Electron Cyclotron Heating in Tokamaks

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ABSTRACT

Tokamaks may be efficiently heated at electron cyclotron resonance by launching of the extraordinary wave from the inside of the torus with a simple waveguide aperture. For typical tokamak parameters, 90% of the incident energy is absorbed in one traversal of the resonant surface. There is an effective maximum density for linear heating at the point where the plasma frequency is equal to the cyclotron frequency. The bulk of the plasma electrons is heated, increasing the perpendicular energy with no tendency to accelerate a high energy tail of the velocity distribution. The ordinary wave may be launched from outside the torus with somewhat smaller heating efficiency. Heating rates and velocity diffusion coefficients are presented.

I. INTRODUCTION

In a series of experiments on the TM-3 tokamak at the I. V. Kurchatov Institute of Atomic Energy in Moscow, Alikaev and coworkers have shown that linear electron cyclotron heating is an effective means of heating electrons in high density plasmas.^{1,2} They obtained evidence for effective absorption of the incident microwave power over a wide range of plasma parameters, including experiments with no upper hybrid resonance in the plasma. In addition, very little electron heating was observed when the upper hybrid resonance was located within the plasma volume but the electron cyclotron resonance was not. The temperature increase obtained with cyclotron heating was comparable to that obtained with ohmic heating. The entire distribution of electrons was heated with no observable acceleration of a hot electron tail.

In contrast, experiments on the Spherator at the Princeton Plasma Physics Laboratory³ indicated that a nonlinear interaction was responsible for heating near the upper hybrid resonance. This interaction had a threshold in incident power and produced the typical sideband spectrum of parametric instabilities. The plasma density was high; that is, the plasma frequency, $\omega_p = (4\pi ne^2/m)^{1/2}$, was about equal to the electron cyclotron frequency, $\Omega = eB/mc$. The electron temperature was very low, $T \approx 1$ eV, in the Spherator experiment. From these and other similar experiments with high density plasmas, one is led to a tentative conclusion: nonlinear interactions occur near the upper hybrid resonance when the electron temperature is low, and linear absorption occurs at cyclotron resonance when the electron temperature is initially high. This is also the conclusion of the present calculation.

For low density plasmas, electron cyclotron heating has been successfully used for many years. In the ELMO mirror machine and the ELMO Bumpy Torus (EBT) at Oak Ridge National Laboratory (ORNL), electron temperatures of $T \approx 700$ keV have been achieved in steady state with quite modest power inputs.⁴⁻⁶ The linear theories of cyclotron heating at low density have accounted quantitatively for the observed heating rates.^{7,8} In these theories, the dispersion of the incident wave was neglected and the electric fields in the plasma were taken to be homogeneous and isotropic. These assumptions are reasonable for large multimode cavities containing low density plasma, with only a fraction of the wave energy absorbed during each traversal of a resonant surface by a wave, but are not valid at high densities.

In the present calculation, these faults of the low density theories are corrected; the overall treatment is similar in that the emphasis is on electron heating rather than on wave damping. In Sect. II, wave dispersion and polarization are discussed in cold plasma theory. Two results are particularly relevant. First, the extraordinary wave must be launched from the inside of the torus, while the ordinary wave may be launched from any point. Second, the linear combination of electric fields that accelerates electrons, which is $E_x - iE_y$ in a local coordinate system with the static magnetic field in the z direction, vanishes at cyclotron resonance. The conclusion is that linear cyclotron heating can only occur with finite temperatures that will broaden the spatial cyclotron resonance or at low density.

In Sect. III, a Fokker-Planck equation is presented for the electron velocity distribution driven by the external fields. The fields are

evaluated from the warm plasma dispersion. The electron heating is found to be effective for the entire velocity distribution, despite an essentially linear dependence on temperature.

In Sect. IV, numerical results are given for waves launched from a waveguide aperture. Heating rates in a tokamak are found to depend essentially on the ratio ω_p/Ω , on the ratio of wavelength to the major radius of the torus, and on the electron temperature. The heating becomes very small in the parameter range where $\omega_p/\Omega > 1$, although the extraordinary wave propagates for all densities such that $\omega_p/\Omega < 2^{1/2}$.

