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STOP BANDWIDTHS OF NONLINEAR BEAM-BEAM RESONANCES

G. PARZEN

December 8, 1979

ACCELERATOR DEPARTMENT

BROOKHAVEN NATIONAL LABORATORY

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ABSTRACT

A general expression is given for the stop bandwidths, $\Delta\nu_N$, of nonlinear beam-beam resonances, which is expanded in powers of $y(s)$, the vertical beam orbit, and which is valid under certain assumptions regarding the orbits and charge distributions near the interaction regions. This result is applied to obtain results for the rms $\Delta\nu_N$ due to random vertical orbit errors, and due to random errors in β_y at the crossing points. Numerical results are given for the ISABELLE storage accelerator.

I. INTRODUCTION

Results are derived for stop bandwidths, $\Delta\nu_N$, of nonlinear resonances due to the beam-beam interaction of horizontally crossing beams. A general result is first given for $\Delta\nu_N$, which involves only the charge distributions ρ (x,y) and not the fields themselves, which is based on the assumptions that $\beta_y(s)$ can be regarded as constant in the interaction region, and that the beam paths near the crossing points may be curved vertically but are straight lines crossing at an angle α horizontally. Assuming that the charge distribution is Gaussian vertically, the result is expanded in powers of y(s), the vertical orbit near the crossing points.

These general results are applied to obtain results for the, rms $\Delta\nu_N$, due to random vertical orbit errors, and due random errors in β_y at the crossing points. Numerical results are given for the ISABELLE storage accelerator.

II. THEORY and GENERAL RESULTS

The half-width of the stop band of the nonlinear resonance $N\nu_y = q$ is given by the often derived formula[1-3]

$$\Delta\nu_N = \frac{F(N)}{2^{N-2}} \frac{1+\beta^2}{\beta} \frac{1}{4\pi} \int ds \frac{\beta_y}{B\rho} (\sqrt{\beta\epsilon})^{N-2} \frac{1}{(N-1)!} D_y^{N-1} E_y \exp(iN\phi), \qquad (2.1)$$

where E_y is the field acting on the test beam due to the other beam and $D_y^n \overset{Y}{=} \partial^n/\partial y^n$. The factor F(N) is often omitted and is given by

$$F(N) = 1 \quad , N = 2$$
$$= 0.9 \quad , N = 3$$
$$= 0.5 \quad , N = 4$$
$$= \pi^{(N-2)/2} , N \geq 5 \qquad (2.2)$$

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1. E. Keil, CERN Report CERN/ISR-TH/72-25 (1972)
2. P.M. Hanney & E. Keil, CERN Report CERN/ISR-TH/73-55 (1973)
3. G. Guignard, CERN Report CERN 78-11 (1978)

It is assumed in this note that the factors β_y and exp $(in\phi)$ do not vary rapidly and can be treated as constant and evaluated at the crossing point of the two beams. It can be shown[4] that under certain conditions

$$\int ds\ D_y^{n-1}\ E_y = 4\pi \int ds\ D_y^{n-2}\ \rho, \qquad (2.3)$$

where $\rho(x,y)$ is the charge density distribution of the beam creating the field E_y. The required conditions are that the position of the test beam relative to the other be given by

$$x = \alpha s,$$
$$y = y(s), \qquad (2.4)$$

where the beams are crossing in the horizontal plane and α is the angle between them which is assumed to be small. The result, Eq.(2.3) follows from the invariance of the interaction integrals when the test beam is displaced by an amount Δs in the longitudinal direction of the other beam. Equation (2.4) says that the test beam moves along a straight line horizontally, but its path can be curved vertically.

