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GENERATION OF INTENSE COHERENT SOFT X-RAY
WITH ELECTRON MICROBUNCHES INDUCED AND FROZEN BY LASERS

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ABSTRACT

We describe a new improved version of Transverse Optical Klystron Harmonic Generator that uses three lasers to replace the undulators in the modulator and radiator and freeze the electron microbunching. We show that intense soft X-rays can be generated.

INTRODUCTION

The transverse optical klystron (TOK) harmonic generator has been studied by many authors¹⁻³. In a TOK harmonic generator the electrons are made to pass through the radiator undulator when they reach the maximum microbunching to generate coherent radiation constructively from different microbunches. After the maximum microbunching the faster electrons leave the slower electrons behind, thus the bunches disappear. The faster electrons will catch up to the slower electrons of the bunches in front and form bunches again.

During half of those bunching-debunching periods the radiation is out of phase from the other half of periods, as we will show later, thus contributing to the radiation only destructively. The output harmonics power is proportional to the square of length of the coherent radiation section. To increase this length one must try to eliminate the contribution during these half periods in some way, for example, by deviating the electron beam away from the axis. This would be complicated. A more difficult problem, however, is due to the energy spread of the electron beam. In a practical situation, the energy spread will limit the coherent collective radiation time interval to a few bunching periods, because, as we will show later, the bunching peak density decays exponentially as the beam moves forward. Undulators with transverse gradient field have been proposed to keep the bunches from dissolving⁴, but at the expense of an increased transverse beam size, that would reduce the amount of radiated power. In this paper we suggest the use of an external powerful laser to form deep buckets, and, before we send the e-beam into these buckets, to prepare a microbunched beam first in a "modulator" as in the TOK, which is similar to the radiator except for the lower laser intensity and a much shorter length. We will show that with proper choice of the length of the modulator and the laser intensity of both the modulator and the radiator, we can send the bunches

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into the center of the potential well, and keep them frozen if the timing and phase of the lasers in both sections are chosen properly. So we can expect to obtain higher harmonics and higher intensity.

In an attempt to improve the TOK harmonic generator, the idea of replacing the undulator by a laser beam in a low energy e-beam FEL⁶⁻⁷ is very attractive. The large number of periods and the requirement of a low energy e-beam, together with the frozen microbunches, seem to provide the scheme for a relatively small coherent soft X-ray source.

Let us describe the new method by giving an example as shown in fig.1. In the modulator a KrF laser ($\lambda_b = 0.248\mu\text{m}$) and a CO₂ laser ($\lambda_w = 10.6\mu\text{m}$) interact with an electron beam ($\gamma = 3.34$).

Fig. 1.

We have chosen the electron energy so that in the electron rest frame, the two lasers have the same wavelength and are forming a standing wave. One can easily show that the electrons tend to move to the small amplitude regions in this standing wave, which corresponds to the bottom of a sinusoidal potential well of the "ponderomotive" force. The electrons move like a pendulum in this potential well. In the beginning they are uniformly distributed (fig 2.a), but most of them move to the bottom almost simultaneously after 1/4 period of the "pendulum", and reach the maximum bunching (fig 2.b). We arrange the device so that we bend the e-beam into the radiator exactly at this time. Here the electrons fall into the potential well of another pair of lasers of

Fig. 2a,b

the same wavelength λ_b, λ_w (actually the new laser beams come from the same lasers to assure the phase relation of the bunching). If the second pair of laser beams is more intense, i.e., the potential well is deeper, and if we adjust the phase of the laser beams relative to the lasers in the modulator, i.e. we move

the relative position of the bottom of the potential well, so that the bunches just formed in the modulator fall into the bottom of the potential well, then the bunches would not disappear any more, they are "frozen".

Because the bunches are separated from each other by about λ_b , if there is a harmonic component with the wavelength λ_{bh} in the radiation from a individual electron, and $\lambda_b = n\lambda_{bh}$, where n is an integer, then the radiation from different bunches adds constructively. The harmonic radiation in an undulator is proportional¹⁰ to $[J_{(n-1)/2(K)} - J_{(n+1)/2(K)}]^2$. For an undulator, the K value can be of order of one. However, if we replace the undulator by a laser beam of even very high intensity, the K value is still much smaller than one, so the Bessel function factor is very small when $n \neq 1$. So even though the microbunches are frozen, the harmonics radiation is still weak.

