical values of the evaporation and condensation coefficients are very sensitive to the surface conditions of the material being studied and to the experimental methods being used to determine the interface temperature of the liquid. The work of Ref. [20] also indicates that small amounts of impurities in the surface give results significantly smaller

than unity.

Because of the uncertainties in these data, the evaporation coefficient as well as the sodium-bubble interface absorptivity were considered as parameters in the present study, and the values were varied over several orders of magnitude to cover the range of measurements indicated.

Results of the example calculation are presented in Figs. 5 and 6, where the amount of fuel vapor condensation were plotted as a function of time with the condensation coefficient and the sodium-bubble interface absorptivity as parameters.

Based on these figures, a number of conclusions can be made:

1. The effects of nonequilibrium mass transfer become significant when the condensation coefficient is less than 0.1. Though 0.1 may look small, it is expected to be realistic in the present application. There are several reasons for low values of the condensation coefficient. As reported in the literature, Ref. [21], the condensation coefficient decreases drastically as the system pressure increases, which is the case here. Another reason is the effective condensation coefficient defined by Eq. (3.18) is a product of the true condensation coefficient and an effective area ratio. Since the effective area ratio is less than unity, the effective condensation coefficient is less than the true condensation coefficient.
2. From the figures we see that the effects of nonequilibrium mass transfer become more meaningful at the later stage of the bubble rise where the temperature difference between the liquid fuel and the gaseous mixture has been increased.

3. Thermal radiative cooling is found to be very effective in attenuating the fuel vapor content of the bubble; depending on the value of the liquid sodium-bubble interface absorptivity, a great reduction of fuel vapor is found to be possible.
4. If the condensed fuel falls out of the bubble, we can further conclude that the thermal radiation, which condenses out most of the fuel vapor, can effectively prevent and eliminate most of the fuel from leaking out of the reactor vessel.
5. Though the calculation does not include the effect of possible wide range of distribution of the particle sizes as indicated in Refs. [22 - 25] and the effect of particle motion within the bubble, it clearly demonstrates the importance of nonequilibrium interfacial mass transfer and of radiative cooling in assessing radiological consequences from LMFBR accidents and indicates the problem deserves further research in these prominent areas.

Currently, a research is being conducted to improve the formulation which will be capable of the fuel droplet fall-out from the bubble during its buoyancy.

Initial Bubble and Cover-Gas Conditions
For The Example Calculations

Time Dependent Variables	Symbols	Initial Values
Fuel Vapor Mass (kg)	m_{fg}	202.0
Liquid Fuel Mass (kg)	m_{fl}	2445.0
Sodium Vapor Mass (kg)	m_{fj}	0.0
Steel Vapor Mass (kg)	m_{sg}	0.0 (constant)
Fission Gas Mass (kg)	m_{ig}	11.0 (constant)
Gas Mixt. Temperature (K)	T_g	4640.0
Liquid Fuel Temperature (K)	T_{fl}	4640.0
Sodium Surface Temperature (K)	T_{jl}	773.0
Cover Gas Temperature (K)	T_c	2613.0
Bubble Pressure (Pa)	P_g	2.17×10^6
Fuel Sat. Pressure (Pa)	P_{fg}^*	1.95×10^6
Fuel Partial Pressure (Pa)	P_{fl}^+	1.95×10^6
Sodium Sat. Pressure (Pa)	P_{jg}^*	6.19×10^2
Sodium Partial Pressure (Pa)	P_{jl}^+	0.0
Cover Gas Pressure (Pa)	P_c	2.17×10^6
Bubble Radius (m)	R_b	1.77
Fuel Particle Radius (m)	R_d	0.5×10^{-3}
Thermal Bound. Layer Thickness (m)	δ	0.0
Cover Gas Volume (m ³)	V_c	2.8
Bubble Velocity (m/s)	v_b	0.0
Bubble Displacement (m)	y_b	0.0

