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SOME EFFECTS OF THE TRANSVERSE-STABILITY REQUIREMENT ON THE DESIGN OF A GRATING LINAC*

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ABSTRACT

The transverse stability of the grating linac proposed by Palmer is analyzed. It is shown that an open structure such as a grating is always unstable transversely as long as it is uniform. The structure can be made stable by utilizing the strong focusing principle. This is achieved by periodically interrupting the grating shape. We analyze the strong focusing grating linac, and find that the stability requirement places a non-trivial constraint on the phase acceptance of the system.

I. Introduction

The laser driven grating linac¹ has emerged as an attractive candidate for the acceleration of electrons to ultra-relativistic energies. Preliminary investigations indicate that such an accelerator might be technically feasible with an average gradient of 1 GeV/m. Thus it becomes important to study the beam dynamics in such a system. In this paper we analyze the transverse motions of the particles in order to determine possible restrictions on the design of a grating linac.

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The geometry of a grating linac differs fundamentally from that of a conventional linac in that the former is an open structure in which the accelerating field decreases rapidly away from the grating surface, while the latter is a closed system with a cylindrical symmetry around the axis. For a conventional linac in extreme relativistic limit, it is well known that the transverse force is negligible and the focusing requirement is not severe. The situation is entirely different in the case of a grating linac; indeed, we find that the transverse force is large and unstable in such a structure. Therefore, the grating linac as originally proposed will not work.

However, the transverse force is similar to the field inside a quadrupole: thus if the force is defocusing in one direction, it is focusing in the other direction. Furthermore, the focusing or defocusing effect depends on the phase of the particle under consideration. Therefore, an obvious solution of the problem is to employ the concept of the strong focusing principle² by changing the phase periodically so that a particle sees the focusing and the defocusing field alternatively. Our analysis here shows that such a scheme works in principle if some non-trivial requirements are met. In particular we find that the net focusing requirement in both directions implies a limited acceptance in particle phase.

Sec (II) discusses the transverse stability from a general point of view. Sec (III) deals specifically with the grating linac. In Sec (IV), we analyze the transverse stability of a strong focusing grating linac by means of a simple model and find the stable phase acceptance region.

II. General Discussion

Consider an extremely relativistic electron traveling along the z-direction through a general accelerating structure. The force acting on the particle is (M.K.S. unit)

$$\underline{F} = -e (\underline{E} + c \hat{z} \times \underline{B}) + \dots \quad (1)$$

In the above, terms which are smaller by a factor $1/\gamma = \sqrt{1 - v^2/c^2}$ are neglected. The Panofsky-Wenzel theorem³ states that, for a traveling wave with a phase factor $e^{i(\omega t - kz)}$, the transverse part of the force is simply related to E_z as follows:

$$\underline{F}_\perp = -e \frac{i}{k} \nabla_\perp E_z \quad (2)$$

In the above, the phase velocity of the wave must match the particle velocity which is almost the light velocity c . Thus

$$\frac{\omega}{k} = c + O\left(\frac{1}{\gamma}\right) \quad (3)$$

The fields then satisfy the two dimensional Laplace equation:

$$\nabla_{\perp}^2 E_z = \left(\left(\frac{\partial}{\partial x} \right)^2 + \left(\frac{\partial}{\partial y} \right)^2 \right) E_z(x, y) = 0. \quad (4)$$

For a system which is uniform along the z-direction, Eqs (2) and (4) imply that the transverse motion is unstable if E_z has an appreciable transverse variation. To see this, assume that the motion is stable around (x_0, y_0) . Eq (2) implies that the point must be a minimum (or a maximum depending on the phase) of the function $E_z(x, y)$. However, E_z is a solution of the Laplace equation (4). Thus an extremum of E_z cannot be a minimum or a maximum but can only be a saddle point. Therefore, if the motion is stable in the x-direction, it must be unstable in the y-direction and vice versa.

In general F contains a constant force which must be cancelled out by an external force. This amounts to changing E_z by

$$E_z \rightarrow E_z' = E_z - ax - by. \quad (5)$$

Since E_z' is still a solution of the Laplace equation, our conclusion remains valid.

