The Computational-Physics Program
of the National MFE Computer Center

Arthur A. Mirin

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I. INTRODUCTION

Since June 1974, the MFE Computer Center has been engaged in a significant computational physics effort. The principal objective of the Computational Physics Group is to develop advanced numerical models for the investigation of plasma phenomena and the simulation of present and future magnetic confinement devices. In addition, the group interacts with the systems programming staff to improve services; it fills the roles of internal critic and advisor by representing a user viewpoint.

The group currently consists of six Ph.D. physicist/mathematicians - D.V. Anderson, C.D. Kerbel, M.G. McCoy, A.A. Mirin, A.I. Shestakov and D.E. Shumaker. In addition, students in the Department of Applied Science (U.C. Davis) are engaged in fusion research under the guidance of senior group members.

The computational physics group is involved in several areas of fusion research. One main area is the application of multidimensional Fokker-Planck, transport and combined Fokker-Planck/transport codes to both toroidal and mirror devices. Another major area is the investigation of linear and nonlinear resistive magnetohydrodynamics in two and three dimensions, with applications to all types of fusion devices. The MHD work is often coupled with the task of numerically generating equilibria which model experimental devices. In addition to these computational physics studies, investigations of more efficient numerical algorithms are being carried out.

One of the principal objectives of the computational physics group is to provide support for experimental and theoretical work within the MFE community. At present, this support falls into the following areas:

- Stability of spheromak configurations
- Equilibrium and stability of FRTP
- Compact torus transport
- Fokker-Planck/transport analyses of advanced toroidal devices
- Fokker-Planck/transport analyses of DITE
- Fokker-Planck and transport analyses of tandem mirror systems
- RF current drive and electron heat transport in tokamaks

DISCLAIMER
A summary of our program follows.

II. COMPUTATIONAL STUDIES

A. MHD Equilibria and Stability

1. TIME INDEPENDENT MAGNETOHYDRODYNAMICS

In simple 1-D configurations the calculation of plasma equilibrium profiles is rather trivial. For more realistic two and three dimensional plasmas the determination of the equilibrium magnetic fields and the plasma pressure is a complicated non-linear problem involving the solution of elliptic partial differential equations. One of the goals of this work is to obtain configurations like those seen in experiments; accordingly some of our calculations use boundaries and external coil sets which accurately represent the devices under study. A secondary goal is to compute equilibria of sufficient accuracy so that the results can be used by stability codes with some confidence. In this connection high order finite element or finite difference representations have been used. These equilibria are also used to explore single particle effects in equilibrium magnetic fields.

Most of our codes solve Ampere's law given some non-linear dependence of the currents on the fields. Other codes use a dynamic evolutionary approach with artificial damping to obtain steady flow equilibria. Still others merely numerically evaluate known analytical equilibrium profiles. Several of these codes are briefly described in Table 1.

Our time-independent MHD publications are summarized in Appendix A1.

2. TIME DEPENDENT MAGNETOHYDRODYNAMICS

A principal technique for determining macroscopic plasma behavior is through the solution of the time dependent MHD equations. Of particular interest is the nonlinear evolution and saturation of fluid instabilities. Accurate simulation of such phenomena requires the solution of the full set of MHD equations, which comprises a coupled system of eight nonlinear partial differential equations. This is a formidable task for any computer system. In order to make these computations tractable, approximations have often been made, including reduction in dimensionality, linearization, restriction to a particular geometry and the assumptions of infinite conductivity and/or low "beta" ("beta" is the ratio of plasma pressure to magnetic pressure). The infinite conductivity assumption (called ideal MHD) greatly simplifies the numerics, since the MHD equations are then hyperbolic rather than parabolic. Moreover, instabilities
Driven by plasma resistivity tend to grow more slowly than those driven by magnetic energy or pressure, thereby requiring longer running times. The low beta assumption allows an ordering in which the problem is reduced to the solution of two scalar equations.

At the MFECC, the emphasis has been on constructing resistive MHD codes which are applicable to all plasmas, independent of beta. Both linear and nonlinear codes have been written which solve the full set of resistive, "finite-beta" MHD equations in two and three dimensions (see Table 2). Moreover, most of these codes use an implicit time discretization, so that there is no restrictive non-physical upper bound on the size of the timestep.

Our MHD research is in the following areas:

- A family of reversed field equilibria that model the FRTP is being generated. Our aim is to study the effects of increased rectangularity of the flux surfaces on stability. The overall object is to examine the discrepancy between the predicted fast instability of earlier computations and the long quiescent times exhibited by the experiments.

- It has been demonstrated both experimentally and computationally that the spheromak is unstable to shift and tilt modes unless the plasma is surrounded by a tight-fitting conducting shell. Our stability codes are being used to ascertain whether having a resistive plasma on the open field lines (line tying) improves stability.

- Ideal ballooning modes in FRTP and FRM have been studied with the energy principle in the kinetic energy norm. Growth rates for MHD configurations in the large toroidal wave number limit are computed. Coupled Sturm-Liouville-like equations on each flux line are solved for the eigenfunctions and eigenvalues, which give the physical displacements and growth rates, respectively. The addition of FLR stabilizing effects is under way.