The way to overcome this difficulty is to add a third laser WH (Nd glass) with $\lambda_w = n\lambda_{wh}$ ($\lambda_{wh} = 1.06 \mu\text{m}$), thus artificially introducing a n 'th harmonic transverse oscillation in addition to the fundamental frequency oscillation induced by the CO₂ laser. In this way the harmonic radiation will increase dramatically, not limited by the Bessel function factor anymore.*

In the following we give a more quantitative discussion. In section 2 we derive the equations of motion of the electron in the four light beams; the result is the traditional pendulum equations. Using these equations, in section 3, we discuss the phase space evolution both in the modulator and the radiator, we derive a formula to calculate the harmonic expansion of the modulated density of the microbunched e-beam, and in particular the expansion coefficient that determines the radiated harmonic power. Then we describe the calculated bunching coefficient - laser intensity relation, which shows a maximum bunching when we vary the laser intensity in the modulator. In section 4 we discuss how the bunching stability is related to the laser bandwidth, and then we derive a formula for the radiation power, which turns out to be related to the second order correlation function of the lasers. In section 5 we discuss the influence of the transverse velocity of the electrons and of the e-beam emittance. In section 6 ,

*The effect of the third laser can be viewed in another way: the laser light is backscattered from the e-beam with frozen microbunches, just as from a moving grating². If in the e-beam rest frame the laser wavelength and the space between bunches satisfy the Bragg condition, then there is strong reflection, and in the laboratory frame the backscattered light with wavelength λ_{bh} is Doppler shifted, and a very intense harmonic radiation is generated. It is easy to show that $\lambda_{bh} = \lambda_b/n = 248 \text{ \AA}$. Essentially, this can be viewed as a coherent Compton backscattering from the frozen microbunches.

we give a numerical example. We conclude with a discussion of the scaling relation, some potential future improvements and problems to be solved.

THE EQUATIONS OF MOTION

We have four light beams in the radiator, labeled by W, WH, B, BH, as shown in fig. 1. For simplicity, we refer to ref.5, assuming the same approximation conditions, neglect space charge force, radiation reaction force, and follow the classical approximation. The differences are: we have a low energy electron beam, and we replace the undulators by lasers. The derivation is not essentially changed.

We use the same notation, except with labels for different light beams. For example, we have for the electric field of laser beam W:

$$\vec{E}_W = E_{W0} (\hat{x} \cos \phi_W + \hat{y} \sin \phi_W) , \quad (2.1)$$

$$\phi_W = -\frac{2\pi}{\lambda_W} z - \omega_W t + \phi_{W0} . \quad (2.2)$$

$$\phi_b = \frac{2\pi}{\lambda_b} z - \omega_b t + \phi_{b0} , \quad (2.3)$$

With (2.1), we can obtain the transverse electron velocity:

$$\vec{\beta} = \sum_i \frac{K_i}{\gamma} (-\hat{x} \sin \phi_i + \hat{y} \cos \phi_i) \quad (2.4)$$

Here the index goes through the 4 light beams,

$$K_i = e\lambda_i E_{i0} / (2\pi m_0 c^2) . \quad (2.5)$$

This and the expressions like (2.1) give $\vec{E} \cdot \vec{\beta}$, the energy exchange:

$$\vec{E} \cdot \vec{\beta} = \sum_{i,j} \frac{K_i E_{j0} - K_j E_{i0}}{2\gamma} \sin(\phi_j - \phi_i) \quad (2.6)$$

We choose the e-beam energy to resonate only with the laser pair W and B, so among those phase differences between light beams we retain only $\phi_b - \phi_W$ that changes slowly, while the others change very fast and do not contribute to the energy exchange. We also assume all the K values of the laser beams are small so $\vec{\beta}$ is very small too. This holds, for even for a 200GW/mm² CO₂ laser it is only $K_W = 0.028$.

With this approximation, and neglect the term $K_b E_{w0}$ ($\lambda_b \ll \lambda_w$), we obtain the same pendulum equation just as in a FEL:

$$\ddot{\phi} = - \frac{e K_w E_{b0}}{m_0 c \gamma^2} \sin \phi \quad (2.7)$$

$$\dot{\phi} = 4 \omega_w \varepsilon \quad (2.8)$$

Here, ϕ is the phase difference between laser W and B:

$$\phi = \phi_b - \phi_w + \pi = 2\pi \left[1/\lambda_b + 1/\lambda_w \right] z - (\omega_b - \omega_w)t + \phi_{b0} - \phi_{w0} + \pi, \quad (2.9)$$

