MASTER

SINGLE PASS COLLIDER MEMO CN-343

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TITLE: Dispersive Effects of Transverse Magnet
Displacements in Rolled Arc Achromats *

Summary

It was first reported¹ last year that certain coherent transverse displacements of Arc magnets linearly perturb the dispersion function in addition to perturbing the beta function. This finding was of interest since it had long been known but has been only recently published^{2,3} that the *matched* dispersion function was undisturbed in the first approximation by such translations, either random or coherent. On the other hand, it is also known that in the Arcs the dispersion function is not always matched due to the necessity of rolling the achromats about the beam axis to follow the site terrain. Here we investigate the effect of transverse displacements of combined function magnets where the dispersion is *not matched* due to roll. We show that this dispersion function is perturbed by displacement of combined function magnets either singly or coherently. In the latter case we examine as was done in Ref. 1 the effect of a systematic (or DC) offset of magnets. This type of error can occur due to systematics in the placement or the readout of Beam Position Monitors or equivalently by correcting the orbit of a beam of the wrong momentum with respect to the Arc magnet excitation.

Background and Introduction

It has long been recognized that gradient errors occurring at the second harmonic of the betatron frequency can cause resonant optical effects. The first application of this idea to SLC optics⁴ was by R. Stiening. It occurred to one of us (THF) that due to the sextupole component in the combined function magnets of the Arcs, patterned gradient errors can be generated by forcing the beam to follow a path with a chosen spatial frequency, amplitude and phase. This is done by calculating an offset for the trajectory at each Beam Position Monitor (BPM) for a given phase and amplitude of the desired harmonic then allowing the orbit

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correction algorithm (using magnet movers in a one to one correspondence to BPMs) to force the orbit through these points.

Such *Harmonic Orbit Distortions* can be applied to either a perturbed (with random alignment and field errors) or an unperturbed Arc lattice. A study of the effects of these harmonic orbits was undertaken and some of the results reported (Ref. 1). These earlier results which were obtained by simulating a *whole Arc* are partly summarized here:

- In an achromat the spatial frequencies that can be imposed on the orbit are $\frac{k}{3}f_\beta$ where f_β is the frequency of the free betatron oscillation and $k = 0, 1, 2, 3, 4$. The fold-over or Nyquist frequency corresponds to $k = 5$ and is determined by the number of equally spaced BPMs (10 in each plane). Thus the second harmonic ($k = 6$) of f_β is the alias of $\frac{4}{3}f_\beta$. It should be noted that the free betatron oscillation is modulated in both frequency and phase whereas in the above studies the constrained position of the beam at each BPM was calculated as a simple harmonic in path length.
- Orbit distortions at two frequencies were found to affect both the beta function, β , and the dispersion function, η . These two were the zero frequency ($k = 0$) corresponding to a *systematic offset* of BPMs and the frequency corresponding to $k = 4$ which has as its alias the second harmonic ($k = 6$). For the whole Arc small amplitudes (of the order of 20 microns) produced significant effects on dispersion (of the order of 10%).
- It was pointed out that the observed susceptibility of the Arc optics to the DC or systematic errors required that care should be taken to eliminate this type of error in both alignment and in the BPM electronics. Next it was pointed out that since the horizontal dispersion function has approximately the same value ($35\mu/ppm$) at all Arc BPMs, steering a beam of the wrong energy through the Arcs is equivalent to steering a beam of the correct energy through BPMs with a DC or systematic error. Also noted was that since there exists a one to one correspondence between magnet movers and BPMs in either of these cases a DC or systematic signal would show up in a harmonic analysis of the magnet mover positions. This last observation was confirmed by one of us (JJM, Ref. 5) where it was reported that simulations using MURTL showed that the optical corrections available in the Final Focus System (FFS) were incapable of correcting the dispersion error caused by steering a beam with a relative energy error $\geq 5 ppm$.

It was shown in this report that the signature of such an energy mismatch is observed as a DC component on the magnet movers of 28 microns per part per thousand error in energy.

We will proceed to show that *some* of these earlier observations stem from the deviation from the matched dispersion condition which is caused by rolling the achromats. Specifically, we reexamine the effect of the $k = 0$ harmonic on dispersion.

