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OF NON-SPHERICAL SPHEROMAK

BY

K. YAMAZAKI

MASTER

PLASMA PHYSICS
LABORATORY PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

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INTERNAL TILTING MODE STABILITY OF NON-SPHERICAL SPHEROMAK

Kozo Yamazaki*

Princeton University, Plasma Physics Laboratory

Princeton, New Jersey 08544

Abstract

Fixed boundary tilting mode stability is analyzed for spheromak with arbitrarily shaped cross section. A prolate spheromak can be stabilized against tilting mode by adding a conducting shell of triangular or trapezoidal half-cross section.

* Permanent address: Institute of Plasma Physics, Nagoya University, Chikusa-ku, Nagoya 464, Japan.

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Introduction

From a viewpoint of simplifying the reactor engineering, there has been a growing interest in the spheromak which has no external field coil linking the plasma topologically. Extensive studies of the stability of this magnetic configuration have been reported in Refs. [1-3].

As far as magnetically driven modes are concerned, it has been shown by Rosenbluth and Russac [2] that tilting is a unique mode for compact torus plasmas and a horizontally elongated spheromak has better stability properties than the spherical spheromak which is marginally stable against this mode.

The purpose of this paper is to investigate the tilting mode stability of the arbitrarily-shaped nonspherical spheromak and to find out the optimum magnetic configuration. For this purpose we adopt Rosenbluth and Russac's approach and introduce the boundary condition representing arbitrary plasma shapes. We will show that the triangular or trapezoidal deformation from the exact semicircular cross section is helpful for internal tilting mode stability as well as for the oblate elongation of spheromak.

Tilting Stability Analysis

The minimum energy state for the magnetic configuration is represented by the following force-free state

$$\vec{j} = k \vec{B} \quad (1)$$

where k is a spatially constant, and its magnetic field components in the spherical coordinates (r, θ, ϕ) are given by

$$B = \sum_{n=0}^{\infty} \sum_{m=0}^n a_n^m b_m^n e^{in\phi} \left(\frac{m(m+1)}{\sin\theta} P_m^n \frac{j_m}{kr} - \sin\theta P_m^n \frac{1}{kr} \frac{d}{dr} r j_m \right. \\ \left. \sin\theta P_m^n j_m' + \frac{1}{\sin\theta} P_m^n \frac{1}{kr} \frac{d}{dr} r j_m \right) \quad (2)$$

where $P_m^n(\cos\theta)$ and $j_m(kr)$ are Associated Legendre function and Spherical Bessel Function, respectively.

Here we will deal with only plasmas bounded by a conductive shell, because for simplicity we can choose J.B. Taylor's stability method [4] concerning both ideal and resistive internal modes. According to this approach the eigenvalue k is proportional to the total energy and we can check the gross stability by obtaining these eigenvalues for different mode numbers.

We assume the following axisymmetric plasma boundary (Fig. 1) is slightly deviated from the exact sphere,

$$r = r_0 \left(1 + \sum_{l=1}^{\infty} \epsilon_l^S \sin l\theta + \sum_{l=0}^{\infty} \epsilon_l^C \cos l\theta \right) \\ = r_0 \left(1 + h(\theta) \right), \quad h \ll 1. \quad (3)$$

Here all coefficients ϵ_l and ϵ_l^C are supposed to be much smaller than one but the same order of h . The $l=0$ cosine term ϵ_0^C does not lead to deformation of the exact sphere. The $l=1$ sine term $\epsilon_1^S \sin\theta$ gives rise to the pumpkin-type ($\epsilon_1^S > 0$) and almond-type ($\epsilon_1^S < 0$) spheres. Positive and negative cosine coefficients ϵ_2^C show the prolate and oblate spheromaks, respectively, which have already been discussed in Ref. [3]. A slightly prolate rectangular spheromak ($\epsilon_3^S > 0$) and a slightly oblate onion-type spheromak ($\epsilon_3^S < 0$) are

obtained by using the $l = 3$ sine term. Making the $l = 4$ cosine term dominant, we modify the circular cross section to the square-like cross section with a diagonal line which coincides with the spheromak axis ($\epsilon_4^C > 0$, half-cross section is triangular) or with the non-axial diagonal line ($\epsilon_4^C < 0$, rectangular). The $l = 6$ cosine term introduces the hexagonal deformation of the spheromak cross section. All $l = \text{odd}$ sine terms and $l = \text{even}$ cosine terms lead to the asymmetric modification with regard to the horizontal plane. The eigenvalue k for the nonspherical spheromak has such a small deviation from the spherical value $k_0 = 4.493$ as

$$k = k_0 + \delta k.$$

Since the plasma is assumed to be bounded by a conducting shell, magnetic field component normal to the plasma surface vanishes at the plasma surface,

$$\vec{B} \cdot \nabla r_0 = B_r - B_\theta h'(\theta) + O(h^2) = 0, \quad (4)$$

where B_r and B_θ are given by Eq. (2).

