Type II Successful Supernovae,† The Anatomy of Shocks: Neutrino Emission and the Adiabatic Index

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The Anatomy of Shocks: Neutrino Emission and the Adiabatic Index

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ABSTRACT

Hydrodynamic calculations of stellar collapse in Type II Supernova are described using a variable stiffness and compressibility for the nuclear equation of state at high density. Initial models employing a relatively small mass core with low central entropy are necessary to achieve viable shocks; near success the models are sensitive to both neutrino emission and the high density equation of state. The treatment of neutrino production and transport is sketched and recent results reported.

INTRODUCTION

The expected scenario for the catastrophic collapse and explosion of a massive star at the end of its life was first described qualitatively by Burbridge, Burbridge, Fowler and Hoyle [1], then quantitatively by Colgate and collaborators in 1960 [2]. This subject has been explored since then by many workers [3,4,5] with resulting changes in the scenario. The initial conditions for the final collapse were spelled out in most detail by Weaver, Zimmerman and Woosley (WZW) [6], Arnett [3] and most recently by Weaver, Woosley and Fuller (WWF) [7] who by including a new treatment of electron capture have reduced the entropy and core size, which as we will see is helpful. Further quantitative progress then requires development of detailed hydrodynamic calculations; qualitative considerations alone are apt to be misleading. Recent work by Mazurek, Cooperstein and Kahana (MCK) [8] and subsequently by Wilson [9] have established that a successful explosion may not be trivially achieved. Using the WZW initial conditions for the initial iron core, $M_{\text{core}} = 1.6 M_\odot$,
S(central)/k = 1 per nucleon, T(central) = .69 MeV etc., MCK obtained stagnant, accretion shocks. Efforts to resuscitate the shock by trapping all neutrinos (following a suggestion by Bethe) were unsuccessful and one concluded the culprit might be nuclear dissociation on the outward shock trajectory, i.e. after bounce. Burrows and Lattimer [10] suggested a lower central entropy was favorable and as WWF [7] indicated this was also likely. Work by Cooperstein, Bethe and Brown [11] (CBB) using a revised version of the MCK code suggest strongly that a reduction in core mass can lead to viable explosions. We will discuss the likely reason this change in initial model is so helpful, i.e. the reduction in the absolute amount of matter of high density traversed by the shock, in a somewhat different context. We also want to address another issue we believe to be important, the stiffness of nuclear matter at densities near to and especially above that for normal nuclear matter saturation, $\rho_0 = 2.4 \text{ gm/cm}^3$.

We do this again within the context of the MCK hydrodynamics as modified by CBB. It is interesting that one's perception of the reasons for success and failure change as the calculations approach closer to success. The degree of cancellation involved amongst the characteristic energies of the system explains this confusion. The initial Fe-core is close to a relativistic $T_3 = 4/3$ gas, the static virial theorem then yields a total energy $E = K + U_{\text{grav.}}$ close to zero. This cancellation exists to perhaps $1/5 \text{ foe} = 1/5 \times 10^{51}$ ergs, though $K, U_{\text{grav.}}$ are separately of the order of 100 foes and a successful shock may deliver less than $1/2$ foe to the outer core regions. Thus in the original MCK calculations nuclear dissociation was identified as the major agent in failure, complete neutrino trapping being unable to convert failure into success.

In our present calculations excessive neutrino loss seems to play a more important role in the stagnation of large core mass models. Thus we will for the purposes of the organizers of this workshop discuss the bare fundamentals of neutrino production, diffusion and emission as laid out by many authors [9,12,13,16]. This subject is, incidentally, by no means well treated in our modeling, diffusion being eliminated in favor of a sharp transition from trapping to free streaming. Wilson [9] has done a better job of this important feature.