Unmatched Beta and Dispersion Functions in Rolled Achromats

If the Arcs lay in a single plane we know (Refs.2, 3) that in the η matched case transverse displacement of the combined function magnets, singly or in groups, produce a deflection which is non-dispersive to first order in the magnitude of the displacements.

In fact, however, many of the achromats are rolled about the beam axis by angles as great as 10 degrees. With an otherwise perfect system this would have no effect overall since the transfer matrix of an achromat is unity. But rolls induce beta and eta mismatch and then the deflection produced by transverse displacements is no longer dispersion free and, as will be shown, contribute to anomalous dispersion throughout the Arc.

In Figs. 1 and 2 we plot the matched β - and η - functions for a single planar achromat. Figs. 3 and 4 illustrate mismatched β - and η - functions of the same achromat rolled by an angle 10° . Note that the change of η_y is of the order of magnitude of the matched η_x , while the change of the η_x is tiny (two orders of magnitude smaller for the case considered).

Effect of Mismatched Dispersion

As in Ref. 3. consider a differential slice of an arc magnet. In a coordinate system where x, z is the bend plane and y is the beam direction, the magnetic field can be represented as follows,

$$B_y = B_0 + B'_0 x + \frac{1}{2} B''_0 (x^2 - y^2), \quad (1)$$

$$B_x = B'_0 y + B''_0 xy, \quad (2)$$

where the coefficients B_o , B'_o and B''_o represent the dipole, quadrupole and sextupole strengths of a combined function magnet, respectively. Let

$$x = \Delta x + \eta_x \delta \equiv \Delta x + (\eta_o + \Delta \eta_x) \delta, \quad (3)$$

and

$$y = \Delta y + \eta_y \delta \equiv \Delta y + \Delta \eta_y \delta, \quad (4)$$

where $\Delta x, \Delta y$ are transverse displacements, η_x, η_y are the actual dispersion functions, η_o is the matched η -function and $\Delta \eta_x$ and $\Delta \eta_y$ are the differences between the actual and matched functions.

Using these equations the horizontal deflection, $\Delta \theta$ and the vertical deflection, $\Delta \phi$ of a ray with $p = p_o(1 + \delta)$, passing through a slice are given by,

$$\Delta \theta = \frac{ds}{p_o(1 + \delta)} \left\{ B_o + B'_o \left(\Delta x + \eta_x \delta \right) + \frac{B''_o}{2} \left(\Delta x^2 + 2 \Delta x \eta_x \delta + \eta_x^2 \delta^2 - \Delta y^2 - 2 \Delta y \eta_y \delta - \eta_y^2 \delta^2 \right) \right\} \quad (5)$$

$$\Delta \phi = \frac{ds}{p_o(1 + \delta)} \left\{ \left[B'_o \left(\Delta y + \eta_y \delta \right) + B''_o \left(\Delta x \Delta y + \Delta x \eta_y \delta + \Delta y \eta_x \delta + \eta_x \eta_y \delta^2 \right) \right] \right\} \quad (6)$$

Retain only terms linear in Δx and Δy ; average the dispersion over the length of the moved magnet, l , and separate the remaining terms in dispersive and non-dispersive groups. Then one finds

$$\Delta \theta \approx \frac{l}{p_o(1 + \delta)} \left\{ \left(B'_o + B''_o \bar{\eta}_o \delta \right) \Delta x - B''_o \delta \left(\Delta \bar{\eta}_x \Delta x + \Delta \bar{\eta}_y \Delta y \right) \right\} \quad (7)$$

$$\Delta \phi \approx \frac{l}{p_o(1 + \delta)} \left\{ \left(B'_o + B''_o \bar{\eta}_o \delta \right) \Delta y - B''_o \delta \left(\Delta \bar{\eta}_x \Delta y - \Delta \bar{\eta}_y \Delta x \right) \right\} \quad (8)$$

Note that terms $B''_o \bar{\eta}_o \delta \Delta x$ in $\Delta \theta$ and $B''_o \bar{\eta}_o \delta \Delta y$ in $\Delta \phi$ have been added and subtracted to facilitate the use of the non-dispersive condition $B''_o \bar{\eta}_o / B'_o = 1$.

