HEAT TRANSFER IN HIGH TEMPERATURE FLUIDIZED BEDS WITH
IMMERSED TUBES FOR COAL COMBUSTION SERVICE

Final Report for Project
Encompassing the Period
October 1, 1977-March 31, 1981

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
a	= tube radius
a, b, c, A, B, C	= parameters appearing in approximate Channel Nusselt number expression; $A(r^+, \theta)$, $B(r^+, \theta)$ functions appear in pressure field equation
a_1, a_2, a_3, a_4	= velocity profile constants
A_2, A_3	= surface areas in radiative heat transfer analysis
$Ar = \frac{gd_p^3(\rho_s - \rho_f)}{\rho_f \nu_f^2}$	= Archimedes Number
b_1, b_2, b_3, b_4	= temperature profile constants
b, b_i	= bubble radius
B_i^R	= radiative heat transfer parameter
c	= geometry parameter for Joukowski transformation
c_f	= skin friction coefficient
c_{f2D}	= two-dimensional skin friction coefficient
c_R	= Chapman-Rubensin constant, $\left(\frac{T_W}{T_B}\right)^{1/2} \left(\frac{T_B + 110 \text{ K}}{T_W + 110 \text{ K}}\right)$
C_1	= specific heat of stainless steel shim stock
d_p	= surface mean particle diameter
d_p^+	= dimensionless particle diameter, d_p/D
D	= cylinder diameter

<u>Symbol</u>	<u>Description</u>
D_n	= complex coefficients in series solution for bubble velocity potential
$\hat{e}_y$	= unit vector
e_B	= bubble boundary emissivity
e_p	= particle boundary emissivity
e_W	= cylinder wall emissivity
F, G, H	= complex potential functions
g	= gravitational acceleration and drag parameter
G_{2H}	= thickness ratio, Δ_2/δ_H
h_o	= scale factor at $y^+ = 0$
h_θ	= local heat transfer coefficient
h_1	= local bed-to-tube heat transfer coefficient (for error analysis in Appendix A)
h_2	= $1/R_{GAGE}$
h_g	= local bed-to-tube heat transfer coefficient due to pure gas convection (bubbles contacting tube)
h_{Tg}	= local bed-to-tube heat transfer coefficient due to pure gas convection (bubbles contacting tube) for an isothermal surface
$\bar{h}$	= spatial-average bed-to-tube heat transfer coefficient
$\bar{h}_T$	= spatial-average bed-to-tube heat transfer coefficient for an isothermal surface
$\bar{h}_{MAX}$	= maximum spatial-average bed-to-tube heat transfer coefficient
H_{12}	= thickness ratio, δ_1/δ_2

<u>Symbol</u>	<u>Description</u>
H_{1V}	= thickness ratio, δ_1/δ_V
H_{2V}	= thickness ratio, δ_2/δ_V
i	= unit imaginary number, $\sqrt{-1}$
k_f	= thermal conductivity of fluidizing gas at the bed temperature
k_1, k_2	= Stokes' region edge location parameters (also, k_1 = thermal conductivity of stainless steel shim stock)
K_i	= overall voltage gain on data channel i
K_g	= gas thermal conductivity
K_T	= universal constant in turbulence model
k_β	= drag coefficient
ℓ_∞	= turbulence length scale at edge of boundary layer
L	= characteristic length
L_c	= average particle spacing and length of channel
M	= boundary layer momentum parameter
n	= normal coordinate and index
N	= number of terms in power series approximation for bubble velocity potential
N_{HV}	= boundary layer thickness ratio, δ_H/δ_V

$$\overline{Nu}_{MAX} = \frac{d_p \bar{h}_{MAX}}{k_f} = \text{maximum spatial-average Nusselt Number}$$

$$Nu_D, Nu_p = \text{Nusselt number based on } D \text{ or } d_p, \frac{D}{K_g} \frac{q_w}{(T_B - T_W)}$$

$$\overline{Nu}_D, \overline{Nu}_p = \text{spatial-average Nusselt Number}$$

<u>Symbol</u>	<u>Description</u>
Nu_{pm}	= maximum Nusselt number for quasi-linear behavior
$Nu_{D_1}^R, Nu_{D_2}^R$	= effective Nusselt number due to radiation for surfaces (1) and (2) respectively
$(Nu_p)_{\text{Stokes edge}}$	= Nusselt number at edge of Stokes' region
Nu_{p_0}	= Stokes' region contribution to Channel Nusselt number
Nu_{D_0}	= Nusselt number without turbulence
Nu_{p_θ}	= local Nusselt number
$(Nu_p)_{2D}$	= two-dimensional Nusselt number
P	= tube spacing
p^+	= dimensionless pressure, $p/\rho u_{mf}^2$
Pr	= Prandtl number
q_1	= heat flux at top surface (bed side) of stainless steel shim stock
q_2	= heat flux at lower surface (tube side) of stainless steel shim stock
q_θ	= local bed-to-tube heat flux
q_w	= heat transfer rate per unit area
$\vec{Q}_g, \vec{Q}_s$	= average velocities of gas and solid phases, respectively
Q_g^+	= dimensionless average gas velocity, Q_g/u_{mf}
r	= radial coordinate on circle plane
r_p	= particle radius

<u>Symbol</u>	<u>Description</u>
r^+	= dimensionless radial coordinate, r/D
R	= tube radius
R_o	= radial coordinate of bubble center
R_1	= inner boundary of Stokes' region on unit circle
R_{GAGE}	= thermal resistance of Micro-Foil Heat Flow Sensor
Re_D	= Reynolds number based on cylinder diameter, $\frac{u_{mf} D}{v_\infty}$
$Re_{L\infty}$	= Reynolds number based on L and u_∞ , $\frac{u_\infty L}{v_\infty}$
Re_p	= Reynolds number based on particle diameter, $\frac{u_{mf} d_p}{v_\infty}$
Re_z	= Reynolds number based on z , $\frac{u_\infty z}{v_\infty}$
s, t	= real and imaginary parts of ζ
S	= Stokes' region temperature parameter
$S(0), S_o$	= S at $\theta^+ = 0$, $-\frac{C_R}{T_B^+} (\frac{1}{2} + T_W^+)$
S_p	= half of particle spacing at wall
t	= time
t_c	= time constant
$t_{w\theta}$	= local surface temperature
$\bar{T}_w$	= spatial-average surface temperature at locations of heat flux measurements
T_w	= temperature of stainless steel shim stock (used in Appendix A and Appendix C)
T_{w2}	= temperature at interface between Micro-Foil Heat Flow Sensor and tube (used in Appendix A and Appendix C)

<u>Symbol</u>	<u>Description</u>
T_B^+	= dimensionless bed temperature, $T_B/(T_B - T_W)$
T_b (or T_B)	= bed temperature
T_{surface}	= particle surface temperature
T_{tube}	= tube temperature
T_W^+	= dimensionless wall temperature, $T_W/(T_B - T_W)$
T^*	= reference temperature
u^+	= dimensionless axial velocity, u/u_{mf}
u_{mf} (or U_{mf})	= minimum fluidization velocity
u_o (or U)	= superficial velocity
u_∞^+	= dimensionless velocity at edge of boundary layer, u_∞/u_{mf}
u'	= turbulence intensity, R.M.S. velocity fluctuation/ u_m
V_{ti}^j	= transducer voltage output for data channel i at time j
V_{OUTi}^j	= voltage output on data acquisition system for data channel i at time j
V_{OSi}	= offset voltage on data acquisition system for data channel i
$\vec{V}_B$	= bubble velocity
v^+	= dimensionless normal velocity, v/u_{mf}
W	= boundary layer energy parameter and complex potential
x, y	= coordinates
x_c^+	= corner location

<u>Symbol</u>	<u>Description</u>
x_s^+	= Stokes' region edge location
x^+, y^+, z^+	= dimensionless coordinates, ()/D
z^+	= complex variable, $x^+ + iy^+$
z_o^+	= bubble center location
$z_{o_1}, z_{o_2}, z_{o_3}$	= singularity location on physical plane

<u>Greek Letters</u>	<u>Description</u>
α	= parameter in Joukowski transformation and Channel Nusselt number parameter
β	= angle of intersection of bubble with tube (must be $\frac{\pi}{2}$) or Channel Nusselt number parameter
β_T	= relative eddy diffusivity of edge of boundary layer, $\epsilon_{T\infty}/\nu_\infty$
β_1, β_2	= drag parameters
δ	= boundary layer thickness
δ_H	= thermal boundary layer thickness
δ_V	= velocity boundary layer thickness
δ_1	= displacement thickness or stainless steel shim stock thickness
δ_2	= momentum thickness
δ_{2D}	= two-dimensional boundary layer thickness
Δ_2	= enthalpy thickness
ϵ	= voidage
ζ	= transform variable, $s+it$
$\zeta_{o_1}, \zeta_{o_2}, \zeta_{o_3}$	= singularity locations on ζ plane

<u>Greek Letters</u>	<u>Description</u>
ϵ_{mf}	= voidage at minimum fluidizing conditions
ϵ_T^+	= dimensionless eddy diffusivity, ϵ_T/ν_∞
ϵ_∞	= ambient voidage
ϵ_θ	= local voidage
η_H	= thermal boundary layer similarity parameter, ξ/δ_H
η_V	= velocity boundary layer similarity parameter, ξ/δ_V
θ	= angular coordinate
θ_0	= angular position of bubble center
θ^+	= dimensionless temperature parameter, $\frac{T - T_W}{T_B - T_W}$
μ_T	= temperature profile parameter
ν_∞	= kinematic viscosity at edge of boundary layer
ν_f	= kinematic viscosity of fluidizing gas at the bed temperature
ξ	= transformed normal coordinate
ρ	= gas density
ρ_s	= density of particle
ρ_f	= density of fluidizing gas at the bed temperature
ρ_l	= density of stainless steel shim stock
σ	= Stefan-Boltzman constant
Σ	= summation
τ	= parameter in Joukowski transformation and voidage distribution parameter
$\bar{\tau}_e$	= emulsion residence time
ϕ	= angular coordinate on circle plane
ϕ_B, ϕ_p	= complex velocity potentials

<u>Greek Letters</u>	<u>Description</u>
ϕ_c	= corner location on circle plane, $\phi_o/2$
ϕ_o	= Stokes' region extent on circle plane
ϕ_s	= particle sphericity
Φ_s	= Stokes' region matching parameter
Ω_v	= velocity profile shape factor

<u>Subscripts</u>	<u>Description</u>
$()_i$	= quantity for ith bubble
MAX	= maximum value or condition which gives maximum error magnitude
RAN	= random calibration and data acquisition errors

<u>Superscripts</u>	<u>Description</u>
$()^+$	= dimensionless quantity
$()^*$	= quantity divided by cylinder radius also quantity based upon reference temperature

<u>Other</u>	<u>Description</u>
$()^-$	= complex conjugate of $()$ or spatial-average of $()$
∇	= gradient operator
∇^2	= Laplacian operator
$\doteq$	= approximately equal to
$\Delta()$	= uncertainty in measurement of the quantity $()$

I. INTRODUCTION

The research project entitled "Heat Transfer in High Temperature Fluidized Beds with Immersed Tubes for Coal Combustion Service" was carried out in the Mechanical Engineering Department at Oregon State University between October 1, 1977 and March 31, 1981. This report is the final, summary document concerning this research. It is intended to be as complete as possible without requiring the reader to peruse earlier reports for details.

The project involved two parts: analytical and experimental. The progression of work, and, thus, the organization of this report includes the following major divisions:

- A. Analytical studies involving studies of heat transfer, hydrodynamics of gas flow within the bed, and some considerations of bubble and adjacent tube influences on the heat transfer.
- B. Experimental apparatus including the instrumented tubes for heat transfer measurements, digital data acquisition, and the high temperature fluidized bed test facility.
- C. Experimental results.
- D. Experimental validation of the analytical model.

In the sections to follow these areas will be discussed in detail followed by a short chapter in which some thoughts are presented regarding future work.

II. ANALYTICAL STUDIES

Analytical effort has concentrated upon further development of the heat transfer model valid for large particle fluidized beds first proposed by Adams (1977). The close coupling between hydrodynamics and heat transfer has been evident from the start of the analysis and the analytical approaches used in both areas are described in this chapter.

2.1 HEAT TRANSFER MODEL

Heat transfer in a gas fluidized bed has been described by Botterill (1975) to be the result of the combined effects of gas convection, particle convection, and thermal radiation. In large particle fluidized beds, the particles remain relatively isothermal during their residence time near the heat transfer surface so that the particle convective component of heat transfer approaches an asymptotic value determined by the contact resistance between the solid particles and the heat transfer surface. The isothermal nature of large particles is illustrated in Figure 2.1 where the time required to reduce relative particle temperature by 10% and 20% is shown with typical residence time data. Meanwhile, the high superficial velocities required to fluidize large particles result in a large fraction of the heat transfer being due to gas convection. In order to model this situation, two phases are identified, the emulsion phase consisting of solid particles and gas and the bubble phase consisting of large gas voids nearly free of solid particles. The heat transfer at a given location will then depend strongly upon which of these two phases is present.

When the heat transfer surface is in contact with the emulsion phases, heat transfer is considered to be the result of the flow of gas within the interstitial voids bounded by the solid particles and heat transfer surface. The solid particles provide geometrical and thermal boundary conditions on this flow and thus influence the convective heat transfer to the surface. Typically, the thermal boundary condition changes with time as the particle surface cools and the geometrical boundary condition is highly variable because of the variety of possible

Figure 2.1 Dolomite particle cooling at an immersed surface

shapes of the solid particles. Thus the particle convective and gas convective contributions for the emulsion phase are coupled. Since the gas convective component is dominant for large particles, average geometrical and thermal boundary conditions are assumed, and the gas convective heat transfer for the average conditions is analyzed in detail. In this way, both gas convective and particle convective contributions are obtained simultaneously. The average geometry selected for this analysis is the interstitial channel shown in Figure 2.2. This channel is bounded by the heat transfer surface at the bottom and by cylindrical surfaces with diameter equal to the average particle diameter on the sides. The top of the channel is arbitrarily taken at one particle radius from the heat transfer surface. The base of the channel is assumed to be square with the dimension L_c determined by the local emulsion phase voidage (the average distance between particles). To obtain a thermal boundary condition, it is assumed that only the portion of the particle surface which contacts the wall is affected by the wall and that the temperature along this surface varies linearly from the wall temperature to the bed temperature.