It will be assumed that $\rho(x,y)$ is Gaussian in the vertial direction and is given by

$$\rho(x,y) = \rho_1(x)\ \exp\ (-y^2/2\sigma_y^2)\ /\ (2\pi\sigma_y)^{\frac{1}{2}}. \qquad (2.5)$$

One thus finds for $\Delta\nu_N$

$$\Delta\nu_N = \Delta\nu \cdot \frac{f(N)\ e^{iN\phi}}{2^{(N-2)/2}\ (N-1)!}\ \alpha \int ds\ \rho_1(x)\ D_\mu^{N-2} e^{-\mu^2} \qquad (2.6)$$

$$\mu^2 = y^2/2\sigma_y,$$

$$\Delta\nu = \frac{1+\beta^2}{\beta}\ \frac{\beta_y}{B\rho}\ \frac{I}{v}\ \frac{1}{(2\pi)^{\frac{1}{2}}\ \sigma_y\alpha}$$

$$f(N) = 1, \qquad N = 2$$

$$f(N) = 0.9\ \pi^{\frac{1}{2}}, \qquad N = 3$$

$$f(N) = 0.5\ \pi, \qquad N = 4$$

$$f(N) = 1, \qquad N \geq 5,$$

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4. M. Cornacchia & G. Parzen, Forthcoming BNL Report (1979)

and $(\beta\epsilon/\pi)^{\frac{1}{2}} = 2\ \sigma_y$ was used. $\Delta\nu$ is the linear beam-beam ν-shift.

We will now expand the integrand in powers of y, and keep just the lowest terms. One can show that for n odd

$$D_y^n\, e^{-y^2} = e^{-y^2}\left[(2y)^n - \frac{n\,(n-1)}{1!}\,(2y)^{n-2} + \frac{n\,(n-1)\,(n-2)\,(n-3)}{2!}\,(2y)^{n-4}\ldots - \frac{n!}{(\,(n-1)/2)!}\,2y\right] \tag{2.7}$$

and for n even

$$D_y^n\, e^{-y^2} = e^{-y^2}\left[(2y)^n - \frac{n\,(n-1)}{1!}\,(2y)^{n-2} + \frac{n\,(n-1)\,(n-2)\,(n-3)}{2!}\,(2y)^{n-4}\ldots - \frac{n!/2}{(n/2-1)!}\,(2y)^2 + \frac{n!}{(n/2)!}\right]$$

Keeping just the lowest powers in y we find for <u>odd N</u>

$$\Delta\nu_N = \Delta\nu\,\frac{f(N)}{2^{(N-1)/2}}\;\frac{1}{(\,(N-1)/2)!}\,A, \tag{2.8}$$

$$A = \alpha\int ds\ \rho_1(x)\ y/\sigma_y \cdot \exp\,(in\phi).$$

<u>For even N</u>

$$\Delta\nu_N = \Delta\nu\,\frac{f(N)}{2^{(N-2)/2}}\;\frac{1}{[\,(N-2)/2]!}\;\frac{1}{N-1}\cdot A,$$

$$A = \alpha\int ds\ \rho_1(x)\left[1 - (N-1)\,\frac{y^2}{2\sigma_y^{\,2}}\right]\exp\,(in\phi).$$

ϕ is the phase, $\nu\,\phi = \int ds\ (1/\beta_y)$

If y(s) = 0 at the crossing points, then Eq. (2.8) gives nonzero results only for the even resonances, which is the case in the absence of vertical orbit errors.

III. $\Delta\nu_n$ DUE TO VERTICAL ORBIT ERRORS AT THE CROSSING POINTS

Vertical orbit errors at the crossing points make the beam-beam interactions different at the crossing points and generate resonances which lie close to the working line in ν_x, ν_y space. The results given by Eq. (2.8) have to summed over all the crossing points. In computing the integral over s, y(s) may be considered as constant and evaluated at the crossing points.

For N odd, the rms value of the stop band half-width is given by:

$$\Delta\nu_{N,\ rms} = \Delta\nu \frac{f(N)}{2^{(N-1)/2}\ [(N-1)/2]!} \cdot N_{cp}^{\frac{1}{2}}\ y_{rms}/2\sigma_y, \qquad (3.1)$$

where N_{cp} is the number crossing points and y_{rms} is the rms value of the vertical orbit error at the crossing points.