γ_r is the resonant energy: $\gamma_r^2 \approx \lambda_w / 4\lambda_b$; ε is the electron energy, $\varepsilon = (\gamma - \gamma_r) / \gamma_r$. In deriving (2.8), we use the approximation that $\beta_{\parallel}$ is a constant:

$$\beta_{\parallel}^2 = 1 - 1/\gamma^2 - \sum_{ij} \frac{K_i K_j}{\gamma^2} \cos(\phi_i - \phi_j) \approx 1 - 1/\gamma^2 - \sum_i K_i^2 / \gamma^2 \quad (2.10)$$

We cut those terms with $\cos(\phi_i - \phi_j)$ because they are either very small or oscillate extremely fast, so their contribution to $\beta_{\parallel}$ is negligible. A more precise value of γ_r is given by (2.10) and:

$$\lambda_b / \lambda_w = (1 - \beta_{\parallel}) / (1 + \beta_{\parallel}). \quad (2.11)$$

Using this, we get $\gamma_r = 3.34$.

Equations (2.7) (2.8) show that in our approximation, the evolution is determined only by lasers W and B, both in the modulator and the radiator. Let us now calculate the density modulation Fourier coefficients.

DENSITY MODULATION CALCULATION

(2.7-8) are a set of Hamiltonian equations, with Hamiltonian:

$$H = 2\omega_w \varepsilon^2 + \frac{2e K_w E_{b0}}{m_0 c \gamma_r^2} \sin^2 \frac{\phi}{2} \quad (3.1)$$

In phase space, electrons move along curves with constant H, the separatrix (bucket height) has a maximum $\varepsilon = 2\sigma_{\varepsilon}\mu$, where:

$$\mu = e \sqrt{\lambda_b \lambda_w} \sqrt{E_{b0} E_{w0}} / (2\pi m_0 c^2 \sigma_{\varepsilon}) \quad (3.2)$$

$2\sigma_{\varepsilon}$ is the energy spread of a Gaussian distribution.

We can see the physical meaning of μ : μ is the largest energy in unit of $2\sigma_\epsilon$ for an electron to be confined within the separatrix. When H is small, the electron moves around a circle in the ϵ, ϕ space, with period $T = \lambda_w / (4\mu\sigma_\epsilon c)$, so T is the bunching-debunching period.

Given the initial phase ϕ_0 and the energy ϵ_0 , we can solve (2.7) (2.8), and express ϕ and ϵ by Jacobian elliptic functions. For a initial distribution which is Gaussian in energy and uniform in phase, as shown in fig. 3.a, we calculate the distribution for $t = T/4, T/2, 4T$, respectively, and plot these data in phase space (fig. 3.b-d). Here we use σ_ϵ as energy unit. When $t = T/4$ the bunch is narrow (all close to $\phi = 0$), but the bunch is wider at $t = T/2$, mainly because those electrons that receive large energy modulation now rotate around a large circle, and reach a larger ϕ . Since an electron with a larger amplitude has a longer period, the distribution of the rotation angle gradually becomes uniform over all directions in the phase space, the oscillation of the bunch size becomes small, it is almost stable at $t = 4T$, but bunching is not dense.

Fig. 3a,b,c,d

However, if we use a two steps bunching, the situation could be different: We start from a smaller μ in the modulator, say $\mu = \mu_1$: We have a correspondingly lower separatrix as shown in fig 4.a. When we reach the maximum bunching at the end of the modulator (which is the beginning of the radiator), let us suddenly increase μ to μ_2 (higher separatrix), but keep the phase the same. Then, since most electrons are already close to the center, only a few electrons receive a large energy modulation when $t = T/4$ after the second step (the radiator). Figs. 4.b-d show the evolution in the radiator. It appears that the bunching is now much more dense at equilibrium.

To study the evolution of the charge density, we observe that ϕ can be expressed as a function of ϕ_0, ϵ_0 , and t : $\phi = \phi(\phi_0, \epsilon_0, t)$. From (2.7) and (2.8) we can see that if the initial value

Fig. 4a,b,c,d

ϕ_0 is increased by 2π . ϕ will increase by 2π at time t . So, if we consider z as a function of ϕ at fixed time t , and express the charge density functions $\rho(z)$ as a function of ϕ , then $\rho(z(\phi))$ is a function with period 2π . We take Fourier expansion:

$$\rho = \rho_0 \left(1 + \sum_{n=1}^{\infty} A_n \cos n\phi \right) \quad (3.3)$$

The Fourier coefficient A_n can be shown to be:

$$A_n = \frac{1}{\pi \sqrt{2\pi\sigma_\epsilon}} \int_{-\infty}^{\infty} e^{-\frac{\epsilon^2}{2\sigma_\epsilon^2}} d\epsilon_0 \int_{-\pi}^{\pi} \cos n\phi d\phi_0 \quad (3.4)$$

