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A CRITICAL STUDY OF TYPE II SUPERNOVAE:
EQUATIONS OF STATE AND GENERAL RELATIVITY

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ABSTRACT

The relevance of relativistic gravitation and of the properties of nuclear matter at high density to supernova explosions is examined in detail. The existing empirical knowledge on the nuclear equation of state at densities greater than saturation, extracted from analysis of heavy ion collisions and from the breathing mode in heavy nuclei, is also considered. Particulars of the prompt explosions recently obtained theoretically by Baron, Cooperstein, and Kahana are presented.

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KEYWORDS

Supernovae, High density nuclear matter, Relativistic heavy ions

INTRODUCTION

I will be principally interested in hydrodynamic simulation of supernovae explosions generated in massive stars which have burned to an exclusively iron core near the Chandrasekhar limit in mass. I will also mention in conclusion the $9 M_{\odot}$ initial model of Nomoto and collaborators (1984) which has a core containing more variation in species. Ed Baron, Jerry Cooperstein and myself (Baron, Cooperstein, and Kahana, 1985b; Baron, 1985) have found a way to generate explosions by a prompt shock mechanism, but in the process have stirred up something of a hornet's nest in the nuclear physics community. Over the years, two major classes of mechanisms have been proposed. One approach seeks an ejection of the stellar mantle and envelope by an explosive shock created in a core bounce after collapse. A second approach would have the exterior regions blown off by the energetic neutrinos produced somewhat after collapse. Both of these scenarios are present in the classic papers of Burbidge, Burbidge, Fowler, and Hoyle (1957), and in the intervening years early work by Colgate and collaborators (1960, 1966) was carried out along both lines. Most recently, several authors (Mazurek, Cooperstein, and Kahana, 1980, 1982; Wilson, 1980; Arnett, 1982; Hillebrandt, 1982) have examined the collapse and subsequent shock formation with apparently negative results. Artificial initial models (Cooperstein, 1982; Cooperstein, Bethe, and Brown, 1984) for the degenerate iron core can lead to explosions, but attempts to begin with the realistic evolutionary models of Weaver and Woosley and collaborators (WNZ, 1978; WZF, 1985; WW, 1985) have uniformly ended with a stalled, accreting shock.

The mass range $10 M_{\odot} \leq M \leq 100 M_{\odot}$ is expected to contain all type II progenitors. Recent work by Nomoto and collaborators (1984) on a $M = 9 M_{\odot}$ stellar model, although clearly a possible Type II progenitor because of the hydrogen present, seems to be better placed in a special category, yielding a collapsing core containing appreciable amounts of carbon, neon, and oxygen, and silicon as well as "iron." Initial quiet evolution in the more massive stars, over 10-100 million years, ends with a core consisting only of elements near iron in atomic number, supported by degenerate electron pressure. Collapse is first triggered by photodisintegration of iron, then by the pressure drop following rapid β -capture, and finally halted by the stiffening of nuclear matter at high density. Bounce occurs when densities at the outer edge of the homologously collapsing portion of the core reach nuclear saturation values. A shock is inevitably formed, at or near the radius where infall and sound velocities match, i.e., at the sonic point (see Fig. 5). In question then is the outward propagation of this shock through the remaining dense material in the core, while additional material continues to rain in.

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The eventual fate of the shock is determined by the initial shock energy and by the losses suffered along its path. The initial energy in the shock is clearly borrowed from the gravitational well, and the losses result from dissociation of nuclei and from neutrino escape. Interestingly, although the collapsing core mass is almost all non-relativistic nuclei or nuclear matter it is supported by the pressure of a highly relativistic, degenerate, lepton gas. The adiabatic index C_p/C_v for such a system is $\Gamma = 4/3$, and the non-relativistic virial theorem tells us that the total core energy starts close to zero. The entropy is initially low and remains low throughout collapse (Bethe and collaborators, 1979); thus the process is essentially adiabatic and the total energy stays close to zero throughout. At maximum compression, this zero core energy is divided between approximately 100×10^{51} ergs (100 foe^1) of negative gravitational energy and 100×10^{51} ergs of internal energy. Since only perhaps one foe emerging in the shock is sufficient to unbind the mantle and envelope (bound by a few tenths of a foe) as well as produce the required visual display, one must be prepared to carry out an accurate calculation on a delicately balanced system.

The early work of Mazurek, Cooperstein, and Kahana (1978, 1982), confirmed in calculations by Wilson (1980), Hillebrandt (1982), Arnett (1982), began with the 10-25 $M_\odot$ models of Weaver, Woosley, and Zimmerman (WWZ, 1978) and found stalled shocks. It was argued that excessive neutrino escape and dissociation were responsible for the enfeeblished shock. However, the WWZ models ascribed a mass of 1.51 $M_\odot$ to the initial iron core. The shock could simply not traverse so much dense material in the highly compact core. In further work by Weaver, Woosley, and Fuller (WWF, 1982) the omission of important S-capture channels was corrected, the initial electron fraction Y_e lowered, and hence the effective core mass (recall $M(\text{Chandrasekhar}) \sim 5.6 Y_e^3$) was reduced to near 1.36 $M_\odot$. We shall see that, though the WWF core is more diffuse than that of WWZ, we were still unable to successfully simulate Type II supernovae.

In the present lectures I wish to outline and critically evaluate the steps my collaborators E. Baron and J. Cooperstein and myself followed towards achieving a prompt explosive mechanism by a hydrodynamic shock. Wilson and collaborators, in a series of interesting calculations (Wilson and Bethe, 1985; Mayle, 1985), have indicated that a stalled, accreting, shock may be revived at later times by neutrino heating. Arnett has further suggested (1985) that convection may also play a role in reviving exhausted shocks. I will make just a few comments about convection later, but have essentially no difficulties with Wilson's calculations except to note neutrino resuscitated shocks appear to be on the weak side and to add that where a prompt explosion occurs no revival is necessary.

