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PROBING THE NUCLEAR STRUCTURE WITH HEAVY-ION REACTIONS

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Abstract

Nuclei display distortions in both ordinary space and in gauge space. It is suggested that it is possible to learn about the spatial distribution of the Nilsson orbitals and about the change of the pairing gap with the rotational frequency through the analysis of one- and two-nucleon transfer reactions induced in heavy-ion collisions.

1. Surface and Pairing Collective Modes

The $I(I+1)$ -law displayed by the low-lying levels of the spectra of many nuclei has long been recognized to be the fingerprint of very coherent nuclear motion.¹ The particles align in the average field, filling first the magnetic substates with $|m| \sim 1/2$ or with $|m| \sim j$ (cf. in Fig. 1). These configurations define privileged directions z' in space. Rotational invariance is regained by allowing the system as a whole to rotate along an axis perpendicular to z' . A typical example of the resulting spectrum is shown in Fig. 2.

Rotational motion in ordinary space is an important example of coherent motion in finite many-body systems. Other degrees of freedom which are not amenable to a classical description can be understood by analogy to spatial rotations. The central tools to carry out this analogy are provided by the concept of average field and of elementary modes of excitation.

In Table 1 we use them to introduce the basic properties of pairing rotations. The violation of angular momentum and of particle number conservation (spontaneous symmetry breaking) is the mechanism responsible for the existence of quadrupole and pairing collective modes.

In the BCS state² the distinction between particles and holes is blurred, and a privileged orientation ϕ_0 in gauge space is defined. Because there is no restoring force in the ϕ variable, the system as a whole rotates restoring particle number conservation, as required by the original Hamiltonian. The members of these pairing rotational bands are the ground states of a series of superfluid isotopes (isotones).

It is not so much the regularity of the energy pattern which qualifies the collectivity of a mode, but the specificity with which the mode is excited by some external field and the strongly enhanced values of the associated cross sections (cf. Fig. 3). In particular, Coulomb excitation³ and two-particle transfer reactions⁴ are the specific probes for studying spatial and pairing rotations, respectively. In these processes, the nucleus increases its rotation frequency

$$\omega_{\text{rot}} = \dot{\omega} = \frac{1}{\hbar} \frac{\partial H_{\text{rot}}}{\partial I} = \frac{\hbar}{\mathcal{J}} I, \quad \phi_{\text{rot}} = \dot{\phi} = \frac{1}{\hbar} \frac{\partial H_{\text{pair}}}{\partial N} = \frac{\lambda}{\hbar}$$

as a whole, without changing its intrinsic structure.

Fig. 1

Fig. 2

The picture of a nucleus which displays distortion in gauge space is closely related to the model one has of a superconductor in the theory of condensed matter. In this case, it is not possible to observe the individual states of the rotational bands. This is because, due to the macroscopic size of the system, one can hardly study the transfer of single Cooper pairs at a time. However, the Josephson junction⁵ displays in a clear way the collective rotational degrees of freedom in gauge space. This junction is built out of two superconductors separated by a thin insulator layer through which pairs of electrons can tunnel. Each of the superconductors can be viewed as a rotor where the number of particles plays the role of the angular momentum. The rotors are coupled together through the exchange of pairs of electrons which corresponds to the exchange of angular momentum. The pairs are created and annihilated by the two-particle transfer operators and the coupling Hamiltonian can be written as

$$H \sim e^{2i\phi_L} e^{-2i\phi_R} e^{2i\delta} + \text{h.c.} \quad (1)$$

$$\sim \cos 2(\phi_L - \phi_R + \delta),$$

where L and R label the superconductor to the left and to the right of the dioxide layer. The quantities ϕ_L and ϕ_R are the gauge angles of the two superconductors and δ a phase shift acquired by the electrons when tunneling through the barrier. The rate at which the Cooper pairs are exchanged is given by

$$\dot{N}_L (= \dot{N}_R) = \frac{i}{\hbar} [H, N_L] = -\frac{1}{\hbar} \frac{\partial H}{\partial \phi_L} \sim \sin 2(\phi_L - \phi_R + \delta). \quad (2)$$

The rotational frequency of the rotors in normal space corresponds to the chemical potential of the superconductors

$$\dot{\phi} = \frac{1}{\hbar} \frac{\partial H}{\partial N} = \frac{\lambda}{\hbar} \quad (3)$$

Introducing $\phi = \dot{\phi}t = \frac{\lambda}{\hbar}t$ in the expression for N_L , we get

$$\dot{N}_L \sim \sin \left\{ \frac{2}{\hbar} (\lambda_1 - \lambda_2)t + 2\delta \right\}. \quad (4)$$

Thus, if there is a difference in the Fermi energy between the two superconductors which can be forced by applying an external voltage, there will be an a.c. current running between the superconductors. In terms of the voltage difference $V_1 - V_2$ one can write

$$\dot{N}_L \sim \sin 2 \frac{e}{\hbar} (V_1 - V_2)t + 2\delta. \quad (5)$$

The frequency of the a.c. current is proportional to the externally applied voltage, and the charge carried by the Cooper pair is $2e$. The phenomenon described by

Table 1

Nuclei display Spontaneous
Symmetry Breaking In

Ordinary Space
(Classical analogue exists)

Gauge Space
(Quantal phenomenon)

The ground state wavefunction

$$|0\rangle = \sum_I d_I |I\rangle$$

$$|0\rangle = \sum_N d_N |N\rangle$$

describes a system which
has not a definite

Angular Momentum I

Number of Particles N

and the nucleus
as a whole rotates
(Goldstone boson)

in ordinary space

in gauge space

$\hat{I}$: angular mom.

