THE MECHANICS OF MICROMACHINING

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Abstract

The current state of machining theory is examined for relevance to micromachining. Of particular interest are those features of the theory which are important to optical surface finishes and surface characteristics. The relation of transverse strain or side flow to the nature of the machining marks is one example of interest. Correlation with measurements of machining parameters and surface finishes is given.

Basic Concepts of Cutting

Understanding the mechanics of cutting begins with macromachining. Macromachining involves depths of cut on the workpiece of more than 0.001 inch (25 microns) while micromachining generally involves smaller depths of cut. Our real interest is micromachining, but the process can be understood by first examining macromachining.

Discussion of macromachining is thoroughly treated by Amarego and Brown⁽¹⁾ and for detailed derivation that source should be studied. Only the results along with some features of special interest to micromachining will be given in this paper. The first complete analysis of the cutting process was advanced by Merchant⁽²⁾. Later, some of the limiting mechanical assumptions were removed by Oxley⁽³⁾ using a slip line theory. Merchant's analysis is inherently simple and the cutting process is illustrated in Figure 1.

Merchant's analysis postulates cutting as a shearing process taking place in a thin plane shear zone. Inherent assumptions are that:

- (a) The tool tip is sharp so that no contact area is presented which contributes to contact stress or friction.
- (b) Deformation is two dimensional plane strain so that no side flow component exists.

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- (c) Stresses on the shear plane are uniformly distributed.
- (d) Resultant cutting forces on the shear plane and the rake face are exactly opposed and in equilibrium so that no balance of moments is involved.

These assumptions imply related assumptions about the material behavior which will be examined later. A force diagram permits a determination of the relation between the shear angle, friction angle, back rake angle and the dimensions of the cut and the force components. The force in the direction of cutting parallel to the workpiece surface is:

$$F_c = \frac{b d_c \tau_s \cos(\beta - \alpha)}{\sin \phi \cos(\phi + \beta - \alpha)} \quad (1)$$

The force component normal to the workpiece surface is:

$$F_t = \frac{b d_c \tau_s \sin(\beta - \alpha)}{\sin \phi \cos(\phi + \beta - \alpha)} \quad (2)$$

where b is the width of the cut, d_c is the depth of cut, τ_s is the shear stress, and θ , β , α are the shear, friction, and rake angles. Using the minimum energy principle, recalling that the shear stress τ_s and friction angle are assumed to be uniformly distributed or constant, the following angular relation obtains:

$$\phi = \frac{\pi}{4} - \frac{(\beta - \alpha)}{2} \quad (3)$$

Shearing and friction processes are heat generating processes, so that thermal terms in addition to mechanical terms are needed in the minimum energy principle. Since the friction angle and shear stress are not necessarily constant and the minimum energy expression is incomplete, it is not surprising that equation 3 does not agree exceptionally well with experimental results.

A useful relation from the geometry of the shear plane and cutting process enables calculation of the shear angle from the chip thickness. That is:

$$\tan \phi = \frac{d_c \cos \alpha}{(d_r - d_c \sin \alpha)} \quad (4)$$

where d_c and d_r are the depth of cut and chip thickness while α is the back rake angle. Once again, the relation follows from Merchant's assumed plane shear surface. However, for a curved shear surface, an approximate average shear angle could be obtained using a chord line across the curved surface. Many experimental methods have been developed to measure the average chip thickness.

Finally, within the context of the assumptions the total strain and the strain rate may be calculated. They are:

$$\gamma = \frac{\cos \alpha}{\sin \phi \cos (\phi - \alpha)} \quad (5)$$

and,

$$\dot{\gamma} = \frac{\cos \alpha}{\cos (\phi - \alpha)} \frac{V}{\Delta y} \quad (6)$$

where V is the velocity of the workpiece in the direction of cutting and Δy is the thickness of the shear zone. At all but very low velocities, the strain rate is very high and the shear zone is thin. In fact Recht⁽⁴⁾ found that the strain rate could be high enough to cause melting along the shear plane for materials with a low thermal conductivity. Thus, Merchant's assumptions approach greater physical reality at high cutting speeds. Generally, Merchant's analysis requires stresses and friction coefficients which are higher than those measured by conventional materials testing.

