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MASTER

AN ALGORITHM FOR DETERMINING EF COILS FROM FIXED-BOUNDARY EQUILIBRIA APPLIED TO ARIES-III†

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Abstract: An algorithm that determines the external EF coils required to produce equilibria calculated by high-resolution, fixed-boundary codes is described. The algorithm permits the specification of the poloidal-field-null location. The EF-coil positions on a specified poloidal surface located just outside of the TF coils are optimized to minimize the stored magnetic energy. Results of the application of this algorithm to ARIES-III are also presented.

INTRODUCTION

The ARIES-III commercial tokamak reactor design¹ requires operation in the high-beta ($\beta_T = 0.24$), second-stability regime to achieve undriven plasma power balance and the economic attractiveness² needed for this $D-3He$ fueled power plant. Typically, a free-boundary equilibrium code is used to solve for the EF-coil currents, given the plasma description (major and minor radii, elongation, triangularity, etc.) and coil positions. However, at high beta, free-boundary equilibrium calculations are difficult because sufficient resolution to model accurately the profiles cannot be provided in (R, Z) coordinates. In addition, the relaxation required by the numerical solution of the Grad-Shafranov equation in a typical free-boundary code can consume an unreasonable amount of computational time for a large number of mesh points. A high-resolution, fixed-boundary equilibrium calculator, JSOLVER,³ was developed to resolve accurately the strongly distorted plasma current-density profiles necessary for such high-beta equilibria and to assess accurately the associated stability properties. An algorithm was then developed to determine the external EF-coil positions and currents required to produce the JSOLVER equilibrium. A Green's function technique quickly calculates the flux requirements of the EF coils on the plasma 95% flux surface (the plasma boundary for the fixed-boundary equilibrium calculation). The position of the separatrix is controlled by specifying the location of a poloidal-field null. The externally produced flux is uniquely specified by requiring minimum current in the EF coils for fixed coil positions. Singular value decomposition then is used to obtain the EF-coil currents subject to the above flux constraints. The EF-coil positions are optimized to minimize simultaneously the stored magnetic energy and the difference between the external flux produced by the EF coils and the external flux required by the equilibrium. The EF coils are constrained to a specified poloidal surface just outside of the TF coils during the optimization of the coil positions. The application of this algorithm to ARIES-III produces an EF-coil set with seven coils, a maximum coil current of 14.2 MA, stored magnetic energy of 15.2 MJ, and maximum force on a coil of 1.17 GN for a plasma toroidal beta $\beta_T = 0.24$ and current $I_p = 29.9$ MA. Results are also presented for a sequence of equilibria that simulate a startup scenario.

MODEL

EF-Coil Currents

After the operating point and a series of equilibria that define an access path to the operating point are determined using a fixed-boundary equilibrium code, the external coil currents and positions that are necessary to produce the equilibrium must be calculated. The method used herein to determine the EF coils starts with the results of the high-accuracy, fixed-boundary equilibrium solution of the Grad-Shafranov equation produced by JSOLVER³ for the poloidal magnetic flux on the plasma surface, χ . This flux consists of contributions from both plasma and EF-coil currents that are necessary for the equilibrium to exist,³ i.e.,

$$\chi(\bar{x}_s) = \chi_p(\bar{x}_s) + \sum_{j=1}^{N_{EF}} G_x(\bar{x}_s, \bar{x}_j) I_j ; \quad (1)$$

where the plasma contribution to the total flux on the plasma surface is

$$\chi_p(\bar{x}_s) = \oint \frac{dl}{2\pi R} G_x(\bar{x}_s, \bar{x}) \frac{\partial \chi}{\partial n} ; \quad (2)$$

dl is the integration element tangent to the plasma surface in a poloidal cross section; $\partial \chi / \partial n$ is the normal derivative of the surfaced averaged poloidal flux; $\bar{x}_s$ is an R - Z vector in cylindrical coordinates denoting a location on the plasma surface; and I_j is the current in the j^{th} of N_{EF} coils that are approximated here as simple hoops. The Green's function in Eqs. (1) and (2) denotes the poloidal flux at the observation point $\bar{x}$ produced by a hoop of unit current at $\bar{y}$ and is given by⁴