The second subject considered is the equation of state of nuclear matter, in particular for densities greater than normal saturation. Virtually nothing is known about dense nuclear matter directly from experiment. What one uses is gleaned by theoretical extrapolation; information about two body forces being translated by many body calculations into knowledge of nuclear matter. Some possible constraints are considered but for the most part we simply vary the parametrization of the equation of state within reasonable limits. From our present calculations, when compared to CBB, we conclude the stiffness of high density matter is also an important element with perhaps optimum choices for the incompressibility and adiabatic index existing.
Hydrodynamics of Stellar Collapse and Neutrino Escape

Many others at this workshop will discuss both the general outline and practical details of the hydrodynamic equations. For completeness and definition of terms we present some of the formalism with the usual Lagrangian coordinate, the mass $M(R)$ within a radius $R$ [14]

$$\frac{dM}{dR} = 4\pi R^2 \rho(R) .$$  \hspace{1cm} (1)

One has the equation of motion

$$\frac{du}{dt} = - \frac{GM(R)}{R^2} - 4\pi R^2 \frac{dp}{dM}$$  \hspace{1cm} (2)

and the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial M} (4\pi R^2 u) = 0$$  \hspace{1cm} (3)

where $u$ is the radial velocity of a mass element, $p$ the pressure and $\rho$ the matter density. A mechanical energy conservation equation, essentially the Bernouilli equation, can be obtained as an integral of (2)

$$\frac{dE'}{dt} = \frac{\partial w}{\partial t}, \text{ with } \frac{\partial w}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial \rho}$$  \hspace{1cm} (4)

The pressure functional $w$, which for adiabatic changes is the specific enthalpy is given by

$$w = \int \frac{dp'}{\rho'} = \epsilon + p/\rho$$  \hspace{1cm} (5)

with

$$\epsilon = \int \frac{p'}{\rho'^2} \rho'$$  \hspace{1cm} (6)

the specific internal energy, also for adiabatic changes. We can then rewrite (4) in the Eulerian form

$$\frac{\partial E}{\partial t} + u \frac{\partial}{\partial x} (E + p/\rho) = \frac{p}{\rho^2} \frac{\partial \rho}{\partial t}$$  \hspace{1cm} (7a)

$$E = 1/2 u^2 + E_{\text{grav.}} + \epsilon$$  \hspace{1cm} (7b)

with the mechanical energy flux containing the characteristic pressure term.
To help interpret (4) and (7) and to treat non-adiabaticity such as neutrino transport, we introduce the first law of thermodynamics in the fluid frame

\[
\frac{dc}{dt} = \frac{dQ}{dt} - p \frac{d(1/\rho)}{dt} = \frac{dQ}{dt} + \frac{p}{\rho^2} \frac{dp}{dt}
\]  

with \(dQ\) the heat created in our mass element. For quasi static conditions the heat added is related to the temperature \(T\) and specific entropy (per nucleon) by

\[
dQ = T ds
\]

Broadly speaking transport by neutrinos is divisible into three categories, trapping, diffusion, i.e. an intermediate situation and free streaming. The neutrino description is controlled by the cross sections for the processes in Table I, and, of course, by the thermodynamic parameters. In particular the density of matter and its composition are crucial, trapping occurring in inner regions over dynamic times. The mean free path for neutrinos was calculated by Lamb and Pethick [15], for \(\sin^2 \theta_{\text{weak}} = 1/4\), to be

\[
\left( \frac{\lambda_n}{10^6 \text{cm}} \right)^{-1} = \left( \frac{12}{10^2 \text{g/cm}^3} \right) \left( \frac{1}{6} \frac{N^2}{A} X_h + X_n + \frac{5}{6} X_p \right) \left( \frac{\epsilon_n}{10 \text{MeV}} \right)^2
\]

where \(X_h, X_n, X_p\), are the heavy nuclei, neutron, proton fraction respectively, \(N/A\) the average neutron number and mass of \(X_h\), and \(\epsilon_n\) the neutrino energy. The characteristic dependence on neutrino energy \(\epsilon_n\) in (10) favors the emission of low energy neutrinos. For conditions in the typical hydrodynamical calculations at a density \(\rho = 10^{12} \text{ gms}\) and neutrino energy \(\epsilon_n = 10 \text{MeV}\) one finds \(\lambda_n = 1.0 \text{km}\), much less than typical distances in the core. More accurately trapping occurs for a density of a few \(\times 10^{11} \text{ gms/cm}^3\).