Oxley used a curved shear surface and incorporated a non-uniform stress distribution which also included a balance of moments between the chip against

the tool rake face and the shear plane. That the shear angle is sensitive to the applied moment at the rake face was demonstrated by Pekelharing⁽⁵⁾ who externally applied forces to the chip and changed both the shear angle and the chip curl. Oxley's approach also enabled accounting for strain hardening and more realistic strain rate effects.

Oxley's approach used a slip line theory and included realistic trajectories or paths of elements flowing from the workpiece through the shear zone to the chip. Recognition of the difference in combined stress at the free surface and the tool tip is then possible. Also, a more realistic "friction" stress distribution between the tool and the chip may be incorporated. The resulting relation is:

$$(\phi + \beta - \alpha) = \tan^{-1} \left[\frac{1}{2} + \frac{\pi}{4} - \phi + \left(1 + \frac{h'}{h}\right) \frac{\cos 2(\phi - \alpha)}{2 \tan \beta} - \frac{\sin 2(\phi - \alpha)}{2} \right] \quad (7)$$

where h and h' are the total length of chip contact with the workpiece and the portion of chip contact for which the yield point in shear obtains. Thus, a situation in which part of the chip was effectively welded to the tool could be included in what is called the friction stress. The strain hardening is shown by a related equation of Oxley and Stevenson⁽⁶⁾ which is:

$$C n = 1 + \frac{\pi}{2} - 2\phi - \tan(\phi + \beta - \alpha) \quad (8)$$

c is a constant, 5.8 for some materials. The results are illustrated in Figure 2. A much better agreement with machining tests was obtained, but still, laboratory mechanical properties data for the workpiece material could not be used. In part, the latter failure is attributable to the thermal effects of the shearing and friction processes which alter the stress-strain-strain rate relationships in the material during machining. By far the largest portion of the heat, about 75%, is convected away by

the chip while about 20% is conducted to the tool and 5% to the workpiece. These proportions vary with the thermal properties of the workpiece and tool materials as well as the cutting conditions.

In addition, the plane strain assumption is not strictly true and the chip is generally wider than the width of the cut through lateral flow. Also, no tool is sharp to a zero edge radius so that contact stresses and friction exist at the edge causing some material to be plastically deformed under the tool edge producing residual stress in the workpiece surface.

Relation of the Surface Finish to Cutting Theory

There is a well known geometrical relation between tool geometry, feed rate, and surface roughness. This may be called an ideal roughness since it implies a perfect tool geometry leaving no burrs or sideflow, no rewelded chip material, no vibration, no tool chatter, and no roughness from material grain randomness. The peak to valley roughness under such ideal conditions is:

$$\begin{aligned} \sin \phi_0 &= \frac{f_r}{2r_n} \\ h_i &= r_n (1 - \cos \phi_0)^2 \\ \text{when } \phi_0 \ll 1, \quad h_i &= \frac{f_r^2}{8r_n} \end{aligned} \quad (9)$$

where f_r and r_n are the feed rate and tool nose radius. Such an ideal surface roughness is not fully achieved. It should be noted though that the ideal roughness does have a period equal to the feed rate and its magnitude decreases as the square of the feed rate. Also, from this criterion alone, the larger the tool nose radius the smaller the amplitude of the roughness. In practice, an unlimited increase in tool nose radius can be detrimental and some of the additional sources of roughness will now be examined.