II. DISPERSION, PROPAGATION, AND POLARIZATION

In a plasma the propagation of electromagnetic waves is determined by the equation

$$\vec{n} \times (\vec{n} \times \vec{E}) + \frac{\vec{n}}{c} \cdot \vec{E} = 0, \quad (1)$$

where $\vec{n} = \vec{k}c/\omega$ is the vector index of refraction. With the static magnetic field in the z direction, the nonzero components of the dielectric tensor in a cold electron plasma are

$$\begin{aligned} \epsilon_{xx} = \epsilon_{yy} &= 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2}, \\ \epsilon_{xy} = -\epsilon_{yx} &= i \frac{\omega_p^2}{\omega^2 - \Omega^2} \frac{\Omega}{\omega}, \end{aligned} \quad (2)$$

and

$$\epsilon_{zz} = 1 - \frac{\omega_p^2}{\omega^2}.$$

The ion contribution to dispersion is negligible in the frequency range near cyclotron resonance. The absolute value of the electron cyclotron

frequency is Ω . The equation is to be interpreted as the first term of an eikonal expansion, with $\vec{n}$, $\vec{E}$, and $\vec{c}$ taken as slowly varying functions of space, while the frequency is real and constant.

The two solutions of the determinantal equation, the extraordinary (X) mode and the ordinary (O) mode, have properties best displayed on a Clemmow-Mullaly-Allis (CMA) diagram,⁹ reproduced in Fig. 1. For the present analysis the salient points are:

(1) The X wave may enter the inside edge of a plasma in a toroidal magnetic field (a point $\omega_p^2/\omega^2 = 0$, $\Omega^2/\omega^2 > 1$) and propagate directly through cyclotron resonance (the line $\omega = \Omega$) to upper hybrid resonance [the line $\omega = (\omega_p^2 + \Omega^2)^{1/2}$]. This preferred path is shown by an arrow in Fig. 1. Cyclotron resonance may always be reached for local densities such that $\omega_p^2 \leq 2\Omega^2$, which is the high density cutoff for the X mode.

(2) When launched from the outside of a torus (a point $\omega_p^2/\omega^2 = 0$, $\Omega^2/\omega^2 < 1$), the X wave meets a cutoff before reaching upper hybrid resonance or cyclotron resonance.

(3) The O wave meets no plasma cutoffs and only cyclotron resonance in the entire region with $\omega_p^2/\omega^2 < 1$; this wave may be launched from any point on the plasma surface.

Figure 2 shows a cross section of a tokamak with a toroidal magnetic field and a parabolic density profile. The locations of the upper hybrid resonant surface, the cutoff surface for the X mode, and the cyclotron resonant surface are shown.

In addition, the cold plasma dispersion relation may be used to show directly that:

Fig. 1.

Fig. 2.

(4) At cyclotron resonance ($\omega = \Omega$), the dielectric tensor components ϵ_{xx} , ϵ_{xy} , ϵ_{yy} , and ϵ_{yx} become infinite, but the index of refraction and the electric fields remain finite and are smoothly varying through this surface.

(5) The electric field combination $E^- = E_x - iE_y$ vanishes at cyclotron resonance in a cold, dense plasma,¹⁰ because from Eq. (1) one finds that $(E_x - iE_y)\omega_p^2/(\omega^2 - \Omega^2)$ is finite at this surface. This field combination accelerates electrons at cyclotron resonance, so cyclotron heating can only occur when the cyclotron resonance is broadened by finite electron temperature. This effect is analyzed in Sec. IV. These conclusions are illustrated in Fig. 3, which shows the electric field components and the index of refraction n_x for the X wave launched into a plane-stratified plasma with a magnetic field decreasing in the x direction, $B_z = B_0 R_0 / (R_0 + x)$. The plasma density is parabolic about $x = 0$. This configuration approximates the principal plane of a tokamak of major radius R_0 . In this plane the parallel index of refraction n_z is constant. Field ratios are determined from Eq. (1), and field magnitudes are determined by requiring that the component of the Poynting vector in the x direction be constant, with absorption neglected. Rotational transform is not included in this model.

(6) The wave energy does not propagate very far along the static magnetic field lines before crossing cyclotron resonance. Several ray, or Poynting vector, trajectories in the x-z plane for the X wave are plotted in Fig. 4. The central density is less than the critical density, $\omega_{p0}^2 < \omega^2$. The ray reaches the resonant surface at $x = 0$ within a short axial distance even when the wave is launched at grazing incidence with $n_z = 1$.

Fig. 3.

Fig. 4.