The heat transfer due to flow within the interstitial channel is obtained using an inviscid core/boundary layer model of the interstitial gas flow as shown in Figure 2.3. The inviscid core velocity is assumed to vary linearly from zero at the entrance to the channel to twice the average interstitial velocity at the exit. Thus the inviscid flow is assumed to be "stagnation-like" as the result of the interaction between the gas and solid particles. Surrounding the inviscid core is a three-dimensional boundary layer flow which is further divided into regions of two-dimensional flow near the channel center and three-dimensional but

Fig. 2.2 Interstitial Channel.

A. Physical Plane

B. Circle Plane

Fig. 2.3 Stokes Flow Regions

Stokes' flow near the cusped corners. The geometrical parameter defining these regions, x_s , is established by requiring the heat transfer to the wall be continuous across the channel and is determined effectively by the two-dimensional boundary layer thickness. Also, the effect of turbulence generated by the particle wakes is included in the analysis.

The situation for bubble phase contact is illustrated in Figure 2.4. The heat transfer in this case will be due to gas convection as well as radiation from the particle string bounding the bubble. The gas convective heat transfer will also be influenced by free stream turbulence generated by the wakes of the particles surrounding the bubble.

Many analytical models have been developed for heat transfer to surfaces immersed in small particle fluidized beds in which unsteady conduction from the particles is the dominant mode of heat transfer (see for example Botterill (1975)). Most of these models are based upon consideration of a packet of material (solid particles and gas) which is swept up to the heat transfer surface and exchanges energy by unsteady conduction. Differences in the various models are primarily differences in modeling the characteristics of the packet. This packet was treated as a continuum by Mickley and Fairbanks (1955) and their model was refined and modified by a number of investigators; for example Yoshida, et al. (1969), Wasan and Ahluwalia (1969), Chung, et al. (1972), Broughton and Kubie (1975), and others. A model based upon discrete particles dispersed throughout the gas was developed by Botterill and Williams (1963). Ziegler, et al. (1964) also used a discrete particle model but assumed that the gas phase is at the wall temperature and heat is transferred from the particles by convection to the gas phase. Basu (1978) used this

Fig. 2.4 Contacting Bubble.

same discrete particle model in his analysis of the effect of combustion on heat transfer and Gabor (1972) based his model on unsteady conduction in alternate states of solid and gas. A key parameter in all of these models is the residence time distribution of the packet and the continuum approaches require thermal properties of the packet as well.

Very little analytical work has been directed toward heat transfer in large particle beds in which gas convection is the dominant mode, though a few gas convection based models have been developed. One of the first of these was based upon "scouring" of the gas boundary layer at particle contact points (a two-dimensional view of the interstitial boundary layer development) and was developed by Levenspiel and Walton (1954). Also, Baskakov, et al. (1973) have developed an empirical model for this regime based upon experimental data obtained by Baskakov and Suprun (1972). Recently, Botterill and Denloye (1976) extended a packed bed model, based upon one-dimensional flow and an effective conductivity, to estimate heat transfer to a vertical cylinder due to gas convection. None of these models adequately treats heat transfer due to the three-dimensional interstitial flow in the presence of isothermal, hot particles and due to flow within bubbles contacting the immersed surface. Empirical approaches which include some of these effects have been developed by Catipovic, et al. (1980) and Glicksman (1979).

2.1.1 Analysis of Two-Dimensional Boundary Layer Flow

The convective heat transfer due to flow in the two-dimensional portion of the interstitial channels and within a contacting bubble is obtained from an integral solution of the two-dimensional boundary layer equations. The effect of interstitial turbulence on the solution is much

like the effect of freestream turbulence on the heat transfer to objects immersed in an air stream for which similarity solutions have been obtained, for example Smith and Kuethe (1966) and Traci and Wilcox (1975). The effect is included here in an integral solution by introducing a transformation which effectively stretches the normal coordinate. Since gas property variations have a small effect on heat transfer (see Levy (1954) or Kays (1966)), the constant property form of the low speed boundary layer equations will be used and the effect of turbulence will be included through an eddy diffusivity term in the time-averaged equations. In dimensionless form, they are

$$\frac{\partial v^+}{\partial y^+} + \frac{\partial u^+}{\partial z^+} = 0 \quad (2.1)$$

$$v^+ \frac{\partial u^+}{\partial y^+} + u^+ \frac{\partial u^+}{\partial z^+} = - \frac{dp^+}{dz^+} + \frac{1}{Re_D} \frac{\partial}{\partial y^+} ((1+\epsilon_T^+) \frac{\partial u^+}{\partial y^+}) \quad (2.2)$$

$$v^+ \frac{\partial \theta^+}{\partial y^+} + u^+ \frac{\partial \theta^+}{\partial z^+} = \frac{1}{Re_D Pr} \frac{\partial}{\partial y^+} ((1+\epsilon_T^+) \frac{\partial \theta^+}{\partial y^+}) \quad (2.3)$$

where ϵ_T^+ is the dimensionless eddy diffusivity and eddy diffusivities of heat and momentum are assumed to be equal and isotropic. Also the boundary conditions are

$$\text{at } y^+ = 0 \quad (\text{cylinder wall})$$

$$u^+ = 0 \quad (2.4a)$$

$$u^+ = 0 \quad (2.4b)$$

$$\theta^+ = 0 \quad (2.4c)$$

$$\epsilon_T^+ = 0 \quad (2.4d)$$

thus

$$\frac{\partial}{\partial y^+} ((1+\epsilon_T^+) \frac{\partial u^+}{\partial y^+}) = \frac{dp^+}{dz^+} \quad (2.4e)$$

and

$$\frac{\partial}{\partial y^+} \left((1+\epsilon_T^+) \frac{\partial \theta^+}{\partial y^+} \right) = 0 \quad (2.4f)$$

at $y^+ = \delta$ (edge of boundary layer)

$$u^+ = u_\infty^+ \quad (2.5a)$$

$$\theta^+ = 1 \quad (2.5b)$$

$$\frac{\partial u^+}{\partial y^+} = 0 = \frac{\partial^2 u^+}{\partial y^{+2}} \quad (2.5c)$$

$$\frac{\partial \theta^+}{\partial y^+} = 0 = \frac{\partial^2 \theta^+}{\partial y^{+2}} \quad (2.5d)$$

$$\epsilon_T^+ = \epsilon_{T\infty}/v_\infty = \beta_T \quad (2.5e)$$

The integral transformation $d\xi = \frac{dy^+}{1+\epsilon_T^+}$ is introduced to absorb the interstitial turbulence in the normal coordinate and Equations 2.1-2.3 are integrated across the boundary layer following the von-Kármán-Pohlhausen approach (see Schlichting (1968)) to obtain

$$\frac{d}{dz^+} \left\{ \int_0^{\delta} V u^+ (u_\infty^+ - u^+) (1+\epsilon_T^+) d\xi \right\} + \left\{ \int_0^{\delta} V (u_\infty^+ - u^+) (1+\epsilon_T^+) d\xi \right\} \frac{du_\infty^+}{dz^+} = \frac{1}{Re_D} \frac{\partial u^+}{\partial \xi} (0) \quad (2.6)$$

$$\frac{d}{dz^+} \left\{ \int_0^{\delta} H u^+ (1-\theta^+) (1+\epsilon_T^+) d\xi \right\} = \frac{1}{Re_D Pr} \frac{\partial \theta^+}{\partial \xi} (0) \quad (2.7)$$

These equations are further reduced to

$$\frac{d}{dz^+} (M u_\infty^+) = 2 \left(\frac{H_{2V}}{u_\infty^+} \frac{\partial u^+}{\partial \xi} (0) \right) - \left(\frac{3}{2} + H_{12} \right) M \frac{du_\infty^+}{dz^+} \quad (2.8)$$

$$\frac{d}{dz^+} (W u_\infty^+) = 2G_{2H} \frac{\partial \theta^+}{\partial \xi} (0) - W \frac{du_\infty^+}{dz^+} \quad (2.9)$$

where $M = \text{Re}_D \delta_2^2$, $W = \text{Re}_D \text{Pr} \Delta_2^2$ and

$$\delta_1 = \int_0^{\delta_v} \left(1 - \frac{u^+}{u_\infty^+}\right) (1 + \epsilon_T^+) d\xi \quad (2.10)$$

$$\delta_2 = \int_0^{\delta_v} \frac{u^+}{u_\infty^+} \left(1 - \frac{u^+}{u_\infty^+}\right) (1 + \epsilon_T^+) d\xi \quad (2.11)$$

$$\Delta_2 = \int_0^{\delta_H} \frac{u^+}{u_\infty^+} (1 - \theta^+) (1 + \epsilon_T^+) d\xi \quad (2.12)$$

$$H_{2v} = \frac{\delta_2}{\delta_v} \quad (2.13)$$

$$H_{12} = \frac{\delta_1}{\delta_2} \quad (2.14)$$

$$G_{2H} = \frac{\Delta_2}{\delta_H} \quad (2.15)$$

Appropriate profiles for u^+ , θ^+ and ϵ_T^+ must now be introduced and fitted to the boundary conditions so that the integral parameters can be evaluated. Following Pohlhausen's analysis, the following profiles are assumed

$$\frac{u^+}{u_\infty^+} = a_1 \eta_v + a_2 \eta_v^2 + a_3 \eta_v^3 + a_4 \eta_v^4 \quad (2.16)$$

$$\theta^+ = b_1 \eta_H + b_2 \eta_H^2 + b_3 \eta_H^3 + b_4 \eta_H^4 \quad (2.17)$$

where

$$\eta_v = \xi/\delta_v \text{ and } \eta_H = \xi/\delta_H$$

and the eddy diffusivity is assumed to be linear in n_v . The boundary conditions (Equations 2.4 and 2.5) are used to establish the profile parameters. Then evaluation of the integrals reveals the true dependent variables to be the velocity profile shape parameter, $\Omega_v = \text{Re}_D \delta_v^2 \frac{du_\infty^+}{dz^+}$ and the thickness ratio, $N_{HV} = \frac{\delta_H}{\delta_v}$. Numerical solution of Equations 2.8 and 2.9 provides Ω_v and N_{HV} as a function of the turbulence parameters β_T . Then the Nusselt number is

$$\begin{aligned} \text{Nu}_D &= \frac{\partial \theta^+}{\partial y^+} \\ &= \frac{2}{\delta_H} = \frac{2}{N_{HV} \left[\frac{\Omega_v}{\text{Re}_D \frac{du^+}{dz^+}} \right]^{1/2}} \end{aligned} \quad (2.18)$$

The turbulence parameter, β_T , is related to the interstitial turbulence intensity through

$$\beta_T = K_T \frac{u' u_\infty \ell_\infty}{v_\infty} \quad (2.19)$$

where K_T is an empirical constant and ℓ_∞ is the length scale for the turbulence. Following Traci and Wilcox (1975), the turbulence length scale is assumed to be

$$\ell_\infty \sim \frac{L}{\sqrt{\text{Re}_{L_\infty}}} \quad (2.20)$$

where L is an appropriate characteristic length (D for a cylinder in an air stream and d_p for a fluidized bed). Thus

$$\beta_T = K_T u' \sqrt{\text{Re}_{L_\infty}} \quad (2.21)$$

The empirical constant, K_T , is obtained by matching predictions for stagnation point heat transfer for a cylinder in an air stream with experimental results. As shown in Figure 2.5, calculations performed using the integral analysis with $K_T = 0.13$ provide results which agree well with experimental data.

For the stagnation-like flow in the interstitial channels, the integral equations reduce to the following nonlinear algebraic equations

$$a_1 - (2H_{2V} + H_{1V}) \Omega_{V_{stg}} = 0 \quad (2.22)$$

$$b_1 - N_{HV_{stg}}^2 \text{Pr} \Omega_V G_{2H} = 0 \quad (2.23)$$

This set of equations has been solved iteratively using Newton's method for a wide range of the parameter β_T . From the numerical results, N_{HV} is approximately independent of the intensity of interstitial turbulence (varying between 1.5 and 1.6 for a wide range of $\beta_T = 0.13 u' \sqrt{2\text{Re}_p Q_g^+}$ and for $\text{Pr} = 0.7$) and $\Omega_{V_{stg}}$ can be approximated by

$$\Omega_{V_{stg}} \approx 1.405 + 5.6 e^{-0.462\beta_T} \quad (2.24)$$

to within 5% of the numerical results. It is further suggested that the approximation

$$N_{HV_{stg}} \approx 1.34 \text{Pr}^{-0.4} \quad (2.25)$$

be used so that the proper variation in stagnation point Nusselt number with Prandtl number is obtained (see, e.g., Kays (1966)). Also, the two-dimensional Nusselt number based upon particle diameter for the channel reduces to

Fig.25. Effect of Free Stream Turbulence on Stagnation Point Heat Transfer for a Cylinder

$$(Nu_p)_{2D} = \frac{2\sqrt{2}}{N_{HV_{stg}}} \frac{Q_g^+ Re_p^{1/2}}{S_p (\frac{r_p}{r_p}) \Omega_{v_{stg}}} \quad (2.26)$$

since the velocity gradient is $\frac{du^+}{dz^+} = \frac{DQ_g^+}{S_p}$ where S_p is half of the particle spacing at the wall.

In the case of flow inside contacting bubbles, the velocity gradient of the freestream can have arbitrary values so that the integral equations (2.8 and 2.9) must be solved numerically. The integration of these equations is performed using the IMSL Routine DGEAR.

2.1.2 Stokes' Region Analysis

The Stokes flow assumption for the cusped corners of the interstitial channel reduces the analysis of heat transfer in these regions to solution of the steady conduction equation for the region shown in Figure 2.3. The edge of this region along the cylinder wall is established by requiring

$$(Nu_p)_{\text{Stokes edge}} = (Nu_p)_{2D} \quad (2.27)$$

with $(Nu_p)_{2D}$ obtained from the two-dimensional boundary layer analysis described above. The remaining boundaries of the region are assumed to be natural coordinates produced by the conformal mapping of a unit circle onto the channel cross section as shown in Figure 2.3B. Furthermore, the region is assumed to form a curvilinear rectangle on the unit circle with a width in the radial direction equal to half the length in the angular direction. These assumptions are based upon experimental results for the development of the boundary layer along a right angle corner obtained by Zamir and Young (1970). The validity of these assumptions and the two-region analysis described here is demonstrated for flow along a right

angle corner in Figure 2.6. This figure provides a comparison of the two-region model with finite difference calculations by Rubin and Grossman (1971) and experiments by Zamir and Young (1970) for the skin friction coefficient.