We can easily rewrite this equation up to the order of h ,

$$\begin{aligned} b_{1,j_1}'(k_0 r_0) \left\{ 2 P_1^n(\cos\theta) \frac{\delta k}{k_0} + h(\theta) \right. \\ \left. + \sin\theta P_1^{n'}(\cos\theta) h'(\theta) \right\} \\ + \{ P_m^n(\cos\theta), m \geq 2 \text{ terms} \} = 0. \end{aligned} \quad (5)$$

Following Ref.[2], we use the orthogonality of the Legendre functions and obtain the following eigenvalues for both $n = 0$ and $n = 1$,

$$\frac{\delta k}{k_0} \bigg|_{n=0} = -\frac{9\pi}{32} \epsilon_1^S + \frac{3\pi}{32} \epsilon_3^S - \sum_{m=0}^{\infty} \frac{9}{(4m^2 - 1)(4m^2 - 9)} \epsilon_{2m}^C, \quad (6)$$

$$\frac{\delta k}{k_0} \bigg|_{n=1} = -\frac{15\pi}{64} \epsilon_1^S - \frac{3\pi}{64} \epsilon_3^S + \sum_{m=0}^{\infty} \frac{3(2m^2 - 3)}{(4m^2 - 1)(4m^2 - 9)} \epsilon_{2m}^C. \quad (7)$$

The constraint $k_{n=0} < k_{n=1}$ for internal tilting mode stability gives rise to the relation

$$\frac{\pi}{128} (\epsilon_1^S - 3\epsilon_3^S) + \sum_{m=0}^{\infty} \frac{m^2}{(4m^2 - 1)(4m^2 - 9)} \epsilon_{2m}^C > 0, \quad (8)$$

then we can show the following criterion up to the $l = 8$ term,

$$1.10 \epsilon_3^S + \epsilon_2^C < 0.37 \epsilon_1^S + 0.57 \epsilon_4^C + 0.14 \epsilon_6^C + 0.07 \epsilon_8^C + \dots \quad (9)$$

Stability properties of each component of the deformation are given in Fig. 1, where stable and unstable modifications are shown by S and U, respectively, and other modifications are marginally stable. All asymmetric deformations with respect to the horizontal plane are marginally stable, which

means that in these asymmetric cases the magnetic energy of the $n = 0$ configuration is the same as that of the $n = 1$ configuration to the order of h . For $l = 2$ cosine components the oblate case ($\epsilon_2^C < 0$) is stable, which was pointed out in Ref. [2]. Other slightly oblate and stable components are pumpkin-type ($\epsilon_1^S > 0$) and onion-type ($\epsilon_3^S < 0$) modifications. Non-oblate stable components are $l = \text{even}$ and $l \geq 4$ cosine modifications with positive coefficients. The triangular half-cross section ($\epsilon_4^C > 0$) is favorable and the hexagonal modification with the diagonal line on the axis ($\epsilon_6^C > 0$) is also stable against internal tilting modes.

In the actual experiment of the spheromak and compact torus these higher order effects may be important, and internal tilting stability of arbitrarily shaped spheromak can be treated by using Eq. (8).

Conclusion

The author applied Rosenbluth and Bussac's stability approach with Taylor's principle to the arbitrarily shaped slightly non-spherical spheromak and obtained the following conclusions.

(1) When the fixed plasma boundary is given by Eq. (3), the internal tilting mode stability can be checked up to the order of h by using Eq. (8).

(2) For sine component modifications, only $l = 1$ and 3 terms lead to the stable shaping, that is, pumpkin-type ($\epsilon_1^S > 0$) and onion-type ($\epsilon_3^S < 0$) spheromaks are stable and effectively oblate. Deformations of $l \geq 4$ sine components are marginally stable against internal tilting mode.

(3) For cosine component modifications, the triangular half-cross section ($\epsilon_4^C > 0$) and the trapezoidal half-cross section ($\epsilon_6^C > 0$) are stable as well as the oblate cross section ($\epsilon_2^C < 0$).

(4) All asymmetric modifications with respect to the horizontal plane are marginally stable.

(5) Finally, it should be noted that even in the prolate spheromak case we have a possibility of stabilizing the internal tilting mode by introducing higher order deformation.

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$l \backslash h(\theta)$	$\epsilon_l^S \sin l\theta$		$\epsilon_l^C \cos l\theta$	
	$\epsilon_l^S > 0$	$\epsilon_l^S < 0$	$\epsilon_l^C > 0$	$\epsilon_l^C < 0$
1				
2				
3				
4				
5				
6				

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Fig. 1. Non-spherical plasma boundaries of spheromak, $r = r_0(1 + h(\theta))$, $h \ll 1$. S and U denote stable and unstable shapes, respectively. Other shapes are marginally stable.