Side Flow and Extrusion

Merchant's first assumption if true would eliminate one source of additional roughness. But no tool has an infinitesimal edge radius making it absolutely sharp. In fact, the ultimate in sharpness would be a radius corresponding to the interatomic distance in the tool material. For most tool materials this is approximately 3 Angstroms. Even a somewhat more compact atomic spacing would achieve little gain in surface finish because you cannot split or cut the atoms in the workpiece. Thus a finite tool sharpness radius exists which results in contact stresses and a contact area on the workpiece. Furthermore, the tool edge being elastic also deforms becoming duller. Abdelmoneim and Scrutton^(7,8) considered the behavior of an artificially rounded tool edge. A region exists which is a plastic zone under the tool edge. Furthermore, a small region possesses a large negative rake angle which tends to plough the material rather than cut it. In grinding and polishing operations, ploughing is the primary effect since the abrasive grains generally have large negative rake angles. Considered in a hydrodynamic analogy, there is a stagnation point where material arriving above that point flows into the chip and material below that point flows under the tool undergoing a plastic deformation which leaves a small amount of surface residual stress. Under some conditions of speed and feed, material builds up on the stagnation point making a false edge of workpiece material. The build-up continues until the stagnant material becomes unstable and breaks off. Generally, part of the build-up goes into the chip, the rest being welded to the workpiece. Under conditions of a built-up edge, a periodic roughness appears on the workpiece and an exactly analogous roughness appears on the rake face side of the chip.⁽⁹⁾ In machining with diamond tools, the sharpness of the edge is sufficient that conditions of a built-up edge seldom exist. The important thing is that a plastic region does exist and that leads to side flow and extrusion.

Analysis of Side Flow

To analyze the side flow consider contact of a surface with a double curvature representing the tool sharpness and nose radius with a curved

groove in the workpiece. This is illustrated in Figure 3. Analyzing the contact stresses as in Seely and Smith⁽¹⁰⁾, the maximum octahedral shearing stress occurs at a distance below the contact surface. Thus, if the force is sufficiently large, a whole region exists in a plastic state of stress. The region of contact can thus be analyzed in terms of the mechanical properties and the geometry. Using the notation of reference 10 two parameters are needed which are:

$$\begin{aligned} A &= \frac{1}{2} \left[\frac{1}{r_n} - \frac{1}{(r_n + \epsilon)} \right] \\ B &= \frac{1}{2} \left[\frac{1}{r_s} - \frac{1}{(r_s + \epsilon)} \right] \end{aligned} \quad (10)$$

where r_n is the tool nose radius, r_s is the tool edge sharpness radius, and ϵ is the thickness of the plastic region under the tool. It should be noted that the negative sign is used between terms in the bracket of A and B because the groove made by the tool has a negative radius.

Several properties depend upon the ratio B/A which in this case is very large.

$$\frac{B}{A} = \frac{r_n (r_n + \epsilon)}{r_s (r_s + \epsilon)} \quad (11)$$

For tools with a nose radius greater than 0.0001 inches (2.5 microns) and a tool at the ultimate sharpness of 3 Angstroms, the ratio is in the vicinity of $B/A \geq 10^8$.

At such large ratios, the value of the parameter c_s in reference 8 is constant at 0.78.* Therefore, the width of the contact region is:

$$b = \frac{z_s}{0.78} = \frac{\epsilon}{0.78} \quad (12)$$

where z_s is the depth to the maximum shear stress location below the surface of contact and is thus equated to the boundary of the fully plastic layer.

* These various c constants are calculated from elliptic integrals and are properties of the contact geometry.

Another contact parameter from reference 10 is:

$$\Delta = \frac{1}{(A+B)} \left[\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right] \quad (13)$$

where ν_1, E_1 are Poisson's ratio and Young's Modulus for the workpiece while the subscript 2 refers to the tool material, in this case Diamond. Since the ratio of equation 11 is so large, it is beyond the graph for the parameter C_g of reference 10. However, that curve, C_g vs B/A is approximately a logarithmic relation and can be expressed as:

$$\ln C_g = 1.0417 - 0.45728 \ln (B/A) \quad (14)$$

The maximum octahedral shearing stress is then,

$$\tau_G = C_g \left(\frac{b}{\Delta} \right) \quad (15)$$

and the parameter C_g is a constant over a very large range of B/A with a value of 0.27. Equating τ_G to the yield stress in shear (which neglects strain hardening) equations 12, 13, and 15 can be used to solve for the ratio (ϵ/ϵ_s) . The depth of displacement of the tool into the workpiece by plastic deformation is then:

$$S = \frac{C_g}{\pi} P \left(\frac{A+B}{b/\Delta} \right) = \left(\frac{C_g}{2\pi C_b} \right) \left(\frac{\epsilon}{\epsilon_s} \right)^2 \left(\frac{1}{0.78} \right)^2 \left(\frac{1}{1 + \frac{\epsilon}{\epsilon_s}} \right) \quad (16)$$

Equations 16 and 14 can be solved for the tool force P by determining the displacement δ and solving for P above. C_b is obtained from a logarithmic expression similar to equation 14 except the coefficients on the right are -0.44083 and -0.17718.