The direction of $\vec{k}$ or the wave velocity is indicated by the short arrows. The wave vector is perpendicular to the ray vector at the upper hybrid resonance. A focusing effect is seen with the rays which were launched parallel to B making an antiparallel approach to the upper hybrid surface.

(7) For densities above the critical density, some X wave energy is reflected, with rays near perpendicular incidence propagating to the center. The case with $\omega_{po}^2 > \omega^2$ is illustrated in Fig. 5 for the X wave. This ray tracing technique has also been applied to the O wave. As expected, the rays always propagate through cyclotron resonance when $\omega_{po}^2 < \omega^2$. A three-dimensional analysis of the ray trajectories in tokamak geometry has been made and agrees quantitatively with these results.¹¹

III. ELECTRON HEATING RATES

A quasilinear equation for the electron velocity distribution driven by external electromagnetic fields may be derived by iteration of the Vlasov equation.¹² This equation has the Fokker-Planck form

$$\begin{aligned} \frac{\partial F}{\partial t} = & \pi \left(\frac{e}{2m\omega} \right)^2 \sum_{\text{modes}} \sum_{N=-\infty}^{\infty} \left\{ \frac{1}{v_1} \frac{\partial}{\partial v_1} \right. \\ & \times \left[N\Omega |A|^2 \delta(\omega - N\Omega - k_z v_z) \left(\frac{N\Omega}{v_1} \frac{\partial F}{\partial v_1} + k_z \frac{\partial F}{\partial v_z} \right) \right] \\ & \left. + \frac{\partial}{\partial v_z} \left[k_z |A|^2 \delta(\omega - N\Omega - k_z v_z) \left(\frac{N\Omega}{v_1} \frac{\partial F}{\partial v_1} + k_z \frac{\partial F}{\partial v_z} \right) \right] \right\}, \end{aligned} \quad (3)$$

with

$$A = v_1 E^- e^{i\alpha} J_{N-1} + v_1 E^+ e^{-i\alpha} J_{N+1} + 2v_z E_z J_N. \quad (4)$$

The argument of the Bessel functions, J_N , is $k_1 v_1 / \Omega$; α is the azimuthal angle of the $\vec{k}$ vector; and $E^\pm = E_x \pm iE_y$. The electron cyclotron

Fig. 5.

frequency Ω is positive, and terms with negative N and negative ω are to be included.

The distribution F may vary slowly in space and time, but on a time scale much longer than the cyclotron period. The interaction occurs on a resonant surface, with the exact position determined by the Doppler shift from parallel motion. The phase of electrons passing through this surface is taken to be random. This random element may be the result of electrons drifting through resonance in a collisionless plasma or the result of collisions. For example, in a toroidal magnetic field without rotational transform, an electron drifts on a surface of constant magnetic field and accelerates continuously. However, a collisional randomization is sufficient to produce velocity diffusion at the rate of Eq. (3).

Strictly, the orbits of electrons should include the slow drift velocity and the rotational transform of the magnetic field. The delta function in Eq. (3) should have a term $\vec{k} \cdot \vec{v}_d$ instead of $k_z v_z$. However, because the sum over modes of the microwave field contains an integral over a broad spectrum in k_z , these effects are completely negligible. In a tokamak, the energy gained by electrons on a millisecond time scale is spread over the drift surface on a microsecond time scale, so that the energy may be considered to be deposited uniformly on the drift surface. For these frequencies, there is little distinction between trapped and untrapped electrons in the heating process. In a collisionless plasma, the velocity diffusion will affect the number and the velocity distribution of the trapped electrons, but this aspect of the problem is not treated here.

Cyclotron heating rates are comparable to ohmic heating rates, and in present-day tokamaks collision frequencies are large enough to maintain a

Maxwellian electron distribution with a local temperature $T(\vec{r})$ and density $n(\vec{r})$. The perpendicular wavelengths are always much larger than the electron cyclotron radius and smaller than scale lengths, so a small argument expansion may be used for the Bessel functions. A plane-stratified model of the plasma is used in which density, temperature, and the magnetic field vary with x (this is appropriate for radiation in the midplane of the torus). The magnetic field is taken to be

$$B_z = B_{T0} R_0 / (R_0 + x) = B_T (R - x) / R, \quad (5)$$

where B_T is the toroidal magnetic field at cyclotron resonance and

$$\Omega(x) = \omega(1 - x/R). \quad (6)$$

Inclusion of the rotational transform will only slightly change the shape of the resonant surface.