Inserting this condition and taking the derivative with respect to δ at the point $\delta = 0$, the anomalous $\Delta\eta'$, for x and y planes, respectively, become

$$\Delta\eta'_x = \frac{-B''_o l}{p_o} \left(\Delta\bar{\eta}_x \Delta x + \Delta\bar{\eta}_y \Delta y \right) \quad (9)$$

$$\Delta\eta'_y = \frac{-B''_o l}{p_o} \left(\Delta\bar{\eta}_x \Delta y - \Delta\bar{\eta}_y \Delta x \right) \quad (10)$$

As previously noted for a 10° roll $\Delta\bar{\eta}_x$ is small compared to $\Delta\bar{\eta}_y$ so the main contribution to both $\Delta\eta'_{x,y}$ comes from the terms proportional to $\Delta\bar{\eta}_y$. That means that $\Delta\eta'_x$ is produced by a vertical displacement Δy and vice versa. For large displacements the quadratic terms neglected in expressions (9) and (10) produce nonlinear anomalous dispersion.

Simulations Results

Calculations using the computer code MURTL have been compared with calculations using equations (9) and (10) and are found to be in good agreement for the displacement of a single magnet. Two cases were considered, one where the effect of the displacement was compared immediately after the magnet and the other where the effect was measured at the end of the achromat. In the latter case there is a contribution to the anomalous dispersion due to the offset in each downstream magnet because of the free betatron oscillation induced by the displaced magnet. If this orbit oscillation is suppressed then the results obtained by MURTL and TRANSPORT agree with those obtained for $\Delta\eta'$ from equations (9) and (10) and for $\Delta\eta \simeq l\Delta\eta'/2$ multiplied by the respective first order transfer matrices.

At first it may appear that the effect of the free betatron oscillation is inconsistent with the results cited in Ref. 1 where no effect at this frequency was observed. However, without repeating those earlier calculations we have confirmed that *a free betatron oscillation which is initiated externally to a rolled achromat does not affect the dispersion at the end of that achromat although internally anomalous dispersion is present*, whereas, a free oscillation initiating within does effect the dispersion both internally and at the end.

Using MURTL we proceed to calculate the cumulative anomalous dispersion at the end of an achromat due to displacement of a single magnet for two cases: a) where the dispersion is matched and b) where the dispersion is mismatched

due to the roll of the achromat. As in Ref. 3 the model used in these simulations is that of a single Arc achromat starting from the symmetry point of a focusing magnet. Each of the 20 magnets has been divided into two equal halves. These half-magnets are numbered sequentially from 1 to 40. Particular half-magnets No. 6 and No. 8 were chosen as examples of defocusing and focusing movers previously examined in Ref. 3. As mentioned, the measured changes of η and η' at the end of the achromat are the cumulative result of the initial and subsequent kicks due to orbit displacement in the downstream magnets. Whether the effect is most noticeable as a change in η or η' depends upon the relative phase shift from the moved magnet to the end.

Figs. 5 through 8 represent anomalous $\Delta\eta_y, \Delta\eta'_y, \Delta\eta_x$ and $\Delta\eta'_x$ measured by MURTL E at the end of the achromat as functions of vertical Δy and horizontal Δx displacements of the focusing half-magnet No. 8. Similarly, Figs. 9 through 12 give the results of MURTL E calculations for vertical Δy and horizontal Δx displacements of the defocusing half-magnet No. 6.

Note that no linear (in magnet displacements) dispersion is produced under η -matched conditions (solid lines). On the other hand a strong linear perturbation appears for a mismatched η -function due to roll (dashed lines). Furthermore, in all cases, the linear behavior is consistent with equations (9) and (10) in that changes in the horizontal dispersion are due to vertical displacements and vice versa.

Effects of Systematic Displacements

We now turn to the effect of systematic error and confirm the observations reported in Ref. 1. Our model is a single achromat in which the magnet displacements were induced in two ways. The first one is to offset BPMs by a constant amount and to apply the orbit correction algorithm. The second is to offset the momentum of the incoming beam and again to apply the orbit correction. In both of these cases a systematic offset of the moved magnets is produced.