Of primary interest is the burr or side flow produced by the flow of material out of the contact zone under the tool edge. Therefore the following model is postulated.

The material at any point in the plastic contact region will flow toward the nearest free surface. The contact region is elliptical, but the minor axis is so small compared with the major axis that it could be considered a very narrow rectangle. Taking the side flow tendency as the reciprocal of the distance from the edge, the amount of flow at any point becomes

$$\frac{da_s}{dx} = \frac{a\delta}{2} \left[2\frac{x}{a} - \frac{x^2}{a^2} \right]^{\frac{1}{2}} \quad (17)$$

where δ is the depth of the tool displacement into the material, a is the length of the semi-major axis and x is the distance from the center. Integrating, the area of material flowing to the side is:

$$a_s = \frac{a\delta}{2} \left[\left(1 - \frac{x}{a}\right) \left(2\frac{x}{a} - \frac{x^2}{a^2}\right)^{\frac{1}{2}} - \sin^{-1}\left(1 - \frac{x}{a}\right) \right] \quad (18)$$

and results in a burr area of $a_s = \frac{\pi}{4} a\delta$. It should be noted that this equation inherently assumes perfect plasticity and incompressible flow, being based on conservation of mass.

Since the material escapes by shearing in the lateral direction, the deformation of the free surface is parabolic with the origin at the depth corresponding to the yield point from equation 12 and following.

Therefore the back side of the side flow burr has a contour which is:

$$h = A_0 z^2 \quad (19)$$

where h is the lateral distance from the undisturbed surface, z is the depth into the workpiece, and A_0 is a constant. Integrating, the area of the side flow burr is obtained and the height of the side flow results from substituting equation 19. Thus,

$$h = A_0^{\frac{1}{3}} (3a_s)^{\frac{2}{3}} \quad (20)$$

Using equation 18, both the area and the height of the side flow region are determined.

Selvan and Radhakrishnan⁽¹¹⁾ made an experimental study of the side flow in cutting Vee grooves. Comparison of reference 11 results using equations 18 and 20 are summarized in Figure 4. Also, their results were used to check the validity of the parabolic deformation in equation 19 and the comparison is depicted in Figure 5.

Applying the side flow calculations to micromachining and circular nosed tools, Figures 6 and 7 show the influence of tool sharpness and tool nose radius on the side flow when using a 0.0001 (2.5 micron) feed rate and depth of cut on aluminum. Also, the role of the yield stress is shown in Figure 8. Using equations 18 and 20, the burr is higher than it is wide. Although strain hardening was not included in the model, strain hardening and thermal expansion considerations may be seen to cause the side flow burr to stand up after the tool has passed. Since the layer adjacent to the tool has undergone considerable strain while the material at the root is just at the yield point, elastic relaxation produces a moment. At higher speeds, the higher strain rates cause the material to exhibit less of the influence of strain hardening but the temperature during shearing would be higher. Again, when the material cools, the hotter material near the cut surface contracts more. Thus, the height of the peak above the surface depends on both the thermal and mechanical properties of the material which make it stand up. The existence of high peaks was observed by Jim Bryan of the Lawrence Livermore Laboratories using a Zeiss interferometer and has been dubbed "Jim Bryan's grass." The best specimen we have measured thus far had a surface finish of 16Å rms and was made at Rockwell's Rocky Flats Plant. That specimen also exhibits Jim Bryan's grass. Calculations from the theory result in an estimated side flow peak producing a predicted rms value of about 15Å if the side flow material were standing almost vertically. It should be noted that under conditions in which the side flow material does not stand up, the region on the back side forms a very fine crevice over the previously cut surface. Such a crevice would entrap coolant material or other foreign