Our development (Baron, Cooperstein, and Kahana, 1983, 1985a, 1985b) depends critically on the equation of state used for nuclear matter at high density and on the introduction of relativistic gravitation into the collapse phase. I will therefore spend considerable time on a discussion of the equation of state (EOS) and will only develop the hydrodynamics sufficiently to familiarize the uninitiated and to outline the incorporation of general relativity. The neutrino transport schemes employed, although of considerable importance, will be referred to only in passing.

In addition to the uncertainties engendered by the hydrodynamic input physics, our calculations, and those of others, are subject to the theoretical volatility of the initial models. Over the past decade we have seen the mass of the collapsing core decrease from 1.51 $M_\odot$ to 1.36 $M_\odot$, and then for stellar masses $\geq 18 M_\odot$ have been faced with cores greater than 2 $M_\odot$. The reasoning leading to this latter bifurcation in core types is equally as compelling as in the earlier analyses. Unfortunately, it is unlikely that a prompt explosion can occur for such heavy cores. Just what is needed in the way of explosions for generating the necessary nucleosynthesis is also uncertain. A successful explosion must be followed out in time to the point where evolution is essentially complete. Only then can a consistent picture of nucleosynthesis emerge.

THE EQUATION OF STATE

Role in Supernovae Studies

Driven by the exigencies of the supernova problem, we have in earlier work (Kahana, Baron, and Cooperstein, 1983; Baron, Cooperstein, and Kahana, 1985a) employed as simple an equation of state as permitted. Such an approach is justified not only from calculational necessity, i.e. reduction in computing time, but also from informational reality. Considerable theoretical knowledge and experimental constraint can be brought to bear on the nuclear equation of

¹G. E. Brown is responsible for the unit 1 foe = 10^{51} ergs.

state at densities below the saturation value $\rho_0(x)$, $x = Z/A$, for normal nuclear matter. At higher density, however, little is known from laboratory experiment. A second point in favour of simplicity is the restricted sensitivity of the collapse calculations to various components in the equation of state. Baldly put, the collapsing stellar core near maximum compression senses some average compressibility at saturation and some average adiabatic index over a range of densities from ρ_0 to perhaps $3-4 \rho_0$, and little else. Too sharp a rise in pressure just at saturation density may be exceedingly harmful. The equation of state we employ reflects the limited sensitivity.

Another relevant issue to keep in mind, missed by many earlier workers, is the actual environment in the stellar core at collapse. In particular, the charge to mass ratio, $Z/A = Y_0$ drops to the value $x = .32$ because of β -capture.

For densities greater than $\rho_0(Z/A)$, equal to 2.4×10^{14} gm/cm³ for $Z/A = 1/3$, we suggested the simple form

$$P_N(\rho) = \frac{K_0(Z/A) \rho_0(Z/A)}{9\gamma} [u^\gamma - 1], \quad u = \rho/\rho_0(Z/A) \quad (1)$$

for the cold nuclear pressure. Here K_0 is the nuclear modulus of incompressibility at saturation, i.e., $u = 1$, and at the charge to mass ratio Z/A , while γ is the limiting adiabatic index at high density. The corresponding nuclear energy per baryon is

$$E/A = \epsilon_{\text{bulk}} + E_D + \frac{\pi^2 \gamma^2}{4\epsilon_{F,0}} \cdot u^{-2/3} \quad (2)$$

$$\text{with} \quad \epsilon_{\text{bulk}} = \left[-16 + 29.3 \left(1 - \frac{2Z}{A} \right)^2 \right] \text{ MeV} \quad (3)$$

and a cold compressional energy

$$E_D = \int_{\rho_0}^{\rho} d\rho \frac{P_N}{\rho^2} = \frac{K_0}{9\gamma(\gamma-1)} [u^{\gamma-1} + \frac{\gamma-1}{u} - \gamma] \quad (4)$$

The thermal energy in (1) is that of a non-interacting Fermi gas, incorporating correlation effects only through the effective mass in the Fermi energy

$$\epsilon_{F,0} = p_F^2/2m^*(p_F) \quad (5)$$

Introducing the cold leptonic pressure and effective lepton fraction

$$P_L = \frac{1}{4} hc(3\pi^2)^{1/3} (\rho Y_{\text{eff}})^{4/3} \quad (6)$$

$$Y_{\text{eff}}^{4/3} = Y_{\text{electron}}^{4/3} + 2^{1/3} Y_\nu^{4/3} \quad (7)$$

and obtains for the combined (cold) pressure

$$P = P_L + P_N \quad (8)$$

and adiabatic index

$$\Gamma(\rho) = \frac{\rho}{P} \frac{dP}{d\rho} = \frac{1}{1 + P_N/P_L} \left[\frac{4}{3} + \frac{K_0 \rho_0}{9P_L} u^\gamma \right] \quad (9)$$

One notes that the cold adiabatic index at saturation

$$\Gamma(1) = 4/3 + K_0/K_L, \quad K_L = \frac{9P_L(\nu_0)}{\rho_0} \quad (10)$$

is independent of γ . Figure 1 displays P_N from eq. (1) as a function of density and parametrically of γ .

Fig. 1. Pressure versus densities are given for the Baron, Cooperstein, Kahana high density equation of state for two values of the parameter γ (the high density adiabatic index) in eq. (1). For comparison, the same curve is shown for a very stiff equation of state (Brick Wall) in an artificially constructed model.

Density and Incompressibility of Asymmetric Matter

Recall also, as I have indicated above, that in the collapse environment β -capture has reduced the electron fraction, i.e. Y_e to near $1/3$. The incompressibility K_0 is a strong function of the asymmetry of nuclear matter, a point first made in Baron, Cooperstein, and Kahana (1985) and missed by many authors in this subject. Using knowledge of the symmetry energy, ECK arrived at the simplified expressions

$$\begin{aligned} K_0(Z/A) &= K_0(1/2) [1 - a(Z/A - 1/2)^2] \\ \rho_0(Z/A) &= \rho_0(1/2) [1 - b(Z/A - 1/2)^2] \end{aligned} \quad (11)$$

which for $a=2$, $b=3/4$ match well the calculations of Kolehmainen and collaborators (1985). Since

$$\frac{K(\rho)}{9} = \left[\frac{\partial P}{\partial \rho} \right]_{\rho_0} = \rho_0^2 \left[\frac{\partial^2 E}{\partial \rho^2} \right]_{\rho_0} + \left(\frac{\partial E}{\partial \rho} \right)_{\rho_0} = 0 \quad (11a)$$

it is clear that somewhere between $Z/A = 1/2$ and completely asymmetric neutron matter, $Z/A = 0$, $K_0(Z/A)$ must vanish; the saturation minimum becomes a point of inflection and then disappears. Most of the softening we require from the Blaizot (1980) value for $K_0(1/2)$, that I later discuss, results from the extrapolation exhibited in Eq. (11).

