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HASTIE'S RIPPLE-TRAPPING
COEFFICIENT, $G(\alpha)$

MASTER

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GENERALIZATION OF CONNOR AND HASTIE'S
RIPPLE-TRAPPING COEFFICIENT, $G(\alpha)$

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ABSTRACT

In their calculation of transport due to ripple trapping, Connor and Hastie employ a factor $G(\alpha)$ to take into account the poloidal variation of the depth and shape of the local magnetic wells. They assume, however, that the toroidal-field ripple strength, δ , is independent of the poloidal angle. This letter generalizes G to treat any arbitrary poloidal variation of δ , and also high-beta, non-circular plasma configurations of arbitrary aspect ratio.

DISCLAIMER

In this letter we extend previous work by Dnestrovskij, et al. [1] and by Uckan et al. [2], [3] on the effects of poloidal variation of the magnetic field ripple in tokamaks. In particular we generalize their calculations to the case of high-beta, non-circular plasma configurations. We base our calculation on Connor and Hastie's [4] ripple-trapping coefficient, $G(\alpha)$, rather than on the less accurate result, $J(\alpha)$, from Stringer [5]. To calculate the generalized $G(\alpha)$ for arbitrary $\delta(\theta)$, we use the techniques developed by Goldston and Towner [6], which are valid so long as the ripple intensity, $\delta(\theta)$, does not vary significantly over a ripple period along a magnetic field line, or

$$\frac{\partial}{\partial \theta} \ln \delta \ll Nq/2\pi \quad .$$

Connor and Hastie have a factor A in each of their transport coefficients. Their Eq. (27) gives:

$$A = \frac{2\sqrt{2N}}{3\pi} \int_0^{2\pi} d\theta \sin^2 \theta \int_a^b d\phi (1 - B/B_{\max})^{3/2}$$

where

$$B(a) = B(b) = B_{\max} \quad .$$

Furthermore, they define $G(\alpha)$ by:

$$A = \frac{64}{9} \delta^{3/2} G(\alpha)$$

where

$$\alpha = \epsilon/Nq\delta \quad .$$

If we define $\delta_0(r)$ as $\delta(r, \theta=0)$, and use δ_0 in the evaluation of the transport coefficients, we may substitute a more general $G(r)$ for the $G(\alpha)$ by Connor and Hastie. Since ripple transport is often most important near the plasma boundary, it is also valuable to retain terms of the order of $\epsilon = r/R_0$. We arrive at:

$$G(r) = \frac{3\sqrt{2} N}{32 \pi \delta_0^{3/2}} \int_0^{2\pi} d\theta \sin^2 \theta (1 + \epsilon \cos \theta) \int_a^b d\phi (1 - B/B_{\max})^{3/2}.$$

where the ϕ integral takes account of the explicit variation of δ with θ . To evaluate $G(r)$, we follow Goldston and Towner, and model the field near θ as:

$$B \approx B_0 (1 + \alpha^* \delta x + \delta \sin x)$$

where $x = N\phi$ $\alpha^* = \epsilon |\sin \theta| / Nq\delta$

$B_{\max}$ is then given by

$$B_{\max} \approx B_0 (1 + \alpha^* \delta x + \delta (\pi - \cos^{-1} \alpha^*) + \delta \sqrt{1 - \alpha^{*2}})$$

a is given by

$$a = x_{\max} = \pi - \cos^{-1} \alpha^*$$

and $b = x_b$ is the solution to the transcendental equation:

$$\alpha^* x_b + \sin x_b = \alpha^* (\pi - \cos^{-1} \alpha^*) + \sqrt{1 - \alpha^{*2}}$$