$\hat{N}$: number of part.

restoring symmetry

The associated Regge trajectory

contains the levels
of a

quadrupole

pairing

rotational band

the members of this band
are specifically excited in

inelastic processes and
Coulomb excitation

$$\langle I+2 | \hat{Q} | I \rangle^2 \sim Q_0$$

$$(\sim 200 B_{sp})$$

in two-nucleon
transfer processes

$$\langle N+2 | \hat{T} | N \rangle^2 \sim \left(\frac{\Delta}{G} \right)^2$$

$$(\sim 10^2 \sigma_{2p})$$

The distortion of the
system is measured by

the quadrupole moment

the pairing gap

Q_0

The Mayer-Jensen particles
which carry a definite

angular momentum

$$R a_{jm}^\dagger R^{-1} = \sum_{m'} j_{m'm}(\omega) a_{jm'}^\dagger$$

$$R = e^{-i \vec{I} \cdot \hat{n} \omega}$$

transfer quantum number

$$a_{jm}^\dagger \rightarrow e^{-i\phi} a_{jm}^\dagger$$

$$= e^{-i\phi}$$

become

Nilsson particles

$$a_{i\Omega}^\dagger = \sum_j w_{i\Omega}^j a_{jm}^\dagger$$

Bogoliubov-Valatin particle

$$\alpha_{jm}^\dagger = U_j a_{jm} - V_j a_{jm}^\dagger$$

and the intrinsic state
 $|0\rangle$ can be written as

Nilsson state

$$|0\rangle = \prod_{i\Omega} a_{i\Omega}^\dagger | \rangle$$

BCS state

$$|0\rangle = \prod_{jm} \alpha_{jm} \sim \prod_{jm} (U_j + V_j a_{jm}^\dagger a_{jm}^\dagger)$$

Eq. (5) is known as the a.c.-Josephson effect, and its experimental confirmation⁶ has provided the most accurate values of $e/\hbar$ to date.

1a. Microscopic Structure of the Deformed State

The microscopic structure of the intrinsic state of a deformed nucleus⁷ is determined by the interplay between the shell structure and the spontaneous symmetry-breaking phenomenon. The occupancy of the different orbitals gives a quantitative measure of this interplay, and can be determined in one-nucleon transfer reactions (cf. Fig. 4).

It is noted that distortions in space and in gauge space coexist. In fact, the quadrupole and the pairing field producing effects compete with each other. Pairing correlations are most prolific in a spherical situation because the degeneracy of the different orbitals is large. Even in a shape-deformed nucleus, the moment of inertia is considerably smaller than $\mathcal{J}_{\text{rig}}$ which is obtained by aligning the particles in the deformed field. Making use of a very simplified picture, one can say that when the nucleus rotates as a whole, only the mass at the tips will be dragged by the average field as the nucleons are in a superfluid state.

In a single-particle transfer process, one can obtain the spectroscopic factor S_j

$$C_j^2 S_{\ell j}^2 = \langle A+1 | a_{\ell j m}^\dagger | A \rangle^2; C_j^2 = \begin{cases} 1 & \text{(stripping)} \\ (2j+1) & \text{(pick-up)}, \end{cases} \quad (6)$$

The operator $a_{\ell j m}^\dagger$ creates a particle in the orbital $\ell j m$. The initial state $|A\rangle$ is assumed to be the ground state of an even-deformed nucleus. To calculate (6), one has to rotate $a_{\ell j m}^\dagger$ into the intrinsic system in both gauge and normal space, i.e.,

$$\begin{aligned} a_{\ell j m}^\dagger &= \sum_{\Omega} D_{\Omega m}^j(\omega) e^{-i\phi} a_{\ell j \Omega}^\dagger \\ &= \sum_{\Omega} D_{\Omega m}^j(\omega) e^{-i\phi} \sum_i W_{\ell j}^{i\Omega} a_{i\Omega}^\dagger \\ &= \sum_{i\Omega} D_{\Omega m}^j(\omega) e^{-i\phi} W_{\ell j}^{i\Omega} (U_i \alpha_{i\Omega}^\dagger + V_i \alpha_{i\Omega}). \end{aligned} \quad (7)$$

where $W_{\ell j}^{i\Omega}$ is the amplitude of the spherical state ℓj in the Nilsson state $i\Omega$. The spectroscopic amplitude can now be calculated,

Fig. 3. (a),(b)(from ref. 1).

Fig. 3. (c) The Sn pairing rotational band (from ref. 4).

$$\begin{aligned}
(S_{\ell j})^{1/2} &\sim A' \langle I_f M_f K_f | a_{\ell j m}^\dagger | I_i M_i K_i \rangle_A \\
&= \int d\phi e^{i(A'-(A+1))\phi} \int d\omega D_{M_f K_f}^{I_f^*}(\omega) D_{m \Omega}^j(\omega) D_{M_i K_i}^{I_i}(\omega) \\
&\quad \times W_{\ell j}^{i \Omega} U_j^2 \\
&\sim \delta(A'-(A+1)) \langle I_i M_i j m | I_f M_f \rangle \langle I_i K_i j \Omega | I_f K_f \rangle \\
&\quad \times W_{\ell j}^{i \Omega} U_i^2.
\end{aligned} \tag{8}$$

The quantities

$$S_{\ell j} \sim \left(W_{\ell j}^{i \Omega} U_i^2 \right)^2, \tag{9}$$

can be obtained from the one-particle transfer differential cross sections associated with the different members of the rotational band based on the Nilsson orbital $i\Omega$. They give the fingerprint of the Nilsson orbital (cf. Fig. 4(c)). Associated with the fact that all the angular momenta transferred in the reaction is carried by the particle, an integration over the Euler angles ω is performed in (8). Thus, the spectroscopic factor (9) does not provide direct information on the spatial distribution of the Nilsson orbitals (cf. Fig. 4(e)).

Because shape deformations are intimately related with a non-uniform distribution of mass in space, the topology of the Nilsson orbitals is an important prediction of the model. Possibilities to learn about the three-dimensional distributions of single-particle orbitals have been opened by the advent of heavy-ion machines capable of bringing two heavy nuclei into contact, as will be discussed below.

2. Mapping the Nuclear Surface

The most conspicuous features of the heavy-ion reactions which can be studied with the new generation of heavy-ion machines are collected in Table 2.

Let us make use of them to discuss what can be learned about the following "gedanken eksperiment": the collision at zero impact parameter between a deformed target and a spherical projectile at a bombarding energy close to that of the Coulomb barrier⁸ (cf. Fig. 5(a)).

We assume that the interaction between the two ions is the sum of a monopole-monopole and monopole-quadrupole Coulomb term. It can be shown (cf., e.g., Ref. 9, Section II.7) that the angular momentum transferred to the rotor in a sudden collision is (for details cf. Appendix)

$$\Delta L \sim 2\hbar q \sin 2\theta_0, \tag{10}$$

Fig. 4. (a) Neutron orbits in prolate potential ($82 < N < 126$) (from ref. 1).
 (b) Spectrum of ^{175}Yb (from ref. 1).

Fig. 4. (c) Fingerprints from intensities of (dp) reactions leading to $^{175}_{70}\text{Yb}$ (from ref. 1). (d) Effect of pair correlations on transfer intensities (from ref. 1).