- Our nonlinear 3D code is being applied to the behavior of resistive modes in compact toroids. Whether or not a general equilibrium relaxes to a force free state, and if it does, how long it takes and how the relaxation is affected by MHD modes, is under investigation.

- The evolution and saturation of magnetic islands in both Cartesian (sheet pinch) and cylindrical (diffuse) geometries have been modeled. The nonlinear evolution of the resistive interchange instability (or "g-mode") in the reversed field z-pinch has also been studied.
Our time-dependent MHD publications are summarized in Appendix A1.

B. PLASMA TRANSPORT

Transport codes are used to evaluate macroscopic plasma parameters (e.g., density, temperature) on a timescale comparable to the lifetime of the plasma. They differ from other fluid codes in that the fast timescale physics is integrated out of the problem.

At the MFEC, transport codes are being applied to tandem mirrors, tokamaks and compact toroids. A list of our transport codes and their applications appears in Table 3.

Over the past year we have made a great deal of progress in the following areas:

- Radial transport in tandem mirrors
- Fokker-Planck/transport studies of tokamaks
- Radial transport in compact toroids
- Anomalous electron transport in tokamaks

A brief summary follows.

- Radial Transport in Tandem Mirrors

A tandem mirror machine consists of a long, solenoidal cell with minimum-$B$ mirrors (plugs) at either end which act to electrostatically confine the central cell ions. The rate at which charged particles and heat diffuse in radius is of crucial importance. Of special significance is the expected enhanced transport of ions due to resonance between their azimuthal drift and axial bounce motions, which results from the presence of a non-axisymmetric magnetic field in the transition regions between the end plugs and the solenoid. A radial transport code, TMT, has been written in order to investigate this and other related phenomena. This multispecies code computes radial profiles of densities and temperatures in both the central solenoid and the end plugs along with a self-consistent electric field. Classical and neoclassical effects on transport are taken into account. This code is being used to study the Tandem Mirror Experiment (TMX) and to help design larger tandem mirror devices.

- Fokker-Planck/Transport Studies of Neutral Beam-Heated Tokamaks

Neutral beam-heated tokamaks are characterized by the presence of one or more energetic ion species which are quite non-Maxwellian along with a warm Maxwellian bulk plasma. For scenarios in which there is a large energetic ion population, it is very important to represent these energetic species by means of velocity space distribution functions and to follow their
evolution in time by integrating the Fokker-Planck equations. It is essential to utilize the full nonlinear Fokker-Planck operator to assure that the slowing down and scattering of these energetic species is computed accurately and realistically.

Our Fokker-Planck/transport code (FPT), in addition to solving radial transport equations for the bulk plasma densities and temperatures, solves nonlinear Fokker-Planck equations in 2D velocity space for the energetic ion distribution functions. The FPT code is unique in that it is the only tokamak transport code which does not either linearize the Fokker-Planck operator or ignore it altogether. Also, neutral beam deposition and neutral transport are computed using appended Monte Carlo codes developed at Princeton.

During the past year, FPT has been applied principally to DITE and it is currently being applied to advanced devices.

*Transport in Compact Toroids*

The compact torus experiments (field reversed mirrors, field reversed theta pinches and spheromaks) differ from the tokamak in that the plasma extends to the axis of rotational symmetry. Interest in the compact torus is due to the fact that it is likely to attain high beta and the fact that there is an engineering advantage of not having any structure through the center of the torus.

A transport code, FRT, has been written to describe the evolution of the plasma and magnetic field in a compact torus. This code differs from the tokamak transport codes in that the plasma extends to the axis of rotational symmetry and the 2-D equilibrium calculation contains a separatrix. The system evolves by alternating between the solution of four 1-D transport equations and the calculation of a 2-D equilibrium. The independent variable is the poloidal magnetic flux. Impurity radiation, neutral beam heating and anomalous electron energy loss are included.

The code has been used to model the Beta II experiment at LLNL and to simulate the neutral beam heating of a similar-sized plasma. Simulation of other compact toroids is under way.

*Anomalous Electron Transport in Tokamaks*

Recently, there has been considerable interest in the effects of anomalous electron transport due in part to magnetic surface destruction. Since this heat loss represents a major problem in toroidal confinement, there has been a need for a program which would simulate this loss and allow for credible estimates of the effects of electron transport on lower hybrid heating, D.C. conductivity and soft x-ray spectra.
The computer program (LDL) which has consequently been developed is 2-D with independent variables velocity magnitude and radial position. It combines a velocity and radially dependent heat source and a 1-D Fokker-Planck treatment of electron collisions with a loss operator simulating diffusion in velocity-radius. This program is in its initial stages of application to the Doublet experiments at General Atomic Corporation.

Our transport publications are summarized in Appendix B.

C. FOKKER-PLANCK

The Fokker-Planck equation is needed to treat plasmas in which the charged particle velocity space distribution functions are non-Maxwellian. In a magnetic mirror device, charged particles will tend to leak out the ends of the device, resulting in a "loss cone" in velocity space. In tokamaks or mirrors where there is neutral beam injection, the ion distribution functions will be characterized by an energetic component (in addition to the maxwellian background). These are two situations which require the use of Fokker-Planck codes.

