LA--10958-MS

DE88 007169

*Arcing and rf Signal Generation
During Target Irradiation
by a High-Energy, Pulsed
Neutral Particle Beam*

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ARCING AND rf SIGNAL GENERATION DURING TARGET IRRADIATION
BY A HIGH-ENERGY, PULSED NEUTRAL PARTICLE BEAM

by

R. T. Robiscoe

ABSTRACT

We present a theory describing the dynamics of arc discharges in bulk dielectric materials on board space-based vehicles. Such "punch-through" arcs can occur in target satellites irradiated by high-energy (250 MeV), pulsed (100 mA x 10 ms) neutral particle beams. We treat the arc as a capacitively limited avalanche current in the target dielectric material, and we find expressions for the arc duration, charge transport, current, and discharge energy. These quantities are adjusted to be consistent with known scaling laws for the area of charge depleted by the arc. After a brief account of the statistical distribution of voltages at which the arc starts and stops, we calculate the signal strength and frequency spectrum of the electromagnetic radiation broadcast by the arc. We find that arcs from thick (~1 cm) targets can generate rf signals detectable up to 1000 km from the target, by a radio receiver operating at frequency 80 MHz, bandwidth 100 kHz, and detection threshold -105 dBm. These thick-target arc signals are 10 to 20 dB above ambient noise at the receiver, and they provide target hit assessment if the signal spectrum can be sampled at several frequencies in the nominal range 30-200 MHz. Thin-target (~1 mm) arc signals are much weaker, but when they are detectable in conjunction with thick-target signals, target discrimination is possible by comparing the signal frequency spectra.

I. OBJECTIVES

The objectives of this report are (A) to study rf (radio frequency) signals generated by arcing on board a target vehicle struck by a high-energy, pulsed NPB (neutral particle beam) and (B) to examine the feasibility of hit assessment and target discrimination by remote detection of the arcing signals. NPB irradiation can cause charging of dielectric layers on board a target, followed by both surface flashover and bulk discharge arcs. The strength and frequency spectrum of bulk discharge arcs depend on the length of the arc, and thus on the thickness of the target dielectric. This fact suggests that a distant observer can discriminate between arc signals emanating from "thick" vs "thin" targets. We construct a theory of bulk discharge arcs in highly stressed dielectric layers, and we estimate the detectability of arcing signals at a remote rf receiver. We close with a preliminary assessment of "thick" vs "thin" target discrimination by analysis of the target arcing signals.

II. SUMMARY

In Calculations, Sec. III-A, we show that target irradiation by a single NPB pulse (beam energy ~ 250 MeV, pulse ~ 100 mA $\times$ 10 ms, divergence ~ 1 μ rad), at source-target distances ~ 1000 km, can deposit enough charge on board target dielectric surfaces to induce one or more arc discharges. We assume the charge is deposited by stripping of the NPB, and we concentrate on bulk discharge arcs rather than surface flashovers. Also, we ignore effects due to x-ray production but do consider possible changes in the target dielectric material upon passage of the stripped NPB nuclei. In Sec. III-B, we describe the bulk discharge arc as a capacitively limited avalanche current flowing in the target dielectric material, a current which is fed by the motion of charges on the NPB-irradiated dielectric surface. In Sec. III-C, we show that certain simple assumptions about the surface currents feeding the arc account for several laboratory observations on how various arc parameters (arc duration, charge transport, peak current, etc.) scale with the charged area available at the inception of the arc. The calculations of Secs. III-B and III-C fix the dynamics of a bulk discharge arc occurring in an NPB-irradiated target dielectric.

In Sec. III-D, we develop a simple probabilistic picture of the arc statistics, i.e., of the fields (voltages) above and below threshold at which the arc starts and stops. Inhibition of the surface currents feeding the arc

appears to be less important in determining when the arc starts and stops than the probability of ionizing collisions which maintain the arc. We analyze the distribution of arcs over various arc start and stop voltages, and we conclude that "typical" arcs from target dielectrics (~1 cm thick) may show peak currents ~500 A and may have durations as long as 0.5 μ s.

In Sec. III-E, we calculate the fields and energy radiated by bulk discharge arcs terminating at an anode which is--in effect--a metal surface. We do not treat radiation from the stripped NPB nuclei or from arcs that pass charge through the anode surface. We estimate arc broadcast fields and signal strengths observable at a remote rf receiver and show that a nominal thick-target arc signal is detectable at target-observer distances up to 1000 km, by a receiver operating at frequency ~80 MHz, bandwidth ~100 kHz, and threshold sensitivity (-)105 dBm. The arc signal here is ~15 dB above ambient thermal and background noise. In Sec. III-F, we compare the frequency spectra of signals from arcs in "thick" and "thin" targets. Over a frequency range 0-1 GHz, spectral outputs from thick targets decrease with frequency, while they increase for thin targets. This difference can be used to discriminate between arc signals emanating from thick and thin targets, if the observer's rf receiver can be scanned over a nominal frequency range 30-200 MHz. We close this report, in Sec. III-G, with comments on how various aspects of the present calculation might be extended or improved.