Fig. 4(e)

Table 2

a) Very short wavelength of relative motion

b) Large values of angular momenta in relative motion

c) Strong localization of the ion-ion interaction

$$U_{aA}^N(s) = \iint dx dy e(s')$$

$e(s)$: int. en. per unit area

s : surface-surface distance

d) Strong time-dependent Coulomb and nuclear fields

(reactions between nuclei in excited states)

where Θ_0 denotes the original orientation of the symmetry axis of the rotor with respect to the incoming beam, and where

$$q = \frac{Z_a e Q_0}{4\pi v b^2}, \quad (10a)$$

is fixed by the charge eZ_a of the projectile, the static quadrupole moment Q_0 of the target, the distance of closest approach b , and the relative velocity v at $t = -\infty$. Typical values of $2q$ for reactions involving very heavy ions are $\sim 15-20$.

As expected from classical arguments, the maximum angular momentum transfer is achieved for $\Theta_0 = \pi/4$ or $\Theta_0 = 3/4 \pi$. These are the orientations for which the difference between the Coulomb force acting on the two poles is maximum due to the $1/r^3$ coupling term, and consequently, when the torque exerted on the top is largest. For $\Theta_0 = 0$ and $\Theta_0 = 90^\circ$ no angular momentum is given to the rotor. The relation between ΔL and Θ_0 is schematically shown* in Fig. 5(b).

We now consider a transfer process of a nucleon from the spherical system a to the deformed system A. In general, the particle will carry only few units of angular momentum and most of the torque exerted on the rotor will be due to Coulomb interaction. Let us assume that in the transfer process the high-spin states of a given rotational band are preferentially excited. This result will indicate that the Nilsson orbital upon which this band is built is localized around $\Theta_0 = 45^\circ$ and 135° on the nuclear surface (cf. Fig. 5(c)). If in the transfer process only the low-spin states of a band are reached, one can expect that the associated Nilsson orbital is concentrated on the equator or on the poles.

Using the standard labeling for the Nilsson levels $[Nn_z\Lambda\Sigma]$, the orbitals with $n_z = N$ are located at the poles, have large intrinsic quadrupole moments, and are strong downsloping; i.e., their energy decreases as a function of the deformation. The orbitals with $n_z = 0$ have negative quadrupole moments and are spatially located around the equator. As a function of the deformation, the energy of these orbitals increases. Single-particle states with $n_z \sim \frac{1}{2} N$ have small quadrupole moments, their energy basically does not change as a function of the deformation, and the associated wavefunctions are localized midway between the equator and the poles. To obtain quantitative information on the spatial distribution of the Nilsson orbitals, the full three-dimensional scattering process has to be solved and the eventual spin-(scattering angle) correlations calculated. The main results of such a calculation are summarized in the next section.

*The existence of two different initial orientations leading to the same ΔL gives rise to quantal interference phenomena. Although these phenomena carry nuclear structure information which is interesting in its own right, we are going to deal only with average properties. It is noted that the interference effects are averaged out if a smoothing of $\Delta L \sim 2\hbar$ is introduced in the quantal results (cf. Appendix).

Fig. 5

2a. Scattering of a Deformed Nucleus in Three Dimensions

The Hamiltonian describing the process is¹⁰

$$H = T_{aA} + H_{\bar{a}} + H_A + V_{aA} \quad (11)$$

where T_{aA} represents the relative kinetic energy, $H_{\bar{a}}$ and H_A are the intrinsic Hamiltonians for the projectile and target, and V_{aA} is the interaction between the two ions. For simplicity, we ignore the intrinsic degrees of freedom of the nucleus $\bar{a}$. We consider the nucleus A to be an axially symmetric rotor with moment of inertia $\mathcal{J}$ with respect to a direction of rotation perpendicular to the symmetry axis. Thus,

$$H_A = \frac{1}{2\mathcal{J}} P_{\theta}^2 + \frac{P_{\phi}}{\sin^2\theta} \quad (12)$$

The coordinate system used and the definition of the different variables is illustrated in Fig. 6. The ion-ion interaction is

$$V_{aA} = \frac{Z_a Z_A e^2}{r} + \frac{Z_a e Q_c}{2r^3} P_2(u) + U_{aA}^N(r, u). \quad (13)$$

The first two terms are the monopole-monopole and monopole-quadrupole contributions to the Coulomb interaction. The variable u denotes the cosine of the angle between the symmetry axis of the rotor and the coordinate of relative motion. The quantity Q_0 is the intrinsic quadrupole moment of nucleus A .

The nuclear interaction U_{aA} is parametrized by a deformed Woods-Saxon function

$$U_{aA}^N(r, u) = \frac{V_0}{1 + \exp \frac{r - R_0 - R_0(u)}{a}}, \quad (14)$$

with

$$R^A(u) = R_0^A \left(1 + \frac{5}{4\pi} \beta P_2(u) \right). \quad (15)$$

The nuclear deformation parameter β is related to Q_0 by

$$\beta = \frac{4\pi}{3A_A e(R_0)^2} \frac{5}{4\pi} Q_0^A. \quad (16)$$

Making use of the Hamiltonian H and of the definitions (12)-(16), one can obtain the classical equations which govern the time dependence of the coordinates r , θ , ϕ , Θ , Φ and their corresponding momenta. They form a set of coupled first-order differential equations which are integrated numerically. The initial conditions are specified by the center-of-mass energy E , the impact parameter ρ , which is fixed in the x - y plane, and a pair of angles θ_0 and ϕ_0 defining the initial orientation of the rotor which we assume at rest at $t = -\infty$.