For N even, $\Delta\nu_N$ is proportional to y^2. The quantity A in Eq. (2.8) becomes

$$A = (N-1)\ \Sigma_k\ e^{iN\phi_k}\ y_k^2\ /\ 2\sigma_y^2,$$

where the sum over k is over all the crossing points.
For the moment it will be assumed that N is not a multiple of N_{cp} and $\Sigma_k \exp\ (iN\phi_k) = 0$. Then one can write

$$A = (N-1)\ \Sigma_k\ e^{iN\phi_k}\ (y_k^2 - \langle y^2\rangle)\ /\ 2\sigma_y^2,$$

where $\langle y^2\rangle$ is the average value of y_k^2, and the average value of A^2 is given by

$$\langle A^2 \rangle = \frac{(N-1)^2}{4\sigma_y^4}\ \Sigma_{kk'} \langle\ e^{in(\phi_k - \phi_{k'})}\ [y_k^2 - \langle y^2\rangle]\ [y_{k'}^2 - \langle y^2\rangle]\rangle.$$

Since $[y_k^2 - \langle y^2\rangle]$ and $[y_{k'}^2 - \langle y^2\rangle]$ are uncorrelated,

$$\langle A^2\rangle = \frac{(N-1)^2}{4\sigma_y^4} \sum_k \langle [y_k^2 - \langle y^2\rangle\ ]^2\rangle$$

$$= \frac{(N-1)^2}{4\sigma_y^4} \cdot N_{cp}\ [\ \langle y^4\rangle - \langle y^2\rangle^2],$$

and

$$A_{rms} = (N-1) \cdot N_{cp}^{\frac{1}{2}} \left[\langle y^4 \rangle - \langle y^2 \rangle^2 \right]^{\frac{1}{2}} / 2\sigma_y^2$$

Now $\langle y^2 \rangle = y_{rms}^2$ and for a Gaussian distribution $\langle y^4 \rangle = 3\, y_{rms}^2$. Thus,

$$A_{rms} = (N-1)\, N_{cp}^{\frac{1}{2}} \cdot 1.41\, y_{rms}^2 .$$

For N even, and N not a multiple of N_{cp}, the rms value of $\Delta\nu_N$ is given by

$$\Delta\nu_{N,rms} = \Delta\nu \frac{f(N)}{2^{(N-2)/2}} \frac{1.41}{[(N-2)/2]!} N_{cp}^{\frac{1}{2}}\, y_{rms}^2 \;/\; 2\sigma_y^2 \qquad (3.2)$$

If N is even, and N is a multiple of N_{cp}, then $\Sigma_k \exp(iN\phi_k) = N_{cp}$, and $\Delta\nu_N$ has a nonzero average value.

$$\langle \Delta\nu_N \rangle = \Delta\nu \frac{f(N)}{2^{(N-2)/2}} \cdot N_{cp}\, y_{rms}^2 \;/2\sigma_y^2, \qquad (3.3)$$

And around this average value, $\Delta\nu_N$ has a statistical fluctuation whose rms value $[\Delta\nu_N - \langle \Delta\nu_N \rangle]_{rms}$, is given by Eq. (3.2).

For this case, one can introduce a pseudo $\Delta\nu_{N,rms}$ defined by the requirement that 2 $\Delta\nu_{N,rms}$ gives roughly the peak $\Delta\nu_N$, and the pseudo $\Delta\nu_{N,rms}$ is given by

$$\Delta\nu_{N,rms} = \Delta\nu \frac{f(N)}{2^{(N-2)/2}} \frac{(0.5\, N_{cp} + 1.41\, N_{cp}^{\frac{1}{2}})}{[(N-2)/2]!}\, y_{rms}^2 \;/2\sigma_y^2,$$

where N is even and a multiple of N_{cp}.

IV. $\Delta\nu_N$ DUE TO RANDOM ERRORS IN β_y AT THE CROSSING POINTS

Random errors in β_y at the crossing point can excite resonances close to the working line. The linear beam-beam ν-shift, $\Delta\nu$, will be different at each crossing point. If one assumes that σ_y is due entirely to betatron oscillation, then $\Delta\nu \propto \beta^{\frac{1}{2}}$ and

$$\frac{\Delta\nu}{\nu} = \frac{1}{2} \frac{\Delta\beta_y}{\beta_y}. \qquad (4.1)$$

Only even resonance are generated and from Eq. (2.8), the rms value of the stop band width is given by

$$\Delta\nu_{N,rms} = \Delta\nu \frac{f(N)}{2^{(N-2)/2}} \frac{1}{[(N-2)/2]!} \frac{1}{N-1} N_{cp}^{\frac{1}{2}} \frac{1}{2} \left(\frac{\Delta\beta_y}{\beta_y}\right)_{rms}, \tag{4.2}$$

where $(\Delta\beta_y/\beta_y)_{rms}$ is the rms value of the error in the vertical β-function at the crossing points.