Diffusion of neutrinos will occur outwards from the trapped regions according to [4]

\[
\frac{\partial n_\nu}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{1}{3} c \lambda_\nu \frac{\partial n_\nu}{\partial r} \right)
\]

Bethe, Brown, Applegate and Lattimer (BBAL [4]) use the latter to deduce an approximate diffusion time \(\approx 100 \text{ microseconds}\) compared to the much shorter dynamical millisecond times. Calculations, with the exception of an interesting long-time computation of Wilson [9], do not yet proceed to times where diffusion and deleptonization of the core are important. In the outer region of
<table>
<thead>
<tr>
<th>Process</th>
<th>$\nu_e$</th>
<th>$\nu_\mu, \nu_\tau$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Capture</td>
<td>X</td>
<td></td>
<td>Dominates pre-shock production of neutrinos</td>
</tr>
<tr>
<td>$e^- + (A,Z) + \nu_e + (A,Z-1)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e^- + p + \nu_e + n$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron Pair Annihilation:</td>
<td>X</td>
<td>X</td>
<td>Significant only after shock forms. Emission is comparable to electron capture for $s \approx 7$ to 8.</td>
</tr>
<tr>
<td>$e^+ + e^- + \nu + \bar{\nu}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay of Excited Nuclei:</td>
<td>X</td>
<td>X</td>
<td>Dominant in pre-shock production of $\nu_\mu$ and $\nu_\tau$.</td>
</tr>
<tr>
<td>$(A,Z)^* + (A,Z) + \nu + \bar{\nu}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron Bremsstrahlung:</td>
<td>X</td>
<td>X</td>
<td>Negligible relative to nuclear deexcitation process above.</td>
</tr>
<tr>
<td>$e^- + (A,Z) + e^- + (A,Z) + \nu + \bar{\nu}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Photo and Plasma Neutrinos:</td>
<td>X</td>
<td>X</td>
<td>Negligible</td>
</tr>
<tr>
<td>$\gamma + e^- + e^- + \nu + \bar{\nu}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma + \nu + \bar{\nu}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meson and Lepton Decays:</td>
<td>X</td>
<td>X</td>
<td>Negligible</td>
</tr>
<tr>
<td>$\pi, \mu, \tau$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*extracted from MCK [8]
the collapsing and post-shocked core with \( \rho < \rho_{\text{trapping}} \) neutrinos will free stream. Fig 1 taken from MCK [8] indicates the general time behavior of the number of escaping neutrinos.

A more careful treatment implemented by Cooperstein [16], is to divide the stellar mix into two components, neutrinos and the rest, matter plus radiation. This two fluid model, with a higher neutrino temperature \( T_{\nu} \), since neutrinos are produced preferentially by electrons near the top of the Fermi sea, permits one to retain the hydrodynamic features consistently. In the high matter-density regions larger interaction rates for all processes and their inverses will rapidly lead to equilibrium and trapping. In the outer regions of low density the two fluids are weakly coupled and the neutrinos approach free streaming.

The basic equations are obtained by rewriting (2) and (8) in the form [16,17].

\[
\begin{align*}
\frac{du}{dt} + \frac{GM(r)}{r^2} + \frac{1}{r} \frac{3p}{3r} &= G_{\nu} \\
\frac{dE}{dt} - \frac{p}{\rho} \frac{d\rho}{dt} &= Z_{\nu}
\end{align*}
\]

\( p, \rho, E \) describing the matter and \( G_{\nu}, Z_{\nu} \) the neutrino momentum and energy coupling. These latter quantities are connected by an energy transport equation devised by Caster [17]

\[
\frac{dE_{\nu}}{dt} = \frac{P_{\nu}}{\rho} \frac{d\rho}{dt} + \left[ \frac{3P_{\nu}}{\rho} - E_{\nu} \right] \frac{u}{R} - Z_{\nu}
\]

\[
- \frac{3}{2M} \left[ \frac{4\pi R^2}{N_0} F \right]
\]

with \( F \) the total frequency averaged, energy distribution of neutrinos and antineutrinos out of the mass element, while \( N_0 \) is Avogadro's number. The second term of Eq. (14) vanishes in the opaque limit of isotropic radiation \( P_{\nu}/\rho = 1/3 \ E_{\nu} \) but not in the intermediate and transparent regions, where \( P_{\nu}/\rho \) between 1/3 \( E_{\nu} \) and \( E_{\nu} \) is more appropriate (see Burrows and Mazurek [20]). A final equation describes lepton conservation

\[
\frac{d(\hat{Y}_{\nu} + \hat{Y}_e)}{dt} = - \frac{1}{N_0} \frac{3}{\partial N_R} (4\pi R^2 N)
\]

where \( \hat{Y}_{\nu} = Y_{\nu} - Y_{\nu}, \hat{Y}_e = Y_e - Y_e \) and \( N = N_{\nu} - N_{\nu} \) is the neutrino number flux. Fermi-Dirac distributions are used for the neutrinos with the chemical potential \( \mu_\nu \). A detailed discussion of the two fluid model is given in reference [16] but is not pursued
here. Indeed detailed calculation by Cooperstein indicate only little differences with a much simpler trapping + free streaming approach (see Fig 2).