From the result of the numerical integration, one obtains the final scattering angle θ and the angular momentum I given to the rotor. A double-differential

Fig. 6

inelastic cross section $d^2\sigma/d\theta dI$ can thus be constructed by sampling all impact parameters and initial orientations with the proper statistical weights

$$2\pi\rho\Delta\rho \frac{\sin\theta_0 \Delta\theta_0 \Delta\phi_0}{4\pi}$$

associated with the steps $\Delta\rho$, $\Delta\theta_0$, and $\Delta\phi_0$ used in the calculations. To take into account the existence of many other channels not explicitly included in the calculations, we introduce an imaginary potential iW which follows the deformation of the system. For simplicity, we use

$$W(r,u) = \frac{W_0}{V_0} U_{aA}^N(r,u). \quad (17)$$

Typical values for (W_0/V_0) are ~ 0.2 . The form chosen for W reflects the fact that in heavy-ion reactions the main contribution to the absorption comes from transfer processes.¹¹

Because the nuclear potential is relatively weak compared to the Coulomb interaction for the very heavy systems considered here, one can treat the effects of W as a local mean-free path for the classical motion (cf., e.g., Ref. 9). Accordingly, we weigh each trajectory with a probability

$$P = e^{-\frac{2}{\hbar} \int_{-\infty}^{\infty} W(r,u) dt}, \quad (18)$$

where the integral is carried out along the trajectory. One thus obtains

$$\left(\frac{d\sigma}{d\theta dI} \right)_{\text{inel+CE}} = P \frac{d\sigma}{d\theta dI} \quad (19)$$

2b. Space Localization of the Single-Particle Orbitals

The double-differential cross section associated with transfer processes is built in a similar way as in the case of inelastic scattering, namely by integrating the equations of motion and accumulating the events at each scattering angle. In this case the number of events is multiplied by the transfer probability P_{transf} . Assuming one can treat the transfer process in first-order perturbation theory, an approximation which seems well justified in the present case, P_{transf} is given by the square of the integral of the transfer form factor along the trajectory

$$P_{\text{transf}} = \left(\frac{1}{\hbar} \int_{-\infty}^{\infty} f_T(r,u) dt \right)^2. \quad (20)$$

Note that the assumption has been made that the transfer process has optimum Q -value.

Calculations have been made where the transfer strength along the trajectory

Fig. 7

differs from zero only when the angle between the relative distance vector and the rotor axis falls into pairs of rings 15° wide on the surface of the rotor at equal distances from the poles (cf. Fig. 5(c)). This defines six zones covering the nuclear surface. The results of the calculations are shown in Fig. 7. The associated inelastic cross section (19) is shown in Fig. 7(a). Very conspicuous correlations are found between the localization of the transfer form factor and the double-differential cross sections (cf. Figs. 7(b)-7(e)).

Actual transfer reactions will not occur in the particular zones used in the illustration. However, the correlation between scattering angle for any given form factor can be constructed in terms of the basic patterns of correlations associated with a set of narrow zones. Thus, measurements of $d^2\sigma/d\theta dI$ may be used to map out the amplitude of the single-particle wave functions along the surface of deformed nuclei.

2c. Study of the Pairing Phase Transitions in Rapidly Rotating Nuclei

When a deformed nucleus is subject to rapid rotations, changes in the $I(I+1)$ rotational pattern can be observed. In particular, in the moment of inertia of the Yrast band as a function of the rotational frequency can take place (cf. Fig. 8).

In strongly rotating nuclei, the Coriolis force acts as an external magnetic field in a superfluid system,* and it is expected that for some critical value of the rotational frequency, it becomes too expensive to rotate in the superfluid phase. In fact, $\mathcal{J}_{\text{sup}} \sim \frac{1}{2} \mathcal{J}_{\text{rigid}}$, where $\mathcal{J}_{\text{sup}}$ is the moment of inertia displayed by the ground-state rotational band at low I . The quantity $\mathcal{J}_{\text{rigid}}$ is the rigid moment of inertia and coincides with that of the independent particle motion. It is as in the superfluid phase, only the mass at the tips of the deformed nucleus is dragged by the walls, as will also be the case of a spheroidal bucket filled with superfluid HeII. Rotation breaks time-reversal invariance. The Coriolis force $H_C \sim \hbar \vec{\omega}_{\text{rot}} \cdot \vec{j}$ acts in opposite ways on each partner of a Cooper pair and for a value of H_C of the order of $2\Delta (\sim 1.7 \text{ MeV})$, two quasi-particle states are excited.¹² This will happen first for Cooper pairs based on states with high- j and low- Ω ¹³ (cf., e.g., Fig. 9). For the case of ^{162}Er the candidate is the Cooper pair built on the [660 1/2] Nilsson orbital (cf. Fig. 4(e)). The observed critical frequency $(\omega_{\text{rot}})_{\text{crit}}$ is $\sim \frac{250}{\hbar} \text{ keV}$ (cf. Fig. 8). Thus, $H_C \sim 250 \text{ keV} \times 6 \sim 1.5 \text{ MeV} (\sim 2\Delta)$ as expected. One can thus view the backbending phenomenon as resulting from the crossing of the ground-state band and of a band displaying a finite mass at $\omega_{\text{rot}} = 0$ and a rigid moment of inertia (cf. Fig. 10). It is noted that the fact that the moment of inertia of the rotational band based on the [660 1/2] two-quasiparticle state is close to $\mathcal{J}_{\text{rig}}$ does

* In this case, however, because of the shell structure, it is expected that the pairing phase transition will be connected with a rich variety of phenomena (band crossing, gapless superconductivity, pairing isomers, etc.).

Fig. 8. Ground-band level energies in even-even rare-earth nuclei (from Rev. of Mod. Phys. 47 (1975) 43).

not necessarily imply that $\Delta = 0$. The orbits which decouple from the rotational motion are strongly concentrated at the poles, leading to a large change in $\mathcal{J}$.

Although much has been learned during the past years, there are still important questions which need to be answered before achieving a consistent picture of back-bending. In particular, which is the dependence of Δ with ω along the Yrast line. In other words, which is the type of transition which takes place in going from the superfluid to the normal phases. Theoretically, one needs to have a good description of the shell model at high frequencies and to understand, among other things, the role of the multipole pairing components of the residual force and of the fluctuations of the pairing gap associated with pairing vibrations.¹⁴

Experimentally, there is only one possibility of measuring directly the pairing gap. It is through the transfer of a pair of particles (cf. Appendix Ref. 14 and Table 1). Furthermore, it has to be in a reaction induced in a heavy-ion collision. This is because reactions induced by light ions will not be able to excite the high-spin states we are interested in.

In the framework of the model discussed in Sections 2A and 2B, the two-nucleon double-differential cross section can be calculated by multiplying the events associated with each scattering angle by the two-nucleon transfer probability P_2 . This quantity is obtained by integrating the two-particle transfer form factor along the different trajectories. It is noted that for a transition between a normal and a superfluid nucleus $P_2 \sim (\Delta(I)/G)^2$, where $\Delta(I)$ is the gap parameter and G the strength of the pairing force. From the ratio between $(d\sigma/d\theta dI)_{\text{transf}}$ and $(d\sigma/d\theta dI)_{\text{inel+CE}}$, one can thus extract $\Delta(I)$.