Assuming top-bottom symmetry, we have

$$G(r) = \frac{3\sqrt{2}}{16\pi} \int_0^\pi d\theta \sin^2 \theta (1 + \epsilon \cos \theta) \delta_0^{-3/2} \times \int_{x_{\max}}^{x_b} dx \left(1 - \frac{1 + \alpha^* \delta x + \delta \sin x}{1 + \alpha^* \delta (\pi - \cos^{-1} \alpha^*) + \delta \sqrt{1 - \alpha^{*2}}} \right)^{3/2}$$

for δ small, and

$$\alpha^* \delta = \epsilon |\sin \theta| / Nq$$

small, we have

$$G(r) = \frac{3\sqrt{2}}{16\pi} \int_0^\pi d\theta \sin^2 \theta (1 + \cos \theta) \left(\frac{\delta(\theta)}{\delta_0} \right)^{3/2} \\ \times \frac{\int_{x_{\max}}^{x_b} dx \left(\alpha^* (\pi - \cos^{-1} \alpha^*) + \sqrt{1 - \alpha^{*2}} - \alpha^* x - \sin x \right)^{3/2}}{C(\alpha^*)}$$

The integral $C(\alpha^*)$ is performed numerically (Fig. 1), and then the θ -integral must be performed numerically as well. Note that α^* varies implicitly with θ (due to the variation of δ with θ) as well as explicitly. For regions of $\alpha^* > 1$, $C=0$. Unfortunately, owing to the dependence of α^* on q , G must be evaluated at each time step in a time-dependent transport code.

To simplify the problem, if necessary, we may approximate $C(\alpha^*)$. Stringer's approximation is to assume a sinusoidal well shape, a well length of $N\Delta\phi = 2\pi$, and a well depth given by $\delta^* = (B_{\max} - B_{\min})/B_0$. This is equivalent to

$$C = \left(\frac{5^*}{2^3} \right)^{3/2} \frac{16\sqrt{2}}{3}$$

This approximation results in a $G(\alpha)$ which is typically high by a factor of ~ 1.6 . We may improve the estimate by including the variation of the length of the ripple well, setting

$$N\Delta\phi = 2(x_{\min} - x_{\max})$$

where $x_{\min}$ is the location of the local minimum in $|B|$.

$$x_{\min} = \pi + \cos^{-1} \alpha^*$$

We arrive at

$$C = \left(\frac{\delta^*}{2\delta}\right)^{3/2} \left(\frac{2\cos^{-1} \alpha^*}{\pi}\right) \frac{16\sqrt{2}}{3} = \frac{32\sqrt{2}}{3\pi} \cos^{-1} \alpha^* \left(\sqrt{1-\alpha^{*2}} - \alpha^* \cos^{-1} \alpha^*\right)^{3/2}.$$

This analytic approximation is also shown in Fig. 1.

Finally, the generalization to non-circularity and high beta is quite straightforward. The θ -integral in the definition of G represents a flux-surface average and the $\sin^2\theta$ factor arises from the cosine of the angle between $\vec{v}_d$ and $\nabla\psi$.

Thus, the more general form of G is

$$G(\psi) = \frac{3\sqrt{2} (R B_{\text{vac}})^2}{16 \int d\chi} \int d\chi \frac{R (\vec{B} \times \vec{\nabla} B \cdot \vec{\nabla} \psi)^2}{R_0 B^6 |\vec{\nabla} \psi|} \left(\frac{\delta(\chi)}{\delta_0}\right)^{3/2} C(\alpha^*)$$

where χ is the poloidal variable such that $(B/B_p)(\partial\chi/\partial l)$ along a field line is unity, and R_0 is defined such that $4\pi R_0 \int d\chi$ gives the surface area of the flux surface under consideration. The transport coefficients that multiply G are to be evaluated using the vacuum toroidal field, $R B_{\text{vac}} = \text{const}$. Gradients of plasma parameters must be expressed as $\partial n/\partial\psi$, $\partial T/\partial\psi$, etc., since G has the dimensions of $|\nabla\psi|$. α^* in this geometry is now given by

$$\alpha^* = \frac{R |B_p \cdot \vec{\nabla} B|_{\text{sym}}}{N \delta B_T^2}$$

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