We notice that the formula of TOK without frozen bunches can be derived with the help of (3.4). Suppose that at the end of the TOK modulator the electron energy is modulated, but the phase is still unchanged. then we have:

$$\phi = \phi_0 \quad (3.5)$$

$$\epsilon = -\epsilon_1 \sin \phi_0 + \epsilon_0 \quad (3.6)$$

Now suppose that the e-beam goes into a free drift space, then (2.7) and (2.8) become:

$$\dot{\epsilon} = 0 \quad (3.7)$$

$$\dot{\phi} = 4\omega_w \epsilon \quad (3.8)$$

Hence, at time t :

$$\phi = 4\omega_w (-\epsilon_1 \sin \phi_0 + \epsilon_0) + \phi_0 \quad (3.9)$$

Substitute this into (3.4), we obtain:

$$A_n = 2e^{-\frac{1}{2} n^2 (4\omega_w \sigma_\epsilon t)^2} J_n(n \cdot 4\omega_w \epsilon_1 t) \quad (3.10)$$

This is a generalization of the well known klystron formulae.^{1,3,8} The Bessel function shows a bunching-debunching period of $2\pi/(4\omega_w \epsilon_1 n)$ in the asymptotic region, with alternating positive and negative intervals. The bunching intervals with negative A_n are out of phase from the others. The additional exponential factor is due to the energy spread, it shows that after $m = \epsilon_1/(2\pi\sigma_\epsilon)$ bunching-debunching periods, A_n becomes vanishingly small, as we mentioned in the introduction.

We can show that for a fixed $\tilde{t}=t/T$ in the radiator (e.g., $t=4T$, or, 4 bunching-debunching periods) A_n depends on only the value μ_1 and μ_2 , as previously defined, and not on σ_ϵ .

A_n was calculated numerically and the results are plotted in Figs. 5-9. A_4 vs. t is shown in fig. 5-6 to show how the oscillation gradually damped, and how for the same $\mu_2=10$, the

Fig. 5. $\mu_1 = 0.8$, $\mu_2 = 10$

Fig. 6. $\mu_1 = 2.9$, $\mu_2 = 10$

asymptotic A_4 is much larger at $\mu_2=2.9$ than at 0.1. Figure 7 shows A_4 vs. μ_1 at $t=\infty$, for a fixed μ_2 values; A_4 exhibits maximum at $\mu_1=2.9$ for a fixed $\mu_2=10$. This is easy to understand: when

Fig. 7.

$\mu_1=0$ or when $\mu_1=10$, the result should be the same as if there were no modulator, and A_4 should be smaller, so there must be a maximum in between. For various μ_2 we have calculated the maximum A_4 , and the corresponding optimum μ_1 , as shown in Fig. 8. Figure 9 shows another plot for the 10'th harmonic. Figures 8 and 9 are useful in determining the parameters of the device and to calculate the power output.

Fig. 8.

Fig. 9.

BUNCHING STABILITY AND RADIATION POWER RELATED TO THE COHERENCE OF THE LASERS

Until now we assumed that the lasers are completely coherent, i.e. the phase term ϕ_{b0} and ϕ_{w0} in (2.2-3) is constant. In real cases, the phase is changing gradually, so the potential well we mentioned in the introduction is moving randomly. The changing phase introduces another term into the equation of motion (2.8) which becomes $\ddot{\phi} = 4\omega_w \epsilon + \dot{\phi}_{b0} - \dot{\phi}_{w0}$. If $\dot{\phi}_{b0} - \dot{\phi}_{w0} \ll 4\omega_w \epsilon$, we can neglect it; in this case the bunching is stable. Physically, this means that when the velocity of the center of the potential well is much smaller than the electron phase oscillation velocity, the electron phase oscillation will adiabatically follow the potential

well. We want to find the relation between this inequality and the laser bandwidth. Because there is no dispersion in the vacuum, the phase term is of the form: $\phi_{bo}(t-z/c) - \phi_{wo}(t+z/c)$. Hence,

$$\dot{\phi}_{bo} = \frac{d}{dt} \phi_{bo}(t(1-\beta_{||})) = (1-\beta_{||}) \frac{\partial}{\partial t} \phi_{bo}(t) \approx (1-\beta_{||}) \Delta\omega_b \quad (4.1)$$

$$\dot{\phi}_{wo} = \frac{d}{dt} \phi_{wo}(t(1+\beta_{||})) = (1+\beta_{||}) \frac{\partial}{\partial t} \phi_{wo}(t) \approx (1+\beta_{||}) \Delta\omega_w \quad (4.2)$$