In the above discussion, we have used the standard pairing theory and characterized the pairing distortion by a single parameter. That is, we have assumed pairing to act only in s-waves. However, both in nuclei and in other Fermi liquids, pairing acting in channels with $\ell \neq 0$ can play an important role (cf. Table 3).

In situations where Nilsson orbitals with negative quadrupole moments are close to the Fermi surface, the pairing gap can display an important state dependence. This is because, due to angular momentum violation, the quadrupole component of the pairing force can contribute to the pairing gap.¹⁵ This quantity is given by

$$\Delta_v = G_0 \sum_v U_v V_v + G_2 q_v \sum_v q_v U_v V_v. \quad (21)$$

The quantity $q_v = 3n_z - N$ is the static quadrupole moment associated with the orbital $v = i\Omega$. The quantity $\Delta_2 = G_2 \sum_v q_v U_v V_v$ measures the quadrupole deformation around the Fermi surface, and G_2 is the quadrupole pairing coupling constant. Let us consider a situation like the one displayed in Fig. 11 when an orbital with negative quadrupole moment (i.e., the [505 11/2] Nilsson level) is close to the Fermi surface and is embedded in a set of downsloping states ($q_v > 0$). In this case, $\Delta_2 > 0$ and

Fig. 9

Fig. 10. (a) Energy trajectories for $^{90}_{64}\text{Gd}^{154}$ in comparison with the experimental data. (b) The same as (a) but in terms of different variables (from Phys. Lett. 50B (1974) 295).

Table 3

Pairing Correlations

l -wave	System	Effect
s	electrons in meta' nuclei	condensation
p	^3He	condensation
d	nuclei	fluctuations

$$\Delta_{i\Omega} \sim \begin{cases} 0 & [550 \ 11/2] \\ \Delta & \text{other states.} \end{cases} \quad (22)$$

That is, the pairs of particles in the [550 11/2] feel a normal and not a superfluid system (pairing isomer), a phenomenon that has its counterpart in condensed matter and is known as gapless superconductivity.¹⁶

This mechanism has recently been used¹⁷ to explain anomalies observed in the critical rotational frequencies at which backbending takes place. The standard situation encountered is depicted in Fig. 12(a). The difference $\hbar\omega_{\text{crit}}(\text{even}) - \hbar\omega_{\text{crit}}(\text{odd})$ is interpreted in terms of the decrease of the pairing gap in odd-nuclei due to the blocking of an orbital close to the Fermi surface. For the case of ¹⁶²Er and ¹⁶¹Er, the situation is as shown in Fig. 12(b). In this case, the odd particle occupies the [505 11/2] orbital, and no change in Δ is expected in going from the even to the odd system.

Again, the specific way to test the value of the pairing gap at backbending is in a two-nucleon transfer reaction induced in a heavy-ion collision.

3. Variety of Nuclear Phenomena

In the previous section we have concentrated our attention on a particular aspect of nuclear structure which can specifically be probed in heavy-ion collisions. In this section we briefly list other nuclear structure problems for which heavy-ion reactions seem to be an ideal research tool (cf. Table 4).

3a. Excitation of Giant Resonances

The large Coulomb and nuclear fields available in heavy-ion collisions opens the possibility to study specific energy regions of the nuclear response by tuning the bombarding energy (cf. Fig. 13). In particular, one expects that heavy ions will become useful tools to study giant resonances with a good ratio of signal to background.¹⁸

3b. Excitation of Pairing Rotational Bands

Based on geometrical arguments one would expect, for medium heavy nuclei, two-nucleon transfer cross section $\sigma(2n)$ to be a factor $1/A \sim 10^{-2}$ smaller than single-nucleon transfer cross sections $\sigma(n)$. On the other hand, reactions involving two superfluid nuclei are estimated to be enhanced by a factor $(\Delta/G)^2 \sim 10^2$ as compared with the transfer of two uncorrelated nucleons, suggesting the possibility of multi-pair transfer.¹⁹

3c. Identification of High- λ Single-Particle Orbitals

The action integral

Fig. 11

Fig. 12

Table 4

Some of the possibilities opened by new
generation of heavy-ion machines

local	QE	Mapping out the surface of deformed nuclei (Nilsson orbitals in 3D)
large U^C	QE	Study of the pairing phase transition in strongly rotating nuclei
large U^C short	QE	Excitation of giant resonances with good ratio of peak to background. (Excitation of magnetic modes)
massive	QE	Coherent excitation of pairing rotational bands
short to massive	hard coll.	Study of zero point motion induced by collective vibrations (isosc. surface, isov. N-Z imbalance)
large U^C	QE	Coulomb excitation of rotational bands to high- spin states
large	QE	Excitation of single-particle orbitals with high spin ($Q \ll 0$)
large	F	Study of nuclear shapes as a function of angular momentum and temperature (superdef., fission, γ -decay GDR)
large	F	Search for shape isomers (molecular states) in strongly rotating nuclei
massive	QE	Study of alpha correlations

Fig. 13. (a) Double differential inelastic cross section associated with the reaction $^{16}\text{O} + ^{208}\text{Pb}$. In (a) the experimental spectrum at the grazing angle $\theta_{\text{lab}} = 14^\circ$ is given. In (b) and (c) the predicted "direct reaction" double differential cross sections are shown in mb/(MeV $\times$ sr). The darkly shaded area corresponds to the events in which at least two harmonic oscillator quanta were absorbed (from ref. 18).

Fig. 13. (d) Probabilities of one-phonon Coulomb excitation as a function of the energy of the mode for $i = 1, 2$ and 3 . In all cases a transition matrix element of one single particle unit was used for a trajectory leading to a distance of closest approach of 15 fm . The different curves for each multipolarity are labelled by the bombarding energy in MeV/nucleon (from ref. 18). (e) Coulomb excitation cross sections for some of the modes considered in the calculations. Only contribution from trajectories leading to distances of closest approach larger than 15 fm were included. The cross sections associated with the giant dipole (13.16 MeV) and the giant isovector quadrupole mode (23 MeV) of ^{208}Pb are also shown. They exhaust 86% and 55% of their corresponding energy-weighted sum rules, respectively (from ref. 18).