Utilizing Kirchoff's transformation (Arpaci (1966)) for thermal conductivity varying linearly with temperature and neglecting turbulence, the energy equation for the Stokes region is

$$\nabla^2 S = 0 \quad (2.28)$$

where

$$S(\theta^+) = \frac{S(0)}{(0.5 + T_W^+)} \left(\frac{1}{2} (1 - \theta^{+2}) + T_W^+ (1 - \theta^+) \right) \quad (2.29)$$

Also the boundary conditions on the circle plane are

$$\theta^+(1, \phi) = (1 - \frac{2\phi}{\phi_0})^{\mu_T} \quad 0 \leq \phi \leq \frac{\phi_0}{2} \quad (2.30a)$$

$$\theta^+(1, \phi) = 0 \quad \frac{\phi_0}{2} < \phi \leq \phi_0 \quad (2.30b)$$

$$\theta^+(R_1, \phi) = 1 \quad (2.30c)$$

$$\frac{\partial \theta^+}{\partial r}(r, \phi) = \frac{\partial \theta^+}{\partial r}(r, \phi_0) = 0$$

where the parameter μ_T is determined so that the temperature variation along the curved portion of the physical boundary is approximately linear. The parameter μ_T can be approximated from the general relation for a straight-sided corner mapping as

$$\mu_T \approx \frac{\ln(x_c^+ - x^+)}{\ln(\phi - \phi_c)} \quad (2.31)$$

with $x^+(\phi)$ determined from the mapping function.

Fig. 2.6 Skin Friction Coefficient for Flow Along a Right Angle Corner

Equation 2.28 is solved using separation of variables and the Fourier coefficients are obtained from the first two boundary conditions. The resulting expressions for the coefficients contain confluent hypergeometric functions and analytic expressions for these are obtained by using the asymptotic forms (see Adams (1977)). The solution obtained in this manner is used to obtain Nusselt number at the edge of the Stokes region (the matching parameter) and the total energy conducted to the wall. Results of computations over a broad range of the parameters R_1 , T_W^+ , and μ_T provided the approximation

$$(Nu_D)_{\text{Stokes edge}} \approx - \frac{1.4 S_0}{\phi_s} \quad (2.32)$$

where $\phi_s = -h_0 \ln(1 - (\phi - \phi_c))$ and is determined as a function of x^+ from the conformal mapping. Also, the variation in total energy is as shown in Figure 2.7.

The mapping function, required for determination of the parameter x_s , is obtained numerically using Theodoren's method (1933). Values of x_s for three particle spacings are shown in Figure 2.8 and μ_T was found to be 0.11 to 0.15 for particle spacings of one to 1.5 diameters. Detailed derivations and the computation scheme can be found in Adams (1977).

When the results of the two-dimensional and Stokes' flow analyses described above are combined, the channel Nusselt number can be expressed as follows

$$Nu_p = \left(\frac{x_s}{s_p}\right) Nu_{p2D} + Nu_{p0} \quad (2.33)$$

where Nu_{p0} depends upon the total energy parameter approximated from

Fig. 2.7 Stokes Region Total Energy Parameter

Figure 2.8 Stokes region matching function.

numerical results as

$$Nu_{p_o} \approx \frac{2 S_o}{\left(\frac{s_p}{r_p}\right)} \left\{ 3.81 - 1.60 \left(\frac{s_p}{r_p} - 1 \right) \right\} \quad (2.34)$$

While the two-dimensional fraction, $\frac{x_s}{s_p}$, can be approximated from the results of numerical calculations as

$$\frac{x_s}{s_p} \approx 0.76 - \frac{k_1 S_o}{(Nu_p)_{2D}} \text{ for } (Nu_p)_{2D} \geq (Nu_p)_m \quad (2.35a)$$

$$\frac{x_s}{s_p} \approx \frac{k_2}{S_o} (Nu_p)_{2D} \text{ for } (Nu_p)_{2D} < (Nu_p)_m \quad (2.35b)$$

where

$$k_1 = 7.36 - 10.81 \left(\frac{s_p}{r_p} - 1 \right) \quad (2.36)$$

$$k_2 = 0.00904 + 0.01587 \left(\frac{s_p}{r_p} - 1 \right) \quad (2.37)$$

and

$$(Nu_p)_m = \frac{2k_1 S_o}{0.76 - (0.578 - 4k_1 k_2)^{1/2}} \quad (2.38)$$

The agreement with this approximation is within about 15% for $1 \leq \frac{s_p}{r_p} \leq 1.27$ or for $0.435 \leq \epsilon \leq 0.65$ where ϵ is the void fraction at the wall and the approximation

$$\frac{s_p}{r_p} = \frac{0.75}{\sqrt{1-\epsilon}} \quad \text{is used.} \quad (2.39)$$

After the above approximations are assembled, the Nusselt number for the interstitial channel flow is found to be

$$Nu_p \approx \begin{cases} a \sqrt{Re_p} + b + Nu_{p_o} & Re_p \geq Re_{p_m} \\ c Re_p + Nu_{p_o} & Re_p < Re_{p_m} \end{cases} \quad (2.40)$$

$$Nu_p \approx \begin{cases} a \sqrt{Re_p} + b + Nu_{p_o} & Re_p \geq Re_{p_m} \\ c Re_p + Nu_{p_o} & Re_p < Re_{p_m} \end{cases} \quad (2.41)$$

where

$$a = 0.76 \alpha \quad (2.42)$$

$$b = -k_1 S_o \quad (2.43)$$

$$c = \frac{k_2 \alpha^2}{S_o} \quad (2.44)$$

$$\alpha = \frac{2.11}{Pr^{-0.4}} \left[\frac{Q_g^+}{\left(\frac{s_p}{r_p}\right) (1.405 + 5.6e^{-0.0849 u'} \sqrt{Re_p Q_g^+})} \right]^{1/2} \quad (2.45)$$

The maximum Reynolds number, Re_{p_m} , for quasi-linear behavior satisfies the nonlinear equation

$$Re_{p_m} Q_g^+ = A + B e^{-C \sqrt{Re_{p_m} Q_g^+}} \quad (2.46)$$

where

$$A = 1.405 \beta \quad (2.47)$$

$$B = 5.6 \beta \quad (2.48)$$

$$C = 0.0849 u' \quad (2.49)$$

with

$$\beta = \left[\frac{1.34 Pr^{-0.4} k_1 S_o \sqrt{\frac{1}{2} \frac{s_p}{r_p}}}{0.76 - (0.578 - 4k_1 k_2)^{1/2}} \right]^2 \quad (2.50)$$

Typically, $S_o \sim 1$, $Pr \sim 0.7$, $u' \sim 0.2$ so that $Re_{p_m} Q_g^+$ ranges between 5000 and 3500 for void fractions of 0.45 to 0.65 (typical surface values). The

accuracy of the various approximations described above is illustrated by the comparison between approximate and exact results for local Nusselt number shown in Figure 2.9.

The case of a packed bed of large particles can be modeled by assuming that the portion of the Stokes' region bounding the particle surface is at the wall temperature. Then no heat transfer occurs by conduction between the particle surface and the wall and the contribution due to conduction from the inviscid core reduces to

$$(Nu_{p_o})_{\text{packed}} = \frac{2 S_o}{\left(\frac{s_p}{r_p}\right)} \quad (2.51)$$

While the Stokes' region matching criterion is modified such that the parameters k_1 and k_2 appearing in the approximations for the Stokes' region boundary are

$$(k_1)_{\text{packed}} = (k_1)_{\text{fluidized}} / 1.4 \quad (2.52)$$

$$(k_2)_{\text{packed}} = [(k_2)_{\text{fluidized}}] \times 1.4 \quad (2.53)$$

Therefore,

$$(k_1)_{\text{packed}} = 5.26 - 7.72 \left(\frac{s_p}{r_p} - 1\right) \quad (2.54)$$

$$(k_2)_{\text{packed}} = 0.0127 + 0.0222 \left(\frac{s_p}{r_p} - 1\right) \quad (2.55)$$

2.1.3 Radiation Heat Transfer

The radiation heat transfer due to emission from the hot particle surfaces is obtained using the net radiation method (Siegel and Howell

Figure 2.9 Local Nusselt number comparison for horizontal tube.

(1972)) and assuming that the particle strings adjacent to the cylinder wall and at the bubble boundary form isothermal gray surfaces as shown in Figure 2.10. The net radiation heat transfer is converted to effective Nusselt numbers to obtain

$$\text{Nu}_{D_1}^R = \text{Bi}^R \frac{e_p (T_B^{+4} - T_W^{+4})}{1 + e_p \left(\frac{1}{e_W} - 1 \right)} \quad (2.56)$$

$$\text{Nu}_{D_2}^R = \text{Bi}^R \frac{e_B (T_B^{+4} - T_W^{+4})}{\frac{A_2}{A_3} + e_B \left(\frac{1}{e_W} - \frac{A_2}{A_3} \right)} \quad (2.57)$$

where $\text{Bi}^R = \frac{\sigma D}{K_g} (T_B - T_W)^3$ and the emissivity of the surface formed by the particles adjacent to the wall, e_p , is adjusted for cooling using the results of Baskakov et al. (1973).

Fig. 2.10 Enclosures for Radiative Exchange Calculation

2.2 HYDRODYNAMIC MODEL-DETERMINATION OF GAS VELOCITY

Two key parameters in the heat transfer analysis are the interstitial gas velocity at the heat transfer surfaces, required for determination of the emulsion phase heat transfer, and the gas velocity distribution within contacting bubbles, necessary for bubble phase heat transfer calculations. These parameters are obtained for a horizontal tube in a two-dimensional bubbling bed by using a combination of complex analysis and numerical solutions of reduced forms of the hydrodynamic equations of Anderson and Jackson, (see Davidson and Harrison (1971)) for the emulsion phase together with a potential flow solution for flow inside contacting bubbles.

Order of magnitude analysis suggests that the equation governing the interstitial gas flow can be reduced to (see Adams (1977))

$$\nabla p^+ = -k_\beta (\vec{Q}_g^+ - \vec{Q}_s^+) \quad (2.58)$$

with

$$\nabla \cdot \vec{Q}_g^+ = \nabla \cdot \vec{Q}_s^+ = 0 \quad (2.59)$$

where k_β , the dimensionless drag coefficient is approximated using the Ergun equation (see Kunii and Levenspiel (1962)) as

$$k_\beta = \frac{1}{d_p^+} \frac{\beta_1}{Re_p} + \beta_2 \quad (2.60)$$

with

$$\beta_1 = 150 \left(\frac{(1-\epsilon)}{\epsilon \phi_s} \right)^2 \quad (2.61)$$

$$\beta_2 = \frac{1.75}{\epsilon_\infty} \left(\frac{1-\epsilon}{\epsilon \phi_s} \right) \quad (2.62)$$

Note that β_2 as given above is based upon the linearization $|Q_g^+| \approx \frac{1}{\epsilon_\infty}$.

By combining Equations 2.58-2.62 above, the equation governing the pressure field is obtained as

$$\nabla \cdot (g \nabla p^+) = 0 \quad (2.63)$$

where

$$g = \frac{\epsilon}{\frac{\beta_1}{Re_p} + \beta_2} \quad (2.64)$$

Note that when $g = \text{const}$ (uniform voidage case), the pressure field satisfies Laplace's equation.

Equation 2.63 was solved numerically to obtain the effect of voidage variations upon the interstitial gas flow for the case of a tube immersed in a bed without bubbles. Analytical solutions were obtained for the uniform voidage case with bubbles present and these results were used to establish gas velocity distributions inside contacting bubbles. In both cases, Equation 2.58 was used to determine the interstitial gas velocity with k_g based upon actual voidage.

2.2.1 Numerical Solution for the Pressure Field Without Bubbles

The pressure equation with variable voidage has been solved using a finite difference scheme for the case of a tube immersed in a bed free of bubbles. This was done to obtain more accurate results for the lower stagnation region of the tube where large variations in voidage have been observed to occur.

Applicable boundary conditions for Equation 2.63 and the chosen geometry are

$$\frac{\partial P^+(r_g^+, \theta)}{\partial r^+} = 0 \quad \text{at the cylinder wall} \quad (2.65)$$

and

$$(\nabla P^+)_{\infty} = \left(\frac{dP^+}{dY^+}\right)_{\infty} \hat{e}_y \quad (2.66)$$

The ambient pressure gradient, $\left(\frac{dP^+}{dY^+}\right)_{\infty}$, in the bed was obtained by expressing Equation 2.58 for locations far from the cylinder surface. This results in the expression

$$\left(\frac{dP^+}{dY^+}\right)_{\infty} = \frac{-1}{\epsilon_{\infty} d_p^+} \left[\frac{\beta_1(\epsilon_{\infty})}{Re_p} + \beta_2(\epsilon_{\infty}) \right] \quad (2.67)$$

To complete the mathematical formulation of the pressure field and interstitial velocity problem it was necessary to specify the voidage throughout the bed. If there are no gas bubbles near the cylinder, the voidage will be constant except for a small region very near the cylinder wall, so the following expression for the voidage distribution has been used:

$$\epsilon = e^{\tau(R^+ - r^+)} [\epsilon(R^+, \theta) - \epsilon_{\infty}] + \epsilon_{\infty} \quad (2.68)$$

The rate at which the voidage approaches its ambient value, at positions radial from the cylinder wall, is determined by the magnitude of the constant τ . Throughout this study, it has been assumed that most of the transition from the near cylinder wall voidage to the ambient bed voidage takes place within one particle diameter of the cylinder surface. In terms of the dimensionless particle diameter, the constant τ can be expressed as

$$\tau = \frac{1}{d_p^+} \quad (2.69)$$

Substantial deviations from the above value of τ produced little change in the calculated pressure field, average interstitial velocity, or

therefore, the heat transfer solution for the lower stagnation point region.

While more rigorous expressions for the voidage variation near a solid surface do exist, they are generally based on the assumption of constant-sized spherical particles. Limestone or dolomite particles, to be used in fluidized bed combustion, typically have a sphericity of about 0.65. Also, the particles in an operating fluidized bed are not of uniform size. For these reasons, and considering the stochastic nature of the particle packing at the cylinder wall, the use of a more complex expression for the voidage variation was considered unwarranted.