We finish this section with some recapitulation and some new results, both within the simple framework. For trapping we use

\[ G_v = -\frac{4\pi R^2}{3} \frac{\partial}{\partial M} (E_v\rho) \]  

which amounts to adding pressures, while

\[ Z_v = \frac{d\epsilon}{dt} \langle \epsilon_v \rangle_p + \frac{d\epsilon}{dt} \langle \epsilon_v \rangle_h + Z_{v,e^-} \]  

where \( \langle \epsilon_v \rangle_p, \langle \epsilon_v \rangle_h \) are average neutrino emission energies in electron capture on protons and heavies and \( Z_{v,e^-} \) is a collisional term.

In the earlier MCK simulations complete trapping did not add sufficient pressure behind the shock to prevent stagnation. The lower mass models used in CBB have led to considerable reduction in both nuclear dissociation and neutrino losses. In this situation the difference between success and failure is very nearly bridged by neutrino escape. Figs. (3) and (4) show the time dependences of escape energy with sharp rises just before and after bounce. The \( \nu \)-loss is appreciable before shock formation and the later increase in slope is probably narrowly contained in space as the shock traverses regions with \( \rho < \rho_{tr} \). Table (2) gives an overall accounting of these losses. The failures, using initial models with \( M_{core} = 1.5 M_\odot \), involve 1 foe additional \( \nu \)-losses. Surely the doubling of post-shock escape energy is occasioned in the failures by the commensurate increase in matter "thickness" from the radius of bounce outwards. The difference in neutrino losses between success and failure after bounce, of approximately 1/2 foe, is by no means inconsequential in light of the CBB total energy of < 1/2 foe finally available for the shock. A gross test of the sensitivity of the rather crude neutrino treatment can be obtained by varying \( \rho_{tr} \) slightly.

The Nuclear Matter Equation of State

Eventually we intend to revise the nuclear equation of state (E.O.S.) for all densities greater than those appropriate to the Saha equations. Many workers Bonche and Vautherin [18], Lamb, Lattimer, Pethick and Ravenhall [19] have derived detailed equations of state for densities \( \rho < \rho_o \), but in the spirit of BBCW [20] and CBB we wish to work with a simplified form amenable to rapid hydrodynamic computation. We would, however, like to remove the thermodynamic inconsistencies in the crossover regions [20] as these may lead to impedance mismatching for the shock. In particular, in the present work we address the stiffness of the EOS at very high density. Weakening the EOS
SUCCESS: \( \Gamma = 3.3 \times 10^{-4} = 220 \)

FAILURE: \( \Gamma = 3.3 \times 10^{-4} = 220 \)

Fig. 3
FAILURE: $\Gamma = 3, K_0 = 220$

SUCCESS: $\Gamma = 3, K_0 = 220$

Fig. 4
Table II  Comparison of neutrino escape energies for several successes and failures. Energy losses are given separately just before bounce and at late times in the evolution. Also given are minimum total energies at the edge of the hydrostatic core which are an indication of what is available for transfer to the shock.

<table>
<thead>
<tr>
<th></th>
<th>Brick Wall</th>
<th>BCK $\gamma=2 K_o=220$</th>
<th>BCK $\gamma=3 K_o=220$</th>
<th>BCK $\gamma=2 K_o=220$</th>
<th>BCK $\gamma=3 K_o=220$</th>
<th>BCK $\gamma=2 K_o=220$</th>
<th>BCK $\gamma=3 K_o=220$</th>
<th>Bick Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_V$ (before bounce)</td>
<td>.75</td>
<td>.75</td>
<td>.75</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>$E_V$ (total)</td>
<td>1.25</td>
<td>1.20</td>
<td>1.20</td>
<td>2.2</td>
<td>2.2</td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{TCT}$ (minimum)</td>
<td>-1.86</td>
<td>-2.53</td>
<td>-2.33</td>
<td>-1.43</td>
<td>-1.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(late time)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 5
Nuclear equation of state: F. Serr
ought to permit a greater compression of central densities, a building up of energy by the inner "piston", and an eventual increase of energy transferred from the final hydrostatic core to the shock. Our results bear out this hope in a highly encouraging fashion. Our standard of comparison is what we refer to as the brick wall success of CCB, where unintentionally these authors used an extremely large effective adiabatic index; treating high density matter virtually as an incompressible fluid. Our conclusions, if borne out in further study, also contradict the contentions of Yahil and Lattimer [21] that the degree of compressibility of high density nuclear matter is irrelevant.