Fig. 14

$$\chi = \frac{1}{\hbar} \int_{-\infty}^{\infty} f(r(t)) e^{-\frac{i}{\hbar} (\Delta E t - \mu \phi(r(t)))} dt$$

measures the strength with which the reaction $a+A \rightarrow b+B$ associated with the form factor $f(r(t))$ takes place. Assuming $\chi^2 \ll \chi$, the process can be treated in perturbation theory and the cross section written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{aA \rightarrow bB} \sim \chi^2 \left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}}$$

The integral in χ is carried over a classical trajectory (grazing trajectory) and ΔE and μ are the energy and angular momenta transferred in the reaction process (cf. Fig. 14(a)). The exponential factor measures the matching between the entrance and exit channels classical trajectories of relative motion. The optimal matching is achieved when the condition

$$\Delta E - \mu \dot{\phi} = 0$$

is fulfilled.

Let us consider the following reactions:

	Q (MeV)	$\mu(\Delta L)$ ($\hbar$)
$^{149}\text{Sm}(^{16}\text{O}, ^{15}\text{O})^{150}\text{Sm}$	- 9.8	-10
$^{149}\text{Sm}(^{12}\text{C}, ^{11}\text{C})^{150}\text{Sm}$	-12.8	-12
$^{149}\text{Sm}(^{13}\text{C}, ^{12}\text{C})^{150}\text{Sm}$	0.9	- 2

The last two columns give the optimum Q-value and the angular momenta needed to fulfill the matching condition $\Delta E - \mu \dot{\phi} = 0$.

The two first reactions at bombarding energies of a few MeV per nucleon are optimal to excite single-particle orbitals with high angular momentum. One expects that the first reaction will predominantly excite orbitals where $j = \ell + 1/2$ and the second those where $j = \ell - 1/2$ (cf. Fig. 14(b) and 14(c)). Patterns of the type shown in Fig. 14(d) have already been observed.²⁰

d. Coulomb Fission and Beyond

With the availability of projectiles like ^{208}Pb , one can expect to multiply Coulomb excite the β -vibrations of, e.g., ^{234}U and eventually force the system into fission (cf. Fig. 15).

Evidence has recently been reported²¹ on non-equilibrium effects in sequential fission induced by energetic collisions between heavy ions. This result suggests the evidence of a new phenomenon in which a direct descent towards fission occurs from the initial shape distribution of the primary fragments. The time between the

Fig. 14(d). Expected relative cross sections for active single-particle levels as a function of excitation energy for the reactions $^{166}\text{Er}(^{16}\text{O}, ^{15}\text{O})$ and $^{166}\text{Er}(^{12}\text{C}, ^{11}\text{C})$ calculated with the distorted-wave Born approximation code PTOLEMY. Unit spectroscopic factors were assumed for all states (from Phys. Rev. Lett. 46 (1981) 1565).

Fig. 15. Cross section for the multiple Coulomb excitation of the beta vibration in the deformed nucleus ${}^{234}\text{U}$. In the lower part of the figure the differential cross section in barns/sr is given for backward scattering of various projectiles at the Coulomb barrier. The calculation was made in a pure vibrational model including the finite energy loss. In the upper part of the figure is given a schematic picture of the position of the multiphonon beta-vibrational states used in the calculation (from ref. 9).

consecutive scission acts is estimated to be of the order of 10^{-21} sec.

3e. Coulomb Excitation of High-Spin States

Coulomb excitation below the Coulomb barrier is specifically suited to excite the rotational states (cf. Fig. 16). Because of the slow variation of the Coulomb field in space, it can only excite modes of low multipolarity. Similarly for bombarding energies $E < E_B$ the slow variation of the Coulomb field in time makes it impossible to excite states at high frequency, since the process in this limit becomes rapidly adiabatic.

In Fig. 17 we show results of a Coulomb excitation measurement of ^{238}U making use of ^{208}Pb as a projectile.

3f. Study of Shapes of Nuclei Just Before Fission

The isovector giant dipole resonance is influenced in a major way by the coupling to the surface degrees of freedom. In particular, in deformed nuclei the resonance splits in two peaks, as protons and neutrons can vibrate out of phase along the symmetry axis or in a plane perpendicular to it (cf. Fig. 18).

Dipole modes can be excited not only from the ground state of a nucleus, but also from the ensemble of states associated with a compound nucleus at finite temperature.²² Measuring the γ -decay from the dipole resonance,²³ it is possible to construct the associated strength function and from its structure eventually infer the shape of nuclei subject to extreme conditions of rotation up to the fission threshold.

3g. Search for Shape Isomers in Strongly-Deformed, Rapidly-Rotating Nuclei

The liquid-drop model of nuclei has played an important role in the study of nuclear fission. To the interplay of surface tension and Coulomb repulsion the centrifugal force adds a new dimension and any nucleus will fission at sufficiently high angular momentum. The maximum amount of angular momentum a nucleus can accommodate before fissioning in two equal masses is $\sim 80\hbar$.

To obtain a detailed picture of the low-energy nuclear spectrum the shell structure has to be considered. The existence of low-lying collective modes of the fission products can lead to structure in the smooth potential energy surface predicted by the liquid drop,²⁴ as can be seen in Fig. 19. Due to the collective low-lying octupole vibration observed in ^{40}Ca and to the high angular moments with which the system rotates, two minima appear in the potential energy surface plotted as a function of the relative distance of the di-nuclear system $^{80}\text{Zr} = ^{40}\text{Ca} + ^{40}\text{Ca}$ and of the octupole deformation of the individual nuclei.

The Coulomb repulsion between the two pear-shaped ^{40}Ca systems can be balanced in two ways. Through a strong surface-surface attraction or by pulling away the centers of charge as much as possible. The first situation is achieved by having the two nuclei in contact at the base. This configuration is very compact and

Fig. 16. Coulomb excitation of deformed nuclei. The expected maximum angular-momentum transfer ΔL to rotational motion in ^{238}U and ^{160}Gd is plotted as a function of the projectile charge at a bombarding energy below the Coulomb barrier so that no interference from nuclear reactions is expected. With uranium ions on a uranium target, one may thus expect to populate states of spin 36 in the ground-state rotational band. In estimating ΔL , the energy of excitation has been neglected. This is expected to reduce the maximum angular momentum by a few units (from ref. 9).

Fig. 17. Level scheme for ^{238}U showing the ground state band up to spin 30 and the octupole band up to spin 23. The transition energies have an error of 0.4 keV for $J \leq 24$, with increasing spin this error increases up to a value of 1.0 keV for $J = 30$ (from Physica Scripta, Vol. 24 (1981) 337).

Fig. 18. Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergere, A. Oepretre, and A. Veyssiere, Nucl. Phys. A172 (1971) 437. The solid curves represent Lorentzian fits (from ref. 1).