Our overall conclusion is that both hit assessment and thick- vs thin-target discrimination by analysis of AGEMPs (arc-generated electromagnetic pulses) at locations remote from the target appear promising. The AGEMP signals occur at readily detectable radio frequencies, are strong enough to be "heard" at distances exceeding 100 km, and carry an unmistakable spectral signature in the limit of thick vs thin targets. Possible countermeasures to the detection of these signals are not discussed. Also, work remains to be done on comparison of bulk discharge arc signals with those from surface flashover and blowoff arcs and on the mechanism by which a bulk discharge arc shuts off.

III. CALCULATIONS

A. Surface Charging by NPB Irradiation

When a high-energy NPB (neutral particle beam) strikes a target surface, there is a good chance the electrons are stripped from the beam and deposited

near the surface, while the positive ions (usually protons) penetrate much farther into the target material. If that material is an insulator, an electron charge layer can build up, and various arcing phenomena can occur on board the target, such as surface flashover, discharges between isolated points on the target, and bulk discharges through the surface dielectric layer (Frederickson 1982). We shall concentrate on bulk discharge arcs (so-called "punch-through" breakdowns), which become an important source of radio "noise" as the NPB energy increases and the stripped electrons are embedded more deeply in the target material. In this section, we calculate the NPB irradiation time necessary to reach the threshold field for bulk discharge breakdown.

The geometry of the NPB vehicle, target, and observer is shown in Fig. 1. The NPB accelerator fires a beam pulse (typically neutral hydrogen atoms at kinetic energy $\sim$ several hundred MeV) with parameters

$$\begin{aligned} \text{pulse instantaneous current: } I_B &= 100 \text{ mA,} \\ \text{pulse (temporal) length: } T_B &= 10 \text{ msec,} \\ \text{beam angular divergence: } \Delta\phi &= 10^{-6} \text{ rad.} \end{aligned} \quad (1)$$

The pulse repetition rate is about 3 Hz (3% duty cycle). Typical distances may be: D [accelerator to target] $\sim$ 500-1000 km, and R [target to observer] $\sim$ 100-1000 km.* We assume that a fraction of the NPB strikes the target surface (at incident angle α) and that at some time during the NPB irradiation an arc discharge occurs in the bulk of the target material. Finally, an observer detects the radiation from this arc at angle θ relative to the arc axis. Note that since the incident angle α is not predetermined, then neither is the viewing angle θ .

Figure 2 shows the target surface configuration during NPB irradiation. The beam pulse current I_B is incident at angle α , and stripped electrons form a charge layer (virtual cathode) within a few hundred μm of the surface. The dielectric layer, of thickness in the range from $d = 0.1 \text{ mm}$ (foil) to $d \sim$ few cm (heat shield), is penetrated by the ongoing high-energy positive ions, and the ions exit either into a vacuum or onto a metal surface. The positive ions are thus lost to further surface charging, and they will affect an arc

* Numerical examples used in this report are based on "Science and Technology of Directed Energy Weapons: Report of the American Physical Society Study Group," April 1987.

discharge of the stripped electron layer only indirectly, by the transient charge density they create within the dielectric, and by possible x-ray production upon exit. In Appendix A, we shall consider possible conductivity changes in the target material induced by the passing positive ions. However, we shall not consider the effect of ambient x-rays upon initiating or maintaining a bulk discharge of the stripped electron layer: x-rays, if they are produced, originate from deep within the target anode material and have little chance of backscattering into the surface dielectric.

We calculate the field buildup in the target dielectric material upon NPB irradiation as follows.

Fig. 1. Geometry of NPB accelerator, target, and observer. The accelerator-target distance may be $D \sim 1000$ km; also, the target-observer distance $R \sim 1000$ km. The NPB pulse instantaneous current equivalent is $I_B \sim 100$ mA, and the beam diverges at $\Delta\phi \sim 1 \mu\text{rad}$. The beam strikes the target at incident angle α , an arc occurs, and the observer--oriented at (unknown) angle θ w.r.t. the arc axis--attempts to detect the arc EMP.