TABLE II

Material Properties Used in the Calculations

Property	Fuel	Sodium	Argon	Xenon	Steel
Liquid Mass Density (kg/m ³)	8500.0	830.0	---	---	6500.0
Vapor Const. Pressure Specific Heat (J/KgK)	510.0	2598.0	520.4	158.3	620.0
Vapor Const. Volume Specific Heat (J/KgK)	490.0	1830.0	312.0	95.0	492.0
Ratio of Specific Heats (dimensionless)	1.04	1.42	1.67	1.67	1.26
Liquid Specific Heat (J/KgK)	504.0	1280.0	---	---	750.0
Latent Heat of Evaporation (J/Kg)	1.93×10^6	4.82×10^6	---	---	8.88×10^6
Liquid Thermal Conductivity (J/msK)	2.5	67.0	---	---	32.8

Figure 5. Fuel Vapor Condensation vs. Time (C_f is parameter)

Figure 6. Fuel Vapor Condensation vs. Time (α_j is parameter)

APPENDIX A

DERIVATION OF THE EXTENDED RAYLEIGH EQUATION, Eq. (4.1)

As the HCDA bubble expands or contracts in the sodium pool during its buoyancy, it interacts with a finite amount of liquid sodium as indicated in Fig. 7. For simplicity, we approximate the liquid sodium by a spherical shell shown in Fig. 7. The continuity equation governing the kinematic field of the liquid sodium which is given by

$$\frac{\partial \rho_{jl}}{\partial t} + \nabla \cdot \rho_{jl} \vec{v}_{jl} = 0 \quad (A.1)$$

will be solved together with the interfacial condition in order to obtain the velocity field in the liquid sodium. The interfacial condition expressing the interfacial mass transfer as given by Ref. [26] is

$$\dot{m}_{jg} = \rho_{jl} (\vec{v}_{jli} - \vec{v}_i) \cdot \hat{n}_j \quad (A.2)$$

In these equations, $\hat{n}_j$ indicates the unit normal vector at the bubble surface directed outwardly from the liquid sodium, $\vec{v}_{jl}$ the velocity vector in the sodium pool, $\vec{v}_{jli}$ the value of $\vec{v}_{jl}$ at the interface, and $\vec{v}_i$ the interfacial velocity of which radial component is given by

$$\vec{v}_i \cdot \hat{n} = \frac{dR_b}{dt} \quad (A.3)$$

In view of Eq. (A.3), Eq. (A.2) may be solved to obtain the radial component of $\vec{v}_{jli}$. Thus,

$$v_{jli})_r = -\vec{v}_{jli} \cdot \hat{n}_j = \frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{jl}} \quad (A.4)$$

Integration of Eq. (A.1) from R_b to $r > R_b$ and substitution of Eq.

(A.4) lead to

$$v_{jl})_r = \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{jl}} \right) \left(\frac{R_b}{r} \right)^2 \quad (A.5)$$

which is the radial velocity component in the sodium pool.

Now superimposing the velocity field expressed by Eq. (A.5) over the velocity field obtained for the inviscid flow over a sphere, we obtain the velocity components developed in a compressible inviscid liquid flowing around an expanding and evaporating bubble. Thus, the radial and tangential velocity components in the sodium pool can be expressed

by

$$v_{jl})_r = \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jq}}{\rho_{jl}} \right) \left(\frac{R_b}{r} \right)^2 + \left(\frac{R_b}{r} \right)^3 v_b \cos \theta \quad (A.6)$$

and

$$v_{jl})_\theta = \frac{1}{2} \left(\frac{R_b}{r} \right)^3 v_b \sin \theta \quad (A.7)$$

where the radial distance r , and the angle θ is measured from the center of bubble (see Fig. 5).

It is interesting to note that the velocity components given by Eqs. (A.6) and (A.7) take into account the bubble motion, bubble expansion and as well as the possible interfacial mass transfer at the bubble surface.

Now attention is directed toward the dynamic field of the sodium pool. As a governing equation for the dynamic field either the momentum equation or the mechanical energy equation can be used because they are not independent from each other. Here we shall use the mechanical energy equation which for an inviscid fluid can be expressed by

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho_{jl} v_{jl}^2 \right) + \nabla \cdot \left(\frac{1}{2} \rho_{jl} v_{jl}^2 \vec{v}_{jl} \right) = \nabla \cdot (p_{jl} \vec{v}_{jl}) \quad (A.8)$$

By inserting the velocity field given by Eqs. (A.6) and (A.7) into Eq. (A.8) we obtain upon integration from the bubble surface, $r=R_b$, to the equivalent radius of the container, $r=R_e$, and after some algebraic manipulations the following expression.