The development in this section runs contrary to the oft-stated opinion that asymmetric matter is "stiffer" than symmetric matter. This statement is presumably based on the true but misleading changes in effective forces expected as one goes towards neutron-rich matter. The only unique density at which to cite a compression modulus is the saturation value $\rho_0(Z/A)$. Clearly from eq. (11a) this incompressibility decreases as $Z/A \rightarrow 0$, i.e., asymmetric matter is softer at saturation than is symmetric matter; this effect is heightened by the $[\rho_0]^{1/2}$ factor in (11a) coupled to the behaviour of $\rho_0(Z/A)$ exhibited in (11).

Nuclear Compressibility Information from Low-lying Excitations: Landau Theory

I would now like to consider briefly just what information we have about $\Pi(\rho)$ and $K_0(Z/A)$ at the densities achieved in the central core at bounce. For nuclear matter it is frequently convenient to follow the Landau development for Fermi-liquids, although perhaps more fundamental information derives from Brueckner-like calculations with both finite and zero range forces. My development here follows that of Blaizot (1980). The energy functional

$$E(\rho) = V \epsilon(\rho), \quad V \text{ the volume}$$

is considered for small fluctuations in the 1-particle density matrix ρ about the isolated ground state value. An effective single particle Hamiltonian and a particle-hole interaction

are defined by

$$h(\rho) = \frac{\delta E}{\delta \rho} \quad (12)$$

$$\text{and} \quad U_{ph}(\rho) = \frac{\delta h}{\delta \rho} = \frac{\delta^2 E}{\delta \rho^2} \quad (13)$$

Introducing the Wigner (Fourier) transform,

$$\delta \rho_{\underline{k}}(\underline{R}) = \int d^3 \underline{r} \, e^{-i \underline{k} \cdot \underline{r}} \delta \rho(\underline{r} + \frac{\underline{r}}{2}, \underline{R} - \frac{\underline{r}}{2}) \quad (14)$$

one obtains the energy

$$\epsilon_{\underline{k}}^0 = h^0(\underline{R}, \underline{k}) = \frac{\delta E}{\delta \rho_{\underline{k}}(\underline{R})} \quad (15)$$

and in a zero-range approximation the particle-hole interaction

$$\frac{\delta h(\underline{R}, \underline{k})}{\delta \rho_{\underline{k}}(\underline{R}')} = f(\underline{k}, \underline{k}', \underline{R}) \, \delta(\underline{R} - \underline{R}') \quad (16)$$

Near the Fermi surface $f(\underline{k}, \underline{k}')$ is a function only of the angle $\underline{k} \cdot \underline{k}'$, and it can be expanded in Legendre functions

$$f(\underline{k}, \underline{k}') = 1/N_0 \sum_{\ell} F_{\ell} P_{\ell}(\underline{k} \cdot \underline{k}') \quad (17)$$

with $N_0 = (3/2) \rho_0/c_F$ the density of states at the Fermi surface. Eq. (17) constitutes a definition of the familiar Landau parameters F_{ℓ} in terms of which most nuclear parameters, near saturation, can be expressed. For example, a static density fluctuation $\delta \rho$ produces an energy near equilibrium

$$\frac{E}{A} = \left(\frac{E}{A} \right)_0 + \frac{c_F}{3} (1 + F_0) \frac{(\delta \rho)^2}{\rho_0} \quad (18)$$

whence

$$K_0 = 6 \, \epsilon_F (1 + F_0) \quad (19)$$

is the incompressibility at saturation. Also the effective mass is related to another Landau parameter

$$m^*(p_F) = m \left[1 + \frac{F_1}{3} \right] \quad (20)$$

while the velocity of (first) sound is given by

$$c_S^2 = \left(\frac{K_0}{9m} \right)^{1/2} \quad (21)$$

For stability it appears $F_0 > -1$ and $\frac{F_1}{3} > -1$.

A zero-range density-dependent force

$$v(\underline{r}-\underline{r}') = 1/V (t_0 + t_3 \rho^d) \delta(\underline{r}-\underline{r}') \quad (22)$$

yields the energy functional

$$\epsilon(\rho) = a(\rho/\rho_0)^{5/3} + b(\rho/\rho_0)^2 + c(\rho/\rho_0)^{d+2} \quad (23)$$

$$a = \frac{3\hbar^2 k_F^2}{10m} \rho_0$$

$$b = -c(\rho_0) - a - 1/d \{-\epsilon(\rho_0) + a/3\} \quad .$$

$$\text{Finally then } K_0 = a/\rho_0 - \frac{9c(\rho_0)}{\rho_0} + d\left[-\frac{9c(\rho_0)}{\rho_0} + \frac{3a}{\rho_0}\right] = 167 + 212d \text{ MeV} \quad (24)$$

for $k_F = 1.35 \text{ fm}^{-1}$ and $-\frac{c(\rho_0)}{\rho_0} = 16 \text{ MeV}$. For d varying from $1/3$ to $-1/3$, corresponding effectively to an increasing range of force. K_0 varies between 238 MeV and 96 MeV.