- 1) At accelerator-target distance D , the beam area is: $A_B = \pi(D\Delta\phi/2)^2$. The target area irradiated is: $A_I = A_B \sec \alpha$; A_I may be smaller if the full beam does not strike the target, or if the target area is less than A_B .
- 2) At time ΔT_B into the NPB pulse, the maximum charge available for deposit in the stripped electron layer is $I_B \Delta T_B$. This charge forms a virtual cathode for an effective parallel plate capacitance C across the dielectric layer. Then the voltage across the layer increases as: $V = (I_B/C) \Delta T_B$.
- 3) We use the parallel plate capacitance: $C = m\kappa\epsilon_0 A_I/d$, where κ is the relative dielectric constant of the target material, $\epsilon_0 = 8.85 \times 10^{-12}$ MKS, A_I is the irradiated area, d is the thickness of the dielectric, and m is a factor ($1 \leq m \leq 2$) accounting for the charges on the anode and between cathode and anode (created by the passing positive ions).

Fig. 2. NPB-irradiated target surface configuration. The beam is incident at angle α , at a kinetic energy of several hundred MeV. Electrons are stripped off near the entry surface, and positive ions exit after passing through thickness d (~ 1 -10 mm) of dielectric material. The electron charge layer near the breakdown field E_s , so that a bulk discharge arc may occur through the target dielectric material.

4) The field buildup in the target during NPB irradiation is $E = V/d = (I_B/Cd)\Delta T_B$, assuming the target material has zero conductivity. Then, the irradiation time necessary to reach a field strength E is

$$\Delta T_B = 6.95 [D\Delta\phi]^2 (\text{msec}\alpha) \kappa E/I_B, \text{ ms}, \quad (2)$$

with: D = target distance in units of 1000 km, $\Delta\phi$ = beam divergence in units of μrad , $1 \leq m \leq 2$ (see above), α = NPB angle of incidence, κ = relative dielectric constant of target material, E = field in target in MV/cm, I_B = NPB current in units of 100 mA.

This calculation assumes the electrons do not migrate during the ~ 10 -msec deposit time, an assumption justified by the relatively large relaxation times (10 - 10^5 sec; see Table I) for typical non-stressed insulators. However, the electron conductivity can increase--at the high fields approaching breakdown and by radiation-induced conductivity due to passage of the stripped NPB nuclei. We consider these effects in Appendix A and conclude that under nominal NPB irradiation, increases in the target conductivity do not prevent breakdown from occurring, but they may delay the onset of breakdown by increasing ΔT_B up to 15%. For simplicity, we shall not include such corrections below.

Nominal values in Eq. (2) are: $D = 0.7$ ($\times 1000$ km), $\Delta\phi = 1$ (μrad), and $I_B = 1$ ($\times 100$ mA). Also, as noted in Table I, the effective field at threshold for dielectric breakdown is: $\langle \kappa E_s \rangle = 0.690$ MV/cm; this figure has an average deviation of only $\pm 20\%$ over more than a dozen insulating materials typically found on board the target. With these representative numbers, the NPB irradiation time required to achieve breakdown fields in the target is

$$\Delta T_B [\text{breakdown}] = 2.35 \text{ msec}\alpha, \text{ ms}. \quad (3)$$

α , the NPB angle of incidence on the target surface, is not fixed; we shall take $\alpha < 45^\circ$ as "typical." If $m \sim 2$ (for a net positive charge in the dielectric or on the anode, which is small compared to the electron charge), we find: $\Delta T_B [\text{breakdown}] < 6.65$ msec. Since this irradiation period is smaller than the

TABLE I. Properties of Some Insulators^a

Material	Relative Dielectric Constant κ	Breakdown Field (MV/cm) E_s	Effective Field (MV/cm) κE_s	Bulk Resistivity (Ω -cm) ρ_o	Relaxa- tion Time (sec) $\tau_o = \rho_o \epsilon_o$
1. Allyl Resin	4.0	0.150	0.600	$>4 \times 10^{14}$	>35
2. Epoxy Resin	3.6	0.160	0.575	$10^{16}-10^{17}$	10^3-10^4
3. Nylon	3.5	0.185	0.650	4×10^{14}	35
4. Polyester Resin	~ 3.4	~ 0.175	~ 0.595	10^{14}	9
5. Polyethylene	2.3	0.470	1.080	10^{17}	$\sim 10^4$
6. Polypropylene	2.3	0.255	0.590	6×10^{16}	5×10^3
7. Polystyrene	2.6	0.235	0.610	10^{18}	$\sim 10^5$
8. PVC Plastic	3.2	0.160	0.510	10^{14}	9
9. PVF Plastic	8.4	0.100	0.840	2×10^{14}	18
10. Teflon	2.1	>0.400	>0.840	10^{17}	$\sim 10^4$
11. Alumina	~ 6.5	~ 0.04	~ 0.260	$10^{11}-10^{14}$	0.01-10
12. Neoprene	6.7	0.120	0.800	8×10^{12}	0.7
13. Porcelain	5.5	~ 0.06	~ 0.330	$10^{12}-10^{14}$	0.1-10
14. Ruby Mica	5.4	~ 1.85	~ 9.990	5×10^{13}	4.4
15. Silica Glass	3.8	0.160	0.610	$>10^{19}$	$>10^6$
16. Steatite	5.8	0.120	0.690	10^{14}	9
17. Kapton	2.5	0.225	0.560	2×10^{13}	2
18. Mylar	3.7	1.57	5.80	8×10^{16}	7×10^3