$$G_x(\bar{x}, \bar{y}) = \frac{2\mu_o (R_i R_j)^{1/2}}{k} \left[\left(1 - \frac{k^2}{2} \right) K(k) - E(k) \right] , \quad (3)$$

where

$$k^2 = \frac{4R_i R_j}{(R_i + R_j)^2 + (Z_i - Z_j)^2} ;$$

the radius and the height above the equatorial plane of the observation point are R_i and Z_i , respectively; the radius and the height above the equatorial plane of a hoop current are R_j and Z_j , respectively; $\mu_o = 4\pi \times 10^{-7}$ h/m is the magnetic permeability in vacuum; and $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds, respectively.

Numerical evaluation of Eq. (2) requires knowledge of $\partial \chi / \partial n$ and a means to avoid the singularity when $\bar{x}_s = \bar{x}$. The fixed-boundary equilibrium solution of the Grad-Shafranov equation produced by JSOLVER³ comprises $\chi(\psi)$, $R(\theta, \psi)$, and $Z(\theta, \psi)$; where (ψ, θ) are generalized magnetic flux coordinates. The integration element along a flux surface can be expressed as³

$$dl = \left[\left(\frac{\partial R}{\partial \theta} \right)^2 + \left(\frac{\partial Z}{\partial \theta} \right)^2 \right]^{1/2} d\theta = \frac{J}{R} |\nabla \psi| d\theta ,$$

where the Jacobian is

$$J = R \left[\frac{\partial R}{\partial \psi} \frac{\partial Z}{\partial \theta} - \frac{\partial R}{\partial \theta} \frac{\partial Z}{\partial \psi} \right] .$$

The normal derivative of the poloidal flux is given by³

$$\frac{\partial \chi}{\partial n} = |\nabla \psi| \frac{\partial \chi}{\partial \psi} . \quad (4)$$

Then, Eq. (4) becomes

$$\frac{\partial \chi}{\partial n} = R \left[\left(\frac{\partial R}{\partial \theta} \right)^2 + \left(\frac{\partial Z}{\partial \theta} \right)^2 \right]^{1/2} \left[\frac{\partial R}{\partial \psi} \frac{\partial Z}{\partial \theta} + \frac{\partial R}{\partial \theta} \frac{\partial Z}{\partial \psi} \right]^{-1} \frac{\partial \chi}{\partial \psi} , \quad (5)$$

where all metric quantities are derivable from the equilibrium solution. Using Eqs. (3) and (5), Eq. (2) is evaluated numerically as

$$\chi_p(\bar{x}_j) = \frac{\Delta I_j}{2\pi} \left[\ln \left(\frac{\Delta I_j}{16 R_j} \right) + 1 \right] \frac{\partial \chi(\bar{x}_j)}{\partial n} + \sum_{i \neq j} \frac{\Delta I_i}{2\pi R_i} G_x(\bar{x}_i, \bar{x}_j) \frac{\partial \chi(\bar{x}_i)}{\partial n} ,$$

where the first term on the right hand side results from an approximation to an analytic integration over an integration step size, ΔI_j , in which the singularity occurs.⁵

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Subtracting χ_p from the total flux in Eq. (1) yields the following matrix equation for the current in the EF coils:

$$G_X I = (\chi - \chi_p) . \quad (6)$$

Equation (6) has two shortcomings: no control over the location of the separatrix is possible and the total flux is indeterminable. Specifying the location of the null, $\bar{x}_n$, can be added as a constraint through the following equations:

$$\sum_{j=1}^{N_{EF}} G_{B_R}(\bar{x}_n, \bar{x}_j) I_j = -B_R^p , \quad (7)$$

and

$$\sum_{j=1}^{N_{EF}} G_{B_Z}(\bar{x}_n, \bar{x}_j) I_j = -B_Z^p ; \quad (8)$$

where a magnetic field component at the null that is produced by the plasma is

$$B_m^p = \sum_{j=1}^{N_p} G_{B_m}(\bar{x}_n, \bar{x}_j) I_j ,$$

and $m = R, Z$ denotes the radial and vertical components, respectively. The plasma is simulated by N_p hoop currents positioned at the plasma grid points where JSOLVER calculates the toroidal plasma current. The radial and vertical magnetic-field functions are given, respectively, by⁴