If we could compare the energy of the giant monopole resonance seen in heavy nuclei directly with the sound propagation in uniform nuclear matter, it would be a simple task to extract a saturation compressibility. Unfortunately, even in so large a nucleus as lead, the nuclear surface plays an important role. The angular frequency of the breathing mode for a finite nucleus can be written

$$\hbar\omega = \sqrt{\frac{1}{m} \frac{AK_A}{\langle r^2 \rangle_0}}, \quad \langle r^2 \rangle_0 \text{ the mean square nucleon radius,} \quad (25)$$

with an effective finite nuclear incompressibility modulus

$$K_A = \eta^2 \frac{d^2(E/A)}{d\eta^2} \quad (26)$$

$$\eta = \frac{\langle r^2 \rangle_0}{A}$$

(compare to $K_0 = 9\rho_0^2 \left\{ \frac{d^2(E/A)}{d\rho^2} \right\}_{\rho_0}$ for uniform nuclear matter).

From a mass formula approach Blaizot (1980) finds

$$K_A = 9 \left(\frac{dP}{d\rho} \right)_0 + \frac{Z^2 e^2}{AR_0} - \frac{8\pi\sigma R_0^2}{A} + \text{symmetry terms} + \dots \quad (27)$$

in terms of volume, Coulomb, surface (σ = surface tension) terms, and symmetry terms. For a finite nucleus $\rho_0(A) \neq \rho_0(\infty)$ and even the first term in (27) is not the infinite matter compressibility,

$$\text{i.e.,} \quad 9 \left(\frac{dP}{d\rho} \right)_{\rho_0} = K_0 + \left[\frac{\rho_0(A) - \rho_0(\infty)}{\rho_0(\infty)} \right] K_0 + 9 \left[\frac{\rho_0(A) - \rho_0(\infty)}{\rho_0(\infty)} \right]^2 \rho_0^2(\infty) \cdot \frac{d^3 E}{d\rho^3}. \quad (28)$$

Figure 2, borrowed from Blaizot (1980) to illustrate this point, shows the results of a self-consistent RPA calculation of the position of the giant monopole state (GMR) in ^{90}Zr and ^{208}Pb as a function of K_0 .

Fig. 2. The RPA predictions of Blaizot for the breathing mode in ^{208}Pb and ^{90}Zr compared to the experimental values (dashed lines). The vertical lines indicate the calculations for different forces used in the RPA. BI and DI, which give results closest to experiment, are both finite range forces, while Skyrme forces SK_0 , SIV, and SII all give larger compressibilities and disagree with experiment (see Blaizot, 1980). The infinity in K_0 (and $\rho_0(\infty)$ above) refers to the infinite matter limit.

A major problem in this standard treatment of finite nuclear compressibility is apparent in the relation (19) for $K_0(1/2)$ in terms of F_0 and m^* . The effective mass is evaluated at the Fermi surface, whereas the single particle (hole) states involved in the finite nucleus RPA are appreciably removed from F_F . The well publicized difficulty in reproducing known single particle levels above and below the closed shell nucleus ^{208}Pb , with the currently popular quasi-zero range forces like Skyrme and its variants, is a strong warning in this regard. The two particle correlations in the RPA shift the monopole resonance some 14 MeV downwards, and are an added complication. I personally feel we are a long way from pinning down the incompressibility of nuclear matter at saturation. Nevertheless, I feel Blaizot's summary value $K_0(Z/A = 1/2) = 210 \pm 30$ MeV is an honest and useful estimate.

Relativistic Heavy Ion Collisions

The analyses of relativistic heavy ion experiments performed at Lawrence Berkeley Laboratory (Stock, Harris, and collaborators, 1982, 1985; Gustafsson, Gutbrod and collaborators, 1984) are also viewed as a commentary on the nuclear EOS. Two differing phenomena, pion production in heavy ion collisions or the transverse momentum distributions of collision fragments, are used to deduce information on the state of matter. The extraction of an infinite matter compressibility from such analyses is complicated in this instance by the non-equilibrium nature of the actual collision and perhaps also by the fairly high temperatures achieved.

In the case of pion production (Harris and collaborators, 1985; Kitazoe and collaborators, 1985) the reasoning is simple: energy generated in the nuclear medium by collision is divided into compressional and thermal components, with only the latter available for pions. One imagines a division of nuclear matter into two regions, one collision-free or unshocked, and one shocked, in which all pions are generated. Presupposing chemical equilibrium for the two baryon species N and Δ , one obtains for the pion multiplicity

$$\langle m_\pi \rangle = \frac{\rho_\pi + \rho_\Delta}{\rho_N + \rho_\Delta} = \frac{\rho_\pi + \rho_\Delta}{\rho} \quad (29)$$

The relativistic Rankine-Hugoniot equations applied for a one-dimensional geometry

$$\gamma_1 v_1 \rho_1 = \gamma_2 v_2 \rho_2$$

$$P_1 + \gamma_1^2 v_1^2 (e_1 + P_1) = P_2 + \gamma_2^2 v_2^2 (e_2 + P_2) \quad (30)$$

$$\gamma_1^2 v_1 (e_1 + P_1) = \gamma_2^2 v_2 (e_2 + P_2)$$

are then used to derive the conditions in the shocked region 2 (with velocity field v_2 , pressure P_2 , energy density e_2 , and Lorentz factor γ_2) from the known state of normal nuclear matter $\rho_1 = \rho_0(1/2)$ in the unshocked region 1.

The form of the compressional energy used in the LBL analyses (Stock and collaborators, 1982) is the so-called parabolic dependence

$$e_\rho = \frac{K_0(u-1)^2}{18} \quad (31a)$$

$$\text{with} \quad P_\rho = \frac{K_0 \rho_0}{9} u^2 (u-1) \quad (31b)$$

$$\text{and} \quad \Gamma(\rho) = 3 + \frac{1}{u-1} \quad (31c)$$

One must take care in all descriptions for the pressure to prevent the sound velocity $(c_s/c)^2 = (\partial P/\partial e)_s$ from exceeding light velocity. Eqs. (31) can be compared to BCK, eqs. (1) and (4). From (31c) it appears the parabolic form, for equal K_0 , generally is stiffer than BCK with $\gamma \lesssim 3$.