NOTE. For materials 1-10, the average effective breakdown field is:

$\langle \kappa E_s \rangle = 0.690 (1 \pm 0.20)$, MV/cm. The range is the average deviation.

^a Data taken from Handbook of Chemistry and Physics (1967), p. E-62, and from Reference Data for Radio Engineers (1968), pp. 4-28 to 4-30.

10-msec beam pulse length, we conclude that under typical operating conditions it is possible to achieve dielectric breakdown on board the target by irradiation with a single NPB pulse.

Typically, arc discharges do not fire until an "overvoltage" is reached, i.e., a field exceeding the arc threshold field. If we assume the required overvoltage is 10-15% of threshold, and if the arc fires so as to discharge just the overvoltage, then the scenario which follows from the above assumptions is that the target will suffer an arc discharge at $\Delta T_B \sim 7.5$ msec into the NPB irradiation, and that several similar arcs can occur before the beam pulse cuts off. Of course this scenario can change as the target distance and/or orientation changes, but it appears reasonable to assume at least one arc from the target per NPB pulse.

B. Capacitive Model of Arc Discharge

Some time ago, F. R. Dickey (1952) devised a simple theory describing the rapid voltage drop across a spark gap during an arc discharge. In Dickey's theory, the spark gap is treated as a charged capacitor, and the ensuing arc (formed by an electron cascade) is just that capacitive discharge needed to relieve the existing overvoltage. The theory accounted well for measured transient voltages across STP air gaps (nearly planar, gap widths 1-4 mm) for impulse overvoltages as high as 100% above threshold. Although Dickey's theory characterizes the medium in the gap only rudimentarily (via the medium's ionization coefficient α_i , breakdown field E_s , and average cascade electron velocity v_e), it does have the virtues of simplicity and direct appeal. In this section, we develop a modification of Dickey's theory that provides a dynamical picture of an arc discharge in an isolated dielectric.

Dickey's spark gap model is shown in Fig. 3. The gap has known width d and effective capacitance C . At time $t=0$, the gap voltage V_0 exceeds the breakdown voltage V_s for the material in the gap, and--at that instant--charge q_0 leaves the cathode to initiate an electron cascade. The ensuing arc discharge current is

$$I(t) = (q_0/T_e) \exp\left[\int_0^t \alpha_i v_e dt\right], \quad (4)$$

where α_i is the ionization coefficient of the medium (number of ions created by an avalanche electron per unit length of travel), v_e is the average velocity, and $T_e = d/v_e$ is the transit time across the gap for an avalanche electron. As the arc develops, the capacitor is fed current I_{ext} by an external constant voltage generator through an external resistor R , so that

$$I_{ext} = [V_o - V(t)]/R, \quad (5)$$

where $V(t)$ is the instantaneous voltage across the gap.

Dickey next solved the circuit equation

$$C(dV/dt) = I_{ext} - I(t), \quad (6)$$

Fig. 3. Dickey's model of an arc discharge in a spark gap. The gap has known width d and effective capacitance C . At time $t=0$, the gap voltage V_o exceeds the breakdown voltage V_s for the gap material, and a charge q_o leaves the cathode to initiate the arc. The instantaneous arc current is $I(t)$, and the instantaneous gap voltage is $V(t)$. The arc may be fed by an external current I_{ext} through an external resistor R . T_e is the gap transit time for an avalanche electron at velocity v_e and α_i is the ionization coefficient.

and he showed that when α_i and v_e are treated as constants

$$C[V_0 - V(t)] = q_0 \exp(\alpha_i v_e t) / [\alpha_i d + (T_e/RC)]. \quad (7)$$

Both α_i and v_e are actually functions of the instantaneous electric field $E(t)$, but Dickey found that existing data could be fit reasonably well by just evaluating these parameters at the breakdown field E_s . We shall assume $\alpha_i = \alpha_i(E_s)$ and $v_e = v_e(E_s)$ in what follows.*

When the electron avalanche reaches the anode of the spark gap, at time $t = T_e = d/v_e$, the fast part of the arc discharge is over, and the gap voltage has fallen by: $\Delta V = V_0 - V(T_e)$. Using Eq. (7), we find the initiating charge q_0 which flowed off the cathode to cause the arc that decreased the gap voltage by ΔV ; it is

$$q_0 = (\Delta V) [\alpha_i d + (T_e/RC)] e^{-\alpha_i d}. \quad (8)$$

If we were to look for a fixed voltage decrease ΔV (in a given gap), this relation would specify the arc current in Eq. (4).