With these assumptions the temporal evolution of the electron distribution is

$$\begin{aligned} \frac{\partial F}{\partial t} = & -\frac{\pi e^2 n(x)}{4m\omega T(x)} \left[\frac{m}{2\pi T(x)} \right]^{3/2} \sum_{\text{modes}} \left\{ \frac{1}{v_1} \frac{\partial}{\partial v_1} \right. \\ & \times \left[\Omega(x) |A|^2 \delta(\omega x/R - k_z v_z) \exp(-mv^2/2T) \right] \\ & \left. + \frac{\partial}{\partial v_z} \left[k_z |A|^2 \delta(\omega x/R - k_z v_z) \exp(-mv^2/2T) \right] \right\}, \quad (7) \end{aligned}$$

with

$$A = v_1 \left[E^- + (v_z k_x E_z / \Omega) \right] \quad (8)$$

and $\vec{k} = (k_x, 0, k_z)$. Only the terms with $N = \pm 1$ are included for both positive and negative frequencies. One concludes directly that the perpendicular heating occurs uniformly for all velocities, while the parallel

heating rate is proportional to the square of the perpendicular velocity and so is much smaller. Runaway electrons are not selectively accelerated; an electron with large v_z is heated at the same rate as one with small v_z , and the Doppler shift only changes the location of the resonant surface. These heating rates agree with those calculated by more primitive means.⁸

The perpendicular and parallel heating rates per unit volume are found by integrating over the velocities

$$\frac{d^2 W_{\perp}}{dt dV} = \sum_{\text{modes}} \frac{\omega_p^2}{8} \frac{\Omega}{\omega^2} \frac{R}{\Delta} \frac{\exp(-x^2/2\Delta^2)}{\sqrt{2\pi}} \left| E^- + \frac{n_x}{n_z} \frac{\omega}{\Omega} \frac{x}{R} E_z \right|^2 \quad (9)$$

and

$$\frac{d^2 W_{\parallel}}{dt dV} = \sum_{\text{modes}} \frac{\omega_p^2}{8\omega} \frac{x}{\Delta} \frac{\exp(-x^2/2\Delta^2)}{\sqrt{2\pi}} \left| E^- + \frac{n_x}{n_z} \frac{\omega}{\Omega} \frac{x}{R} E_z \right|^2, \quad (10)$$

where the spatial width of the resonance is

$$\Delta = (k_z^2 R^2 T / m \omega^2)^{1/2} = n_z R (T / mc^2)^{1/2}. \quad (11)$$

For a tokamak with $T = 1$ keV and $R = 1$ m and for a ray with $n_z = 0.5$, one finds $\Delta = 2.2$ cm, indicating that the energy is deposited close to the resonant surface. Equation (10) also indicates that the local parallel heating rate is smaller than the perpendicular heating rate by a factor x/R , and it is asymmetric about $x = 0$. This asymmetry is produced by the Doppler shift; it is washed out by the poloidal drift motion of electrons in a tokamak. There is also a small axial momentum transfer to electrons, which is proportional to k_z and is ordinarily negligible.

IV. HEATING IN A WARM PLASMA

Finite temperature effects are included in the dielectric tensor in order to find the electric fields at resonance. Finite cyclotron radius effects are included to first order in the temperature for those terms that are large near the fundamental resonance. The form of the characteristic denominator, $\omega - \Omega - k_z v_z = \omega x/R - k_z v_z$, results in the appearance of the characteristic plasma dispersion function¹³ $Z(x/\Delta^{1/2})$ and its derivative Z' in the dielectric tensor. The appropriate tensor components with $n_y = 0$ are:

$$\begin{aligned}
 \epsilon_{xx} &= \epsilon_{yy} = 1 - \frac{\omega_p^2}{2\omega^2} \left(\frac{\omega}{\omega + \Omega} - \frac{R}{\Delta^{1/2}} Z \right), \\
 \epsilon_{xy} &= -\epsilon_{yx} = \frac{-i\omega_p^2}{2\omega^2} \left(\frac{\omega}{\omega + \Omega} + \frac{R}{\Delta^{1/2}} Z \right), \\
 \epsilon_{xz} &= \epsilon_{zx} = \frac{-\omega_p^2}{4\Omega\omega} \frac{n_x}{n_z} Z', \\
 \epsilon_{yz} &= -\epsilon_{zy} = i\epsilon_{xz}, \\
 \epsilon_{zz} &= 1 - \frac{\omega_p^2}{\omega^2} \left(1 + \frac{\omega^2}{4\Omega^2} \frac{n_x^2}{n_z^2} \frac{x}{R} Z' \right).
 \end{aligned} \tag{12}$$