$$G_{B_R}(\bar{x}_i, \bar{x}_j) = \frac{\mu_0(z_i - z_j)k}{4\pi R_i^{\frac{3}{2}} R_j^{\frac{3}{2}}} \left[-K(k) + \frac{R_j^2 + R_i^2 + (Z_i - Z_j)^2}{(R_j - R_i)^2 + (Z_i - Z_j)^2} E(k) \right] ,$$

and

$$G_{B_Z}(\bar{x}_i, \bar{x}_j) = \frac{\mu_0 k}{4\pi(R_i R_j)^{\frac{3}{2}}} \left[K(k) + \frac{R_j^2 - R_i^2 - (Z_i - Z_j)^2}{(R_j - R_i)^2 + (Z_i - Z_j)^2} E(k) \right] .$$

Adding the constraints of Eqs. (7) and (8) and recognizing that the flux calculated by JSOLVER at each of N_s grid points on the plasma surface is indeterminable to within a constant, α , Eq. (6) can be rewritten as the following matrix equation:

$$GI = (\chi^* + \alpha u) ,$$

or more explicitly as

$$\begin{pmatrix} \dots & G_X^{1,k} & \dots \\ \vdots & \vdots & \vdots \\ \dots & G_X^{N_s,k} & \dots \\ \dots & G_{B_R}^{N_s+1,k} & \dots \\ \dots & G_{B_R}^{N_s+2,k} & \dots \\ \dots & G_{B_Z}^{N_s+2,k} & \dots \end{pmatrix} \begin{pmatrix} \vdots \\ I_k \\ \vdots \end{pmatrix} = \begin{pmatrix} \chi - \chi_p^1 \\ \vdots \\ \chi - \chi_p^{N_s} \\ -B_R^p \\ -B_Z^p \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ 0 \end{pmatrix} .$$

For the high accuracy required in these equilibrium calculations, $N_s \gg N_{EF}$.

The coils currents necessary to produce this flux distribution for given coil locations are calculated using the method of singular value decomposition⁶ (SVD). To summarize briefly SVD, G is decomposed as follows:

$$G = UWV^T ,$$

where U is a column orthogonal rectangular matrix, W is a square diagonal matrix, and V^T is the transpose of a square orthogonal matrix. Then,

$$G^{-1} = VMU^T ,$$

where M is a diagonal matrix with elements simply related to the elements of W (i.e., $M_j = W_j^{-1}$). Generally, this problem of solving for the coil currents to produce a specified flux condition is ill-conditioned. The merit of using SVD is that the method offers a natural means of conditioning the matrix G . If $\max(W_j)/\min(W_j) \gg 1$, then G is ill-conditioned; where $\max(W_j)$ denotes the maximum of the diagonal elements W_j and, similarly, $\min(W_j)$ denotes the minimum. However, G may be conditioned by setting

$M_j = 0$ for $W_j \ll \max(W_j)$. This conditioning constraint is taken here to be: $M_j = 0$ for $W_j \leq 10^{-6} \max(W_j)$.

The current in the EF coils then is

$$I = G_{cond}^{-1}(\chi^* + \alpha u) , \quad (9)$$

where G_{cond}^{-1} is the conditioned variation of G^{-1} . The current can be minimized with the following condition:

$$\frac{\partial I \cdot I}{\partial \alpha} = 0 ,$$

which yields

$$\alpha = \frac{G_{cond}^{-1} \chi^* \cdot G_{cond}^{-1} u}{G_{cond}^{-1} u \cdot G_{cond}^{-1} u} . \quad (10)$$

The coil currents then are obtainable from Eqs. (9) and (10).