We can see that $(\dot{\phi}_{bo} - \dot{\phi}_{wo})/4\omega_w$ corresponds to an energy spread. The equivalent energy spread be expressed as $\sqrt{\sigma^2 + [(\dot{\phi}_{bo} - \dot{\phi}_{wo})/4\omega_w]^2}$. If the phase term is smaller than $\sigma_\epsilon/2$, and its influence is negligible, using (4.1), (4.2), and (2.11) we obtain the condition for stable bunching:

$$\frac{\Delta\omega_b}{\omega_b} < \sigma_\epsilon, \quad \frac{\Delta\omega_w}{\omega_w} < \sigma_\epsilon \quad (4.3)$$

from this equation it appears that a requirement is that the CO₂ and KrF laser bandwidth is smaller than σ_ϵ .

With the condition of frozen bunches satisfied, we can now calculate the harmonic radiation power. The radiation distribution for a group of N_e electrons is⁹:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{Z_0 e^2 \omega^2}{16\pi^3} \left| \sum_{j=1}^{N_e} \int \hat{n} \times (\hat{n} \times \bar{\beta}_j(t)) e^{i\omega(t - \hat{n} \cdot \bar{r}_j(t)/c)} dt \right|^2 \quad (4.4)$$

$\bar{\beta}_j(t)$ $\bar{r}_j(t)$ are the velocity and position of the electron j . $\hat{n}$ is the direction of the solid angle Ω . Z_0 is the vacuum impedance. If we take the approximation that the trajectory of electrons are different from each other only by their initial position, and suppose the electron distribution at $t=0$ is:

$$\rho(x, y, z) = N_e \rho_{ox}(x) \rho_{oy}(y) \rho_{oz}(z) \left(1 + \sum_{n=1}^{\infty} A_n \cos n\phi\right), \quad (4.5)$$

with $\rho_{ax}(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{x^2}{2\sigma_x^2}\right]$, and similar expressions for

ρ_{ox} and ρ_{oy} . then it can be shown, after a long but straight forward derivation, (for simplicity, we assume $\sigma_x = \sigma_y$ here) that the total radiation can be expressed by the radiation of a single electron (same as an undulator radiation) times a "form factor" $|F(\omega, \hat{n})|^2$, which describes the collective effects:

$$\frac{d^2 I}{d\omega d\Omega} = N_e^2 |F(\omega, \hat{n})|^2 \frac{d^2 I_0}{d\omega d\Omega} \quad , \quad (4.6)$$

$$|F(\omega, \hat{n})|^2 = I_n(\omega) |U_n(n_x, n_y)|^2 \quad , \quad (4.7)$$

$$U_n(n_x, n_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_{ox}(x) \rho_{oy}(y) A_n e^{-i \frac{\omega}{c} (n_x x + n_y y)} dy \quad (4.8)$$

$$I_n(\omega) = \left| \int_{-\infty}^{\infty} \rho_{oz}(z) \cos\left(\frac{2\pi}{\lambda_b} z + \phi_0\right) \exp(-i \frac{\omega}{c} z) dz \right|^2 \quad (4.9)$$

Here $\phi_0(z) = \phi_{b0} - \phi_{w0}$ is the phase in (2.9). If ϕ_0 and ρ_{0z} were constants, then the product of two sinusoidal functions in the integrand of I_n gives a δ function spectrum around $\omega = n\omega_b$. However, ρ_{0z} has a range $2\sigma_z$, the bunch length, and ϕ_0 has a constant range of about the laser coherence length. ϕ_0 is a random phase in a stochastic process, we can use the technique of the ensemble average over all possible functions of ϕ_0 . If we assume the laser coherence can be described by a "second order correlation function"¹¹⁻¹² (which is a little different from the traditional definition):

$$\gamma_{12}^{(n)}(z-z') \equiv \langle \cos(n\phi_0(z) - n\phi_0(z')) \rangle \approx \exp\left(-\frac{(z-z')^2}{2\sigma_{bn}^2}\right) \quad (4.10)$$

and assume:

$$\sigma_z \gg \sigma_{bn} \quad (4.11)$$

we obtain the ensemble average of I_n :

$$\langle I_n \rangle = \sigma_{bn} \exp\left[-(k - nk_b)^2 \sigma_{bn}^2 / 2\right] / (4\sqrt{2}\sigma_z) \quad ,$$

here $k = \omega/c$. σ_{bn} is to be studied. If we assume that the laser W is completely coherent, then $\phi_0(z)$ is determined by $\phi_{b0}(z)$

(1)

only, and $\gamma_{12}(z-z')$ is just the second order correlation function, and $2\sigma_b$ is the coherence length. We can estimate σ_{bn} to be between σ_b and σ_b/n , it seems to depend on the higher order coherence properties of the laser.