To apply Dickey's ideas to an arc discharge in an isolated dielectric, we must modify two features of the above treatment. First, since it is known that these arcs can remove a substantial fraction (~30%) of the charge on the cathode surface (Frederickson 1982), then we should allow a macroscopic initiating charge q_0 to leave the cathode; q_0 cannot leave instantaneously, so we postulate a current leaving the cathode surface and flowing into the site where the arc is struck. Second, for our isolated dielectric sample, we must remove the external power supply in Fig. 3 (i.e., set $R \rightarrow \infty$ and $I_{ext} \rightarrow 0$) and instead allow the cathode surface currents mentioned above to feed the arc discharge.

*Generally, the electron velocity v_e depends relatively weakly on the electric field E ; typically: $v_e \propto E^{1/2}$. This is for gases (see Papoular 1965, Sec. 9.2), but probably holds also in solids. The ionization coefficient in gases varies with field as: $\alpha_i \propto \exp(-E_0/E)$ (Papoular, Sec. 10.2). This variation can be rapid, and this relation might be extended to solids, but at most phenomenologically.

The notion of surface currents flowing into the site of an arc discharge breakdown in an isolated dielectric layer is illustrated in Fig. 4. The layer still can be characterized by a capacitance C , gap spacing d , and instantaneous gap voltage $V(t)$. But now the cathode initial surface charge density ρ_0 --depleted at the arc site by the discharge--is driven into the site by the large potential difference that exists between the initial point of the discharge and the surrounding charges. If those charges move into arc site with uniform radial velocity u , and if ρ_0 remains (nearly) constant, then the cathode feeds the arc with a current of size

Fig. 4. Arc discharge in an isolated dielectric. This schematic is similar to Dickey's model in Fig. 3, except the external current source is removed, and the arc is fed by currents moving across the surface of the (charged) cathode into the (uncharged) arc site. The flow of charge into the arc site occurs at some velocity u ; the assumption that $u = \text{constant}$ (i.e., surface current $i(t)$ linear in time) explains a number of well known experimental observations.

$$i(t) = (2\pi\rho_0 u^2)t = (\rho_0 s_1/T_e)(t/T_e), \quad s_1 = 2\pi(uT_e)^2. \quad (9)$$

$T_e = d/v_e$ is the avalanche transit time; typically $T_e \sim 10$ nsec, for a 1-cm gap (Frederickson 1982). Clearly, on a time scale $\sim T_e$, the surface charge must flow at $u \sim 10^7$ cm/sec in order to deplete even a few mm^2 of the cathode surface.

Since surface flow velocities $u \sim 10^7$ cm/sec are much larger than those permitted for dielectrics under normal conditions, we must assume that under breakdown conditions, the abnormally high u -values (required for adequate arc current feed) result from a large increase in the surface conductivity in and around the arc site. In turn, the increased surface conductivity might be due to an alteration of the dielectric by the large fields induced during breakdown near the arc site or even to the formation of a space charge layer above the surface, which is able to carry sufficiently large currents (Mandell et al. 1979). This picture also might provide a method for deciding when the cathode feed current $i(t)$ --and hence the arc--will stop: as the discharge progresses, and depletes a larger and larger arc site, the fields that drive charge into the site diminish; when these fields have fallen to sufficiently low values, the surface conductivity returns to its normal (small) value, and the arc shuts off.

For the moment, however, we shall not pursue the dynamics of the cathode surface currents in detail. Instead, we adopt a phenomenological approach and assume surface currents of the form

$$i(t) = (\rho_0 s_n/T_e)(t/T_e)^n, \quad (10)$$

where ρ_0 is the cathode initial surface charge/unit area, T_e is the electron avalanche transit time, s_n is a parameter with the dimensions of an area (roughly the area depleted of charge by the arc in time T_e), and the exponent n is some non-negative integer. Other forms of $i(t)$ can be used in the theory below, but the form of Eq. (10) is sufficient for illustration, and the $n=1$ case (i.e., case of flow velocity $u = \text{cnst}$) gives results that are in general accord with observation.