Fig. 19. Contour levels of the effective potential U_{eff} for $\ell = 50$ minimized with respect to all deformation degrees of freedom except α_3 displayed in the $(\alpha_3 r)$ plane for the nucleus $^{80}\text{Zr} = ^{40}\text{Ca} + ^{40}\text{Ca}$. In (a) the liquid-drop parameters were used for all multipoles $\lambda (\leq 5)$. In (b) the octupole restoring force was extracted from a) fig. 7, while C_2 , C_4 and C_5 were assumed to have the same liquid-drop values as in (a). In (c) both C_2 and C_3 were extracted from a) fig. 7, while C_4 and C_5 have the liquid-drop values a) (from Nucl. Phys. A349 (1981) 496).

coincides with the liquid drop minimum. The second configuration is achieved by having the two nuclei in contact at the tip. This configuration leads to a strongly-deformed system (shape isomer). It is an open question whether these strongly rotating shapes isomer exist and which are the optimal bombarding conditions to produce them. In the same context, it is a challenging question to what extent the shape isomer observed in ^{236}U reflects a low-lying collective octupole vibration in ^{118}Pd .

3h. Study of the Zero-Point Motion Induced by Collective Vibration

Collective vibrations play an important role in the damping of energy and angular momenta in deep-inelastic reactions.²⁵

These collective modes induce zero-point fluctuations in the ground state. In classical terms, it means that for each impact parameter the surface of the two nuclei will be in a variety of positions.

Because the ion-ion potential depends on the position of the nuclear surfaces of the two interacting nuclei, one expects large variations of the energy and angular momentum loss. The analysis of data like the one displayed in Fig. 20 seems to offer a unique possibility of measuring the fluctuations in the position of the nuclear surface. Surface fluctuations seem also to play a central role in determining the fission cross section in collisions at energies below and above the Coulomb barrier.²⁶

An isovector vibration induces fluctuations in the ground state ratio N/Z . In an energetic heavy-ion collision, it is possible that the projectile can cut off a part of the target. The mass distribution of this chunk of nuclear matter will carry information on the zero-point fluctuations associated with isovector modes.

3i. Four-Particle Correlations

The possible existence of alpha-correlations in nuclei is a subject that has attracted much attention. One of the intriguing questions is the extent to which the alpha particle is transferred as a whole, rather than a combination of a transfer of a pair of neutrons and a pair of protons.²⁷

Transfer reactions induced by heavy ions allows for the transfer of four particles which can move both in target and projectile in the same potential. In this sense, heavy-ion reactions are the specific probes to study cluster correlation in nuclei. In Fig. 21 we show schematically the predictions of the pairing model for two- and four-particle transfer reactions around ^{208}Pb . Although the pairing spectrum is well established, preliminary attempts to observe the "alpha-vibrations" have not been successful.

Discussions with C. H. Dasso, S. Landowne and A. Winther are acknowledged.

Fig. 20. Energy-angle distribution of the $^{84}\text{Kr} + ^{208}\text{Pb}$ reaction ($E_{\text{lab}} = 494$ MeV)

Scattering in two dimensions

Interaction between
two nuclei: Coulomb

Solve problem: calculate probability
of exciting a given ang. mom.

$$H = \frac{p_r^2}{2m_0} + p_\chi^2 \left[\frac{1}{2m_0 r^2} + \frac{1}{2} \right] + \frac{Z_a Z_A e^2}{r} + \frac{Z_a e}{4r^3} Q_0 (3\cos^2 \chi - 1),$$

$$\begin{cases} \dot{\chi} = \frac{\partial H}{\partial p_\chi} = \left(\frac{1}{m_0 r^2} + \frac{1}{2} \right) p_\chi \\ -\dot{p}_\chi = \frac{\partial H}{\partial \chi} = -\frac{3Z_a e}{4r^3} Q_0 \sin 2\chi \end{cases}$$

Order of magnitude estimates

$$p_\chi \sim \frac{3Z_a e}{4b^3} Q_0 \tau \sim \frac{3Z_a e}{4b^3} Q_0 \frac{b}{v} \sim \hbar \underbrace{\left(\frac{Z_a e Q_0}{\hbar v b^2} \right)}_q,$$

$$p_\chi \sim \hbar q,$$

$$\tau \sim \frac{b}{v}$$

$$\Delta \chi \sim \hbar q \left(\frac{1}{m_0 b^2} + \frac{1}{2} \right) \frac{b}{v}$$

$$\sim q \left(\frac{1}{n} + \xi \right)$$

$v = \text{veloc. at } \infty$

$$b = \frac{Z_a Z_A e^2}{E}$$

$$n = \frac{Z_a Z_A e^2}{\hbar v} = \frac{b}{2\chi} \quad (\text{Sommerfeld par.})$$

$$\chi = \frac{\hbar}{mv} = \frac{\hbar v}{2E}$$

$$E_R = \frac{\hbar^2}{2\mathcal{I}} I(I+1) \quad ; \quad (\Delta E_R)_{0 \rightarrow 2} = \frac{\hbar^2}{2\mathcal{I}} 6 = \frac{3\hbar^2}{\mathcal{I}}$$

$$\xi = \frac{\Delta E}{\hbar} \tau \sim \frac{3\hbar^2}{\hbar \mathcal{I}} \frac{b}{v} = \frac{3\hbar b}{v} \quad (\text{adiabaticity parameter})$$

$$\frac{q}{n} \ll 1$$

$$q\xi \ll 1$$

$$\Delta \chi \sim 0$$

$\chi \sim \theta_0$ throughout the collision.

$$\frac{\partial H}{\partial r} = -\dot{p}_r = -p_\chi^2 \frac{1}{m_0 r^3} - \frac{Z_a Z_A e^2}{r^2} - \frac{3Z_a e}{4 r^4} Q_0 (3\cos^2\chi - 1)$$

$$\frac{\partial H}{\partial p_r} = \dot{r} = \frac{p_r}{m_0}$$

$$m_0 \ddot{r} = \overbrace{p_\chi^2 \frac{1}{m_0 r^3}}^A + \overbrace{\frac{Z_a Z_A e^2}{r^2}}^B + \overbrace{\frac{3Z_a e}{4 r^4} Q_0 (3\cos^2\chi - 1)}^C$$

$$\frac{A}{B} \sim \left(\frac{q}{\hbar}\right)^2 \quad \frac{C}{B} \sim \frac{q}{\hbar}$$