The MFECC has led the nation in the development and implementation of multispecies Fokker-Planck codes employing the complete nonlinear two-dimensional Fokker-Planck operator. Since our codes have been generalized to deal with toroidal and open-ended configurations, many physical effects have been incorporated in them and a variety of physical problems have been studied. A summary of our Fokker-Planck codes appears in Table 4.

One of our main achievements of late has been the development of a user-oriented package, FPPAC, which computes the coefficients of the complete nonlinear 2D velocity-space Fokker-Planck collision operator and time-integrates the corresponding finite difference equations. This package runs from 10 to 14 times as fast on the CRAY-1 as on the CDC 7600. The tremendous gain in speed is due not only to the vectorization efficiency of the CFT compiler, but also to the fact that on the CRAY, one does not have to constantly move data between small and large core. FPPAC has been made available to the MFE community.

A new package, FBOUNCE, which bounce-averages the Fokker-Planck and rf operators is under development. This package is designed for use in small banana width, banana regime toroidal systems. The microwave field may have somewhat arbitrary characteristics, thus allowing a great deal of flexibility in evaluating a variety of the rf current drive schemes which have proliferated in recent years.
Our Fokker-Planck publications are summarized in Appendix C.

D. OTHER AREAS

Although most of our research has dealt with time dependent MHD, transport, and Fokker-Planck equations, we have undertaken some projects in other areas — in particular stellarator/torsatron configurations, Vlasov and particle simulations, convergent neutral beam studies and general numerical methods.

A self-consistent guiding center particle model coupled with a fluid equilibrium code has been used to study ion beam motion in a toroidal geometry. Applications include the study of collisionless beam behavior including strong counterstreaming beams and the steady state modeling of a CIT reactor, in which steady state beam currents and energy reinjection rates are computed.

A single particle orbit following code allows us to make detailed studies of the confinement properties of equilibrium fields in two and three dimensions. Studies of adiabaticity as well as resonant orbital effects have been made. Experimentalists have used the code to design ion probe diagnostics.

A general study of the Incomplete Cholesky Conjugate Gradient Method (ICCG) for solving sparse linear systems has been carried out. This technique has been shown to be considerably faster than other traditional linear solvers such as SOR and ADI. An ICCG solver applicable to general nine point two-dimensional difference operators has been written, and it is available to the HFE community.

A study has been made involving the creation of a very dense plasma by injecting convergent neutral beams into spherical or cylindrical chambers. Calculations of particle distribution functions, densities, ionization rate parameters and ionization probabilities have been carried out for both geometries.

A one-dimensional Vlasov finite difference code for ions and electrons has been written to study the formation of a plasma sheath and to compute steady state distribution functions along with a plasma potential.

A code has been developed to solve nonlinear elliptic equations on domains of very general shape.

Publications in these areas are summarized in Appendices D, E and F.
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<td>D.V. ANDERSON</td>
<td>2D (R,Z), bicubic splines, finite element, tensor pressure; FRTP, tandem mirrors, field reversed mirror</td>
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<td>VEPEC</td>
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<td>3D (X,Y,Z), tricubic splines, finite element, vector potential, tensor pressure; mirrors</td>
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<td>LDL</td>
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Appendix A1 - Time Independent MHD References

Papers


Reports


Abstracts


Appendix A2 - Time-Dependent MHD References

Papers


D.D. Schnack, J. Killeen, and R.A. Gerwin, "The Non-Linear Evolution of Resistive Interchange Modes in Reversed-Field Pinches," accepted for publication in Nucl. Fusion.


Reports


Abstracts


Appendix B - Transport References

Papers


Reports


Abstracts


Appendix C - Fokker-Planck References

Papers


Reports


Abstracts


M.G. McCoy, J. Killeen, K.D. Marx, A.A. Mirin and M.E. Rensink, "Calculations of \( \langle \sigma v \rangle \) for Anisotropic Mirror and Toroidal Distributions," Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, Madison (1976), 2B-11.


R.W. Harvey, J.C. Riordan J.L. Luxon and K.D. Marx, "Studies of Current Due to RF-Induced Runaway in the DIIA Lower Hybrid Experiment," Annual Controlled Fusion Theory Conference, Mount Pocono (1979), 1C43.


Appendix D - Particle/Vlasov References

Papers


Reports

C.G. Tull, "Guiding Center Simulations of Strong Counterstreaming Ion Currents with Applications to the Counterstreaming Ion Torus," UCRL-52436 (1978).


Abstracts


Appendix E - Convergent Neutral Beam References

Papers


Abstracts


Appendix F - Miscellaneous References

A. Review Publications

Papers


J. Killeen, "Computational Problems in Magnetic Fusion Research," in Proc. Tenth IFIP Conf. on System Modeling and Optimization, to be published by Springer-Verlag.

Abstracts


B. Other publications of group members

Papers


Reports


Abstracts