EF-Coil Positions

The coil model has two options: a) calculate the coil currents for fixed coil positions as described above; and b) calculate the coil currents for optimal coil positions as described below. During the coil optimization process, the positions of the coils are constrained to an axisymmetric surface as shown in Fig. 1 with only poloidal movement of the coils permitted to minimize the number of variables optimized. The optimal coil positions are determined by evaluating an object function, O , for each set of coil positions (and calculated currents) and then repositioning the coils to minimize the object function. The object function used herein consists of a term measuring the accuracy of the solution for the currents and a term measuring the square of the stored energy ($W_m \sim R_i I_i^2$) of the coil set and is given by

$$O = \sum_{i=1}^{N_s} \epsilon_i^4 + w \sum_{i=1}^{N_{EF}} (R_i I_i^2)^2 ,$$

where the solution accuracy at the i^{th} grid point on the surface is

$$\epsilon_i = 1 - (\chi - \chi_p^i)^{-1} \sum_{j=1}^{N_{EF}} G_X(\bar{x}_i, \bar{x}_j) I_j ; \quad (11)$$

R_i is the radial location of the i^{th} EF coil; and a weighting factor, w , is chosen during the optimization process to make the two sums in Eq. (11) approximately equal. The exponents on the accuracy and stored-energy terms in Eq. (11) were chosen to magnify the non-uniformity of ϵ_i along the plasma surface and of the currents from coil to coil, respectively.

The optimization algorithm is a fast version of the algorithms described in detail in the Ascent Phase of Ref. 7. The optimization process begins with a first-order method of steepest descent. Samples are taken at plus or minus one step size in each variable about a reference point to form first-order derivatives. These derivatives are then used to form a search vector along the gradient. The sampling direction in the derivative calculation is along the previously calculated gradient. The magnitude of the search vector is scaled to be four step sizes. Four samples are taken along the search vector outward from the reference point. If lower values of the object function are obtained for any of these samples, the lowest-value sample is adopted as the new reference point, and the steepest-descent method is repeated for the new reference point. If no lower-value sample is obtained, then an n -dimensional cube of dimension one step size is constructed with the reference point at one corner. The opposite corner is in the direction of lower O . All of the corners of the cube other than that occupied by the reference point are sampled up to a maximum of 2048 corners. If lower-value samples are obtained, then the minimum-value sample is adopted as the new reference point and the steepest-descent method is repeated. If no lower-value sample is obtained, then the step size is halved. The step size begins at 0.2 m. If the step size is reduced to below 1 mm, then the step size is reset to the initial value, and the steepest-descent method is repeated. Optimization is terminated upon reaching the minimum step size a second time.

During the optimization process, two constraints are active: one constraint prevents overlap of the coil cross sections, and a second constraint excludes coils from specified regions (e.g., regions reserved for beam lines). Some special handling of the object function then is required for infeasible samples that violate either of these constraints. Feasibility checks are made

Fig. 1. The steady-state flux surfaces calculated with the fixed-boundary equilibrium code JSOLVER plus flux surfaces calculated between the outermost surface produced by JSOLVER (dashed line) and the separatrix. The surface (long-short dashed line) upon which the EF coils are constrained during the optimization process is also shown. The EF-coil cross sections are sized using a current density of 15 MA/m² for the maximum current required during the start-up sequence. A TF coil and the supporting bucking cylinder are also shown.

in the optimization routine. For an infeasible sample, the object function is set to the value of the reference point (i.e., zero gradient) and the direction of the sampling in that variable is reversed.

EF-Coil Flux

Also of interest are the flux produced by the EF coils and the flux required by the plasma for startup. These flux calculations require the appropriate single-turn-equivalent currents and inductances. Since the N_{EF} individual EF coils with currents I_i need not have currents of the same sign (i.e., $I_i I_{i+1} < 0$), the following definition of the single-turn-equivalent EF-coil current is used:

$$I_{EF} \equiv \begin{cases} \sum_{i=1}^{N_{EF}} |I_i|, & \text{if } \sum_{i=1}^{N_{EF}} I_i \geq 0; \\ -\sum_{i=1}^{N_{EF}} |I_i|, & \text{otherwise.} \end{cases} \quad (12)$$