$$\begin{aligned}
& \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{jl}} \right) \left[2R_b^2 \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{jl}} \right) \left(1 - \frac{R_b}{R_e} \right) \frac{dR_b}{dt} + R^3 \left(\frac{d^2 R_b}{dt^2} - \frac{1}{\rho_{jl}} \frac{d\dot{m}_{jg}}{dt} \right) \right. \\
& \left. \left(1 - \frac{R_b}{R_e} \right) - \frac{1}{2} R_b^2 \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{jl}} \right) \left(\frac{dR_b}{dt} \right) \right] + \frac{1}{2} \left(1 - \frac{R_b}{R_e} \right) \cdot \\
& \left(v_b \frac{dR_b}{dt} + \frac{1}{3} R_b \frac{dv_b}{dt} \right) R_b^2 v_b - \frac{1}{4} R_b^2 v_b^2 \frac{dR_b}{dt} = \frac{1}{2} \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{jl}} \right) \cdot \\
& \frac{R_b^2 \dot{m}_{jg}}{P_{jl}} - \frac{1}{4} \frac{\dot{m}_{jg} R_b^2 v_b^2}{\rho_{jl}} + \frac{R_b^2}{\rho_{jl}} \left(\frac{dR_b}{dt} - \frac{\dot{m}_{jg}}{\rho_{jl}} \right) (P_g - P_c) \tag{A.9}
\end{aligned}$$

where P_g is the total pressure in the bubble, and P_c is the pressure at $R=R_e$, namely, the cover-gas pressure. In the derivation of Eq. (A.9), the dynamic force balance at the bubble surface, that is,

$$P_g = P_{jl} + \frac{2\sigma}{R_b} \approx P_{jl} \tag{A.10}$$

has been used. Here, P_{jl} is the liquid sodium pressure at the bubble interface, and σ is the surface tension.

Equation (A.9) may be called the extended Rayleigh equation which takes into account the bubble motion, bubble expansion and as well as the possible interfacial mass transfer at the bubble surface. It is interesting to note that Eq. (A.9) reduces to the classical Rayleigh equation as

$$\begin{aligned}
R_e & \rightarrow \infty \\
v_b & = 0 \\
\dot{m}_{jg} & = 0 ,
\end{aligned}$$

and furthermore reduces to the Taylor's equation as given in Ref. [15] as

$$\begin{aligned}
R_e & \rightarrow \infty \\
\dot{m}_{jg} & = 0 .
\end{aligned}$$

Fig. 7. Bubble Expansion During Bubble Buoyancy

APPENDIX B

DERIVATION OF INTEGRAL ENERGY EQUATION, Eq. (4.3)

The application of integral methods to analyze transient heat transfer problems is well established as a result of studies reported by Goodman [27] and by Veinik [28]. The versatility of the method in solving various nonlinear problems of heat transfer with and without phase change is well illustrated in Ref. [29].

As in other integral methods, the formulation and solution of the problem consists of three steps. First, a spatial profile of temperature is assumed which must satisfy the boundary conditions of the particular problem. Second, the energy equation is integrated across the thickness, δ , of the thermal boundary layer yielding the energy integral equation for the thickness of the thermal boundary layer. Its solution yields δ , as a function of time and specifies therefore the functional relation between temperature and time. These three steps will be followed in formulating and solving the present problem.

It has been shown in Refs. [27-29] that for a planar geometry, an assumed parabolic temperature distribution, i.e.,

$$T(r) = a + br + cr^2 \tag{B.1}$$

yields results which are in good agreement with those obtained by exact methods.

This same temperature distribution, e.q., Eq. (B.1) has been used in Refs. [28,30,31] for solving spherically symmetric bubble dynamic problems. However, the application of Eq. (B.1) to geometries other than planar, is surprising since it is known that such an application yields erroneous results. Indeed, it was first noted by Sparrow [32] and demonstrated by Lardner and Pohle [33] that an error of 100% and

higher, results if Eq. (B.1) is applied to problems of polar symmetry, i.e., to cylindrical geometry.