For an arbitrary cathode current $i(t)$ feeding an arc that has just started, the charge in the gap during $0 \leq t \leq T_e$ is

$$Q(t) = \int_0^t i(\tau) d\tau \{ \exp[\alpha_i v_e (t-\tau)] \}, \quad t \leq T_e. \quad (11)$$

This is sum of the charges $dq(\tau) = i(\tau) d\tau$ that have left the cathode in the interval $d\tau$ at time τ and which--having traveled at the assumed constant avalanche velocity v_e --have cascaded by the factor $\exp[\alpha_i v_e (t-\tau)]$. We have assumed that the ionization coefficient α_i is also constant (i.e., field independent). Unlike Dickey's case, where the charge q_0 leaves the cathode instantaneously at $t = 0$ and the avalanche is completely localized in space, the gap charge $Q(t)$ of Eq. (11) is distributed in both space and time. It is not hard to show the spatial distribution for $0 \leq t \leq T_e$ is specified by the charge/unit length

$$\lambda(x, t) = (1/v_e) \exp[\alpha_i x] i(t_{ret}),$$

$$\text{where: } t_{ret} = t - (x/v_e), \quad 0 \leq x \leq v_e t. \quad (12)$$

In turn, λ gives the gap charge of Eq. (11) as $\int_0^a \lambda(x, t) dx = Q(t)$, where the upper limit $a = v_e t$. The temporal development of $\lambda(x, t)$ across the gap, for the power-law current in Eq. (10), is sketched in Fig. 5.

At time $t > T_e$, the leading edge of the cathode current avalanche has hit the anode, as depicted in Fig. 5, and since charge is being removed from the arc, $Q(t)$ of Eq. (11) no longer represents the charge in the gap. Instead, the gap charge becomes

$$Q(t) = \int_0^d \lambda(x, t) dx = \int_{t-T_e}^t i(\tau) d\tau \{ \exp[\alpha_i v_e (t-\tau)] \}, \quad t > T_e. \quad (13)$$

This expression joins smoothly with $Q(t)$ of Eq. (11) as $t \rightarrow T_e$ and behaves properly in the limits of an impulse current: $i(\tau) = q_0 \delta(\tau)$ [for which $Q(t) = 0$ at $t > T_e$], and a steady current: $i(\tau) = i_0$ [for which $Q(t) = \text{const}$ at $t > T_e$].

Note particularly that Eq. (13) accounts for charge flowing in the arc itself, but does not include any charge flowing to the right of the anode ($x > d$). This point is important later, in Sec. III-E, when we calculate the EM radiation from the arc.

Now, for the arc discharge in an isolated dielectric as shown in Fig. 4, our version of Dickey's Eq. (6) is

$$C(dv/dt) = -(1/T_e)Q(t), \quad (14)$$

Fig. 5. Temporal development of arc charge density: a sketch of the arc charge per unit length, $\lambda(x,t)$ of Eq. (12), over the extent of the gap, at various times. We have used the power law cathode current $i(t)$ of Eq. (10). At times exceeding the electron avalanche transit time T_e , the charge fills the gap.

where the gap charge $Q(t)$ is given by Eq. (11) when $t < T_e$, and by Eq. (13) when $t > T_e$. Equation (14) simply states that the dielectric, viewed as a charged capacitor, will--when it arcs--discharge at a rate proportional to the average arc current. We neglect relatively small inductive effects; see Appendix B. The solution to Eq. (14) is straightforward and gives the following results for the voltage drop $\Delta V(t)$, which accompanies an arc discharge fed by a cathode current $i(t)$

$0 < t < T_e$ [and $Q(t)$ of Eq. (11)]

$$k\Delta V(t) = \int_{0-}^t \{ \exp[(k/T_e)(t-\tau)] - 1 \} i(\tau) d\tau,$$

$$\text{where: } \Delta V(t) = V(0) - V(t), \quad k = \alpha_i v_e T_e = \alpha_i d; \quad (15)$$

$t > T_e$ [and $Q(t)$ of Eq. (13)]

$$k\Delta V(t) = (e^k - 1) \int_{0-}^{t-T_e} i(\tau) d\tau + \int_{t-T_e}^t \{ \exp[(k/T_e)(t-\tau)] - 1 \} i(\tau) d\tau. \quad (16)$$

The parameter k defined here can be interpreted as an amplification factor due to the electron cascade, since a unit charge released from the cathode arrives at the anode with strength e^k . Also, for the $t > T_e$ solution of Eq. (16), note that the first term RHS (right-hand side) is the discharge that has already hit the anode, while the second term RHS is the discharge due to the current in transit.