The single-turn-equivalent mutual inductance, $M_{EF,p}$, is defined to preserve the total energy stored between the EF-coil and plasma circuit elements,

$$M_{EF,p} \equiv I_{EF}^{-1} I_p^{-1} \sum_{i=1}^{N_{EF}} \sum_{j=1}^{N_p} G_x(\bar{x}_i, \bar{x}_j) I_i I_j. \quad (13)$$

The single-turn-equivalent self-inductance of the EF coils is $L_{EF} = \frac{1}{2} M_{EF,EF}$. The total toroidal plasma current is defined as

$$I_p \equiv -\sum_{i=1}^{N_p} I_i. \quad (14)$$

However, each coil within a coil set is simulated with an equally spaced 4×4 R-Z grid of equal-current hoops, so that $N_{EF}^* = 16 N_{EF}$. To avoid the singularity in $G_x(\bar{x}_i, \bar{x}_j)$ that arises for overlapping hoops, the coil in the i -series is treated differently from the same coil in the j -series. In the overlapping coil in just the i -series, the number of hoops in the vertical column in which the singularity occurs is doubled and the current per hoop is

halved. The single-turn-equivalent self-inductance of the plasma, L_p , can be calculated in the same way as the self-inductance of the EF coils. However, an equivalent, yet more economical ($N_p^{3/2}$ versus N_p^2 operations) method is used for calculating L_p , specifically,

$$L_p \equiv \frac{1}{2 I_p^2} \sum_{i=1}^{N_p} \left[\chi_p(\bar{x}_i) I_i + \sum_{j < i}^{N_p} G_x(\bar{x}_i, \bar{x}_j) I_i I_j \right]. \quad (15)$$

Equations (12)–(15) then permit the calculation of the flux required by the plasma for start-up, $L_p I_p$; the flux contribution of the EF coils to plasma start-up, $M_{EF,p} I_{EF}$; and the total stored magnetic energy, $W_M = \frac{1}{2} L_p I_p^2 + M_{EF,p} I_{EF} I_p + \frac{1}{2} L_{EF} I_{EF}^2$.

EF-Coil Forces

The forces acting on the EF coils are also calculated to complete the specification of the impact that the equilibrium has on machine design. The forces are obtained from $\vec{F} = I d\vec{l} \times \vec{B}$; where $\vec{F}$ is the force on an infinitesimal element, $d\vec{l}$, along the coil current path; I is the coil current; and $\vec{B}$ is the total magnetic field. For the hoop model of the EF coils, a force component on the n^{th} coil then is

$$F_n^m = 2\pi \sum_{i=1}^{16} \sum_{j=1}^{N_{EF}^* + N_p} R_i I_i G_{B_n}(\bar{x}_i, \bar{x}_j) I_j, \quad (16)$$

where $m = R, Z$ denotes either the radial or vertical component of the force and $k = Z, R$ denotes the vertical or radial field component, respectively; $\sum_{j=1}^{N_{EF}^* + N_p}$ denotes a sum over the 16 hoops used to simulate each of the N_{EF} EF coils plus the N_p hoops used to simulate the plasma; and $\sum_{i=1}^{16}$ denotes a sum over the 16 hoops used to simulate the n^{th} coil. The singularity in $G_{B_n}(\bar{x}_i, \bar{x}_j)$ that arises for overlapping hoops is avoided in the same way as for the case of the mutual-inductance calculation of Eq. (13).

RESULTS

Shown in Fig. 2 are a series of equilibria, calculated with JSOLVER, that depicts the ARIES-III start-up scenario. The final, high beta ($\beta_T = 0.239$) case in Fig. 2 is the steady-state operating point featured in Fig. 1. The EF-coil positions were determined for this high-beta case using the coil optimizer described above. The surface on which the EF-coils were constrained was chosen so that the EF coils did not interfere with the TF coils. The resulting EF-coil positions are given in Table I and the currents are given in Table II. The poloidal flux provided by this EF-coil set matches the external flux required by the equilibrium with an average error of 2.5×10^{-3} over the 51 surface grid points used in the equilibrium calculation. A vertical clearance of more than 2 m exists between EF coil 7 and the equatorial plane for neutral beam line penetrations. This EF-coil and plasma configuration has 15.2 MJ stored in poloidal magnetic fields. The plasma requires 441 Wb to establish the steady-state magnetic configuration, excluding resistive dissipation during start-up. Achieving the steady-state currents in the EF coils provides 389 Wb, leaving a deficit of 52 Wb to be supplied by other means.