As discussed in Refs. [29,32,33], the erroneous results which are obtained when Eq. (B.1) is applied to geometries other than planar, stem from the fact that for these geometries, the volume into which heat diffuses does not remain the same for equal increments of r , as in the planar case. Consequently, a modification of the assumed parabolic profile is necessary. It was suggested therefore in these references that for spherical geometry, the appropriate spatial temperature distribution should be of the form:

$$T(r) = \frac{a+br+cr^2}{r} \quad (B.2)$$

In order to increase the accuracy, we shall assume a cubic profile in the present analysis. Thus,

$$T_j(r) = \frac{a+br+cr^2+dr^3}{r} \quad (B.3)$$

The coefficients a , b , c and d in Eq. (B.3) are to be determined from the boundary conditions for the particular problem under consideration. Denoting by $R_b(t)$, the radius of the HCDA bubble, by $\delta(t)$, the thickness of the thermal boundary layer, by $T_{j\infty}$ the temperature of the liquid sodium at a great distance from the interface and by T_{j1} the liquid sodium temperature at the bubble surface, (see Fig. 8), the boundary conditions consistent with this approach can be expressed by

$$\begin{aligned} T_j(R_b, t) &= T_{j1} \\ T_j(R_b + \delta, t) &= T_{j\infty} \\ \left. \frac{\partial T_j}{\partial r} \right|_{R_b + \delta} &= 0 \\ \left. \frac{\partial^2 T_j}{\partial r^2} \right|_{R_b + \delta} &= 0 \end{aligned} \quad (B.4)$$

Evaluating the constants in Eq. (B.3) by means of Eq. (B.4) we obtain the following temperature distribution

$$T_j = T_{j\infty} + (T_{j1} - T_{j\infty}) \left(\frac{R_b}{r}\right) \left(\frac{R_b + \delta - r}{\delta}\right)^3 \quad (B.5)$$

We turn our attention now to the energy integral equation which can be obtained by integrating the local energy equation. The differential form of the local energy equation suitable for incompressible inviscid fluids is

$$\frac{\partial h_{j1}}{\partial t} + \nabla \cdot \rho_{j1} \vec{v}_{j1} h_{j1} = -\nabla \cdot \vec{q}_j \quad (B.6)$$

By inserting the temperature distribution given by Eq. (B.5) and the velocity distributions given by Eqs. (A.6) and (A.7) into Eq. (B.6) we obtain, upon integration from R_b to $R_b + \delta$ and after some rearrangement, the following expression

$$\begin{aligned} & \delta(\delta + 10 R_b) (T_{j1} - T_{j\infty}) \frac{dR_b}{dt} + R_b(5 R_b + 2\delta) (T_{j1} - T_{j\infty}) \frac{d\delta}{dt} + \\ & R_b \delta(5 R_b + \delta) \frac{dT_{j1}}{dt} = \frac{20 R_b^2}{\rho_{j1} C_{j1}} \{ \dot{q}_{ij} - \dot{m}_{je} [h_{j1}(T_{j1}) - h_{j1}(T_{j\infty})] + \\ & \dot{m}_{jc} [h_{j1}(T_g) - h_{j1}(T_{j\infty})] \} \end{aligned} \quad (B.7)$$

Furthermore, expressing the specific enthalpy by

$$dh_j = C_{j1} dT_j$$

and using this expression in Eq (B.7), we obtain the energy balance in the liquid sodium as follows:

$$\begin{aligned} & \delta(\delta + 10 R_b) (T_{j1} - T_{j\infty}) \frac{dR_b}{dt} + R_b(5 R_b + 2\delta) (T_{j1} - T_{j\infty}) \frac{d\delta}{dt} + \\ & R_b \delta(5 R_b + \delta) \frac{dT_{j1}}{dt} = \frac{20 R_b^2}{\rho_{j1} C_{j1}} [\dot{q}_{ij} - \dot{m}_{je} C_{j1} (T_{j1} - T_{j\infty}) + \\ & \dot{m}_{jc} C_{j1} (T_g - T_{j\infty})] \end{aligned} \quad (B.9)$$

where $\dot{q}_{ij}$ is the heat flux at the bubble surface which can be expressed in terms of the temperature distribution by

$$\dot{q}_{ij} = -k_j \left(\frac{\partial T_j}{\partial r} \right)_{r=R_b} \quad (B.10)$$

By making use of Eq. (B.5), Eq. (B.10) can be expressed by

$$\dot{q}_{ij} = \left(3 + \frac{\delta}{R_b} \right) \left(\frac{k_{jl}}{\delta} \right) (T_{jl} - T_{j\infty}) \quad (B.11)$$

where k_{jl} is the thermal conductivity of the liquid sodium.