Baron, Brown, Cooperstein, and Prakash (1986) have attempted to pursue a fundamental derivation of the high density EOS by introducing necessary relativistic effects. Figure 3 from these authors presents comparative results for $\langle m_\pi \rangle$ from BCK and from the parabolic form. Clearly with $K_0 = 400$ MeV, an even stiffer EOS than afforded by $\gamma = 4$ in BCK, for which the parabolic and BCK forms are roughly equivalent, is required by the data. The stiffer the EOS employed, the less energy available for pions. A cold compressibility of $K_0 = 600$ -800 MeV seems necessary to fit the pion data (Kitazoe and collaborators, 1984).

Fig. 3. Heavy-ion pion production. Comparisons of BCK for $\gamma=2,3,4$ and of the "parabolic" EOS with the data for the number of pions/baryons, $\langle m_\pi \rangle$, produced in heavy ion collisions, from Baron and collaborators (1986). The data is best fitted by an even stiffer EOS with $K_0(1/2) = K_{nm} = 600\text{-}800 \text{ MeV}$.

Baron and collaborators (1986) argue that the nuclear EOS is indeed stiffer at higher temperature, e.g. at the $T = 70 \text{ MeV}$ reached in the experiments. Their argument exploits the weakening in the attractive tensor force occasioned by higher average excitation energies in colliding matter. Similar conclusions, but not so extreme with respect to the degree of stiffness, follow from an analysis of sideways flow in heavy ion collisions (Gustafson and collaborators, 1985; Molitoris and Stocker, 1985).

This is again illustrated in Fig. 4, which summarizes results from both a production and flow in heavy ion collisions. Many problems arise in the extraction of K_0 from $\langle m_\pi \rangle$: finite nuclear size, one-dimensional geometry, pion absorption in unshocked material. Most of these

Fig. 4. Summary of energy versus density curves favoured by data for pion production (hatched area) and transverse flow (the cross-hatched region). Also shown is the soft theoretical EOS obtained by Baron, Brown, and collaborators (1986) at zero temperature. These authors suggest the difference between theory and experiment may be explained by the changes in the EOS produced at the 70-100 MeV temperatures of the experiments.

effects suggest n_0 has been overestimated theoretically, and hence the K_0 extracted is also overestimated. In any case one must take very seriously the problem associated with bringing such a high value for K_0 in line with either the theoretical studies of nuclear matter or with the properties of the breathing mode in heavy nuclei.

This disagreement, already present between the two different analyses of nuclear properties considered, is again exhibited in the hydrodynamic evolution of stellar collapse I now turn to. Ironically, it is only in the distant stellar laboratory that truly uniform nuclear matter obtains in conditions of chemical and thermal equilibrium.

HYDRODYNAMIC SIMULATION: NEWTONIAN

The hydrodynamic scheme I discuss here is based on a Newtonian code developed by Cooperstein (1982) and used extensively in earlier work with my collaborators (Baron, Cooperstein, and Kahana, 1983, 1985a, 1985b). The force equation being studied for a spherically symmetric distribution of matter with velocity field $U(r,t)$ is

$$\rho \frac{dU}{dt}(r,t) = -G \frac{\rho(r) m(r)}{r^2} - \frac{dP}{dr} \quad (32)$$

with $P = P_{\text{matter}} + P_{\text{nepton}}$.

The mass $m(r)$ within the sphere radius r provides a Lagrangian coordinate for the problem. Supplementing (32) one must have some treatment of the only non-equilibrium aspect of this problem, neutrino transport. Wilson (1980) has included a more complete diffusion approach in his calculations, and these provide a normalizing standard. In BCK (1984, 1985a,b) two approximate treatments are employed: (1) trapping at some density plus free-streaming, or (2) leakage (Epstein and Pethick, 1981). The leakage scheme permits neutrinos to escape from all zones in the star, and was conservatively designed to overestimate the escape. Energy is explicitly conserved in all these Lagrangian schemes.

The Newtonian simulations performed following the early work of Mazurek, Cooperstein, and Kahana (1980, 1982) were intended for answering one question: What is the sensitivity of shock stalling to the nuclear EOS above (and just below) saturation density? We recognized early on that little constraint is provided here by laboratory experiment, and also we felt that the shock energy would be clearly enhanced by a softening of the high density matter. Increased gravitational compression of the "spring" in the stiffening matter would leave the final hydrostatic core more bound, and could result in the increased transfer of energy to the shock. The realistic initial models used were those of WWF (1982) with central entropies of 0.67 per baryon (in units of k_B) and with a core mass of approximately $1.36 M_\odot$, such to be preferred to the $1.51 M_\odot$ core of WWZ (1978).

Fig. 5. The sonic point is displayed for two artificially constructed Newtonian models, FAIL and EXT. The dotted lines refer to the model FAIL, which produced a stalled shock, and the solid lines refer to model EXT(TREME), which ended in a successful explosive shock. Note the small mass difference between the formation of the sonic point in the two models (indicated by the arrows on the abscissa). This small mass difference is quite important to the shock propagation.

Other, artificially generated, models were examined to study the effects of a softer EOS and also to study shock energy loss during outward propagation. Table I, reproduced from BCK (1985), illustrates the role of the EOS, while Fig. 5 from the same source shows the relationship to shock propagation of the mass coordinate for the sonic point just after the last good homology. The pressure waves reaching the edge of the homologous core at this time must generate a shock at the sonic point. The latter is more likely to stall if it has to traverse more mass during its passage outwards.

TABLE I: The Effect of Equation of State on Shock Parameters for Two Models

Initial Model	K_0	γ	$R_{\max}$ (km)	$\rho_{\max}$ (g/cm^3) ¹⁴	E_S (foes)
(BCK) I (a) 0.6	220	2	300	4.67	0.30
(b) -	150	2	835	5.54	0.64
(c) -	120	2	(1200)	6.08	0.96
(WWF) II (a) 0.67	220	2	150	4.80	1.47
(b) 0.67	120	2	320	5.60	1.80

Model I is artificially constructed with core mass $M_{\text{core}} = 1.32 M_0$ and Model II is that of WWF (1985). $R_{\max}$ is the largest radius reached by the shock before stalling. In I(c) the shock is marginally successful in ejecting the mantle. $\rho_{\max}$, the maximum central density reached during collapse, is to be compared with $\rho_0[(Z/A) = .32] = 2.3 \times 10^{14} \text{ g/cm}^3$. The energy E_S , a measure of shock energy discussed in BCK (1985), is evaluated when the shock is at the Lagrangian coordinate $M = 1.21 M_0$.