The solution for the pressure field, as given by the variable voidage formulation, Equation 2.63, and the boundary conditions, Equations 2.65 and 2.66, has been obtained numerically for several particle sizes. Equation 2.63 can be expressed in dimensionless polar form as

$$A(r^+, \theta) \frac{\partial p^+}{\partial r^+} + B(r^+, \theta) \frac{\partial p^+}{\partial \theta} + r^+ \frac{\partial^2 p^+}{\partial r^{+2}} + \frac{1}{r^+} \frac{\partial^2 p^+}{\partial \theta^2} = 0 \quad (2.70)$$

Where the functions A and B are defined as

$$A(r^+, \theta) = \frac{1}{g} \frac{\partial(r^+ g)}{\partial r^+} \quad (2.71)$$

$$B(r^+, \theta) = \frac{1}{gr^+} \frac{\partial g}{\partial \theta} \quad (2.72)$$

Finite difference approximations correct to $O[(\Delta r^+)^2]$ and $O[(\Delta \theta)^2]$ were used to reduce Equation 2.70 to a set of linear algebraic equations.

The numerical solution is complicated by the asymptotic boundary conditions expressed by Equation 2.66; which requires a node network of large radial extent, and the rapid variation in voidage near the cylinder

wall; which requires very small radial increments for accuracy. To circumvent these conflicting requirements, and still obtain solutions for the pressure field in a reasonable amount of computer time, a graded finite difference network was used. Fine radial increments near the cylinder wall and relatively coarse radial increments further from the cylinder wall proved to work quite well. A total of 900 nodes were used in the calculation of the pressure field.

The solution of the linear algebraic equations for the pressure field was obtained by the successive overrelaxation method. Rapid convergence was obtained, and no stability problems were evident, with an overrelaxation parameter of 1.4.

The pressure gradient at the cylinder wall was obtained from the derivative of a cubic spline function which was fitted to the pressure values along the cylinder wall. After obtaining the local pressure gradient, the average interstitial velocity at the cylinder wall was obtained directly from Equation 2.58.

As a check on the accuracy of the numerical procedure outlined above, the limiting case of uniform voidage throughout the bed was used. For the case of uniform voidage, the governing equation for the pressure field reduces to Laplace's Equation and the pressure gradient at the cylinder wall can be expressed analytically as

$$\frac{1}{R^+} \frac{\partial P^+(R^+, \theta)}{\partial \theta} = 2 \left(\frac{dP^+}{dY^+} \right)_m \cos \theta \quad (2.73)$$

The numerical solution discussed above yielded a solution for the pressure gradient at the cylinder wall with a maximum error of less than 1.2% when compared with the exact analytical solution. This was considered

adequate accuracy for this investigation. Increasing the total number of node points in the finite difference mesh improved the accuracy of the numerical solution somewhat. However, computer run time and storage requirements also increased substantially as a greater number of nodes were used. While it is not possible to check the accuracy of the numerical solution for the case of variable voidage directly, the error due to numerical roundoff and truncation should be of approximately the same magnitude as for the uniform voidage case.

The results of numerical calculations were compared with the uniform voidage pressure field to establish a correction to the average interstitial velocity in regions of high voidage. Calculations were performed for $\frac{d}{D}$ of 0.05, 0.08 and 0.12 and Re_D of 500, 1000, 2000, 4000, 8000 and 12000. A typical result for the voidage distribution assumed (see Figure 2.11) is shown in Figure 2.12. Correlation of the results provided the following correction factor;

$$\frac{Q_g}{(Q_g)_{\text{const voidage}}} = \frac{1}{1 + \left(\frac{\epsilon}{\epsilon_\infty} - 1\right)^2 \left(0.65 + 12.13 \frac{d}{D} + \frac{991}{Re_D}\right)} \quad (2.74)$$

valid for $\epsilon_\infty = 0.5$, $\phi_s = 0.65$ and $\epsilon > 0.65$. This correction factor then gives the stagnation point velocity gradient without bubbles in the form

$$\left(\frac{dQ^+}{dZ^+}\right)_{\text{stag. pt.}} \approx \frac{2}{\epsilon_\infty} \frac{k_\beta(\epsilon_\infty)}{k_\beta(\epsilon)} \left\{ \frac{1}{1 + \left(\frac{\epsilon}{\epsilon_\infty} - 1\right)^2 \left(0.65 + 12.13 \frac{d}{D} + \frac{991}{Re_D}\right)} \right\} \quad (2.75)$$

Figure 2.11 Voidage distribution around a cylinder (assumed)

Figure 2.12 Interstitial velocity distribution at cylinder wall.

2.2.2 Approximate Solution for the Velocity Field With Bubbles

An approximate solution for the pressure and particle velocity fields with bubbles is obtained by assuming uniform voidage throughout the bed and assuming that the particle flow induced by moving bubbles is equivalent to potential flow. With these assumptions, the pressure field satisfies Laplace's Equation, i.e.,

$$\nabla^2 p = 0 \quad (2.76)$$

with boundary conditions

$$\frac{\partial p}{\partial n} = 0 \quad \text{at tube surface} \quad (2.77a)$$

$$p = \text{constant at bubble boundary} \quad (2.77b)$$

$$\nabla p = -\left(\frac{dp}{dy}\right)_\infty \hat{e}_y \quad \text{far from tube} \quad (2.77c)$$

For the present analysis, it is assumed that particle motion in the vicinity of the tube is induced by rising bubbles which pass the tube. Models of particle motion induced by bubbles range from the potential flow model of Davidson (1961) to solution of the full two-phase flow equations as done by Martin-Gautier and Pyle (1976). The Martin-Gautier and Pyle solution provides no substantial increase in accuracy, when compared to experimental data, over the potential flow solutions. Therefore, the simpler potential flow model will be used here. With this assumption, the particle motion is described using the velocity potential ϕ_p , with

$$\nabla^2 \phi_p = 0 \quad (2.78a)$$

$$\frac{\partial \phi_p}{\partial n} = \vec{V}_B \cdot \hat{n} \quad \text{at bubble boundary} \quad (2.78b)$$

$$\frac{\partial \phi_p}{\partial n} = 0 \quad \text{at a tube surface} \quad (2.78c)$$

$$\nabla \phi_p = 0 \quad \text{far from the bubble inducing the motion} \quad (2.78d)$$

Separate calculation of the pressure field and particle velocity as described above allows determination of the local interstitial gas velocity through Equation 2.58. In this regard, the parameter, K_β , is based upon local surface voidage.

The pressure field and particle velocity potential are obtained using the method of images, to satisfy the boundary conditions. This technique is commonly applied in the aerodynamics of multi-element airfoils, see for instance the recent paper by Halsey (1979). To facilitate the analysis, complex pressure and velocity potential functions are used so that images are established by application of the circle theorem (see, e.g., Batchelor (1967)). When several objects are present, multiple reflections are made, with the number of reflections controlled by the accuracy with which the boundary conditions are to be met. The complex functions representing the pressure field and particle velocity potential are constructed by beginning with the ambient potentials (the potential without bubbles and tubes) and adding complex doublets to represent bubbles and tubes, then images of these doublets and images of the images and so on until suitable accuracy is achieved.

For the particle motion calculation, boundary condition (2.78b) is satisfied by representing the bubble with a moving doublet, then images are used to cancel the effect of adjacent objects (bubbles and tube) on the normal component of velocity at the bubble boundary.

Two types of boundary conditions must be satisfied. Type 1 is the condition of flow tangency for the gas and particles at the tube surface and for the portion of particle motion which is produced by images at the bubble boundary. This condition is achieved by complex functions of the form

$$G(Z) = F(Z) + \bar{F}\left(\frac{b_i^2}{Z - Z_{o_i}}\right) \quad (2.79)$$

in accordance with the circle theorem, where b_i is the radius of the object and Z_{o_i} is the location of the center of the object. The second type boundary condition is that of constant pressure along a bubble boundary; this condition is achieved by functions of the form

$$H(Z) = F(Z) - \bar{F}\left(\frac{b_i^2}{Z - Z_{o_i}}\right) \quad (2.80)$$

which is a modification of the circle theorem. Equations 2.79-2.80 provide the algorithm for addition of images to achieve constant pressure and flow tangency boundary conditions.

A potential flow solution is also obtained for flow inside a contacting bubble. The boundary condition at the surface of the bubble (normal velocity component) is obtained from the emulsion phase analysis described above. The flow tangency boundary condition at the surface of the tube is satisfied by assuming a complex potential of the form (according to the circle theorem)

$$\Phi_B = \sum_{n=0}^N \left(D_n Z^n + \bar{D}_n \left(\frac{a^2}{Z}\right)^{-n} \right) \quad (2.81)$$

The complex coefficients, D_n , appearing in the above series are obtained by truncating the series and making a least squares fit to the boundary condition using the Householder transformation (see, e.g., Ralston and

Rabinowitz (1978)). In this analysis, N was fixed at 8 and 20 values of the boundary condition were used to determine the 8 D_n .

The emulsion phase velocity field outside a contacting bubble requires special consideration because the bubble and tube boundaries intersect and the image method described above cannot be used. However, it has been found that successive application of the Joukowski transformation

$$\zeta = c \left(\frac{\tau+1}{\tau-1} \right) \quad (2.82)$$

where $\tau = \left(\frac{Z+c}{Z-c} \right)^{1/\alpha}$, $\alpha = 2 - \frac{\beta}{\pi}$ and β and c are defined in Figure 2.13, will produce two pseudo circles with an altered region of intersection as shown in Figure 2.13. The transformed geometry can be represented by two contacting circles and the image method can be applied.

Generally it was found that five image reflections were required when bubbles contact the tube and two or three reflections were sufficient for object separations greater than 1/4 tube diameter for boundary condition accuracy to within 5%. Figure 2.14 shows typical velocity distributions obtained using the model. Shown for comparison are distributions without bubbles present and with bubbles present at two locations.

2.2.3 Exact Solution for the Velocity Field With a Single Contacting Bubble

The solution for the pressure field and therefore the velocity field in the case of a single contacting bubble is greatly simplified. For this case, the contacting bubble can be viewed as a boundary of constant velocity potential while the tube is a boundary of constant stream function. Furthermore, these two boundaries are orthogonal intersecting circles as shown in Figure 2.15a and the mapping

Fig.2.13 Use of Joukowski Transformation to Handle Contacting Bubbles.

Fig. 2.14 Polar Plot of Typical Velocity Distributions for $V_B = u_{mf}$ and $b/a = 1$ Without Adjustment for Local Voidage Variations.

a. Physical Plane

b. Transform Plane

Figure 2.15 Transformation $\zeta = 1/z$.

$$\zeta = 1/Z \quad (2.84)$$

reduces these boundaries to two orthogonal lines on the ζ -plane as shown in Figure 2.15b (see, e.g., Churchill (1948)).

With the mapping of Equation 2.84, the ambient flow due to the bed pressure gradient is represented by a doublet at the origin of the ζ -plane since

$$\begin{aligned} W_{\infty} &= -i \frac{u_{mf}}{\epsilon_{\infty}} Z \\ &= -i \frac{u_{mf}}{\epsilon_{\infty}} \left(\frac{1}{\zeta}\right) \end{aligned} \quad (2.85)$$

Then the boundary conditions of flow tangency at the tube wall and flow normalcy at the bubble boundary are met with the arrangement of images shown in Figure 2.16b. The corresponding points on the physical plane are shown in Figure 2.16a. Note that this arrangement produces a tangential velocity at the tube surface which is exactly twice that due to the ambient flow with bubble alone and the normal component of velocity at the bubble boundary is exactly twice that due to the ambient flow with the tube alone.

The resulting complex velocity is

$$\frac{dW}{dZ} = u - iv = -i \frac{u_{mf}}{\epsilon_{\infty}} \left(1 + \frac{a^2}{Z^2} - \frac{b^2}{(Z - Z_0)^2} - \frac{a^2 b^2}{(Z \bar{Z}_0 - a^2)^2}\right) \quad (2.86)$$

Thus, the tangential velocity at the tube wall and outside the contacting bubble is

$$u_{\theta}^+ = \frac{2}{\epsilon_{\infty}} \left\{ \cos\theta - b^2 \frac{(\cos\theta - 2R_o^* \cos\theta_o + R_o^{*2} \cos(\theta - 2\theta_o))}{[1 + R_o^{*2} - 2R_o^* \cos(\theta - \theta_o)]^2} \right\} \quad (2.87)$$

with $R_o^{*2} = 1 + b^2$.

Figure 2.16 Arrangement of singularities

The velocity potential for flow inside the contacting bubble follows directly from the result that the normal velocity component at the bubble boundary is twice that produced by the ambient flow without the bubble present. Following Equation 2.81, the normal component of velocity for the inside could be produced by the complex potential

$$W = - 2i u_{mf} \left(Z - \frac{a^2}{Z} \right) \quad (2.88)$$

Therefore, the tangential velocity at the tube wall is just

$$u_{\theta}^+ = 4 \cos \theta \quad (2.89)$$

This is twice the tangential velocity at the tube surface with no particles in the bed and is independent of bubble size and location.

2.3 PARAMETRIC STUDY OF BUBBLE AND ADJACENT TUBE INFLUENCE ON HEAT TRANSFER.

The hydrodynamic and heat transfer models described in Sections 2.1 and 2.2 have been combined so that the effects of adjacent bubbles or tubes upon local and total heat transfer to a horizontal tube can be established. The calculations presented were carried out for a tube immersed in a bed of 4 mm diameter particles operating at room temperature. The results presented should be representative of bubble and tube influence in the general case for large particle beds. The main input parameters for the study are listed in Table 2.1.

2.3.1 Effect of Bubbles

Figure 2.17 shows the effect of bubble velocity on the heat transfer distribution for the case of a single contacting bubble. Note that local values of heat transfer are effected by bubble velocity, but the total heat transfer remains nearly constant as the bubble velocity is increased from 0 to $2 u_{mf}$ (twice minimum fluidizing velocity). It seems that the alterations to interstitial gas velocity due to particle motion are self compensating. Since bubble velocity depends strongly upon superficial velocity, these results suggest that the Nusselt number ought to be independent of superficial velocity once the bed is fluidized and provided the bubble contact fraction is fixed. Also note that the presence of the bubble decreases the tube Nusselt number by about 20% relative to the level without bubbles present. These results agree qualitatively with experiments reported by Catipovic (1979).

The effect of bubble size and position on the total heat transfer to a single tube is shown in Figures 2.18, 2.19 and 2.20. For these

Table 2.1. Calculation parameters for the 4mm particle case.

Reynolds Number (Re_D)	5900
Bed Temperature	308 K
Wall Temperature	376 K
Bed Voidage	0.5
Surface Voidage	
Lower Stag. Pt.	0.90 ($\theta = 0$ to 0.452 rad)*
Side	0.55 ($\theta = 0.498$ to 2.29 rad)
Stack	0.44 ($\theta = 2.36$ to 3.1416 rad)
Turbulence Intensity	0.2
Prandtl Number	0.7
Particle Dia./Tube Dia.	0.0787

* θ from lower stagnation point, linear interpolation between limits, e.g., 0.452 to 0.498.