The dispersion functions Z and Z' are never larger than 1, and for standard tokamaks $\omega_p^2/\Omega^2 \sim 1$. Near resonance the components ϵ_{xx} and ϵ_{xy} are large (of order R/Δ), while the other components are of order one. As discussed in Sec. II, the electric fields change smoothly through cyclotron resonance, while $E^- = E_x - iE_y$ becomes small, of order $1/\epsilon_{xx} \sim \Delta/R$. The criterion that this effect be negligible is that $\epsilon_{xx} \sim 1$ or, in general,

$$\omega_p^2/\Omega^2 \ll 2^{3/2}\Delta/R = 2^{3/2}n_z(T/mc^2)^{1/2}, \quad (13)$$

which defines a "low density" plasma for cyclotron heating. For perpendicular propagation the resonance broadening depends on collisions rather than on the Doppler shift; however, a finite microwave source produces a broad spectrum in n_z and the Doppler broadening dominates. With an average $n_z = 0.5$, $T = 1$ keV, and $B = 20$ kG, the dispersion can be neglected for $n \ll 2.5 \times 10^{12} \text{ cm}^{-3}$.

Some mirror plasmas with cyclotron heating⁴ are "low density" by this criterion, but the high density approximation, in which $\omega_p \approx \Omega$ and $R/\Delta \gg 1$, is necessary for tokamaks.

A. The heating rate

From Eq. (1), one finds

$$E_z = - \left[(\epsilon_{xz} + n_x n_z) E^- + i n_x n_z E_y \right] / (\epsilon_{zz} - n_x^2) \quad (14)$$

and

$$E^- = -i \left[(\epsilon_{xx} - i\epsilon_{xy} - n_z^2) E_y - i(\epsilon_{xz} + n_x n_z) E_z \right] / (\epsilon_{xx} - n_z^2). \quad (15)$$

Because $\epsilon_{xx} - i\epsilon_{xy}$ is of order one and ϵ_{xx} is of order R/Δ , E^- is smaller than E_y or E_z by a factor Δ/R ; E^- may be neglected in Eq. (14), and

$$E^- + \frac{n_x}{n_z} \frac{x}{R} E_z \approx -i E_y \left[\frac{(\epsilon_{xx} - i\epsilon_{xy} - n_z^2)}{(\epsilon_{zz} - n_x^2)} + n_x^2 \frac{x}{R} \frac{(\epsilon_{xx} - n_z^2)}{(\epsilon_{zz} - n_x^2)} \right] / (\epsilon_{xx} - n_z^2) \quad (16)$$

Because $Z'(\xi) = -2[1 + \xi Z(\xi)]$, one finds $n_x^2 \epsilon_{xx} x/R - n_x n_z \epsilon_{xz} \approx -n_x^2 \omega^2 / 2\omega^2$, where terms of order Δ/R are neglected. The only large term is in the denominator, so to order Δ/R one has

$$E_y + \frac{n_x}{n_z} \frac{x}{R} E_z = \frac{-iE_y 2^{3/2} \Delta [(1 - \alpha/2 - n_z^2)(1 - \alpha) - n_x^2]}{\alpha R Z (1 - \alpha - n_x^2)}, \quad (17)$$

where $\alpha = \omega_p^2/\omega^2 = \omega_p^2/\Omega^2$. Because

$$\text{Im}[1/Z(x/\Delta 2^{1/2})] = \frac{-\pi^{1/2} \exp(-x^2/2\Delta^2)}{|Z(x/\Delta\sqrt{2})|^2}, \quad (18)$$

the perpendicular heating rate of Eq. (9) is

$$\frac{d^2 W_\perp}{dt dV} = \sum_{\text{modes}} \frac{\omega}{4\pi\alpha} \frac{\gamma^{1/2} \Delta}{R} |E_y|^2 \left[\frac{(1 - \alpha/2 - n_z^2)(1 - \alpha) - n_x^2}{1 - \alpha - n_x^2} \right]^2 \text{Im}(1/Z). \quad (19)$$

The perpendicular index of refraction near resonance is

$$n_x^2 = \left\{ 3 - 2\alpha - n_z^2 \pm \left[(1 + n_z^2)^2 - 4\alpha n_z^2 \right]^{1/2} \right\} / 2 \quad (20)$$

to lowest order and is real. The $\pm$ sign refers to the X(+) or O(-) mode.