substances and would be very difficult to remove by ion bombardment or other cleaning methods. It would generally remain in a shadow region, not accessible to ion bombardment. The extent of such crevices might be measurable by determining the amount of a liquid absorbed by the surface. However, other cracks in addition to the side flow crevices would also absorb somewhat, leading to some error. Side flow peaks may be reduced by (a) using materials with a low strain hardening exponent, (b) cutting at higher speeds, (c) using a negative side rake angle on the tool, and (d) using smaller tool nose radii although the latter method does not make a very significant reduction. Figure 9 compares the percentage of side flow material area to total area for various tool nose radii. The side flow burr being thin should be more susceptible to laser heating, so a reduction in the percentage of the surface area made up of such material should lead to a less vulnerable surface. Tool force and the depth of tool contact penetration are shown in Figure 10. Other materials may be considered by changing the moduli in equation 13 and the yield stress value in equation 15 appropriately. Since the temperature is unknown, an approach such as in reference 4 may be used or preferably, "machining" values of the yield stress from an orthogonal cutting test on the material may be used for consistency. The forces involved in the contact region may be calculated from equation 16 and when the depth of cut is sufficiently small that the force in equation 2 approaches the contact force from equation 16, additional contact and friction components must be added to those of equations 1 and 2 removing Merchant's first assumption. Smaller depths of cut would reduce the cutting action to pure ploughing or burnishing of the surface causing most of the material to pass under the tool edge.

The mechanism of some forms of chatter, namely autonomous oscillation through friction excitation was considered in reference 12. Reduced depths of cut and very large tool nose radii increase the tendency to chatter.

A consideration not incorporated in this analysis is the influence of the workpiece grain size and grain orientation. Since for many materials, the properties are not isotropic with grain orientation, randomly oriented

large grains introduce variations from one grain to the next in most of the above equations adding additional roughness. Also, when grain boundaries are relatively weak, whole grains may be torn out and deposited in the side flow. This tearing may be reduced by selecting rake angles from the macromachining theory which produce a combined compressive and shear stress on the shear surface. This approach for brittle materials was reported by Kobayashi⁽¹³⁾. Furthermore, no diamond has a perfectly smooth edge. There are small notches and microcracks plus minute polishing marks.⁽¹⁴⁾ Pekelharing and Gieszen⁽¹⁵⁾ found that each notch in the edge extruded the workpiece material through plastic flow, even when the notch was somewhat above the cutting zone. This adds a burr to the side flow material and roughness to the machined groove. It should be noted that a negative side rake angle on the tool constrains the side flow. Also, for strain rate sensitive materials, higher cutting speeds can reduce the side flow.

All of the theory discussed has assumed the material behaves according to continuum mechanical principles. For surface finishes better than $6\text{\AA}$ P-V much of the theory should be re-examined using a discrete particle atomic concept.

Conclusions and Future Directions

1. Machining parameters such as rake angle, depth of cut, feed rate, tool nose radius, and cutting speed are effective in controlling the cutting forces and reducing figure error.
2. Super smooth diamond edges and very sharp edges are effective in reducing side flow, reducing machined surface residual stress, and producing surfaces which approach ideal roughness values.
3. A compromise is involved in selecting the best tool nose radius since a large radius produces a maximum of smooth cut surface area to burr or side flow area. However, large tool nose radii produce higher burrs, are more likely to induce chatter, and the burr lies at a lower angle to the surface encouraging the entrapment of coolant or foreign substances.

4. Ultimately, the best surface that could be produced would appear like rows of 3 Angstrom balls and only removal of material dislocations would improve it.

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Strain Hardening Exponent n

Figure 2 Comparison of Theories

Figure 3 Tool-Workpiece Contact

Figure 4 Comparison of Contact Theory with Data

Figure 5 Comparison of Contact theory with Data

Figure 6 Influence of Tool Nose Radius on Side Flow

Tool Nose Radius (mm)

0.5 (mm)

1.0

1.5

2.0

2.5

3.0

3.5

4.0

Figure 7 Influence of Tool Sharpness on Side Flow

Figure 8 Influence of Yield Stress on Contact Deformation

Figure 9 Proportion of Burr to Clean Cut Surface