Figure 2.17. Polar Plot Showing Effect of Bubble Velocity on Local Nusselt Number.

Figure 2.18. Effect of Bubble Size and Position on Total Nusselt Number for Bubble at $\theta = \pi/4$ and $V_B = u_{mf}$.

Figure 2.19. Effect of Bubble Size and Position on Total Nusselt Number for Bubble at $\theta = \pi/2$ and $V_B = u_{mf}$.

Figure 2.20. Effect of Bubble Size and Position on Total Nusselt Number for Bubble at $\theta = 3\pi/4$ and $V_B = u_{mf}$.

calculations, the bubble velocity was fixed at U_{mf} , since bubble velocity has a small effect on total heat transfer. Again the bubble interaction effect was found to be small; the maximum change in total Nusselt number is 33% for a bubble three times tube size and nearly contacting the tube.

2.3.2 Effect of Adjacent Tubes

A summary of the effect of adjacent tubes upon total heat transfer in the absence of bubbles is presented in Table 2.2. Generally, adjacent tubes were found to increase the heat transfer due to increases in the local interstitial gas velocity. The maximum increase was obtained with one tube on each side and as close as possible to the heated tube. But when the reference tube was surrounded by four tubes, very little change in Nusselt number was observed. This suggests that tubes should be placed in rows with close horizontal spacing between adjacent tubes and wide vertical spacing between adjacent rows. The benefit of such an arrangement is suggested by the last column of Table 2.2 which shows that a horizontal spacing of 1.5 diameters and a vertical spacing of three diameters could provide an increase of 14% over the single tube level. According to the model, horizontal spacings of less than 1.5 diameters would further improve this benefit but are not practical because of the poor fluidization characteristics (bridging of particles) reported by Lese and Kermode (1972). To our knowledge there have been no experimental studies of optimum tube arrangements in this flow regime though a number of studies have been done in the particle convection dominant regime (see, e.g., McLaren and Williams (1969), Gelperin, et. al., (1969), and the review by Botterill (1975)). However, Catipovic's (1979) results for a staggered array with a pitch of two diameters indicate a slight decrease

Table 2.2. Effect of adjacent tubes.

Arrangement Spacing P/D	Total Nusselt Number

1.25	275.8	259.8	266.4	313.3	256.1	-
1.5	263.3	-	-	286.8	-	247.9
2	-	247.8	244.9	265.5	243.8	263.4
2.5	249.3	-	-	-	-	-
3	-	244.6	244.9	252.2	242.5	275.6
3.5	245.8	-	-	-	-	-
5	244.0	-	-	-	-	-
∞	242.4	242.4	242.4	242.4	242.4	286.8

NOTE: Tubes 1 and 2 are fixed at P/D = 1.5 while the position of 3 and 4 is varied as indicated.

in total heat transfer relative to single tube levels while the present results would suggest a very small increase. The major cause of the decrease observed by Catipovic is believed to be the increased bubble contact fraction for the tube bank, with this consideration, it would appear that the model prediction is at least qualitatively correct.

III. EXPERIMENTAL APPARATUS

Equipment used to measure the time-average local heat transfer rate and time-average local temperature¹ on the surface of a 50.8 mm O.D. tube is described. Also, the Oregon State University High Temperature Fluidized Bed Facility and associated data acquisition equipment are described in this chapter.

3.1 INSTRUMENTED TUBE FOR THE MEASUREMENT OF LOCAL HEAT TRANSFER RATE AND SURFACE TEMPERATURE

3.1.1 Design Considerations

After survey of the relevant literature on heat flux measurement and errors incurred in heat flux measurement (Brown et al., 1961; ASTM, 1978; Bachman et al., 1965; Westkaemper, 1961; Hornbaker and Rall, 1964; Hager, 1965; Woodruff et al., 1967; Schulte and Kohl, 1970), the following points were found to be important in the design of the instrumented tube:

- a. The tube surface must be smooth. Flow of gas and particles around the tube must be undisturbed by surface protrusions. It is particularly important that the surface be smooth near the locations of the heat flux and surface temperature measurement.
- b. In operation, the tube should have a near uniform surface temperature around its periphery. That is, the instrumented tube should provide a thermal boundary.

¹ Since only time-average quantities are reported, the words "time-average" will usually be omitted.

condition for the fluidized bed similar to that provided by a relatively thin wall steam generator or process heater tube. Investigators (see Botterill, 1975) generally agree that the local heat transfer coefficient is nonuniform around the periphery of the tube. In order to maintain a near uniform surface temperature, the tube must be of high thermal conductivity material and/or relatively thin wall thickness. Instruments which grossly violate the uniform surface temperature condition have been used by some investigators, for example, Cherrington et al. (1977). The error incurred is debatable.

- c. Even if extremely overheated, the instrumented tube should remain structurally sound. This would provide some protection in case of coolant pump failure or operator error.
- d. The instrumented tube should be durable enough to withstand many hours of use in the high temperature fluidized bed environment.

3.1.2 Description of the Instrumented Tube

A device which has most of the desirable qualities mentioned in the previous section 3.1.1 is shown in Figure 3.1. The tube itself was machined from thick wall SAE 660 bronze hollow bar stock. This material has a thermal conductivity of approximately $86 \text{ W/m} \cdot \text{K}$.

Longitudinal grooves were milled in the tube wall; these were used as channels for the lead wires. Each channel was covered by a mild steel

Figure 3.1 Instrumented tube.

bar stock, which was machined to conform to the tube contour. Two stainless steel bands held the wire channel covers in place.

Two Micro-Foil Heat Flow Sensors^{2,3}, with copper-constantan surface temperature thermocouples, were bonded to the tube with high temperature thermal conducting epoxy^{4,5}. An illustration of the sensor is shown in Figure 3.2. These sensors are less than 0.076 mm thick, except where the lead wires exit the sensors; where the thickness is slightly greater. The lead wires were inlaid into the tube wall and ran through the wire channels to the outside. Assembled in this manner, protrusion of the sensor from the tube surface was less than 0.080 mm.

The Micro-Foil Heat Flow Sensor is a thermopile type heat flux transducer. Calibration data, for a particular sensor, are provided by the manufacturer. An early transducer of this type was developed and analyzed by Hager (1965). Others describe similar devices (Brown et al., 1961).

To protect the sensors from the fluidized bed, a 0.127 mm thick stainless steel shim stock, cut to appropriate dimensions, was tightly wrapped around the portion of the tube on which sensors were mounted. This shim stock was held in place by a clamp as shown in Figure 3.1.

² Model number 20455-1, RdF Corp., Hudson New Hampshire. Maximum operating temperature is 533 K.

³ It was originally planned to mount seven sensors side-by-side around half of the tube periphery. Long delivery time for the needed sensors eliminated this possibility.

⁴ Omegabond 200, Omega Engineering, Inc., Stamford, Connecticut.

⁵ Due to delays in obtaining the thermal conducting epoxy, one of the sensors was bonded to the tube with M-BOND 610 adhesive from Micro-Measurements, Romulus, Michigan. While performance of this adhesive was satisfactory, the thermal conducting epoxy is to be preferred since it provides a lower contact resistance between the sensor and tube.

Figure 3.2 Approximate dimensions of the Micro-Foil Heat Flow Sensor. Thickness 0.076 mm.

To reduce thermal contact resistance, a high thermal conductivity paste⁶ was applied under the shim stock, between wire channel covers and wire channels, and under the two stainless steel bands.

3.1.3 Comments on Instrumented Tube Design

The clamp, which holds the shim stock in place, protrudes from the tube surface about 3 mm⁷. Air and particle flow around the tube is disturbed by the clamp's presence. However, based on heat transfer results obtained, the error incurred is very small.

A nonuniformity in surface temperature is caused by the presence of the Micro-Foil Heat Flow Sensor. The error in measured heat transfer coefficient is small. A further discussion of this source of error is given in Appendix A (Error Analysis).

For the maximum local heat flux measured during this study, the temperature drop across the stainless steel shim stock was computed to be less than 1.8 K. This was only 0.3% of the bed-to-tube surface temperature difference. The error in surface temperature measurement, due to the presence of the shim stock, was thus negligible and not further considered.

The time constant for the Micro-Foil Sensor and covering shim stock combined, to a step change in surface heat flux, is estimated to be 0.24 seconds (see Appendix C). A time constant of 0.02 seconds is given by the manufacturer, for the Micro-Foil Sensor alone.

⁶ Omegatherm 201, Omega Engineering, Inc., Stamford, Connecticut.

⁷ A clamp which more closely fits the tube contour could be designed. The present design was used for ease of fabrication.

3.2 DESCRIPTION OF DATA ACQUISITION EQUIPMENT

In operation, the Micro-Foil Heat Flow Sensor, with surface temperature thermocouple, produces analog voltages which are directly related to the quantities of principal interest: the local heat flux and local surface temperature. From this information the local heat transfer coefficient was computed.

The analog form of the data was processed to a useful form by the steps shown in Figure 3.3. Equipment used for data acquisition and reduction is described below.

3.2.1 Instrumentation Amplifiers

A total of four data channels were used during the experiment: two for local heat flux and two for local surface temperature. For each channel, a high quality instrumentation amplifier⁸, with adjustable gain and offset, was provided.

3.2.2 FM Data Recorder

A Lockheed Store 4 FM data recorder was used to store the local heat flux and local surface temperature data in analog form. The recorder was thoroughly checked for operational fitness by the Oregon State University Electronics Service Shop. Performance of the unit was excellent throughout the study.

⁸ Model AD522, Analog Devices, Norwood, Massachusetts. The amplifiers were assembled and tested by Gosmeyer (1979).

DATA ACQUISITION

DATA REDUCTION

Figure 3.3 Flow diagram of data acquisition and data reduction process.

3.2.3 MacSym 2 System

The MacSym 2 System⁹ provided a 12-bit analog-to-digital converter¹⁰ (accurate to one part in 4096) and digital processing via a MacBasic¹¹ language compiler. Operation of the system is covered in the reference manuals (Analog Devices, 1980a,b).

As presently configured, the system does not have hardcopy capability. Therefore, no listing of the MacBasic program used in the data reduction process is given. The equations used in the data reduction are, however, given in Appendix B (Data Reduction).

3.3 OREGON STATE UNIVERSITY HIGH TEMPERATURE FLUIDIZED BED FACILITY

A schematic illustration of the Oregon State University High Temperature Fluidized Bed Facility is shown in Figure 3.4. A complete description of the facility, including engineering drawings, was given in previous report FE-2714-5 (Welty, 1978). Principal components are described below.

3.3.1 Air Metering System

To measure the air flow rate into the combustion chamber and test section, an ASME standard (ASME, 1959) venturi meter was used. Calibration of the meter was performed by Gosmeyer (1979), who developed a FORTRAN computer program to compute the superficial gas velocity in the bed from the venturi pressures, flowing air temperature, and bed temperature. The only modification to the program made during this study was

⁹ Analog Devices, Norwood, Massachusetts.

¹⁰ Model AIM03 analog-to-digital converter, Analog Devices, address as above.

¹¹ Version of the BASIC computer language with many additional commands.

Figure 3.4 Schematic illustration of the Oregon State University High Temperature Fluidized Bed Facility.

use of the barometric pressure on the day a given experiment was performed, rather than standard atmospheric pressure.

3.3.2 Propane Burner and Temperature Controller

A proportional type controller was used to regulate the propane flow rate and maintain the desired combustion chamber exit temperature. The combustion chamber is refractory lined. No combustion occurred in the bed itself.

3.3.3 Test Section and Distributor Plate

The refractory lined test section provided mounting ports for the instrumented tube at 0.152 m vertical increments. Cross-sectional dimensions of the fluidized bed are 0.60 x 0.30 m. Pressure taps are provided at 0.102 m vertical increments.

A digital thermometer and chromel-alumel thermocouple (type K) were used to measure the fluidized bed temperature. The thermocouple was located about 0.152 m above the distributor plate and protruded into the fluidized bed about 0.102 m.

A 3.18 mm thick stainless steel (ASTM 310) distributor plate, with 171 holes of 6.35 mm diameter in a square array pattern 31.75 x 31.75 mm, was used. To prevent particle flow back when the bed was not fluidized, a stainless steel (ASTM 316) 16-mesh screen with 0.457 mm wire diameter was placed under the plate. This screen was supported by a second plate identical to the first except having 9.53 mm diameter holes.

Operating experience has shown distributor plate life to be acceptable, provided the combustion chamber exit temperature is limited to 1175 K. Failure, due to metal creep, will occur rapidly if this limit is exceeded.

3.3.4 Cooling System for Instrumented Tube

To maintain the instrumented tube at the desired temperature (about 470 K), a light oil (primary coolant) with commercial designation DEXRON II, was pumped through the coolant passage. A shell-and-tube heat exchanger, with water as a secondary coolant, was used to maintain the primary coolant at a reasonable temperature (about 330 K).

IV. EXPERIMENTAL RESULTS

Experimental results for local heat transfer coefficient and spatial-average heat transfer coefficient, to the surface of a 50.8 mm diameter horizontal tube, immersed in a large particle high temperature fluidized bed, are reported.

Particles of surface mean diameter 2.14 mm and 3.23 mm were fluidized at bed temperatures of 810 K and 1053 K. Documentation of test conditions is also given in this chapter.

4.1 TEST CONDITIONS

4.1.1 Bed Material

A granular refractory material with commercial designation Ione Grain A was used for bed material. Chemical composition of this material is given by the supplier¹² as: 53.5% silica, 43.8% alumina, 2.3% titania, 0.4% other. The particles had a smooth surface and fairly spherical shape from several previous test runs.

No reliable data for the thermal conductivity or specific heat of Ione Grain A are known to the writer. However, Chung et al. (1972) and Ziegler et al. (1964) found the thermal conductivity of the particle to be an insignificant factor concerning heat transfer to surfaces immersed in fluidized beds. For large particle beds ($d_p > 1$ mm), Zabrodsky et al. (1978) found no significant dependence on either the thermal conductivity or volumetric heat capacity of the particles. Other properties directly related to the particles are summarized in Table 4.1. The surface mean

¹² Interpace Corporation, Portland, Oregon.