We concluded from our studies of the EOS in a Newtonian setting that indeed shock energies were enhanced. We mapped out the losses per unit mass traversed by the shock, but were forced to accept the unlikelihood of a prompt explosion in the realistic evolutionary models. One interesting point, however, was the rapid increase in central density associated with the softening of higher density nuclear matter.

HYDRODYNAMIC SIMULATION: GENERAL RELATIVISTIC GRAVITATION

The Schwarzschild radius of some 2 km for a homologous core of $0.7 M_0$, which collapses to 15-20 km in radius, coupled to the cancellation of core internal and gravitational energies, alluded to above, suggests that the effects of general relativity cannot be ignored. The softer EOS further motivates the introduction of relativity, since non-linear gravitational effects are larger for the increased central densities. At first sight the strengthening of gravity inherent in general relativity will lead to deeper digging into the gravitational potential. However, this strengthening may also produce a smaller homologous core, and hence force the shock to traverse more matter on its way out. Indeed, early work by Van Riper (1979) and Takahara and Sato (1984) suggested general relativity might be harmful to shock health, but Van Riper noted the possible sensitivity to the EOS. It is the unique combination of the softer EOS and relativistic gravitation present in our calculations that changes the picture.

The Newtonian force equation (32) is replaced by (see Misner and Sharp, 1964)

$$\Delta_t U = e^{-\lambda} \frac{\partial U}{\partial t} = -\frac{4\pi GR\rho}{c^2} - \frac{Gm}{R^2} - \frac{4\pi R^2}{w} \bar{r} \frac{\partial P}{\partial m} \quad (33)$$

with a gravitational mass

$$\bar{m}(m, t) = \int_0^{R(m)} 4\pi R^2 \rho(1 + E/c^2) dR \quad (34)$$

and where w is a comoving Lagrangian coordinate in terms of which the metric is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{2\lambda} c^2 dt^2 + e^{2\lambda} dw^2 + R^2 d\Omega^2. \quad (35)$$

The factor $\tilde{r}$ in (33) is related to the radial metric coefficient by

$$e^{2\lambda} = \left(\frac{\partial R}{\partial n}\right)^2 \tilde{r}^{-2}, \quad (36)$$

while the specific enthalpy is $u = 1 + E/c^2 + \frac{P}{\rho c^2}$.

This metric must be matched to an exterior Schwarzschild solution at some low-density surface $n = n_g(t)$,

$$ds^2 = - \left(1 - \frac{2G\tilde{M}}{RC^2}\right) c^2 dt^2 + \frac{dR^2}{1 - 2G\tilde{M}/RC^2} + R^2(dn)^2, \quad (37)$$

thus introducing the total mass-energy $\tilde{M}$ within this surface. The stress energy tensor is taken to be that of a perfect fluid (ρ = rest mass density)

$$T^{\mu\nu} = \rho u^\mu u^\nu + P g^{\mu\nu}. \quad (38)$$

Equation (37) must be solved using (38) and the EOS for the pressure, together with the energy equation, defined in the comoving system

$$\frac{dE}{dt} = - \frac{dV}{dt} - e^\phi \dot{S}, \quad V = 1/\rho. \quad (39)$$

Here $\dot{S}$ is the rate of energy loss in escaping neutrinos and e^ϕ a (small) neutrino redshift. Solving the Einstein equations yields explicit forms for ϕ and T , in particular

$$\tilde{r}^2 = 1 + u^2/c^2 - 2G\tilde{M}/RC^2 \quad (40)$$

The zero of internal energy, customarily defined with respect to iron, must here take account of the dynamic, gravitational, rest mass.

The mathematical procedure followed (Baron, 1985) is standard finite differencing: an initial guess is made for the form of $\tilde{r}$, taking $\tilde{M} = m$ in (40); thence $\tilde{m}$, and e^ϕ can be determined, and the acceleration equation (33) solved. $\tilde{r}$ is then updated, the energy equation (39) solved, and the cycle repeated. A leapfrog differencing scheme is used to estimate quantities at times when they are not known. All of this was tested for pressureless collapse (freefall) where an analytic solution exists, and found to be adequately accurate. Pseudo-viscosity, spread over three zones, handled the shock discontinuity. Finer zoning provided further tests of accuracy.

Fig. 6. The radius as a function of time for a collapsing general relativistic model. ($12 M_\odot$ with $\gamma = 3$, $K_0(1/3) = 140$ MeV). The numbers on the right hand side indicate the total enclosed mass in units of $M_\odot$. The dotted line denotes the approximate position of the shock.

Fig. 7. The velocity profiles for Model 41 ($12 M_{\odot}$ model with $\gamma = 3$ and $K_0(1/3) = 140$) at selected later times. Clearly, the shock is successful, propagating to the edge of the radial grid.