In a conventional linac with cylindrical symmetry, $\nabla_{\perp} E_z$ vanishes on the axis and the defocusing force is negligible in the extreme relativistic limit. However, in a one-sided open structure like a grating linac, E_z vanishes necessarily rapidly away from the grating surface. The transverse defocusing effects are then quite large. The difficulty can be overcome by shifting the grating ruling periodically so that the phase of a particle alternates between the focusing and defocusing region. The transverse motion can then be made effectively focusing in both x and y direction. This is known as the strong focusing principle².

III. Application to Grating Linac

We shall now work with an explicit field solution of the grating linac treated by Palmer¹. The $n = 1$ accelerating component which is in phase with particle is given by

$$\begin{aligned} E_z &= -E_0 \cos \phi \cos py e^{-px}, \quad c B_z = -E_0 k \cos \phi \sin py e^{-px}, \\ E_x &= E_0 \frac{k}{p} \sin \phi \cos py e^{-px}, \quad c B_x = E_0 p \sin \phi \cos py e^{-px}, \\ E_y &= 0, \quad c B_y = E_0 \left(\frac{k^2}{p} - p \right) \sin \phi \cos py e^{-px}. \end{aligned} \quad (6)$$

Here

$$\phi = \omega t - kz + \text{const.} \quad (7)$$

In the above, x is the direction pointing away from the grating surface, y is along the grating ruling and z is the particle direction. The electric fields are given in the Palmer's paper¹. The magnetic fields are then determined from the Maxwell's equation. It is straightforward to check that the above field satisfy eqs (2) and (4).

Taking z as the independent variable, the equations of motion are

$$mc^2 \frac{dy}{dz} = e E_0 \cos \phi \cos py e^{-px}, \quad (8a)$$

$$mc^2 \frac{d}{dz} \gamma \frac{dx}{dz} = -e E_0 \frac{p}{k} \sin \phi \cos py e^{-px}, \quad (8b)$$

$$mc^2 \frac{d}{dz} \gamma \frac{dy}{dz} = -e E_0 \frac{p}{k} \sin \phi \sin py e^{-px}. \quad (8c)$$

Assuming that the particle moves around the point $x = 0$, $y = 0$, we expand eq (8) near this point. Introducing a constant magnetic field along y -direction to cancel the constant deflecting force in the x -direction, one obtains

$$\frac{d^2 x}{dz^2} = -\frac{p}{\mathcal{X}k} \sin \phi (1 - px) + \frac{p}{\mathcal{X}k} \sin \phi e, \quad (9a)$$

$$\frac{d^2 y}{dz^2} = -\frac{p}{\mathcal{X}k} \sin y \quad (9b)$$

The last term in (9a) represents the external force, and we have introduced the quantity $\mathcal{X}$ via

$$\mathcal{X} = \frac{mc^2 \gamma}{et_0} \quad (10)$$

In deriving eq (9), we have assumed that γ is a slowly varying function of z .

In a laser driven grating linac, eE_0 is typically 1 GeV/m. At the injection point $mc^2 \gamma \sim 1$ GeV, thus $\mathcal{X} \sim 1$ m. In the following, we shall work with the following values of parameters specified by Palmer¹.

$$\mathcal{X} = \frac{\gamma}{\gamma_{in}} \cdot (\text{m}), \quad \lambda = \frac{2\pi}{k} = 10\mu, \quad p = .2 k, \quad (11)$$

where $mc^2\gamma_{in}$ is the injection energy. Taking $\sin \phi \sim .5$, an external magnetic field of ~ 3.3 KG will produce the last term in Eq (9a).

From Eq (9), one sees that the coefficient of the linear force for the x and y motions are equal in magnitude but opposite in sign. Thus the motion is stable in x and unstable in y or vice versa depending on the sign of $\sin \phi$, in agreement with our general discussion. The focusing or the defocusing strength is characterized by a distance $2\pi \sqrt{k/p^2} \sim 4$ cm for the case (11) at injection point. This is quite serious for an accelerator of 1 Km long.

The structure can be made to focus in both directions by introducing the strong focusing principle. One shifts the grating rulings so that the phase ϕ changes by $\pm \Delta$ periodically. The external field represented by ϕ_e must also change in order to maintain an appropriate relation with the changing ϕ . Ideally, one would like to have $\phi = \phi_e$ for all particles but this is impossible because different particles have different phases. A detailed analysis of the focusing properties taking into account this effect is presented below.