Using fixed EF-coil positions (Table I), the EF-coil currents were determined for each equilibrium in the start-up sequence and are listed in Table II. As the plasma current increases, I_{EF}/I_p generally decreases because the coil positions minimize the stored energy only for the final equilibrium. Accompanying the increase in plasma current is a radially outward shift of the magnetic axis. This shift of the axis requires that the radial current center of the EF coils also shifts radially outward.

To make up the deficit of 52 Wb between what the plasma needs and what the EF coils provide with a purely inductive startup, the EF coils must be precocked with the currents calculated for the back bias (Table I), rather than starting at zero current as required by the equilibrium. Ramping the back bias currents to zero, while preserving the initial current distribution, would provide 52 Wb. The EF-coil current trajectory during start-up then would be a sum of the current trajectories determined from the start-up sequence of equilibria and from ramping the back bias currents to zero. Also shown in Fig. 2 is the ohmic flux distribution at back bias. Although no initiation constraints were imposed in the ARIES-III study on the coil design, the ohmic flux distribution displays the customary poloidal-field null near the vacuum vessel center and uniform flux within the vacuum vessel to within 1.7×10^{-3} , which peaks at the vacuum vessel wall.

Fig. 2. Flux surfaces for a sequence of equilibria simulating start-up, including the initial ($\beta_T = 0$) back bias. The EF coils are sized using a current density of 15 MA/m^2 . A positive current in the EF coils is denoted with an $\times$ in the coil cross section. The plasma current is negative. Also shown are the forces (denoted as arrows) acting on each EF coil.

The EF-coil design must accommodate the largest currents and forces that occur during the start-up sequence. The largest EF-coil currents generally occur at steady state, except for EF coils 2 and 3, which have the largest currents at low beta and back bias, respectively. The cross-section dimensions of square EF coils were determined from an overall current density of 15 MA/m^2 and the largest current in a coil during the start-up sequence. The largest force (1169 MN) that must be reacted occurs in EF coil 7 at steady state.

TABLE I EF-Coil Positions and Dimensions^(a)

Coil	R (m)	Z (m)	$\delta^{(b)}$ (m)
1	2.731	0.990	0.511
2	2.731	1.672	0.763
3	2.767	3.608	0.649
4	5.566	6.576	0.866
5	6.816	6.742	0.426
6	8.643	6.456	0.723
7	12.556	3.052	0.973

(a) Cross sections sized for current density of 15 MA/m^2 and the maximum coil current within startup equilibria sequence.

(b) Coil cross sections are assumed to be square.

TABLE II EF-Coil and Plasma Currents for Plasma at Various Values of Toroidal Beta in the Start-up Sequence

β_T	0.000 ^(a)	0.031	0.064	0.149	0.239
EF-Coil Current (MA):					
I_1	-3.922	1.846	0.310	-0.204	0.001
I_2	0.270	8.725	6.841	0.333	-0.901
I_3	-6.322	1.744	2.752	3.449	4.562
I_4	-3.394	-5.618	-6.439	-8.412	-11.246
I_5	1.804	1.477	1.199	1.539	2.716
I_6	-0.901	4.264	4.991	5.915	7.846
I_7	-0.198	6.221	8.261	11.813	14.195
I_{EF}	-33.622	59.792	61.585	63.329	82.935
Plasma Current (MA):					
I_p	0	-17.745	-19.823	-23.424	-28.859

(a) Back bias condition.

SUMMARY

As reactor designers push tokamak performance (in particular, beta), high-accuracy, fixed-boundary equilibrium codes will play a prominent role in reactor design. The use of these codes will necessitate algorithms to determine EF coils that satisfy equilibrium requirements and reactor design constraints on stored energy, coil interference, and penetrations. One such algorithm has been developed and reported herein. This algorithm satisfies the flux constraints to a tolerance of $\approx 2 \times 10^{-3}$ and the design constraints and also provides additional design information, such as forces and flux.

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