The equation of state for nuclear matter is given by the internal energy density per nucleon

\[ E/A = \varepsilon(\rho, T) \]  

whence the pressure and incompressibility are

\[ p = \rho^2 \frac{\partial E}{\partial \rho}, \; \frac{K}{\partial} = \rho^2 \frac{\partial^2 E}{\partial \rho^2} \]  

The adiabatic index is

\[ \Gamma = \frac{d(\ln p)}{d(\ln \rho)} \]  

For a pure polytrope \( p = \text{constant} \rho^\gamma \) and \( \Gamma = \gamma \). For cold nuclear matter in the present calculations the energy/nucleon we employ is

\[ E/A = -16 + 29.3 (1 - 2x)^2 + \int \frac{P}{\rho^2} \, d\rho \; \text{MeV} \]  

\[ p = \frac{K_0 \rho_0}{\gamma} \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \]

where \( x = Z/A \).

An improved form would use for example the modified Skyrme-force treatment of Serr [22] permitting one to impose the constraints of information deduced from finite nuclei, in particular the position and strength of the giant monopole resonance (GMR) in heavy nuclei. Serr obtains some freedom in choice of bulk properties by exploiting the existence of both surface and bulk compressibilities in finite nuclei. Fig. (4) shows a plot of \( \varepsilon = E/A \) for nuclear matter for three different force choices, and widely differing compressibilities.

For the moment, we simply alter the effective stiffness between \( \gamma = 2 \) and \( 3 \) and hence to infinity via the "brick wall" of CBB. Before discussing the results it is useful to reiterate the arguments of CBB and others [10] which yield qualitative criteria for successful shocks in particular simulations. An alternative way of handling energy conservation is to use a variant of the
virial theorem. In a static hydrostatic situation, for example at the instant before collapse, we can integrate the equation of motion (2) multiplied by the volume $V(r)$ over the stellar core to obtain

$$
[4\pi R^3 p]_{R_o}^{R_s} = \int M_o^M dM (e + \varepsilon) + \int \frac{(3p - \varepsilon)}{\rho} dM
$$

(23)

where the two radii $R_o$ and $R_s$ define the edges of a dense inner region and of the shock respectively. Rearrangement yields the Net Ram Pressure (NRP) difference,

$$
[4\pi R^3 p - E_{TOT}]_{R_o}^{R_s} = \int \frac{(3p - \varepsilon)}{\rho} dM
$$

(24a)

or the ram pressure just inside the shock

$$
4\pi R^3 p_s = E_s + (4\pi R^3 p_o - E_o) + \int \frac{(3p - \varepsilon)}{\rho} dM
$$

(24b)

CBB argue that $E_s = -E_o = 1.25$ foe is the energy loss due to neutrino escape; they evaluate the second term on the right side in (24b) to be 4.0 foes and finally indicate the last term gives a small positive gain to the shock pressure, arising from nuclear dissociation at a temperature $T \approx 6.5$ MeV. Eq. (14b) then results in a net positive pressure $p_s$ to sustain the shock.

Use of the Hugoniot relations

$$
(u_2 - u_1)^2 = (p_2 - p_1) \left[ \frac{1}{\rho_1} - \frac{1}{\rho_2} \right]
$$

(25)

connecting the pre- and post shock velocities, densities and pressures $(u_1, \rho_1, p_1), (u_2, \rho_2, p_2)$ and examination of these quantities as actually calculated hydrodynamically, indicates a barely viable shock in the "brick wall model." CBB find $4\pi p_s R_s^2 = .39$ foe for a shock radius $R_s \approx 500$ km., whereas a ram pressure $4\pi p_2 R^2 = .37$ foes is needed to power the calculated shock.