The model developed in this section has two important features: (1) it gives a dynamic picture of the arc charge transport (this is required for any estimation of the radiation broadcast by the arc); (2) the capacitive discharge equations [Eqs. (15) and (16)] allow fixing the arc peak current and maximum charge transport in terms of a cutoff time T_c , when the voltage decrease ΔV is large enough to cut off the cathode feed current $i(t)$. Although both $i(t)$ and

the cutoff time T_c are phenomenological, reasonable assumptions about them can set limits on the strength of typical arc broadcasts.

C. Arcology - Cathode Currents for Bulk Discharge Arcs

By "arcology" we mean the scientific study of arcs. The model developed in Sec. III-B contributes to the arcology of isolated charged dielectrics by providing the dynamics of the arc current (an electron avalanche fed by a cathode current) and by connecting the arc charge transport to its effect in relieving the electrical stress in the dielectric (by discharging the medium's capacitance). However, our arc discharge model is phenomenological, and so far we do not know what form of cathode current, $i(t)$, to choose, nor do we have any precise idea of when (or why) the arc begins and ends. In this section, we make the model more specific by fitting it to known data on arcs. We show that certain scaling laws (for the dependence of various arc parameters on the irradiated surface area) are reasonably well obeyed for a cathode current that grows linearly in time, as in Eq. (9).

For an arc discharge in a bulk dielectric, we make the following plausible claims in terms of the model of Sec. III-B.

- 1) No discharge occurs until the dielectric's internal field exceeds its breakdown field E_s ; for an arc to occur, the initial medium-as-capacitor voltage must be $V_0 > V_s$, with V_s a threshold voltage.
- 2) After the discharge, the final capacitor voltage $V(t) > 0$ in Eqs. (15)-(16); the total voltage decrease is then $\Delta V(t) < V_0$. Although decreases as large as $\Delta V(t)/V_0 \sim 100\%$ are occasionally seen (Frederickson 1982), it is more common to observe $\Delta V(t)/V_0 \sim 10-50\%$ (Flanagan et al. 1979).
- 3) If the arc relieves the "electrical stress" in a dielectric, it seems reasonable to assume that the voltage decrease is at least as large as the overvoltage $(V_0 - V_s)$, to restore the medium to a pre-breakdown condition; so: $(V_0 - V_s) < \Delta V(t) < V_0$.

- 4) With cathode surface currents $i(t)$ present, the arc will last longer than the electron avalanche transit time T_e , but not as long as the time T_∞ at which the entire cathode is drained of its initial charge [calculable from: $\int_0^\infty i(t)dt = CV_0$]. There will be some time T_c , $T_e < T_c < T_\infty$ at which the arc cuts off.

These claims follow from the basic logic of our discharge model.

More stringent restrictions for our model follow from three "arcological" laws. Experiments show that for surface and bulk (punch-through) discharges in dielectrics, several important arc parameters scale with the charge-irradiated area A_I by simple power laws. The laws are*

Arc Quantity	Scales As
arc duration T_c	$(A_I)^{1/2}$
charge release $q(T_c)$	A_I
peak current $I(T_c)$	$(A_I)^{1/2}$

(17)

The "charge release" here means the charge which leaves the cathode. Our arc discharge model should incorporate these scaling laws.

To begin accounting for these arc discharge features, we assume the arc cutoff time $T_c > T_e$ is determined by a specific voltage decrease ΔV_c across the dielectric's effective capacitance. At time T_c , the cathode feed current $i(t)$ ceases, and at time $(T_c + T_e)$ --when the gap charge clears out--the arc electron current stops. These events constitute the fast part of the arc; they are followed by a slow migration of positive ions back to the cathode, which is of little importance to the arc radiation.** In any case, the cutoff time T_c can

* Balmain and Dubois (1979). Most detailed data on scaling laws for arcs result from studies involving surface discharges. However, bulk (punch-through) discharges occur in conjunction with the surface discharges, and they apparently obey the same scaling laws. See Flanagan et al. (1979).

** An informative account of the full history of an arc discharge can be found in Karkkainen (1976).

be related to the critical voltage decrease ΔV_c by the capacitive discharge equation, Eq. (16) above. For a specific choice of cathode feed current $i(t)$, this relation will determine most of the arc current dynamics.

It is convenient now to refer the characteristic arc times to the electron avalanche transit time T_e . We define the following scale variables:

$$\begin{aligned} y &= \tau / T_e, \text{ dimensionless integration variable;} \\ z &= t / T_e, \text{ dimensionless real-time variable;} \\ z_c &= T_c / T_e > 1, \text{ dimensionless cutoff time;} \\ \tilde{y} &= y T_e = \tau, \text{ integration time;} \\ \tilde{z} &= z T_e = t, \text{ observation time.} \end{aligned} \quad (18)$$

In these terms the discharge equation, Eq. (16), relates the cutoff time z_c to the critical voltage decrease ΔV_c as follows:

$$(k/T_e) C \Delta V_c = (e^k - 1) \int_0^{z_c} i(\tilde{y}) d\tilde{y} - e^k \int_{z_c-1}^{z_c} \{1 - \exp[k(z_c - 1 - y)]\} i(\tilde{y}) d\tilde{y}. \quad (19)$$

For a given choice of cathode current $i(t)$, this equation can be solved for z_c in terms of ΔV_c .