The heating rate of Eq. (19) is the principal result of this calculation. The shape of the resonance line is illustrated in Fig. 6, in which $\text{Im}(Z)$ is plotted for low density plasmas and $\text{Im}(1/Z)$ for high density plasmas. The magnitude and width of the two functions are similar, and absorption occurs within a distance of order Δ from resonance.

B. Wave attenuation

For an energy flux $\vec{S}$ in a plane plasma, one has

$$\nabla \cdot \vec{S} = \frac{\partial S_x}{\partial x} = -\frac{d^2 W}{dt dV}, \quad (21)$$

Fig. 6.

where

$$S_x = \frac{c}{8\pi} \operatorname{Re} \left(\vec{E} \times \vec{B}^* + \frac{1}{2} \vec{E}^* \cdot \frac{\partial \vec{E}}{\partial n} \cdot \vec{E} \right)_x$$

$$\approx \sum_{\text{modes}} \frac{c}{8\pi} n_x |E_y|^2 \left[1 + \frac{(1 - \alpha) n_z^2}{(1 - \alpha - n_x^2)^2} \right] \quad (22)$$

near resonance.

By assuming that the energy associated with each wave vector is conserved, one finds for real $\vec{k}$

$$\frac{1}{S_x(x, \vec{k})} \frac{\partial S_x(x, \vec{k})}{\partial x} = \frac{2\omega}{c} \frac{2^{1/2} \Delta}{R} \frac{1}{\alpha n_x} \operatorname{Im}(1/Z)$$

$$\times \frac{[(1 - \alpha/2 - n_z^2)(1 - \alpha) - n_x^2]^2}{[(1 - \alpha - n_x^2)^2 + (1 - \alpha) n_z^2]} \quad (23)$$

This expression is equivalent to the imaginary part of $-2k_x$ in a wave damping treatment.

For the representation with total index of refraction n and angle θ , so that $n_x = n \sin\theta$ and $n_z = n \cos\theta$, one finds

$$\operatorname{Im}(n) = (2\pi T/mc^2)^{1/2} \frac{\exp(-x^2/2\Delta^2)}{|Z|^2} \frac{\cos\theta}{\alpha} \frac{M^2}{PQ}, \quad (24)$$

with

$$M = (1 - \alpha)(1 - \alpha/2 - n^2 \cos^2\theta) - n^2 \sin^2\theta,$$

$$P = n^2 \sin^2\theta + \alpha - 1, \quad (25)$$

and $Q = 2n^2 \sin^2\theta + 2\alpha - 2 - \sin^2\theta$.

For easy comparison with the results of Akhiezer¹⁴ and Litvak,¹⁵ this result can be laboriously transformed by using the dispersion relation.

One finds

$$\begin{aligned}
\frac{M^2}{P} = & n^4(1 - \alpha + \frac{7\alpha \sin^2\theta}{4}) \\
& - n^2[(1 - \alpha)(1 - \frac{\alpha}{4})(1 + \cos^2\theta) \\
& + (1 + \alpha)(1 - \frac{\alpha}{2}) \sin^2\theta \\
& - \frac{\alpha^2}{4} (1 + \cos^2\theta) \tan^2\theta] \\
& + (1 - \alpha)(1 - \frac{\alpha}{2}) - \frac{\alpha^2}{4} (2 - \alpha) \tan^2\theta .
\end{aligned} \tag{26}$$

By contour integration one finds

$$\int_{-\infty}^{\infty} dx \operatorname{Im}[1/Z(x/\Delta 2^{1/2})] = -\pi \Delta 2^{-1/2} . \tag{27}$$

Because Z has no zeros in the upper half of the complex plane, the integral is converted to an integral over a large half circle in the upper half plane. Along this contour, the first two terms of the large argument expansion give this result. Whenever the line width Δ is small compared to the size of the plasma, the fraction of the energy absorbed as a ray passes through the resonant surface in the high density limit is

$$\begin{aligned}
f = & 1 - \exp \left\{ -\frac{(2\pi)^2}{\alpha} \frac{R}{\lambda} \frac{n_z^2}{n_x} \frac{T}{mc^2} \right. \\
& \times \left. \frac{[(1 - \alpha/2 - n_z^2)(1 - \alpha) - n_x^2]^2}{[(1 - \alpha - n_x^2)^2 + (1 - \alpha)n_z^2]} \right\},
\end{aligned} \tag{28}$$

where λ is the wavelength. The damping coefficient is proportional to the temperature and to the major radius of the torus, as expected. The density dependence is not easy to untangle and is discussed in Sec. V.