IV. A Strong Focusing Grating Linac and Phase Acceptance

In this section, we consider a simple model of the strong focusing grating linac in which the phase ϕ changes discontinuously. $\phi_e(z)$ is a periodic function of period L and is given by

$$\begin{aligned}\phi_e(z) &= -\Delta/2, \quad -L/2 < z < 0 \\ &= \Delta/2 \quad 0 < z < L/2.\end{aligned}\quad (12)$$

ϕ differs from ϕ_e by a constant phase ϕ_0 which characterizes different particles:

$$\phi(z) = \phi_e(z) + \phi_0 \quad (13)$$

Eq (9a) becomes

$$\begin{aligned}\ddot{x} &= \frac{1}{p} (q_1^2 - k^2) - q_1^2 x, \quad -L/2 < z < 0, \\ &= -\frac{1}{p} (q_2^2 - k^2) + q_2^2 x, \quad 0 < z < L/2,\end{aligned}\quad (14)$$

while Eq (9b) becomes

$$\begin{aligned}\ddot{y} &= q_1^2 y, \quad -L/2 < z < 0 \\ &= -q_2^2 y, \quad 0 < z < L/2\end{aligned}\quad (15)$$

In the above, $\ddot{x} = d^2x/(dz)^2$ and

$$k^2 = \frac{p^2}{\hbar^2 k} \sin \frac{\Delta}{2}, \quad q_1^2 = k^2 \frac{\sin(\Delta/2 - \phi_0)}{\sin \Delta/2}, \quad q_2^2 = k^2 \frac{\sin(\Delta/2 + \phi_0)}{\sin \Delta/2}. \quad (16)$$

Eqs (14) and (15) are linear differential equations with periodic coefficients, and can be solved by the standard matrix method². The discussion of the stability of the motion involves two parts: First, Eq (14) is inhomogeneous. Therefore, we have to find a periodic solution to define the equilibrium orbit. If the equilibrium orbit deviates too much from $x = 0$, the particle will either hit the grating surface, while if it deviates too much from $x = 0$ the acceleration becomes small. Second, we have to consider the stability of the homogeneous betatron motion around the equilibrium orbit. We consider the homogeneous part first.

The betatron motion is stable if

$$|\cos \mu_x| < 1, |\cos \mu_y| < 1, \quad (17)$$

where

$$\begin{aligned} \cos \mu_x &= \cosh \theta_2 \cos \theta_1 - \frac{1}{2} \left(\frac{q_1}{q_2} - \frac{q_2}{q_1} \right) \sinh \theta_2 \sin \theta_1, \\ \cos \mu_y &= \cosh \theta_1 \cos \theta_2 + \frac{1}{2} \left(\frac{q_1}{q_2} - \frac{q_2}{q_1} \right) \sinh \theta_1 \sin \theta_2. \end{aligned} \quad (18)$$

Here we have defined

$$\theta_i = q_i L/2 \quad (19)$$

in the above, q_i and θ_i could be imaginary if $\Delta < 0$. The above formulae still apply with the replacement $\sin i \theta = i \sinh \theta$, etc. In addition to (18), we must require

$$-\pi/2 < \pm \Delta/2 + \phi_0 < \pi/2, \quad (20)$$

since otherwise the particle will be decelerated.

The inequalities (18) and (20) are analyzed numerically to find the stable region of the parameter space. It is convenient to present the results in terms of Δ , ϕ_0 and

$$\psi = KL = \sqrt{L \frac{p}{\mathcal{E}k} \sin(\Delta/2)} \quad (21)$$

We wish to determine the range of ϕ_0 corresponding to stable motion for given ψ and Δ . We find

(i) for all values of Δ , the maximum phase acceptance (range of ϕ_0) is obtained near $\psi = \pi$,

(ii) setting $\psi = \pi$, the value $\Delta = .7 \pi$ corresponds to the maximum phase acceptance $|\phi_0| \leq .11 \pi$,

(iii) for comparison, the phase acceptance for the case $\psi = \pi$, $\Delta = .5 \pi$ is $|\phi_0| \leq .06 \pi$.