$|F|^2$ and $d^2 I_0/d\omega d\Omega$ in the forward direction are shown in fig. 10 qualitatively. $d^2 I_0/d\omega d\Omega$ consists of two peaks, the one on the right side is contributed by the electron oscillation induced by the laser beam WH, the peak on the left is contributed by the laser W. So, the effect of the third laser WH is obvious: without it the peak on the right would be very small, hence the harmonic radiation $d^2 I/d\omega d\Omega$ as a product of $|F|^2$ and $d^2 I_0/d\omega d\Omega$ is also very small. For simplicity, we discuss one special case only,

assuming that the peak of $|F|^2$ is much narrower than that of $d^2I_0/d\omega d\Omega$. The same assumption is also applied to the angular distribution.

Fig. 10.

I_n provides a spectral width of c/σ_{bn} in (4.6), the width determined by the coherent collective radiation. $dI_0^2/d\omega d\Omega$ also provides a width of $1/2N_{wh}$, characteristic of the undulator radiation¹⁰ ($N_{wh}=2L/\lambda_{wh}$ is the number of oscillations of an electron in the radiator of length L). We assume the former is much narrower, so we can take $dI_0/d\omega d\Omega$ to be constant within the spectral range. An examination of the integrand in (4.8) also indicates that U_n has a peak around $n_x=n_y=0$, $n_z=1$, with an angular divergence of $\lambda_{bh}/2\sigma_x$, the diffraction limit. The factor $d^2I_0/d\omega d\Omega$ also has an angular divergence of $1/(\gamma N_{wh})$, which is characteristic of the undulator radiation. We assume the former is much narrower so the angular distribution is determined by U_n , and $dI_0/d\omega d\Omega$ is almost a constant within this range. After some lengthy but straight forward derivations, we have:

$$\frac{dP_{bh}}{dS} = Z_0 \left[\frac{jA_n K_{wh} L}{4\gamma} \right]^2 \quad (4.12)$$

where dP_{bh}/dS is the power density; j , the current density. In deriving (4.12), we used Parseval's theorem to calculate $\int |U_n(n_x, n_y)|^2 d\Omega = \int |U_n(n_x, n_y)|^2 dn_x dn_y$. And we also assumed that the laser WH is coherent over the whole length L , so the natural width of $d^2I_0/d\omega d\Omega$ is given by $1/2N_{wh}$. We also made the approximation that A_n is a constant within the e-beam.

The spectral width requirement, as we mentioned in deriving (4.12), can be written as:

$$2\sigma_{bn}/[(1-\beta_{||})L] \gg 2/\pi \quad (4.13)$$

If $2\sigma_{bn}/[(1-\beta_{||})L] \ll 2/\pi$, i.e. the natural line width of $dI_0^2/d\omega d\Omega$ is narrower, with the other conditions the same, we can obtain:

$$\frac{dP_{bh}}{dS} = Z_0 \sqrt{\frac{\pi}{2}} \frac{2\sigma_{bn}}{(1-\beta_{||})L} \left[\frac{jA_n K_{wh} L}{4\gamma} \right]^2 \quad (4.14)$$

(4.13) shows that if L is shorter than $2\sigma_{bn}/(1-\beta_{||})$, the radiation build up coherently over the whole length L , and so, is proportional to L^2 , as in (4.12). Otherwise, it is proportional to L , as in (4.14).

(4.12) also shows that even K_{wh} is smaller than 1, the output power still can be comparable with the traditional TOK with real undulators, because γ is also smaller by a large factor.

Electron beam requirement

If the velocity of an electron deviates from the axis by an angle θ , then the axial velocity component $\beta_{||}$ will be decreased by $-\theta^2/2$, it can be shown that this corresponds to a smaller energy. This deviation from resonance can be taken as an equivalent energy spread. It can be shown that if $\gamma^2\theta^2=2\sigma_E$, the influence of the angular spread to the energy spread would be negligible, and this limit can be taken as the restriction for the angular divergence. Given the emittance of the e-beam it will determine the transverse beam size.