Neglect A

$$m_0 \frac{d}{dt} \dot{r}^2 = \frac{2Z_a Z_A e^2}{r^2} \dot{r} + \frac{3}{2} \frac{Z_a e}{r^4} Q_0 (3\cos^2\chi - 1)$$

$$dt = \frac{dr}{\sqrt{\frac{2E}{m_0} - \frac{2Z_a Z_A e^2}{m_0 r} - \frac{1}{2m_0} \frac{Z_a e}{r^3} Q_0 (3\cos^2\chi - 1)}}$$

$$\Delta L = p_\chi(\infty) = \frac{3Z_a e Q_0}{4} \int_{-\infty}^{\infty} \frac{dt}{r^3} \sin 2\chi$$

$$\sim \frac{3Z_a e Q_0}{4} \sin 2\theta_0 \int_{-\infty}^{\infty} \frac{dt}{r^3}$$

$$\underbrace{\frac{8}{3} \frac{1}{b^2 v}}$$

$$\Delta L \sim 2\hbar q \sin \theta_0$$

$$(\Delta L)_{\max} = \hbar 2q$$

Solutions

$$f = \sin 2\theta_0$$

$$f = \frac{\Delta L}{(\Delta L)_{\max}}$$

$$\boxed{f < 1} \quad (\text{classically allowed region})$$

$$\theta_0^{(1)} = \frac{1}{2} \arcsin f$$

$$\theta_0^{(2)} = \frac{\pi}{2} - \frac{1}{2} \arcsin f$$

$$\boxed{f > 1} \quad (\text{classically forbidden region})$$

Thus θ_0 complex

$$\theta_0 = \frac{\pi}{4} \pm \frac{i}{2} \operatorname{arcosh} f$$

$$\sin 2\theta_0 = \sin \left(\frac{\pi}{2} \pm i \operatorname{arcosh} f \right)$$

$$= \cos(i \operatorname{arcosh} f) = \cosh(\operatorname{arcosh} f) = f$$

Extreme Classical Model

$$\begin{cases} \theta_0^{(1)} = \frac{1}{2} \arcsin f \\ \theta_0^{(2)} = \frac{\pi}{2} - \frac{1}{2} \arcsin f \end{cases}$$

$$P_1(\Delta L) = \frac{1}{\pi/2} \Delta \theta_0^{(1)} = \frac{1}{\pi/2} 2 \left(\frac{\partial \theta_0^{(1)}}{\partial \Delta L} \right) = \frac{4}{\pi} \frac{1}{2} \frac{1}{\sqrt{1-f^2}} \frac{1}{(\Delta L)_{\max}}$$

$$|P_1| = \frac{2}{\pi} \frac{1}{\sqrt{1-f^2}} \frac{1}{(\Delta L)_{\max}}$$

In the same way

$$|P_2| = \frac{2}{\pi} \frac{1}{\sqrt{1-f^2}} \frac{1}{(\Delta L)_{\max}}$$

$$P(\theta_0) = \begin{cases} \left(\frac{\pi}{2}\right)^{-1} & (2D) \\ \sin \theta_0 & (3D) \end{cases} \quad (0 < \theta_0 < \frac{\pi}{2})$$

(prob. for isotropic distr. in space)

$$\int_0^{\pi/2} \frac{1}{(\pi/2)} d\theta = 1 \quad ; \quad \int_0^{\pi/2} \sin \theta d\theta = 1.$$

$$|P_1| + |P_2| = \frac{\sin(\frac{1}{2} \arcsin f) + \cos(\frac{1}{2} \arcsin f)}{(\Delta L)_{\max} \sqrt{1-f^2}} = \frac{\sqrt{1+f}}{(\Delta L)_{\max} \sqrt{1-f^2}}$$

$$P(f) = P(\Delta L) = \frac{1}{(\Delta L)_{\max}} \frac{1}{\sqrt{1-f}}$$

$$f = \frac{\Delta L}{(\Delta L)_{\max}}$$

Semiclassical approximation

$$f < 1$$

$$P(\Delta L) = \left| \sqrt{|P_1|} e^{i\phi_1} + \sqrt{|P_2|} e^{i\phi_2} \right|^2$$

$$\phi_2 = \pm \frac{\pi}{4} - \frac{1}{\hbar} \int_{-\infty}^{\infty} (\chi \dot{p}_\chi + r \dot{p}_r) dt$$

$$\frac{1}{\hbar} \int_{-\infty}^{\infty} \chi \dot{p}_\chi dt \sim 2q\theta_0 \sin 2\theta_0 = 2q\theta_0 f$$

$$\frac{1}{\hbar} \int_{-\infty}^{\infty} r \dot{p}_r dt \sim B + 2q \cos^2 2\theta_0$$

$$\phi_2 = \pm \frac{\pi}{4} - 2q\theta_0 f - 2q \cos^2 2\theta_0 + \text{const}$$

$$P(\Delta L) = \underbrace{|P_1| + |P_2|}_{\text{class. result}} + \underbrace{2\sqrt{|P_1||P_2|} \cos(\phi_1 - \phi_2)}_{\text{interf.}}$$

$$P(\Delta L) = \frac{1}{(\Delta L)_{\max}} \left(\frac{1}{\sqrt{1-f}} + \sqrt{\frac{2f}{1-f^2}} \cos(\phi_1 - \phi_2) \right)$$

$$\phi_1 - \phi_2 = \frac{\pi}{2} + 2q(f \arccos f - \sqrt{1-f^2})$$

$$f > 1$$

$$P(\Delta L) = \left| \sqrt{|P_1|} e^{i\phi} \right|^2 = |P_1| e^{i(\phi - \phi^*)}$$

ϕ complex

$$P(\Delta L) = |P_1| e^{-2 m\phi} \quad \text{choose } m\phi > 0$$

$$\phi = \pm \frac{\pi}{4} - 2qf\theta_0 - 2q \cos^2 \theta_0$$

$$\theta_0 = \frac{\pi}{4} + \left(\pm \frac{i}{2} \operatorname{arcosh} f \right)$$

$$P(\Delta L) = \frac{\sqrt{f}}{(\Delta L)_{\max} \sqrt{2(f^2-1)}} e^{-2 m\phi}$$

$$-2 m\phi = (\Delta L)_{\max} \left[\sqrt{f^2-1} - f \ln \left\{ f + (f^2-1)^{1/2} \right\} \right]$$

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