RESULTS

Our general relativistic calculations have been performed using the initial models of Weaver and Woosley (1985), with dramatic results. I have already referred to these latest models in the introduction, but note here they resemble WWF (1982) in structure for $M \leq 15 M_{\odot}$, having iron cores with masses near $1.36 M_{\odot}$. However, between $M = 15 M_{\odot}$ and $20 M_{\odot}$ a sudden change in core mass appears in the WW analysis, a jump to core masses $\geq 2.0 M_{\odot}$. This discontinuity results from faster rates used for $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ (Rofls, 1986) and depends on the formation or non-formation of a carbon-burning shell. If the carbon-shell forms, its large entropy creates a barrier to further burning ending in iron. This is the case for the lighter initial stellar masses. Table 2 is a summary of numerical experiments carried out for several initial models differing in their parametrization of the EOS. Figure 6 and Fig. 7 contain radius vs time and velocity profiles for the $12 M_{\odot}$, model 41 of Table 2. Reasonable choices for $K_0(1/2)$, very close if not within the range obtained by Blaizot (1980), and for γ values near those expected

Table 2 Description of Calculations

Model	Mass ($M_{\odot}$)	K_0^{sym} (MeV)	$K_0(0.33)$ (MeV)	γ	GR	$\frac{\rho_c^{\text{max}}}{\rho_0(0.33)}$	E_{exp} (foes)	E_{lost} (foes)
32	12	220	220	2	no	1.7	-	2.6
33	12	220	170	2	no	2.0	-	2.1
38	12	180	140	2	no	2.3	0.1	3.2
40	12	180	140	2	yes	12.0	3.2	2.2
41	12	180	140	3	yes	3.1	0.8	3.3
29	15	220	220	2	no	1.7	-	2.1
42	15	180	140	3	yes	3.1	-	2.5
43	15	180	140	2.5	yes	4.1	1.7	3.4
44	15	140	120	3	yes	3.3	-	3.2
45	15	90	90	3	yes	4.0	0.8	3.2

The column labeled GR refers to whether general relativity or Newtonian gravity was assumed. $K_0(0.33)$ refers to the value of the nuclear incompressibility at saturation density when there are twice as many neutrons as protons ($Z/A = 0.33$). For model 33 the form of the incompressibility was taken to be that of eq. 11 with $K_0^{\text{sym}} = 220$ MeV, while for models 38 and 40-43 K_0^{sym} was chosen to be 180 MeV. Model 44 had $K_0(x) = 140(1-1.3(1-x)^2)$ and the other models (29, 32, 45) had $K_0 = \text{constant}$ at the value given in the table. γ refers to the high-density adiabatic index discussed in the text. ρ_c^{max} is the maximum central density achieved just prior to bounce. E_{exp} is an estimate of the explosion energy neglecting the binding energy of the mantle and envelope. No entry in this column means the shock failed to reach the edge of the iron core. E_{lost} is the total neutrino loss when the calculation was stopped (roughly 50 milliseconds after bounce).

from purely theoretical considerations, result in viable prompt explosions with energies adequate to account for any visual display. I wish to reemphasize that most of the softening from the near 200 MeV value of $K_0(1/2)$ comes from the well understood extrapolation to asymmetric matter. Although one continues to characterize the nuclear EOS with $K_0(1/2)$ it is the value near $Z/A = 1/3$ that is most important for supernova calculations and not the history between $Z/A = 1/2$ and $1/3$.

Fig. 8. The sonic points just after bounce for a collapse model with general relativistic hydrodynamics (dashed line) or with Newtonian hydrodynamics (solid line). The Lagrangian mass coordinate is smaller for relativistic gravitation. This hurts the outer propagation of the shock.

"Successful" explosions in these calculations are most succinctly characterized by the maximum central density achieved during collapse (see Table 2). This may well be misleading, since phase changes to quark-gluon or other exotic forms of matter are excluded in our work, e.g., at sufficiently high density one expects hadron matter to coexist with two-flavour quark matter. Nevertheless, a high central density signals that the unfavourable aspects of general relativity, viz. the smaller homologous core (see Fig. 8) and the perhaps stronger gravitational force the outward-moving shock must fight against, have been offset by an increase in the energy transferred to the shock from the gravitational well. An unexpected, but important, further advantage of stronger gravitation is occasioned by the faster collapse of the central core. The shock forms faster and will thus meet the infalling outer core zones at times when the latter are at lower density and falling slower. The passage outward is thus eased considerably relative to that of the Newtonian shock. A clear sign of this effect was the discovery in the calculations that, despite the increase in gravitational energy of the inner zones, the total core gravitational energy decreases in magnitude relative to Newtonian, at times relevant to shock propagation. The effect is further illustrated in the density profiles in Fig. 9.

Fig. 9. Density profiles near bounce for the same collapse model in Fig. 8. In this case general relativity (dashed line) clearly produces a more favourable configuration for shock propagation.

To summarize the results: In Table 2 for the reasonable choices $\gamma = 2.5$ and $K_0(.33) = 140$, corresponding to $K_0(1/2) = 180$ MeV, one finds explosion energies near 2.0 foe in the general relativistic calculations. Lower shock energies are indeed acceptable, and one could probably produce prompt explosions for the 12 $M_\odot$ and 15 $M_\odot$ WW models with $K_0(0.50) \approx 200$ MeV, i.e. very close to the Blaizot (1980) value. These shock energies are not to be taken too seriously at this point, partly because of the uncertainties in our parametrizations and partly because the calculations should be followed further in time. Nevertheless, sufficient energy has been generated to eject the non-core regions of the star and account for the visual display of a supernovae. Equally certain is that the remnant of such an explosion will be a neutron star, although its actual observable mass remains in some doubt.

NEUTRON STAR MASSES

A possible constraint on the K_0, γ used in the nuclear EOS may result from a study of the hydrostatics of neutron star remnants. Inordinately soft nuclear matter would be unable to sustain the 1.4 $M_\odot$ mass observed for known neutron stars. For example, dropping the constant term in eq. (1) yields

$$P = \frac{K_0 \rho_0}{9\gamma} u^\gamma \quad (41)$$

an appropriate non-saturating EOS for neutron matter. Presumably, but unfortunately not necessarily, the parameters in (41) should match those in BCK, eq (1); i.e., K_0, ρ_0 , and γ in (41), are values extrapolated to $Z/A = 0$. The very low density regions of the neutron star, for which $\rho < \rho_0$ are, better described by $\gamma = 5/3$; however, these regions having low mass play only a small role in determining the total mass of neutron stars. The static equilibrium calculations with eq. (41) are well approximated by

$$\frac{M_{\max}}{M_\odot} = \frac{1}{n} \left(\frac{K_0}{130} \right)^{n/2} \left(\frac{2.4 \times 10^{14}}{\rho_0} \right)^{1/2} \quad (42)$$

in terms of the polytropic index n , i.e. $\gamma = 1/n$.