TABLE 4.1 PARTICLE PROPERTIES

density $\rho_s = 2700 \text{ Kg/m}^3$		
screen analysis:		
Screen Opening mm	Mass Fraction	Mass Fraction
Larger	0.0717	0.0016
4.70	0.1217	0.0028
3.99	0.2975	0.0061
3.33	0.4632	0.3648
2.36	0.0377	0.3692
2.00	0.0070	0.1933
1.68	0.0012	0.0466
1.17		
Smaller	0.000	0.0154
$d_p =$	3.23 mm	2.14 mm

diameter, d_p , shown in Table 4.1, was computed as in Kunii and Levenspiel (1969).

4.1.2 Geometry

Packed bed height was 0.50 m for all cases reported. The centerline of the instrumented tube was located about 0.31 m above the distributor plate.

Design of the instrumented tube was covered in Chapter III of this document. Design of the Oregon State University High Temperature Fluidized Bed Facility was also given in Chapter III.

4.1.3 Minimum Fluidization Velocity

Fluidized beds of large particles with wide size spectrum, as used here, have no well-defined minimum fluidization velocity (Botterill and Teoman, 1980; Doicher and Akhnikov, 1979; Babu et al., 1978; Wright et

al., 1970). Cherrington et al. (1977) refer to the "nominal minimum fluidization velocity" when working with such systems.

For this study, the superficial gas velocity was reduced, from that required to sustain bubbling, until the pressure drop across the bed started to decrease sharply. This velocity was found to be repeatable within $\pm 10\%$, and is reported as the minimum fluidization velocity (U_{mf}). Values obtained are shown in Table 4.2.

TABLE 4.2 MINIMUM FLUIDIZATION VELOCITY

T_B K	d_p mm	U_{mf} m/s
810	2.14	1.42
810	3.23	2.07
1052	2.14	1.62
1053	3.23	2.21

The value of U_{mf} plays no direct role in the data reduction or presentation.

4.1.4 Experimental Procedure

The instrumented tube was mounted in the test section, coolant lines attached, and bed filled with particles. The fluidized bed temperature was increased slowly; according to the warm-up schedule given by Welty (1978).

Since only two sensors (heat flux transducers) were mounted on the tube, it was necessary to turn the tube in order to obtain heat flux and surface temperature data around half the tube periphery. Heat transfer data were taken for five positions spaced in angular increments of $\pi/4$ radians. Approximately 90 seconds of data were recorded for each condition.

4.2 EXPERIMENTAL RESULTS

4.2.1 Heat Transfer Data

Data for the local heat transfer coefficient (h_θ), with $d_p = 3.23$ mm and $T_B = 1053$ K, for a range of superficial gas velocities, are shown in Figure 4.1. The excellent agreement between the readings given by sensor 1 and sensor 2, for the $\theta = \pi/2$ radian position, are clearly shown. Good agreement was found for the other cases considered also; the maximum discrepancy was always less than 7%. Typically the difference was even smaller.

Similar plots to that shown in Figure 4.1 were constructed on fine grid graph paper. This allowed interpolating the data on convenient increments of superficial gas velocity. Figures 4.2, 4.3, 4.4 and 4.5 show the local heat transfer coefficient for both particle sizes and bed temperatures considered in this study. Figure 4.6 shows the spatial-average heat transfer coefficient ($\bar{h}$). This average value was obtained by applying the trapezoidal rule for numerical quadrature (see, for example, Hamming, 1973) to the local values of the heat transfer coefficient.

The velocity range covered is from minimum fluidization, or slightly packed, to the velocity at which slugging first occurred, or the highest which air blower capacity would allow. Bed temperature was controlled to within ± 6 K for all cases reported.

Figure 4.7 shows the spatial-average tube surface temperature (T_w). This was computed by applying the trapezoidal rule to the local values of the surface temperature; which were obtained simultaneous with the local heat flux measurements.

Figure 4.1 Local heat transfer coefficient vs. superficial velocity for $d_p = 3.23 \text{ mm}$ and $T_B = 1053 \text{ K}$.

Figure 4.2 Local heat transfer coefficient vs. angular position for $d_p = 2.14 \text{ mm}$ and $T_B = 810 \text{ K}$.

Figure 4.3 Local heat transfer coefficient vs. angular position for $d_p = 3.23 \text{ mm}$ and $T_B = 810 \text{ K}$.

Figure 4.4 Local heat transfer coefficient vs. angular position for $d_p = 2.14 \text{ mm}$ and $T_B = 1052 \text{ K}$.

Figure 4.5 Local heat transfer coefficient vs. angular position for $d_p = 3.23 \text{ mm}$ and $T_B = 1053 \text{ K}$.

Figure 4.6 Spatial-average heat transfer coefficient vs. superficial velocity.

Figure 4.7 Spatial-average tube wall temperature at locations of heat transfer measurements vs. superficial velocity.

4.2.2 Comments

Probably the most obvious trend present in the local heat transfer coefficient data is the very large (factor of ten or more) change in the magnitude of the coefficient at the $\theta = \pi$ radian position as the superficial gas velocity is varied. This is probably due to a relatively cool defluidized stack of particles, which remains on the top of the tube at low gas velocities. Intense bubbling at higher gas velocities displaces this defluidized stack of particles and the local heat transfer coefficient increases greatly. This explanation is consistent with the hydrodynamic information (local voidage, emulsion residence time, visual inspection) available in the literature (Syromyatnikov et al., 1977; Glass and Harrison, 1964; Loew et al., 1979; Hager and Schrag, 1976; Gelperin and Einstein, 1971; Catipovic, 1979).

The local heat transfer coefficient at the $\theta = 0$ position is not a monotonic function of the superficial gas velocity. Rather, it tends to increase slightly as the gas velocity is reduced to or below minimum fluidization. This effect is known to occur in large particle fluidized beds ($d_p > 1$ mm) at near ambient temperature (Catipovic et al., 1978). The result had not been previously reported for high temperature conditions.

At least for the superficial velocity range considered, the distribution of the local heat transfer coefficient appears to approach an asymptotic limit as the velocity is increased. This is particularly noticeable with $d_p = 3.23$ mm and $T_B = 1053$ K (Figure 4.5); where the distribution of the local heat transfer coefficient is nearly identical for $U = 3.0$ m/s to $U = 3.4$ m/s. The trend is less noticeable for the other cases considered.

The spatial-average heat transfer coefficient ($\bar{h}$) also approaches an asymptotic limit as the velocity is increased.

At both bed temperatures considered, the maximum value of $\bar{h}$ was higher for the smaller particles ($d_p = 2.14$ mm) than the larger particles ($d_p = 3.23$ mm). However, the differences are rather small: less than 4% at 1052 K and less than 10% at 810 K.

Difficulties, related to the fluidized bed used in this study, which limit the accuracy of data taken near U_{mf} , or in packed beds, are:

- (1) large temperature gradients (several hundred degrees Kelvin) occur if the bed was packed or at U_{mf} for moderate periods of time (5 minutes);
- (2) distributor plate warpage caused uneven fluidization of the bed at or slightly below U_{mf} . At times, some bubbling appeared at one end of the bed, while the remainder of the bed was definitely packed.

The first difficulty was partially circumvented by taking data as soon as possible (45 seconds) after the desired superficial velocity was set. This allowed data to be taken while the bed temperature was still fairly well-defined. However, it is questionable if steady-state conditions were established under these operating conditions.

Heat transfer results obtained in packed beds or near U_{mf} are unreliable. Neither of the above-mentioned difficulties was significant at higher gas velocities when the bed was bubbling uniformly.

The spatial-average heat transfer coefficient ($\bar{h}$) attains a higher maximum value for the higher bed temperature considered. Radiant heat transfer between bed and tube is increased at high bed temperatures. The thermal conductivity of the gas also increases with increasing bed temperature.

4.2.3 Comparison of Present Results with Other Data

Little heat transfer information is available in the literature for the geometry, particle size range, and bed temperatures considered here.

Fluidized bed combustor studies, employing horizontal tube arrays, and similar size particles to those used in the present study, generally give lower values for $\bar{h}_{MAX}$ than obtained here. A comparison of present data for $\bar{h}_{MAX}$ with fluidized bed combustor studies (Tang and Howe, 1980; Leon et al., 1979; Goblirsch et al., 1980; Wright et al., 1970) is given in Figure 4.8. All data shown were obtained using refractory particles (dolomite, limestone or "crushed refractory") and bed temperatures between 1046 K and 1173 K.

At low bed temperatures (approximately 310 K) tube arrays, at least for tubes internal to the array, give lower values for $\bar{h}_{MAX}$ than single horizontal tubes (Catipovic, 1979; Lese and Kermode, 1972). While no suitable data are available, it is plausible that the reduction of $\bar{h}_{MAX}$ for a tube array may be even larger at high bed temperature; where radiant heat transfer becomes significant (Botterill and Sealey, 1970; Baskakov et al., 1973b). Due to intense bubbling, a given tube in an array will "see" the relatively cool neighboring tubes, as well as the high temperature emulsion. This will give a lower radiant heat transfer for a tube in an array than for a single tube. Also, the fraction of time for which the tube wall is in contact with the emulsion phase is smaller for a tube in an array than for a single tube (Catipovic, 1979). The data of Leon et al. (1979), for a single horizontal tube, compare well with the present study.

Grewal and Saxena (1981) have written a comprehensive paper on the

Figure 4.8 Maximum spatial-average heat transfer coefficient vs. average particle diameter for several studies.

topic of maximum heat transfer coefficients for single horizontal tubes immersed in fluidized beds. They emphasized small particle beds ($d_p < 1$ mm), but present some data for large particle beds (obtained from other literature) as well. For all data considered, they found good correlation between $\overline{Nu}_{MAX}$ and Ar. While Grewal and Saxena recommend their own more complex correlation, the earlier correlation due to Zabrodsky et al. (1974) and shown below fits the available data nearly as well.

$$\overline{Nu}_{MAX} = 0.88 Ar^{0.213} \quad (4.1)$$

Extrapolation of this correlation to values of Ar greater than 10^5 is risky, since that is the highest value of Ar considered by Grewal and Saxena.

For the high Archimedes number range, Baskakov et al. (1973a,b) recommend the following correlations:

$$\overline{Nu}_{MAX} = 0.86 Ar^{0.20}, \quad 10^2 < Ar < 2 \times 10^5 \quad (4.2)$$

$$\overline{Nu}_{MAX} = 0.21 Ar^{0.32}, \quad 2 \times 10^5 < Ar < 10^8 \quad (4.3)$$

Almost all of the data correlated by Zabrodsky et al. (1974) and Baskakov et al. (1973a,b) were obtained at near ambient temperature with air as a fluidizing gas. The above correlations, along with other high Archimedes number data, are compared with results of the present study in Figure 4.9. All gas (air) properties have been evaluated at the bed temperature. Property data were obtained from Touloukian et al. (1970; 1975).

The thermal conductivity and viscosity for the products of complete

Figure 4.9 Maximum spatial-average Nusselt number vs. Archimedes number data for several studies.

combustion of propane were computed using the equation given by Wilke (1950); which was suggested for such use by White (1974). Dissociation was neglected. Botterill and Teoman (1980) also neglected dissociation when performing similar calculations. For the complete range of excess air ratio and combustion chamber exit temperatures encountered in this study, properties of the combustion products were found to be within 4% of those of air at the same temperature. Properties of the constituent gases in the combustion products are presently known to only 5 or 10% accuracy (Touloukian et al., 1970; 1975). Therefore, with no real loss of precision, properties of air have been used in the data reduction. Westmoreland (1954) conducted an experimental study to measure the viscosity of several exhaust gases at elevated temperatures (up to 1373 K). He found the viscosity of the exhaust gases to be within a few percent of that of air at the same temperature.

The presence of radiant heat transfer tends to increase $\overline{Nu}_{MAX}$ over that attainable at the same value of Ar without radiant heat transfer. Data taken at high temperature tend to give higher values of $\overline{Nu}_{MAX}$.

Chandran et al. (1980) present heat transfer data for a horizontal tube in a large particle low temperature pressurized fluid-bed. However, they fail to specify a bed temperature or sufficient information from which the bed temperature could be estimated. Their data could not be used in the comparison shown in Figure 4.9.

In conclusion, available data are insufficient to either validate or invalidate the result of the present study. Further experimental work is indicated.

V. EXPERIMENTAL VALIDATION OF THE ANALYTICAL MODEL

The analytical model described in Chapter 2 has been validated by comparing model predictions with both two-dimensional cold bed and three-dimensional hot bed data. Reasonable agreement between model calculations and experimental results was obtained with only minor adjustments of the input parameters. The details of these comparisons are discussed below.

5.1 COMPARISON OF EXPERIMENTAL RESULTS WITH MODEL PREDICTIONS AT LOW TEMPERATURE

Cold bed measurements made by Catipovic (1979) were used to validate the analytical model for low temperature conditions. These tests were performed under Electric Power Research Institute funding in a two-dimensional bed and provided both local and total instantaneous and time-averaged heat transfer to a horizontal tube for particle diameters of 0.37-6.6 mm.

In comparing experimental data with model predictions, it was required that experimental conditions correspond to the situation modeled. For instance, it was found that experimentally determined heat transfer coefficients at minimum fluidization should not be compared against theoretical values because particle residence times, especially on the upper portion of the tube, are such that the assumed thermal boundary condition is no longer valid. Therefore, experimental results used in the comparisons were obtained at velocities which insured adequate particle replacement at the surface.

5.1.1 Comparison of Time-Averaged Local Coefficients

For the case where the emulsion phase covers the entire tube, velocities just slightly in excess of u_{mf} were used. The values used for void fraction, as well as the corresponding physical situation, are shown in Figure 2.11. The slightly increased voidage on the sides (as compared to ϵ_{mf}) reflects the fact that occasional bubbles pass by the cylinder and insure sufficient particle replacement. The voidage distribution shown essentially represents a time-averaged picture at a velocity

slightly above minimum fluidization. Predicted values of local heat transfer coefficients were therefore compared with time-averaged experimental results.

The average voidage around the tube circumference, resulting from the distribution shown in Figure 2.11, is 0.57. The gas velocity at which this voidage is measured experimentally depends on the particle size. Table 5.1 shows the corresponding superficial velocities for the particles used, as well as some other quantities of interest (which will be used in later comparisons).

Table 5.1. Gas velocity at which the average voidage around the tube is 0.57.

Particle Diameter d_p (mm)	Superficial Gas Velocity u_o (m/s)	u_o/u_{mf}	Re_p
0.37	0.22	1.45	4.58
0.8	0.56	1.22	24.97
1.3	0.77	1.10	56.11
2.0	1.24	1.08	138.16
2.85	1.54	1.06	245.00
4.0	1.90	1.04	426.40
6.6	2.50	1.03	918.60

To validate the hydrodynamic portion of the analytical model, values of local pressure at the tube surface were calculated and compared with experimental data. Figure 5.1 shows the good agreement between predicted and experimental values of local pressure along the cylinder circumference near u_{mf} , for a particle diameter of 4.0 mm.