If the injected beam is uniformly distributed in phase, case (ii) accepts 11 percent of the beam while case (iii) accepts only 6 percent. The ratio of

the accelerating field to the peak available field is roughly $\cos(\Delta/2)$, which is .16 for case (ii) and .71 for case (iii). Thus case (ii) accepts more particle at the expense of less acceleration. Given Δ and ψ , the period length L is determined from Eq (21). Using the values given by Eq (11), one obtains

$$L = 2.1 \frac{Y}{Y_{in}} \text{ cm for case (ii)}. \quad (22)$$

Next we consider the equilibrium orbit by the x-motion, which is the periodic solution of the inhomogeneous equation. One obtains

$$\begin{aligned} x(z) &= f_1 + G(a_1 \cos q_1 z + b_1 \sin q_1 z), \quad -L/2 < z < 0, \\ &= f_2 + G(a_2 \cosh q_2 z + b_2 \sinh q_2 z), \quad 0 < z < L/2. \end{aligned} \quad (23)$$

Here

$$f_1 = \frac{1}{p} \left(1 - \frac{\sin \Delta/2}{\sin(\Delta/2 - \phi_0)} \right), \quad f_2 = \frac{1}{p} \left(1 - \frac{\sin \Delta/2}{\sin(\Delta/2 + \phi_0)} \right),$$

$$G = \frac{1}{2(1 - \cos \mu)} (f_1 - f_2),$$

$$a_1 = \cosh \theta_2 - \cos \theta_1 - 1 + \cosh \theta_2 \cos \theta_1 + \frac{q_2}{q_1} \sinh \theta_2 \sin \theta_1 \quad (24)$$

$$b_1 = \sin \theta_1 (1 - \cosh \theta_2) - \frac{q_2}{q_1} \sinh \theta_2 (1 - \cos \theta_1),$$

$$a_2 = \cosh \theta_2 - \cos \theta_1 + 1 - \cosh \theta_2 \cos \theta_1 + \frac{q_1}{q_2} \sinh \theta_2 \sin \theta_1,$$

$$b_2 = \frac{q_1}{q_2} \sin \theta_1 (1 - \cosh \theta_2) - \sinh \theta_2 (1 - \cos \theta_1).$$

In the above, we have assumed $\Delta > 0$. If $\Delta < 0$, the orbit can be obtained by setting $\Delta \rightarrow -\Delta > 0$ and displacing z by half a period.

Qualitative behavior of the equilibrium orbit can be seen directly from Eq (14). Without loss of generality, we may assume $\Delta > 0$. The inhomogeneous terms of Eq (14) are negative when ϕ_0 is positive, and vice versa. Thus, one may expect that x will generally be negative when $\phi_0 > 0$, and vice versa. When $\phi_0 > 0$ and increases, the focusing in the region $-L/2 < z < 0$ becomes weaker while the defocusing in the region $0 < z < L/2$ becomes stronger. Thus the orbit deviation will become large and negative until ϕ_0 reaches the limit of the homogeneous stability region where it blows up. This

is also apparent from the factor $(1 - \cos \nu)^{-1}$ in the expression of G in Eq (24). On the other hand, when ϕ_0 is negative and decreases, the focusing becomes stronger and the defocusing weak, and we expect that the equilibrium orbit will behave better in that region.

We need to specify the maximum tolerable excursion of the equilibrium orbit. We shall take

$$|x| \leq \frac{1}{p} = \frac{5\lambda}{2\pi} \quad (25)$$

We have evaluated Eq (23) numerically, and find that (25) implies

$$\begin{aligned} -0.11 \pi < \phi_0 < 0.05 \pi & \text{ for case (ii),} \\ -0.06 \pi < \phi_0 < 0.02 \pi & \text{ for case (iii).} \end{aligned} \quad (26)$$

The phase acceptance region is reduced in the $\phi_0 > 0$ region as expected.

The limit of the phase acceptance region for negative ϕ_0 in (26) is due to the linear stability of the y -motion. There is a good reason to believe that this could be extended substantially if one takes into account the full non-linear equation (8). The equilibrium x -orbit, when $\phi_0 < 0$, tends to have a maximum in the region $-L/2 < x < 0$ and a minimum in the region $0 < x < L/2$. Therefore, the factor e^{-px} in Eq (8c) tends to neutralize the mismatch of the focusing and defocusing effects of the factor $\sin \phi$. This point needs to be studied further.

We conclude, therefore, that the strong focusing scheme for the grating linac works in principle. However, the phase acceptance region derived for the linear stability consideration is rather small. One hopes that the analysis of the full non-linear equation improves this limit substantially.

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