The damping rate of Eqs. (24) and (26) is identical to the results of Akhiezer,¹⁴ Litvak,¹⁵ Stepanov,¹⁶ and Alikaev¹⁷ (except for some minor

misprints in these references); the form of Eqs. (23) and (28) is more convenient for calculations. The heating rate of Eq. (19) may also be obtained from the conductivity tensor.

V. RESULTS AND CONCLUSIONS

A. Almost perpendicular incidence

The result of Eq. (28) is particularly advantageous in evaluating limiting cases. Near perpendicular incidence, for small n_z , one finds the fraction of energy absorbed from the X mode to be

$$f_x \approx 1 - \exp \left[-\pi^2 \frac{T}{mc^2} \frac{R}{\lambda} n_z^2 \frac{(2 - \alpha)^2 (1 + \alpha)^2}{\alpha (2 - \alpha)^{1/2}} \right], \quad (29)$$

so that f_x is zero for $n_z = 0$, but rapidly increases to unity for finite n_z . In the same limit, for the O mode

$$f_o \approx 1 - \exp \left[-\pi^2 \frac{T}{mc^2} \frac{R}{\lambda} \frac{\alpha (1 - \alpha)^{1/2}}{1 + n_z^2 (1 - \alpha)} \right], \quad (30)$$

which is maximum at perpendicular incidence. Although these formulas are valid only for a high density plasma, in the range $1 > \alpha \gg n_z (T/mc^2)^{1/2}$ the density dependence is approximately correct.

B. Fractional absorption

The fractional absorption of the X mode, plotted as a function of n_z for several values of density at resonance, is shown in Fig. 7. The absorption at low densities is complete except for a region near perpendicular propagation. For higher densities, the absorption is only complete for large n_z , indicating that a slow wave structure that would radiate in the $n_z > 1$ region would be advantageous. In any case, for $\omega_{po}^2 > \omega^2$ the absorption becomes small. A similar plot for the O mode is given in Fig.

Fig. 7.

8. Here one sees that the absorption increases with density and is confined to the region where $n_z < 1$. A narrow beam near perpendicular incidence is most efficient for this mode. These plots are for a tokamak with $R/\lambda = 92$. For larger major radius and higher frequencies, the damping decrement increases.

C. Waveguide excitation

The simplest, but not the optimum, method of launching the waves is a waveguide aperture opening into the plasma. For a rectangular waveguide of width a and breadth b , the X mode is excited preferentially with the long side mounted parallel to the toroidal field. For a cosine dependence of the electric field E_y , the normalized spectrum in n_z is

$$U_x(n_z) = \frac{\lambda^3}{2\pi^2 a^3} \frac{\cos^2(\pi a n_z / \lambda)}{[n_z^2 - (\lambda/2a)^2]^2} . \quad (31)$$

For the O mode, the guide is oriented with the short side along the field. For a uniform electric field E_z the spectrum is

$$U_o(n_z) = \frac{\lambda}{\pi^2 b} \frac{\sin^2(\pi b n_z / \lambda)}{n_z^2} . \quad (32)$$

The fractional absorption for these spectra, with $a = \lambda/2$, $b = \lambda/4$, $\lambda = 1$ cm, and $R = 92$ cm, is plotted in Fig. 9 as a function of density at the resonant surface. One sees that the X mode is more effective, with 90% of the energy absorbed at low density. The missing fraction is the small n_z part of the spectrum. There is essentially no absorption for $\omega_{po}^2 > \omega^2$, although the X wave can propagate to twice this density.

The absorption of the O mode is smaller but sufficient for practical purposes, because the wave is reflected from the metallic walls and will

Fig. 8.

Fig. 9.

again pass through the resonant surface. In one sense the 0 mode offers an advantage, because the absorption is small at low densities, near the edge of the plasma. A simple waveguide aperture is certainly not the optimum way to launch the 0 mode. The small magnitude of the total absorption is primarily an artifact of the broad spectrum in n_z that this aperture produces.