In the heat transfer comparisons which follow, the value of the interstitial turbulence intensity in the model was set at $u' = 0.2$, as suggested by experiments of Galloway and Sage (1970) for large spherical particles. Also, the ambient voidage was taken as 0.5.

Figure 5.1 Average local pressure distribution around a horizontal tube.

Figure 5.2 shows a typical comparison between theoretical and measured values of local heat transfer coefficients. For a more meaningful comparison, the theoretical coefficients were averaged over the section of the cylinder covered by the platinum sensor. Figure 5.3 shows a detailed graphical summary of this type of comparison for particles used in our experiments. The graphs are plotted in terms of dimensionless groups Nu_{p0} and Re_p since such representation provides a convenient way of assessing the influence of physical properties on heat transfer behavior.

The agreement between experiment and theory is very satisfactory, both in values and trends, supporting the basis of the analytical model. Under the operating conditions considered here, the lower limit of the model's applicability seems to be around 1 mm ($Re_p = 40$); this significantly exceeds Adams and Welty's (1979) conservative estimate of 2-3 mm. As particle diameter decreases below 1 mm, the disagreement between predictions and data becomes significant, reaching 150-200% for 0.37 mm particles (not shown in Figure 5.3). The thermal inertia of smaller particles is not sufficient to keep them isothermal while at the tube wall; therefore, the actual heat transfer coefficient is well below predicted. It must be noted that the actual limit of the model will depend on the physical properties of gas and particles.

Actual and predicted local heat transfer coefficients shown in Figure 5.3 rarely differ by more than 15%.

The agreement is even more striking when overall coefficients are considered. Figure 5.4 compares theory versus experiment for three cases: a) data of this study using a 50.8 mm cylinder at atmospheric pressure, with air as the fluidizing gas; b) data of Canada and McLaughlin (1978)

Figure 5.2 Local heat transfer coefficient for a horizontal tube without bubbles.

$$\theta = 180^\circ$$

$$135^\circ$$

$$90^\circ$$

$$45^\circ$$

$$0^\circ$$

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Figure 5.3 Local Nusselt number for a horizontal tube when the average wall voidage is 0.57.

using a 32 mm tube in a bed of 2.6 mm glass particles, with air at three different pressures; and c) data of Canada et al. (1977) using the same set-up as in b), but with Freon-12 at 1.2 and 2.3 atm. The heat transfer coefficients used for comparison in cases b) and c) are those obtained at $1.1 u_{mf}$.

The model is successful in predicting the effect of gas properties on heat transfer. It also correctly predicts the negligible influence of tube diameter. For fine particles Gelperin et al. (1969) suggested that heat transfer coefficients are independent of the tube diameter for tube diameters greater than 20 mm. No such information has been previously reported for large particles.

5.1.2 Comparison of Instantaneous Local Coefficients in the Presence of Bubbles

The model can predict instantaneous local heat transfer coefficients for practically any configuration of a single bubble clinging to the immersed cylinder. In order to obtain data for comparison with these predictions, superficial gas velocities slightly higher than in the previous section were used.

Plexiglas inserts were placed in the bed to reduce its thickness to 50 mm. In such a narrow bed, bubbles could be seen easily. When in operation, this two-dimensional bed was backlighted and filmed with a Nikon R-10 camera at 18 frames per second. For every frame registered on film, the corresponding heat transfer, voidage, and pressure signals were registered on a Data General Nova 840 computer. These data gave instantaneous values of local heat transfer rates, voidage, and pressure which correspond to the picture on the film frame.

Figure 5.4 Total Nusselt number for a horizontal tube
when the average wall voidage is 0.57.

Several hundred movie frames were examined, and the corresponding predicted and experimental values were compared. In all of the simulations, the voidage distribution was as shown in Figure 2.11, except that the voidage on the sides of the tube was taken as 0.5. As in the comparison of time-averaged values, the model gave good predictions for particle diameters of 1.3 mm and larger, with the agreement getting better as particle size increased. Predicted and measured values of instantaneous local heat transfer coefficients rarely differed by more than 15%, while the instantaneous average values for the entire circumference agreed to within 10%. Some illustrative examples are given in the figures which follow.

Figure 5.5 illustrates the good agreement between local heat transfer coefficients for the case of a bubble engulfing the bottom platinum sensor ($d_p = 4.0$ mm), while Figure 5.6 compares prediction versus experiment for a bubble clinging to the side of the cylinder ($d_p = 2.85$ mm). These comparisons were made for the narrow, two-dimensional bed. Several other cases are given in Catipovic (1979).

The numerous cases with different bubble positions cannot all be presented. However, experimental data confirm the success of the model in predicting the influence of bubble location on instantaneous local heat transfer coefficient for particle diameters above 1 mm.

5.1.3 Comparison of Packed Bed Model with Cold Bed Results

Heat transfer data for a horizontal tube immersed in an atmospheric pressure packed bed were also reported by Catipovic (1979). These data were selected to test the validity of the packed bed model described in Section 2.1.2. Other data for horizontal tubes immersed in large particle/high pressure beds have been reported by Chandron et al. (1980)

Figure 5.5 Instantaneous local heat transfer coefficients for a horizontal tube with a contacting bubble at lower stagnation point

Figure 5.6 Instantaneous local heat transfer coefficients for a horizontal tube with a contacting bubble at the side of the tube.

but packed bed data are not included. Also, Botterill and Denloye (1978) report data for a vertical cylinder immersed in a large particle bed under both packed and fluidized conditions.

Local and total Nusselt number calculations were made for particle sizes in the range 1.3-6.6 mm in a low-temperature bed at atmospheric pressure. The parameters used in the calculations are listed in Table 5.2 and comparison of calculated results with Catipovic's data is made in Figures 5.7 and 5.8. Figure 5.7 illustrates the accuracy of calculated local values for 4 mm diameter particles. It appears from the figure that local fluidization and bubbling may be occurring near the side of the tube ($\theta = \pi/2$), resulting in poor agreement at the higher Reynolds number. The top and lower stagnation values agree reasonably well with the measurements though it is noted that the analytical value for the lower stagnation point is sensitive to the voidage assumed. Figure 5.8 is a comparison of calculated and measured total Nusselt number for the four particle sizes considered. Note that the theoretical curves are nearly coincident and linear with Reynolds number. The best agreement was obtained in the mid-range of Reynolds number and worst agreement when local bubbling and fluidization effects seem to be present. Over the range of Reynolds number from 50 to 1000, the maximum difference between theory and experiment is about 20%.

Table 5.2. Calculation parameters for the 4mm packed bed case.

Bed Temperature	308 K
Wall Temperature	376 K
Bed Voidage	0.40
Surface Voidage	
Lower Stagnation	0.8 ($\theta = -.452$ to 0.452 rad)
Remaining Surface	0.44
Particle Sphericity	0.97
Turbulence Intensity	0.20
Prandtl Number	0.70

5.2 COMPARISON OF MODEL WITH HIGH TEMPERATURE DATA

In the case of the hot bed tests, detailed comparisons of theory and experiment were not possible because of the lack of local hydrodynamic data along the tube surface. Based upon the cold bed studies, the model provided reasonable results for superficial velocities near u_{mf} when bubble effects can be neglected but particle residence times are small. Therefore data for u_o between u_{mf} and $1.1 u_{mf}$ will be selected for comparison purposes.

The main parameters selected for the calculation are listed in Table 5.3. Note that the ambient bed voidage of 0.44 was selected because better theory/data agreement was obtained. The wall emissivity of 0.3 is representative of values for the stainless steel foil used. Also, the calculations were made using the reference temperature method (see, e.g., Kays (1966)) in which all properties and dimensionless parameters are based upon the mean temperature.

$$T^* = (T_B + T_W)/2 \quad (5.1)$$

This approach was found to provide better agreement than the use of the S_o parameter described in Chapter 2.

Figures 5.9 and 5.10 show comparisons between model predictions and experimental data for both local and total Nusselt number. The data spread indicated results from the uncertainty in the value of superficial velocity at which the average wall voidage is 0.57. Generally, best agreement was obtained in the lower stagnation point region and worst near the upper stagnant cap ($\theta = 135^\circ$). Local data for the top of the

Figure 5.7 Theoretical and experimental local Nusselt number for the high temperature case at $U_o = U_{mf}$ to $1.1 U_{mf}$.

Figure 5.8 Theoretical and experimental total Nusselt number for the high temperature case at $U_o = U_{mf}$ to $1.1 U_{mf}$.

Table 5.3 Parameters for the hot bed calculation.

Bed Temperature	1052 K and 810 K
Wall Temperature	448-462 K and 394-397 K
Bed Voidage	0.44
Surface Voidage	
Lower Stagnation	0.9
Side	0.55
Stack	0.44
Particle Sphericity	0.75
Turbulence Intensity	0.2
Prandtl Number	0.72
Emissivities	
Tube Wall	0.3
Bed-Lower Stagnation	1.0
-Side	0.5

tube are not shown because the agreement between data and theory was especially poor there. The total Nusselt number comparison (Figure 5.10) indicates a trend for the theory to overpredict at the lower temperature (810 K) and underpredict at the higher temperature (1052 K). This trend is primarily due to the lack of good agreement between theory and experiment in the upper stagnation region.

Complete comparison of the theoretical model with data at high temperature requires detailed information regarding the hydrodynamic situation. This means measured values of surface voidage, bubble fraction, and emulsion residence time.

limit as the velocity is increased. Small changes in gas velocity near the maximum velocity considered yield small changes in the distribution of the local heat transfer coefficient. The distribution of the local heat transfer coefficient was nonuniform for all cases considered.

- c. The maximum spatial-average heat transfer coefficient ($\bar{h}_{MAX}$) was higher for the smaller particle ($d_p = 2.14$ mm) at both bed temperatures. However, the differences were rather small: 4% at $T_B = 1052$ K and 10% at $T_B = 810$ K.
- d. For both particle sizes used, the maximum spatial-average heat transfer coefficient ($\bar{h}_{MAX}$) was higher at $T_B = 1052$ K than at $T_B = 810$ K. This is probably due to the increased thermal conductivity of the gas and increased radiant heat transfer at the higher bed temperature.
- e. At minimum fluidization, or when the bed was slightly packed, nonuniform fluidization, i.e., uneven bubbling and large temperature gradients in the bed, made heat transfer measurements taken under these conditions unreliable. Neither of these difficulties were present at higher gas velocities.
- f. It was found that available heat transfer results, in the literature, were inadequate to either validate or invalidate the results of the present study.

VI. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

6.1 CONCLUSIONS6.1.1 Experimental Study

An instrumented tube for the measurement of time-average local heat transfer rate and time-average local surface temperature was described in Chapter III. The commercially available Micro-Foil Heat Flow Sensor with surface temperature thermocouple was used. Heat transfer measurements with excellent repeatability were made with the device.

In Chapter IV experimental results for the time-average local heat transfer coefficient to a single horizontal tube in a large particle ($d_p = 2.14$ mm and $d_p = 3.23$ mm) fluidized bed at elevated temperature ($T_B = 810$ K and $T_B = 1053$ K) were presented. The superficial gas velocity ranged from that required for minimum fluidization, or slightly packed, to the velocity where slugging first occurred, or the highest velocity air blower capacity would allow. The following conclusions are suggested:

- a. As the superficial gas velocity is increased from that required for minimum fluidization, a large increase in the local heat transfer coefficient at the top of the tube ($\theta = \pi$) occurs. This is probably due to the presence of a defluidized stack of particles, on top of the tube, which is displaced by intense bubbling at higher gas velocities.
- b. At least for the superficial velocity range considered, the distribution of the local heat transfer coefficient (i.e., h_θ vs. θ) appears to approach an asymptotic

6.1.2 Analytical Study

The comparisons of analytical and experimental results for both cold and hot fluidized beds has further emphasized the importance of simultaneous hydrodynamic and heat transfer information. In the case of the two-dimensional cold bed conditions, reasonable agreement between theory and experiment could be obtained because sufficient hydrodynamic data were available to facilitate the calculation. This information was not available for the hot bed conditions and the data/theory comparisons for these conditions are correspondingly uncertain.

However, the cold bed results do provide an indication of the validity of the model when the effects of thermal radiation can be neglected. In fact, when the superficial velocity is high enough to assure good particle mixing and dominance of the gas convective component of heat transfer, good agreement between theory and experiment is obtained for both local and total instantaneous heat transfer coefficients. This seems to occur when the Reynolds number based upon particle diameter and minimum fluidizing velocity is greater than about 50 and when the superficial velocity is greater than minimum bubbling velocity. Good agreement is also obtained for the packed bed case because the thermal state of the particles can be estimated. Again, the minimum Reynolds number for model validity seems to be about 50 (based upon superficial gas velocity).

The parametric calculations performed to establish the effect of adjacent objects (bubbles and tubes) on gas-convection-dominant heat transfer to a horizontal tube in a two-dimensional bed were also consistent with the above observations. Results of calculations performed using the model indicate that bubble size, position, and velocity have a weak effect upon total heat transfer to the tube, with a maximum variation of about 33%

for the configurations studied. This result is consistent with measurements made by Catipovic (1979) which indicate a weak dependence of total heat transfer upon superficial velocity and therefore bubble interaction. Adjacent tubes were also found to have a weak effect on total heat transfer but the calculations suggest some benefit can be obtained with arrays having close horizontal spacing and wide vertical spacing. For example, a horizontal pitch of 1.5 tube diameters and vertical pitch of three tube diameters provides an increase in total heat transfer of about 14% over single tube levels.

6.2 RECOMMENDATIONS FOR FUTURE WORK

It would be highly desirable to perform a calibration check on the Micro-Foil Heat Flow Sensors for a range of surface temperatures after the sensors are mounted and the instrumented tube assembled. Limitations imposed by time and available funding prohibited such a calibration technique from being developed during the present study.

In the area of heat transfer to immersed tubes in large particle fluidized beds at elevated temperature, much work remains to be performed.

There exists a need for detailed hydrodynamic information (voidage, bubble fraction, emulsion residence time) for regions of the fluidized bed near the tube. This information is required to allow validation of the analytical model of fluidized bed heat transfer developed during this study; i.e., the Adams-Welty Model (Adams, 1977; Adams and Welty, 1979). Ideally, such measurements should be made in conjunction with simultaneous local heat transfer measurements. Separate measurement of radiative heat transfer and total heat transfer, for high temperature conditions, would also be desirable.