From Table 3, based on eqs. 41 and 42, one might conclude $\gamma \geq 2.5$ is required. However, one could easily imagine a density variation in the adiabatic index which kept γ low in the regions important in collapse while having γ rise sufficiently for $\rho \geq 3$ to accommodate known neutron star masses. Such a choice would also go a long way towards explaining the Berkeley heavy ion experiments mentioned earlier. More work on the EOS is required for both the collapse and neutron star phases, and certainly more fundamental theoretical work incorporating, amongst other things, the nuances of phase changes.

Table 3 Maximum Neutron Star Masses

γ	K_0 (MeV)	$M_{\max}(M_\odot)$	$M_{\text{fit}}(M_\odot)$
2	120	0.70	0.72
2	140	0.76	0.78
2	180	0.86	0.88
2	220	0.95	0.98
2.25	120	0.96	0.96
2.25	140	1.02	1.02
2.25	180	1.13	1.13
2.25	220	1.22	1.23
2.67	120	1.37	1.37
2.67	140	1.43	1.44
2.67	180	1.54	1.55
2.67	220	1.64	1.64
3	120	1.67	1.70
3	140	1.73	1.77
3	180	1.85	1.88
3	220	1.94	1.98

The maximum neutron star masses for various values of γ and K_0 for the polytropic equation of state discussed in the text, eq. (41). The column label $M_{\max}$ is the result of a numerical integration, while the column labeled M_{fit} tabulates the values of our fit (eq. 42) to these results. We present the results for the parameter $\rho_0 = 2.0 \times 10^{14}$ g/cm³.

CONCLUSIONS

In conclusion, we note, no Newtonian calculation with WW initial models yields sufficient energy to disrupt the star. The most favourable case for $K_0(.33) = 140$ MeV and $\gamma = 2$ (model 38) yields a shock which propagates off the mathematical grid, but whose .1 foe energy cannot overcome the few tenths of a foe binding of the mantle and envelope. There is a threshold of softness in nuclear matter beyond which the effects of general relativity become helpful to shocks. Shock energies are enhanced over Newtonian values by an order of magnitude reasonably close to this threshold in $K_0(.33)$ and γ .

The WW initial models for stars more massive than, perhaps, $18 M_\odot$, possessing iron cores greater than $2.0 M_\odot$ in mass, will not explode; there is almost no chance for a hydrodynamic shock to be viable in such a massive core. Perhaps the delayed neutrino-heating scenario of Wilson (1985) is then appropriate. However, the use by nature of more than one means of producing type II supernova appears inelegant. Arnett (1985) has put forward yet another mechanism for reviving stalled shocks: convection generated by the gradients in Y_p and entropy behind the shock. Sathya and Brown (1986) have examined this possibility for the model 42 of BCK (Table 2) and conclude that although convection is indeed possible sufficiently far behind the shock, the details of the matter and energy distribution render such convection unfavourable to further progress of the shock.

I have in this presentation stretched the permissible range of parameters in the nuclear EOS only a bit, being at the low end of incompressibilities permitted by the analysis of Blaizot (1980). One must come to terms with the data from relativistic heavy ions, but I have already indicated the analysis of heavy ion collisions is at odds with explanations from the breathing mode.

To finish off this critique of the nuclear EOS I refer to one more piece of work. Motivated by the results found by BCK (1985b), Brown and Osnes (1985) have reexamined symmetric nuclear matter. In particular they consider the Landau sum rule

$$\sum_i \frac{F_i}{1 + F_i/2t + 1} = -3 \sum_i \frac{G_i}{1 + G_i/(2t + 1)} + r_t \quad (43)$$

where G_i are Landau coefficients similar to the F_i but for spin and isospin dependent parts of the interaction. Taking G_i from knowledge of the giant Gamow-Teller (GT) resonance, assuming G_i and r_t are small, i.e., that exchange forces are short range, and also that F_1 is small, these authors obtain $F_0/(1+F_0) = -1.37$ or $F_0 = -0.58$. At the Fermi surface these authors also argue that $m^*/m = 0.9$ rather than the 0.7 generally obtained in Brueckner calculations, and finally obtain $K_0(1/2) = 106$ MeV. This is a surprisingly low value, well below the requirements of the supernova calculations discussed here, but this analysis does illustrate the uncertainties present in the derivation of the EOS.

Undoubtedly we have not yet heard the end of this tale, although I believe an important watershed has been reached. Initial models still retain the capacity for terror, for example a drop in core masses below $1.25 M_\odot$ would dissolve all difficulties, while an appreciable increase in core mass would prevent any prompt shock from succeeding. Finally, the details of nucleosynthesis must be followed hydrodynamically for a successful shock-produced explosion, and thus the initial stellar mass range required to partake in supernova, for an adequate description of the known elemental abundances, would be defined.

LESS MASSIVE STARS

In this last brief addendum to my talks I would like to mention current work on a $9 M_\odot$ initial model being carried out by Baron, Cooperstein, and Nomoto (who is visiting Brookhaven this year) and myself. This model of Nomoto and collaborators (1982) is another source of controversy. In the calculations of Hillebrandt and collaborators at Munich (1983) the $9 M_\odot$ model has been successfully and vigorously "exploded," but in the calculations of Burrows and collaborators at SUNY Stony Brook (1986) the shock fizzles out. To add to the confusion, we have reexamined this model and find it teetering on the edge. The outer core regions exhibit a precipitous drop in density, and a finer zoning sends the shock considerably further out (from 100 to 600 km). Surprisingly, the collapsed core in the calculations is pulsating in radius, well after the bounce and during the outward shock propagation. Some difficulties for shock

propagation result in this model from its rather compact structure. As with Hillebrandt and collaborators (1985), we find a detonation wave, initiated by collapse before the shock forms, and moving out ahead of the shock. This detonation wave is eventually overtaken by the shock. If further changes in the EOS and refining of the initial model eventually produce an explosion, it will certainly not be vigorous, limited in energy to perhaps a few tenths of a foe or less. This state of affairs may be consistent with observations on the Crab.

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