Employing the same initial Cooperstein model, using essentially $M_{core} = 1.25 M_\odot$ and $s$(central) per nucleon = 0.5 we have then reverted to the simple nuclear EOS given by Eq. (22) with $\gamma$ varying and $K_o = 220$ MeV. Tables 3, 4 present some aspects of our calculations. Fig. 6 shows the early and late time behavior of the ram pressures while Fig. 7 presents velocity profiles. Several comments can be made about these results.
Table III  Sensitivity of successful shock parameters to the stiffness of nuclear matter. Comparison is made of the brick wall success of CBB with the $\gamma = 3$, $k_0 = 220$ MeV model of Baron, Cooperstein and Kahana (BCK) as spelled out in equation (22). The initial model is that used by Cooperstein in the CBB hydrodynamic calculations with a small core possessing mass $M_{\text{core}} = 1.25 \, M_\odot$ and an entropy/nucleon between .5 and 1.0. Several shock parameters are shown at a very late stage in the hydrodynamic calculation. The shock has progressed close to the edge of the model core. Two snapshots at slightly different times are shown for the (BCK) calculation.

<table>
<thead>
<tr>
<th>Shock Parameters</th>
<th>Brick Wall (CBB)</th>
<th>(BCK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (sec)</td>
<td>.2176</td>
<td>.2165</td>
</tr>
<tr>
<td>iteration number</td>
<td>11468</td>
<td>12,997</td>
</tr>
<tr>
<td>$R_g$ (km)</td>
<td>738</td>
<td>1060</td>
</tr>
<tr>
<td>$M_g$ ($M_\odot$)</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>$u_2$ ($x 10^9$ cm/sec)</td>
<td>1.16</td>
<td>1.56</td>
</tr>
<tr>
<td>$u_1$ ($x 10^9$ cm/sec)</td>
<td>-.7</td>
<td>-.41</td>
</tr>
<tr>
<td>$\rho_2$ (gm/fm$^3$)</td>
<td>$1.35x10^8$</td>
<td>$1.5x10^8$</td>
</tr>
<tr>
<td>$\rho_1$ (pressure/cm$^3$)</td>
<td>$7.3x10^6$</td>
<td>$3.6x10^6$</td>
</tr>
<tr>
<td>Net Ram (foes)</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Ram Pressure (foes)</td>
<td>.007</td>
<td>.007</td>
</tr>
<tr>
<td>Kinetic energy at shock (foes)</td>
<td>.038</td>
<td>.12</td>
</tr>
<tr>
<td>$E_{\text{TOT}}$ (minimum) (foes) at edge of hydrostatic core</td>
<td>-1.86</td>
<td>-2.33</td>
</tr>
</tbody>
</table>
Table IV  Further comparison of hydrodynamics for varying stiffness in the EOS, near maximum "scrunch" in time.

<table>
<thead>
<tr>
<th></th>
<th>Brick Wall (CBB)</th>
<th>$\gamma = 3, K_o = 220$ BCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iteration</td>
<td>1805</td>
<td>1977</td>
</tr>
<tr>
<td>Time</td>
<td>.19024</td>
<td>.19025</td>
</tr>
<tr>
<td>$\rho$(central) (gm/cm$^3$)</td>
<td>$3.091 \times 10^{14}$</td>
<td>$4.09 \times 10^{14}$</td>
</tr>
<tr>
<td>Shock at:</td>
<td>zone 53</td>
<td>zone 55</td>
</tr>
<tr>
<td>Mass ($M_\odot$)</td>
<td>.875</td>
<td>.892</td>
</tr>
<tr>
<td>at Radius (km)</td>
<td>13.53</td>
<td>13.43</td>
</tr>
<tr>
<td>$T_c$ (MeV)</td>
<td>6.94</td>
<td>8.27</td>
</tr>
<tr>
<td>$T_{max}$ (MeV)</td>
<td>13.6</td>
<td>14.9</td>
</tr>
<tr>
<td>$Y_e$ (central)</td>
<td>.3313</td>
<td>.3313</td>
</tr>
<tr>
<td>$Y_e$ ($s_{max}$)</td>
<td>.3277</td>
<td>.3222</td>
</tr>
<tr>
<td>$S_{max}$</td>
<td>3.318</td>
<td>3.718</td>
</tr>
<tr>
<td>$p_c$ (dynes)</td>
<td>$1.384 \times 10^{34}$</td>
<td>$1.83 \times 10^{34}$</td>
</tr>
</tbody>
</table>
Fig. 6
(1) Direct measures of success, velocities and kinetic energies just behind the shock, indicate a definite sensitivity to the stiffness of the equation of state. Further careful study is clearly warranted to further define the role of the phenomenological parameters $K_0$, $\gamma$ and to introduce a theoretically deeper approach to the E.O.S.