Finally, hydrodynamic and heat transfer measurements should be made for tube arrays; if possible, under large-scale pilot plant conditions.

The following tasks provide a natural extension of the analytical study presented here:

- 1) Further analytical modeling to include the effect of transient conduction within the particles upon the Stokes' region contribution and radiative heat transfer.
- 2) Further parametric studies (e.g., wider range of particle size) for a single tube in a high-temperature bed.
- 3) Theoretical modeling of the hydrodynamics of three-dimensional bubbles near a horizontal tube.

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APPENDICES

APPENDIX A

Error Analysis

The heat flux transducer employed in this study consists of a Micro-Foil Heat Flow Sensor (hereafter referred to as sensor) covered by a 0.127 mm thick stainless steel shim stock. Three sources of error are identified and their magnitudes estimated:

1. Heat conduction in the shim stock along the surface.
2. Disturbance of the thermal boundary layer due to non-uniform surface temperature.
3. The combination of 1 and 2 above along with calibration and data acquisition errors.

In operation, the heat flux signal was found to vary only about $\pm 6\%$ from its time-average value. Therefore, only a steady-state error analysis will be made. Time-average values will be used for all parameters.

A.1 Heat Conduction Along Surface

An analysis is performed to estimate the fraction of energy (heat flux) absorbed at the shim stock's upper surface, which is actually conducted to the sensor.

Since the shim stock is thin (0.127 mm) and made of relatively high thermal conductivity material, the temperature difference across its thickness will be neglected. The sensor is considered to represent a contact resistance (R_{GAGE}) between the shim stock and tube wall. A uniform convective boundary condition is assumed for the upper surface. The boundary condition around the periphery of the sensor is taken to be uniform temperature at T_{w2} . Boundary conditions and coordinate system are given in Figure A.1.

SIDE VIEW WITH BOUNDARY CONDITIONS

Figure A.1 Boundary conditions and coordinate system for heat conduction analysis.

The formulation of the problem is

$$\frac{\partial^2 T_w}{\partial x^2} + \frac{\partial^2 T_w}{\partial y^2} - \frac{(h_1 + h_2)}{k_1 \delta_1} T_w = - \frac{(h_1 T_B + h_2 T_{w2})}{k_1 \delta_1}$$

Boundary conditions are

$$T_w(x_o, y) = T_{w2}$$

$$T_w(x, y_o) = T_{w2}$$

$$\frac{\partial T_w(x, 0)}{\partial y} = 0$$

$$\frac{\partial T_w(0, y)}{\partial x} = 0$$

One quantity of interest is the ratio of heat flux transmitted to the sensor to that absorbed at the upper surface of the shim stock.

$$\frac{q_2}{q_1} = - \left(\frac{h_2}{h_1} \right) \left(\frac{T_w - T_{w2}}{T_w - T_B} \right)$$

Details of the solution, by the classical technique of separation of variables, are left to the reader.

Parameters used are as follows:

$$\delta_1 = 0.127 \text{ mm}$$

$$k_1 = 20.0 \text{ W/m} \cdot \text{K}$$

$$h_1 = 365 \text{ W/m}^2 \cdot \text{K}$$

$$h_2 = \frac{1}{R_{\text{GAGE}}} = 1893 \text{ W/m}^2 \cdot \text{K}$$

$$T_B = 1053 \text{ K}$$

$$T_{w2} = 363 \text{ K}$$

The values of h_1 , T_B and T_{w2} correspond to the $\theta = \pi/2$ radian position on the tube during a typical high temperature experiment. Values of q_2/q_1 over the active area of the sensor (where the thermocouple junctions are located) are shown in Figure A.2. About 98% of the heat flux absorbed at the shim stock's upper surface is conducted to the sensor's active area.

The surface temperature distribution at $x = 0$ is shown in Figure A.3. This surface temperature is required for the thermal boundary layer analysis which follows.

A.2 Thermal Boundary Layer Analysis

For purposes of thermal boundary layer analysis, the surface temperature distribution is idealized somewhat as shown in Figure A.4. The point at which the boundary layer starts is a distance L ahead of the sensor's position.

The superposition technique, based on an integral solution for a step in surface temperature, described by Kays (1966), was used. Leaving details to the reader, only relevant results are presented here. A laminar boundary layer was assumed. The solution applies only when the surface is covered by gas bubbles, i.e., heat transfer by pure gas convection.

Of interest is the ratio of gas convective heat transfer coefficient for the nonuniform surface temperature (h_g) to the gas convective heat transfer coefficient for an isothermal surface (h_{Tg}). The ratio h_g/h_{Tg} for the active area of the sensor is shown in Figure A.5. Values of L used are near maximum for the experimental conditions encountered.

Flat plate flow gives lower values of h_g/h_{Tg} than stagnation point flow. As worst probable case, the $L/y_0 = 6$ flat plate case is taken and spatially averaged over the sensor active area.

Figure A.2 Values of q_2 / q_1 for the active area of the sensor.

Figure A.3 Surface temperature distribution along the line $x = 0$.

Figure A.4 Surface temperature distribution for thermal boundary layer analysis.

Figure A.5 Ratio of gas convective heat transfer coefficient for the nonuniform surface temperature to that for uniform surface temperature for the active area of the sensor.

$$\left(\frac{h_g}{h_{Tg}} \right)_{MAX} \approx 0.75$$

In terms of relative error,

$$\left(\frac{h_g - h_{Tg}}{h_{Tg}} \right)_{MAX} \approx -0.25$$

An estimate of the relative error in the spatial average heat transfer coefficient, due to thermal boundary layer disturbance, is obtained by multiplying the above relative error by the fraction of total heat transfer due to bubbles contacting the tube. This value is estimated from data given by Catipovic (1979) as 0.15. Therefore,

$$\left(\frac{\bar{h} - \bar{h}_T}{\bar{h}_T} \right)_{MAX} \approx (0.15)(-0.25)$$

$$\approx -0.04$$

The relative error in $\bar{h}$ could be as large as 4%. However, it is probably less. By mounting sensors side-by-side around half of the tube periphery, the nonuniformity in surface temperature and the associated measurement error would be largely eliminated.

It should be mentioned that all of Catipovic's heat transfer data were taken in a narrow (0.127 m wide) low temperature fluidized bed. Cranfield and Geldart (1974) note significant hydrodynamic differences between narrow two-dimensional beds and wider three-dimensional beds of large particles ($d_p > 1$ mm). The use of Catipovic's data for the fraction of total heat transfer due to bubbles is a rather crude approximation.

A relative magnitude error of 4% for nonuniformity in surface temperature will be used for the local heat transfer coefficient (h_θ) as well as for $\bar{h}$.

A.3 Estimated Error in the Local Heat Transfer Coefficient

The local heat transfer coefficient is given by

$$h_\theta = \frac{q_\theta}{T_B - T_{w\theta}} = \frac{CV_\theta}{T_B - T_{w\theta}}$$

where

C = calibration constant for the sensor at the given
surface temperature

V_θ = voltage output of the sensor

For random calibration and data acquisition errors, the root-sum-square (RSS) error is given by Doebelin (1975); Thrasher and Binder (1957); and Kline and McClintock (1953).

$$\left(\frac{\Delta h_\theta}{h_\theta} \right)_{\text{RAN}} = \pm \left[\left(\frac{\Delta C}{C} \right)^2 + \left(\frac{\Delta V_\theta}{V_\theta} \right)^2 + \frac{(\Delta T_B)^2 + (\Delta T_{w\theta})^2}{(T_B - T_{w\theta})^2} \right]^{1/2}$$

Using the manufacturers' specifications and operating experience, best estimates of the uncertainty for each quantity for a typical high temperature experiment are shown below.

$$\begin{aligned} \left(\frac{\Delta h_\theta}{h_\theta} \right)_{\text{RAN}} &= \pm \left[(0.05)^2 + (0.05)^2 + \frac{(8.5)^2 + (3.5)^2}{(1053 - 478)^2} \right]^{1/2} \\ &= \pm \left[(0.05)^2 + (0.05)^2 + (0.016)^2 \right]^{1/2} \\ &= \pm 0.08 \end{aligned}$$

Errors in bed temperature and tube wall temperature contribute very little to the random error.

The inherent errors in the experiment due to nonuniform surface temperature and heat conduction along the surface through the shim stock analyzed in Section A.1 and A.2, are now added to the random errors.

$$\begin{aligned} \left(\frac{\Delta h_{\theta}}{h_{\theta}} \right) &= (+ 0.08, - 0.08) + (0, - 0.04) + (0, - 0.02) \\ &= (+ 0.08, - 0.14) \end{aligned}$$

It is improbable that the values of the local heat transfer coefficient reported are more than 8% higher or 14% lower than the actual values.

APPENDIX B

Data Reduction

Equations used in computing time-average values of the local heat flux, local surface temperature, and local heat transfer coefficient are given below. It is assumed the reader is familiar with simple analog data acquisition techniques.

For any data channel (includes instrumentation amplifier, FM data recorder, and analog-to-digital converter incorporated in the MacSym 2 data acquisition system) denoted by a subscript i , the relationship between transducer output voltage (either surface temperature thermocouple or thermopile type heat flux transducer), overall offset voltage, overall channel gain, and voltage output as given on the MacSym 2 system, is

$$V_{ti}^j = \frac{V_{OUTi}^j - V_{OSi}}{K_i} \quad \text{in volts}$$

For example, let channel 3 correspond to the local surface temperature, and channel 4 correspond to the local heat flux for a given condition. The surface temperature, at a time denoted by superscript j , is given by

$$T_{w3}^j = a_1 + a_2 e + a_3 e^2 + a_4 e^3 + a_5 e^4 \quad \text{in } ^\circ\text{C}$$

where $c = V_{t3}^j (10^6) \quad \text{in } \mu\text{V}$

$$a_1 = 0$$

$$a_2 = (2.56613) 10^{-2}$$

$$a_3 = (-6.19549) 10^{-7}$$

$$a_4 = (2.21816) 10^{-11}$$

$$a_5 = (-3.55009) 10^{-16}$$

The above constants are given by Analog Devices (1980b) for copper-constantan thermocouples (type T) with an ice bath reference junction.

Local heat flux is given by

$$q_4^j = \frac{V_{t4}^j (3.154) 10^6}{SC_w^j} \quad \text{in W/m}^2$$

where S = Micro-Foil Heat Flow Sensor calibration factor
at 21°C, in $\mu\text{V}/(\text{Btu/hr}\cdot\text{ft}^2)$

C_w^j = dimensionless correction factor for surface temperature, a function of T_{w3}^j , given by

$$C_w^j = 0.85 - (0.09)D^j + (0.015)(D^j - 1)D^j$$

where $D^j = (T_{w3}^j - 93.667)/55.556$

The above equation for C_w^j and D^j is valid between 93°C and 215°C: the range of surface temperature encountered in this study.

Time-average values of local surface temperature and local heat flux are obtained by sampling, at a rate of about 12 samples per second per channel, and summing the instantaneous readings, then dividing by the total number of samples taken (N).

$$T_{w3} = \frac{1}{N} \sum_{j=1}^N T_{w3}^j$$

$$q_4 = \frac{1}{N} \sum_{j=1}^N q_4^j$$

Local heat transfer coefficient, for the condition considered, is given by

$$h_\theta = \frac{q_4}{T_B - T_{w3}} \quad \text{in W/m}^2\cdot\text{K}$$

Sixty seconds of data were sampled for each condition, i.e., about 720 samples per channel. Use of a larger number of samples did not improve the repeatability of the results.

For those at Oregon State University, the data reduction program used may be inspected by listing the program DATARE.2 on the MacSym 2 system's CRT screen. Some documentation is provided on the program listing in the form of remark statements. The program mentioned is stored on the duplicate master diskette. It is strongly recommended that those conducting similar research write their own data reduction program(s) rather than use the existing one.

APPENDIX C

Response Time of the Heat Flux Transducer

The heat flux transducer employed in this study consists of a Micro-Foil Heat Flow Sensor (hereafter referred to as sensor) covered by a 0.127 mm thick stainless steel shim stock. The time constant, for a 63.2% response to a step change in surface heat flux for the sensor alone is given by the manufacturer as 0.02 seconds. However, the covering shim stock slows the response greatly.

For purposes of estimating the response time of the combination it is assumed the response is dominated by the shim stock. This assumption is justified by the value of the time constant obtained. Similar analyses have been carried out by others (Hager, 1965).

Thermal conductivity of the shim stock is much larger than that of the sensor. The temperature difference across the shim stock thickness is neglected. An energy balance for the shim stock is

$$\rho_1 C_1 \delta_1 \frac{dT_w}{dt} = q_1 - h_2 (T_w - T_{w2})$$

where

$$h_2 = \frac{1}{R_{\text{GAGE}}}$$

The heat flux which enters the sensor is

$$q_2 = h_2 (T_w - T_{w2})$$

Solution for a step change in surface heat flux (q_1) yields

$$T_w - T_{w2} = \frac{q_1}{h_2} \left[1 - \exp \left(- \frac{h_2 t}{\rho_1 C_1 \delta_1} \right) \right]$$

and

$$\frac{q_2}{q_1} = 1 - \exp \left(- \frac{h_2 t}{\rho_1 C_1 \delta_1} \right)$$

The time constant for 63.2% response (t_c) is given by

$$\frac{h_2 t_c}{\rho_1 C_1 \delta_1} = 1$$

$$t_c = \frac{\rho_1 C_1 \delta_1}{h_2} = \rho_1 C_1 \delta_1 R_{\text{GAGE}}$$

Using the shim stock thickness, $\delta_1 = 0.127$ mm, properties of stainless steel, and $R_{\text{GAGE}} = 5.28 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$ (as given by the sensor manufacturer), gives

$$t_c = 0.24 \text{ second}$$

Therefore, the time constant for the sensor and shim stock combination is a factor of 12 larger than for the sensor alone.

It is clear that t_c is too large for the heat flux measurements to be interpreted as instantaneous measurements. Data reduction does not require the value of t_c .

Presence of thermal contact resistance between sensor and tube wall or sensor and shim stock will slow the response further. These resistances have not been included in the calculation since the Micro-Foil Heat Flow Sensor manufacturer provides R_{GAGE} accurate to only one significant digit (in U. S. customary units $R_{\text{GAGE}} = 0.003 \text{ hr} \cdot \text{ft}^2 \cdot ^\circ\text{F/Btu}$). The additional resistances are certainly much less than the above value.