Eq. (B.9) together with Eq. (B.1) will be used to express the energy balance in the liquid sodium pool as the HCDA bubble rises through the liquid sodium.

Fig. 8 Heat Transfer on Sodium Side

10. NOMENCLATURE

A_e	- Cover-gas plenum exit area (m^2)
A_{id}	- Single fuel particle surface (m^2)
A_{if}	- Fuel effective interfacial area (m^2)
A_{ij}	- Bubble exterior surface (m^2)
A_{is}	- Area of the steel structure in the bubble (m^2)
C_{Af}	- Interfacial area coefficient (dimensionless)
C_{fc}	- Fuel condensation coefficient (dimensionless)
C_{fe}	- Fuel evaporation coefficient (dimensionless)
C_f	- Fuel effective evaporation or condensation coefficient (dimensionless)
C_{jc}	- Sodium condensation coefficient (dimensionless)
C_{je}	- Sodium evaporation coefficient (dimensionless)
C_j	- Sodium evaporation or condensation coefficient (dimensionless)
C_p	- Specific heat at constant pressure (J/kg K)
C_v	- Specific heat at constant volume (J/kg K)
g	- Gravitational acceleration (m/s^2)
$h(T)$	- Specific enthalpy at temperature T (J/kg)
k_{lj}	- Liquid sodium thermal conductivity (J/msk)
m	- Mass (kg)
$\dot{m}$	- Mass flow rate per unit area, i.e., mass flux (kg/m^2s)
$\hat{n}$	- Unit normal vector
P	- Pressure (Pa)
P^+	- Saturation Pressure (Pa)
P^*	- Partial Pressure (Pa)

- $\dot{q}$ - Heat flux ($\text{J/m}^2\text{s}$)
- $\dot{q}_{gj}$ - Net radiative heat flux from gaseous phase to liquid sodium ($\text{J/m}^2\text{s}$)
- $\dot{q}_{gs}$ - Net radiative heat flux from gaseous phase to steel structure ($\text{J/m}^2\text{s}$)
- $\dot{q}_{lj}$ - Net radiative heat flux from liquid fuel to liquid sodium ($\text{J/m}^2\text{s}$)
- $\dot{q}_{ls}$ - Net radiative heat flux from liquid fuel to steel structure ($\text{J/m}^2\text{s}$)
- $\dot{q}_{ij}$ - Conductive heat flux at liquid sodium-bubble interface ($\text{J/m}^2\text{s}$)
- R_b - Bubble radius (m)
- R_d - Fuel droplet radius (m)
- $\bar{R}$ - Ideal gas constant (J/kg K)
- r - Radial distance in the spherical coordinate system (m)
- T - Temperature (K)
- t - Time (s)
- V - Volume (m^3)
- v_b - Bubble rise velocity (m/s)
- $\vec{v}_i$ - Bubble surface velocity vector (m/s)
- $\vec{v}_{jl}$ - Liquid sodium velocity vector (m/s)
- $\vec{v}_{jl_i}$ - Liquid sodium velocity vector at sodium-bubble interface (m/s)
- y_b - Bubble displacement from the center of core (m)

Greek Letters

- α_d - Liquid fuel void fraction in the bubble (dimensionless)
- α_j - Surface absorptivity of sodium (dimensionless)
- α_s - Surface absorptivity of steel (dimensionless)
- δ - Thermal boundary layer thickness (m)
- γ - Isentropic exponent (dimensionless)

- ϵ - Emissivity (dimensionless)
- ρ - Mass density (kg/m^3)
- θ - Angle in spherical coordinate system (degree)
- σ - Stefan-Boltzmann constant ($\text{J/m}^2 \text{K}^4 \text{s}$)

Subscripts

- c - Cover gas (argon)
- f - Fuel
- g - Gaseous
- i - Fission gases
- j - Sodium
- f_g - Fuel in gaseous state (fuel vapor)
- f_l - Fuel in liquid state (liquid fuel)
- j_g - Sodium in gaseous state (sodium vapor)
- j_l - Sodium in liquid state (liquid sodium)

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