According to Elias^{6,13}, an electrostatic accelerator with continuous beam of 25 ampere, 9.38 Mev, energy spread $6 \cdot 10^{-5}$, and emittance $\pi 0.8 \text{ mm} \cdot \text{mrad}$ can be realized using present electron beam recovery techniques. In the following example, we assume the same parameters except at a lower energy, 1.69 Mev (including the electron rest energy), i.e. $\gamma=3.34$. Now, from the energy spread $6 \cdot 10^{-5}$ and $\gamma=3.34$ the divergence angle we can tolerate is given by:
 $\theta = \sqrt{2\sigma_E}/\gamma = \sqrt{6 \cdot 10^{-5}}/3.34 = 2.35 \text{ mrad}$. Hence, the emittance $\pi \sigma_x \theta = \pi 0.8 \text{ mm} \cdot \text{mrad}$ gives σ_x as the e-beam radius: $\sigma_x = 0.34 \text{ mm}$.

A Numerical Example

Consider the following case: the radiation of the lasers W (CO_2) and WH (Nd glass) is coupled to the EH_{11} mode of a circular waveguide with a diameter $2a=3.6 \text{ mm}$; the laser W power is $P_W=200 \text{ GW}$; $P_{wh}=5 \text{ GW}$; the laser beam B is Gaussian, and is focused to a $2w_b=0.96 \text{ mm}$ beam waist size with a power $P_b=2 \text{ GW}$.

The electric field intensity $E_{w0}(z)$ of the laser W on the axis is related to P_W by:

$$P_W = J_1^2(u) \pi a^2 [E_{w0}(0)]^2 / Z_0, \quad (6.1)$$

here $u=2.405$ is the first root of Bessel function J_0 , $[J_1(u)]^2=0.270$. (6.1) is different from the formula (19) in the ref.7 by a factor of 2, this is because here we have a circularly polarised light instead of a linear one. From (6.1), we obtain

$E_{w0}(0)=5.19 \cdot 10^9 \text{ V/m}$. The Rayleigh length of the laser B is $2z_b=2\pi w_b^2/\lambda_b=5.84 \text{ m}$. One can show that the laser beam power is related to the power density by¹⁵:

$$\frac{dP_b}{dS} = \frac{[E_{b0}(0)]^2}{Z_0} = \frac{2P_b}{\pi w_b^2(1+z^2/z_b^2)} \exp\left(-\frac{r^2}{w_b^2(1+z^2/z_b^2)}\right) \quad (6.2)$$

If the e-beam is co-linear with the laser beam, with a transverse size $2\sigma_x=0.68 \text{ mm}$, and the radiator length is $L=2 \text{ m}$, then we can obtain the smallest $E_{b0}(r)$ value within a cylinder of 2 m long and radius $\sigma_x=0.34 \text{ mm}$, which is co-linear and centered with the Gaussian beam waist, by setting $z=L/2=1 \text{ m}$ and $r=\sigma_x=0.34 \text{ mm}$ in (6.2). This value of E_{b0} can be used as an estimation of the electric field intensity of the laser B in the e-beam. We found $E_{b0} > 8.03 \cdot 10^8 \text{ V/m}$ within the cylinder.

Using the value of E_{w0} and E_{b0} obtained, we can use (3.2) to obtain the half "potential well" height $\mu_2=35.4$. Using the μ_2 - A_{10} relation in fig.7, we can obtain $A_{10}=0.20$ and $\mu_1=4.9$.

As pointed out by C.M.Tang,¹⁴ the light intensity variation within the laser beam gives a variation of K values, which in turn changes the $\beta_{||}$, as we can see from (2.10). This may influence the resonance condition (2.11). The variation of $\beta_{||}$ is equivalent to an energy spread as we discussed before. If the waveguide diameter is large enough compared with the e-beam size, this variation will be small enough. It is easy to show that when $2a=3.6 \text{ mm}$ and $P_w=200 \text{ GW}$, the equivalent energy spread generated by the transverse light intensity distribution is much smaller than the energy spread σ_E , so we can neglect it.

Now (4.12) can be used to calculate the output radiation power. Notice that there is a factor K_{wh}^2 in the power density, which is proportional to P_{wh} through the factor E_{wh0} in K_{wh} . Using (2.5) and a formula similar to (6.1):

$$P_{wh} = J_1^2(u) \pi a^2 [E_{wh0}(0)]^2 / Z_0. \quad (6.3)$$

we obtain $K_{wh} \approx 2.7 \cdot 10^{-4}$.

The Gaussian distribution of the current density j can be obtained from the total current 25 ampere and the transverse size $2\sigma_x$. If we substitute the A_{10} , K_{wh} , and j , etc., obtained above into (4.12), integrate it along the transverse cross section, we get the total radiation power to be about 11 W . If we define the conversion efficiency as $\eta = P_{bh}/P_b$ then $\eta = 0.6 \cdot 10^{-8}$.