V. NUMERICAL RESULTS FOR ISABELLE

The results given in Sections III and IV can be applied to compute the stop-band half widths, $\Delta\nu_N$ due to random vertical orbit errors, and random errors in β_y at the crossing points for the ISABELLE storage accelerator.

Results are given below at two energies, 30 GeV and 400 GeV. For the even order resonance, the contributions due to random vertical orbit errors, $\Delta\nu_{N,y}$ and due to random errors in β_y, $\Delta\nu_{N,\beta}$ are given separately and $\Delta\nu_N$ is the rms sum of these two contributions. For the odd order resonance, $\Delta\nu_N$ is due to the vertical orbit errors only. It was assumed that the rms error in the vertical orbit at the crossing points is the same as 30 GeV and 400 GeV and is y_{rms} = 0.05 mm. It may be possible to correct the orbit better at 400 GeV because of the smaller size of the beam.

<u>30 GeV</u>

α = 0.011, I = 8 A, β_y = 7.5 m, σ_y = 0.75 mm, y_{rms} = 0.05 mm,

N_{cp} = 6, $(\Delta\beta_y/\beta_y)_{rms}$ = 0.045, $\Delta\nu$ = 0.0058.

N	$\Delta\nu_N$	N	$\Delta\nu_{N,\beta}$	$\Delta\nu_{N,y}$	$\Delta\nu_N$
		2	3.20 E-4	4.46 E-5	3.23 E-4
3	7.55 E-4	4	8.37 E-5	3.47 E-5	9.06 E-5
5	1.18 E-4	6	7.99 E-6	1.03 E-5	1.31 E-5
7	1.97 E-5	8	9.51 E-7	9.21 E-7	1.32 E-6
9	2.47 E-6	10	9.25 E-8	1.15 E-7	1.48 E-7
11	2.47 E-7	12	7.57 E-9	2.15 E-8	2.28 E-8
13	2.05 E-8	14	5.33 E-10	9.57 E-10	1.09 E-9
15	1.46 E-9	16	3.30 E-11	6.85 E-11	7.60 E-11
17	9.18 E-11	18	1.82 E-12	8.03 E-12	8.23 E-12
19	5.10 E-12	20	9.05 E-14	2.38 E-13	2.54 E-13

400 GeV

$\alpha = 0.011$, $I = 8$ A, $\beta_y = 7.5$ m, $\sigma_y = 0.21$ mm, $y_{rms} = 0.05$ mm,

$N_{cp} = 6$, $(\Delta\beta_y/\beta_y)_{rms} = 0.045$, $\Delta\nu = 0.00156$.

N	$\Delta\nu_N$	N	$\Delta\nu_{N,\beta}$	$\Delta\nu_{N,y}$	$\Delta\nu_N$
		2	8.87 E-5	1.61 E-4	1.84 E-4
3	7.24 E-4	4	2.25 E-5	1.19 E-4	1.21 E-4
5	1.14 E-4	6	2.15 E-6	3.55 E-5	3.56 E-5
7	1.89 E-5	8	2.55 E-7	3.16 E-6	3.16 E-6
9	2.37 E-6	10	2.48 E-8	3.94 E-7	3.95 E-7
11	2.37 E-7	12	2.03 E-9	7.39 E-8	7.40 E-8
13	1.97 E-8	14	1.43 E-10	3.29 E-9	3.29 E-9
15	1.41 E-9	16	8.87 E-12	2.35 E-12	2.35 E-10
17	8.80 E-11	18	4.89 E-13	2.75 E-11	2.75 E-11
19	4.89 E-12	20	2.43 E-14	8.15 E-13	8.16 E-13