(2) Considering the cancellations inherent between internal and gravitational energy some care must be exercised in using quantities such as net ram pressure as a measure of success, or in the use of other simple qualitative pictures.

(3) Near success neutrino transport plays an important role distinguishing large core failures from small core successes and treatment of such losses must be improved. At the very least we intend to vary the trapping density to test sensitivity.

(4) The higher temperatures produced by softening the equation of state seems helpful.

(5) Other initial models will be constructed to define the limits in mass, entropy, $Y_e$, etc. for success.

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The authors are grateful to G. E. Brown for useful discussions on the results of the calculations. They are also much indebted to F. Serr for his careful elucidation of the freedom in choices of equation of state, and hope at a further stage to incorporate in a collaborative work the constraints provided by his studies.

The calculations discussed here were performed on the MPE CRAY(D) through the auspices of the Division of Nuclear Physics, Research Division of the U.S.D.O.E. The generous time thus provided has been instrumental in permitting a more complete investigation of the model parameters.

REFERENCES

7. T. A. Weaver, S. E. Woosley and G. M. Fuller, Bull. Am.
Astron. Soc. 14, No. 4, p. 957 (1982) and in preparation for
Ap. J.
Workshop - University of Hawaii, ed. by V. J. Stengler 1980;
also Proc. NATO Advanced Study (1981) Institute on Supernova,
Hydrodynamics (1982); also J. Wilson in Ninth Texas Symposium
Propagation in Supernovae: Concept of Net Ram Pressure",
Phys. Rev. D6 (1972) 941; D. L. Tubbs, and D. N. Schramm,
and Astrophys. and Space Sci. 35 (1975) 117.
14. See for example, Donald A. Clayton "Principles of Stellar
Evolution and Nucleosynthesis (McGraw Hill, New York, 1968);
also A. L. Fetter and D. J. Walecka "Theoretical Mechanics of
16. J. Cooperstein, "The Equation of State and Neutrino Transport
(1978) 1623.
20. H. A. Bethe, G. E. Brown, J. Cooperstein and J. R. Wilson,
21. A. Yahill and J. M. Lattimer, in Supernova, A Survey of
Current Research, M. J. Rees and R. J. Stoneham, eds.
22. F. Serr, private communication.

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FIGURE CAPTIONS

Figure 1 Neutrino particle emission $N_\nu$ as a function of time as calculated by MCK. Table I gives the various processes which generate neutrinos. Comparison should be made to later calculations of Baron, Cooperstein and Kahana (BCK) in Figures 3, 4.

Figure 2 Model differences in specific energy profiles after the core has gone hydrostatic. The principle point of this figure is to indicate the changes occasioned by the fluid model treating matter and neutrino separately. The changes in EPM, the specific internal energy, are seen to be minimal.

Figure 3 Neutrino emission: Escape energies calculated for large core (failure), small core (success) initial models using the EOS of (BCK). The one foe difference in $E_\nu$ is a significant difference.

Figure 4 Similar to Fig. 3 but with $dE_\nu/dt$ plotted instead.

Figure 5 Equations of state calculated by F. Serr for differing Skyrme-like treatments of nuclear matter and finite nuclei. The giant monopole resonance is well described by all calculations despite the strong variation in $K_0$.

Figure 6 Comparison of early net ram pressure profiles, just after bounce, for failure and success. The profiles in mass are presented parametrically in time for snapshots 20 (low NRP) and 50 (high NRP) indicating the buildup of net ram to its maximum interior values. The successful maximum is > 1 foe higher than the failing maximum. The calculations are for $\gamma = 3$ in BCK.

Figure 7 Velocity profiles throughout the hydrodynamic evolution. The successful calculation yields positive velocities out to the edge of the model ($M = 1.25 M_\odot$) at the latest times. The failure stagnates, with negative velocities appearing at $M = 1.25 M_\odot$ and the shock does not propagate to the $M = 1.5 M_\odot$ edge.