We can obtain the bandwidth, coherence length and pulse length requirements. The CO_2 laser bandwidth is determined by the bunching stability condition (4.3), so $\Delta\omega/\omega = \sigma_E = 6 \cdot 10^{-5}$. The

coherence length of KrF laser is determined by (4.13). This gives $2\sigma_{bn} > 2(1-\beta) L / \pi = 9 \text{ cm}$. So (4.11) gives the electron bunch length $2\sigma_z > 2\sigma_{bn} = 9 \text{ cm}$, this means a pulse duration of about 300ps. If $\sigma_b = \sigma_{bn}$, then we need $\sigma_b > 9 \text{ cm}$. The requirement that the Nd glass laser is coherent over the whole length L requires the coherence length to be longer than $\sigma_{wh} = 2L = 4 \text{ m}$.

Summary

We have suggested the use of lasers to freeze electron microbunching in a TOK. We have also discussed the possibility to replace the undulators with laser radiation in a TOK. The method to freeze microbunching can also be applied to the ordinary TOK with undulators. We also suggested the use of a third laser to be sent onto the frozen microbunches, thus enhancing the harmonic radiation. The calculation shows that the suggested device is within present technologies.

The numerical example given in the last section can be scaled to other conditions. The result is given in Table 1. These scaling relations are given by (4.12).

TABLE 1 Numerical Examples

W	B	WH	BH	I	L	$2\sigma_c$	$\pi\sigma_x\theta$	$2\sigma_x$	η
CO ₂ 10.6 μm 12ns*	KrF 0.248 μm 0.3-12ns†	Nd glass 1.06 μm 12ns†	output 248 $\text{\AA}$ 0.3-12ns		0.3-12ns				
200GW	2GW	5GW	11W	25A	2	$6 \cdot 10^{-5}$	$\pi 0.8$	0.68	$0.6 \cdot 10^{-8}$
200GW	2GW	5TW	0.17MW	100A	2	$6 \cdot 10^{-5}$	$\pi 0.8$	0.68	$0.9 \cdot 10^{-4}$
20GW	20MW	5GW	11W	25A	2	$6 \cdot 10^{-6}$	$\pi 0.25$	0.68	$0.6 \cdot 10^{-6}$

* bandwidth $3 \cdot 10^{-5}$ † $2\sigma_{bn} > 9 \text{ cm}$ ‡ $2\sigma_{wh} > 4 \text{ m}$

The requirement of the power of the lasers W and B drops drastically as the energy spread and electron beam emittance drop. If the energy spread is reduced by a factor of 10 and the emittance reduced by 3, then the intensity of one of the lasers W or B can be reduced by a factor 10000 with the same output.

The example in line 2 shows an attractive possibility of an extremely high intensity soft X-ray source. This is because it can be shown that the power of laser WH can be scaled to a very high value, while the undulator field is restricted by the magnets available. The power level 0.17MW, and pulse duration 0.3-12ns is approaching the level of one shot x-ray holography as mentioned by J.C.Solem¹⁶. It seems to indicate the possibility of x-ray holography of a living cell with a space resolution of 250 $\text{\AA}$ and a time resolution of 300ps to 12ns to avoid the hydrodynamic image blurring. The parameters of lasers are within reach of the present technology. The current and energy spread is also near the range of modern electrostatic accelerators¹³. Since we can use pulsed current, we can consider the table 1 of ref.13, the

performance of an electrostatic accelerator using no beam recovery techniques is: 5 MeV, peak current 100 A, charging current 500 mA, time for voltage drop to be within 1/500 is 20 ns. The voltage drop is proportional to the pulse duration, so with shorter pulse length, for example 100 ps, the voltage drop can be much smaller for even a higher current. Whether the energy spread can be the same as in DC current case is not clear.

The example in line 3 shows another interesting possibility for a small X-ray source. When the energy spread and emittance is reduced the whole device can be in a much smaller scale than a storage ring. The example can be compared with the third harmonic generation in gaseous media¹⁷, here a 40 MW 7 ns ArF laser pulse is used to generate 30 W 643 Å radiation, the conversion efficiency is $0.7 \cdot 10^{-6}$. Again, the problem whether the energy spread can be reduced enough in the pulsed case using existing technology is not clear.

There are several problems to be studied further. When the radiator length is longer than the WH laser coherence length, the output power should be calculated, it seems to be proportional to the radiator length instead of the square of it in this case. The more quantitative description of bunching stability, transverse velocity influence, the distribution of the bunching coefficient etc., is to be studied. The details of the section between the modulator and the radiator should also be studied.

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