D. Spatial dependence

The absorption occurs near the resonant zone in this treatment. For large x/Δ one has

$$-\text{Im}[1/Z(x/\Delta 2^{1/2})] \approx \pi^{1/2} \frac{x^2}{2\Delta^2} \exp(-x^2/2\Delta^2) . \quad (33)$$

For $x/\Delta = 5$ one finds this function is 5×10^{-5} of its value at $x = 0$. From Eq. (23), with the minimum value of $\alpha = 8^{1/2} \Delta/R$ corresponding to maximum absorption, one finds that the incident energy is decreasing as $\exp[-5(10)^{-4}(1 + n_z^2)x/\lambda]$, which is completely negligible. For $R = 1$ m and $T = 1$ keV this point is 11 cm from resonance with Δ proportional to $T^{1/2}R$. One concludes that cyclotron heating is well localized.

E. Conclusions

The most serious problem in cyclotron heating of tokamaks is the density limitation. Whenever the central density is high enough that $\omega_{po}^2 > \omega^2$, the wave energy will be deposited along the resonant surface near the wall, with potentially deleterious effects. The only cold plasma mode that propagates in the region $\alpha = \omega_p^2/\Omega^2 > 2$ is the whistler or helicon, and it will not propagate to cyclotron resonance. Finite temperature effects do not change the characteristics of the whistler very much. Because this mode propagates almost along field lines, it is extremely

difficult to launch in a tokamak. One concludes that electron cyclotron heating is probably not possible for $\omega_p^2/\Omega^2 > 2$, and the practical limit is $\omega_p^2/\Omega^2 \approx 1$. Heating at cyclotron harmonics has an even lower density limit. Heating of electrons by parametric oscillations is also difficult in this high density regime, because the pump wave must first propagate with the plasma.

For moderate density plasmas, the experimental results with tokamaks² are in agreement with the theoretical results of the present paper. Heating occurs at cyclotron resonance, increasing the temperature of all the electrons without preferential heating of high energy electrons, and a large fraction of the incident energy is absorbed. From Eq. (28) we predict the heating rate of both modes to be directly proportional to electron temperature, to the major radius of the torus, and to the microwave frequency. There is no positive confirmation of these dependences, but there is some negative evidence. Experiments on the Spherator device^{3,18} indicate a nonlinear interaction at the upper hybrid resonance. The electron temperature was very low, so that the linear heating was small. Furthermore, with low temperature the linear turning point¹⁹ for the X mode occurs very close to the upper hybrid resonance, so that the electric fields become very large and can exceed the threshold for parametric excitation. In a tokamak with ohmic heating, the electron temperature is always large enough for efficient linear heating, and the X mode is turned back well before the upper hybrid resonance.

In addition to the bulk heating of a tokamak plasma with microwaves, which has a direct effect on the transport coefficients of the plasma, the spatial localization of cyclotron heating makes it possible to modify the

temperature profile of the plasma. This profile control is of particular interest as a means of optimizing the transport coefficients. An experiment is being planned at the Oak Ridge National Laboratory to test the results of this calculation, the effects of bulk heating, and the effects of local changes on the temperature profile of a tokamak plasma.

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FIGURE LEGENDS

Fig. 1. The CMA diagram for a cold electron plasma with infinite mass ions.

Fig. 2. The cross section of a tokamak, showing the locations of the cyclotron surface, the upper hybrid layer, and the X mode cutoff.

Fig. 3. The electric fields E_x , iE_y , E_z in 100 V/cm for an energy flux of 100 W/cm² and n_x in dimensionless units. The electron density profile is parabolic with central density less than critical; $n_0 = 8 \times 10^{12}$ cm⁻³, $B_0 = 10.7$ kG.

Fig. 4. The ray trajectories in the principal plane of the torus for the X mode launched from the inside with $\omega_{po}^2 < \omega^2$. Starting from the left $n_z = -1.0, -0.75, -0.5, 0.5, 0.75$, and 1.0, respectively.

Fig. 5. The ray trajectories with $\omega_{po}^2 > \omega^2$. Starting from the left $n_z = -0.75, -0.5, -0.25, 0.25, 0.5$, and 0.75, respectively.

Fig. 6. The spatial form factors for wave damping in dimensionless units.

Fig. 7. Energy absorbed from a single ray as a function of n_z for several values of density, X mode.

Fig. 8. Energy absorbed from a single ray as a function of n_z for several values of density, O mode.

Fig. 9. Fraction of energy absorbed at cyclotron resonance, integrated over a waveguide spectrum, plotted as a function of density.

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