Figs. 13 and 14 show the effect of a vertical systematic error. As expected the horizontal dispersion has a linear response, whereas the vertical dispersion is unaffected.

Figs. 15 and 16 show the effect on vertical and horizontal dispersion of a horizontal systematic error. As expected the vertical dispersion is the one which is affected.

In Figs. 17 and 18 the anomalous dispersion is caused by steering an off-momentum beam through the achromat. Note that the results plotted in Figs. 15 and 17 can be made identical by scaling the abscissa by the expected change in sign and using the scale factor obtained from the value $35\mu/ppm$ of the matched dispersion function at each horizontal BPM.

Fig. 18 (unlike Fig. 16) shows a linear effect on the horizontal dispersion. The reason for this is that for an off-momentum ray in a rolled achromat both horizontal and vertical steering are required. In the horizontal plane all focusing magnets are moved by the same amount (i.e. by $28\mu/ppm$). In the vertical plane displacement of only the first two defocusing magnets are required to correct the vertical orbit. These small vertical displacements affect the horizontal dispersion.

Conclusion

We have shown that for rolled achromats the horizontal (vertical) dispersion function is affected linearly by vertical (horizontal) displacement of combined function magnets either singly or coherently. We have confirmed the finding in Ref. 1 that in the region of interest a DC or systematic translation of these magnets generates a predominantly linear perturbation on the dispersion. Such a coherent translation can be induced by BPM errors or by steering an off-momentum beam.

There exists a possibility that these and other *Harmonic Orbit Distortions* may be useful for correcting optical anomalies occurring in the perturbed Arcs.

REFERENCES

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Fig. 1. β function for single arc achromat, for case where lattice starts in center of horizontally focussing magnet. Magnets split into halves.

Fig. 2. η function for single arc achromat, for case where lattice starts in center of horizontally focussing magnet. Magnets split into halves.

Fig. 3. β function for single arc achromat, where input beam has a mismatch resulting from a roll of 10 degrees. Magnets split into halves.

Fig. 4. η function for single arc achromat, where input beam has a mismatch resulting from a roll of 10 degrees. Magnets split into halves.

Fig. 5. Effect on η_y, η'_y for Vertical displacement of half magnet 8 (F) with and without rolling the achromat.

Fig. 6. Effect on η_z, η'_z for Vertical displacement of half magnet 8 (F) with and without rolling the achromat.

Fig. 7. Effect on η_y, η'_y for *Horizontal* displacement of half magnet 8 (F) with and without rolling the achromat.

Fig. 8. Effect on η_x, η'_x for *Horizontal* displacement of half magnet 8 (F) with and without rolling the achromat.

Fig. 9. Effect on η_y, η'_y for Vertical displacement of half magnet 6 (D) with and without rolling the achromat.

Fig. 10. Effect on η_x, η'_x for Vertical displacement of half magnet 6 (D) with and without rolling the achromat.

Fig. 11. Effect on η_y, η'_y for Horizontal displacement of half magnet 6 (D) with and without rolling the achromat.

Fig. 12. Effect on η_x, η'_x for Horizontal displacement of half magnet 6 (D) with and without rolling the achromat.

Fig. 13. Effect on η_y, η'_y of steering beam for a *Vertical* displacement at entrance of each defocusing magnet, with and without roll.

Fig. 14. Effect on η_x, η'_x of steering beam for a *Vertical* displacement at entrance of each defocusing magnet, with and without roll.

Fig. 15. Effect on η_y, η'_y of steering beam for a *Horizontal* displacement at entrance of each focusing magnet, with and without roll.

Fig. 16. Effect on η_x, η'_x of steering beam for a *Horizontal* displacement at entrance of each focusing magnet, with and without roll.

Fig. 17. Effect on η_y, η'_y of steering an off momentum beam through an achromat with and without rolling the achromat. $\Delta p/p$ is in *ppm*.

Fig. 18. Effect on η_x, η'_x of steering an off momentum beam through an achromat with and without rolling the achromat. $\Delta p/p$ is in *ppm*.