In the same notation, the history of the arc current occurs in three distinct stages; they are

A. $0 \leq z < 1$. (initial avalanche traversing gap):

$$I(\tilde{z}) = \int_0^z \{\exp[k(z-y)]\} i(\tilde{y}) d\tilde{y}; \quad (20A)$$

B. $1 < z < z_c$ (gap populated up to cutoff time):

$$I(\tilde{z}) = \int_{z-1}^z \{\exp[k(z-y)]\} i(\tilde{y}) d\tilde{y}; \quad (20B)$$

C. $z_c < z < z_c + 1$ (gap empties after cutoff):

$$I(\tilde{z}) = \int_{z-1}^{z_c} \{\exp[k(z-y)]\} i(\tilde{y}) d\tilde{y}. \quad (20C)$$

The scaling laws in Eq. (17) can be compared with the arc duration T_c to see if it scales as $(A_I)^{1/2}$, and with the charge release: $q(T_c) = T_e \int_0^{T_c} i(\tilde{y}) d\tilde{y}$, and peak current: $I(T_c) = I(\tilde{z}_c)$, to see if they scale properly.

To illustrate the calculations outlined here, and to gain insight into the sort of surface currents required to feed an arc, we generalize Dickey's problem [Eqs. (4)-(8) above] to a cathode current

$$i(t) = (q_0/\Delta\tau) \exp(-t/\Delta\tau), \quad (21)$$

which supplies total charge q_0 to the arc, but over a finite time interval on the order of $\Delta\tau$; when $\Delta\tau \rightarrow 0$ here, this $i(t) \rightarrow q_0 \delta(t)$, which is Dickey's impulse current. The cutoff relation, from Eq. (19), is

$$C\Delta V_c = [q_0/k](e^k - 1) - [q_0/(k+h)](e^{k+h} - 1) \exp(-T_c/\Delta\tau), \quad (22)$$

where: $h = T_e/\Delta\tau$. Equation (22) shows that when the cathode charge q_0 flows over a finite time, the capacitor discharge $C\Delta V_c$ is always less than the first term RHS, which is Dickey's result in Eq. (7) when $R \rightarrow \infty$. If the cascade factor k is large enough so that $e^k \gg 1$, then Eq. (22) prescribes the arc cutoff time T_c in terms of the critical voltage drop ΔV_c as

$$T_c = T_e + \Delta\tau \cdot \ln\{ [k/(k+h)] / [1-S] \},$$

$$\text{where: } S = (C\Delta V_c/q_0) k e^{-k}, \quad 0 < S < 1. \quad (23)$$

Because the capacitance $C \propto A_I$ (the area initially irradiated with charge), the arc duration (roughly T_c , when $T_c \gg T_e$) does increase with A_I , but does not go as $T_c \propto (A_I)^{1/2}$, contrary to Eq. (17).

Also, the charge release does not scale as in Eq. (17), since

$$q(T_c) = \int_0^{T_c} i(\tau) d\tau = q_0 \{ 1 - [(k+h)/k] (1-S) \exp[-T_e/\Delta\tau] \}, \quad (24)$$

for this model [Eq. (21)]. That portion of $q(T_c)$ which depends on S does vary linearly with A_I , but the A_I dependence is frozen out when $S \rightarrow 1$ or 0 (large or small discharge). Finally, the peak current for this model is

$$I(T_c) = (q_0/k\Delta\tau)(1-S)e^k. \quad (25)$$

This arc quantity decreases with A_I , contrary to Eq. (17). That the critical arc parameters for the cathode current of Eq. (21) do not scale according to the arcological laws of Eq. (17) simply means that a current of this type cannot be a plausible cathode feed for a bulk discharge arc. But the "incorrect" scaling of the Dickey-type current does suggest that the cathode current feeding a bulk discharge arc should increase, rather than decrease, with time.

Thus we analyze power-law cathode currents, as a simple example of currents that increase with time. These are of the form of Eq. (10); in current notation (with $y = \tau/T_e$)

$$i(\tilde{y}) = (\rho_0 s_n / T_e) y^n, \quad n = 0, 1, 2, \dots \quad (26)$$

The adjustable parameter s_n has the dimensions of an area (and is related to the surface current dynamics), and ρ_0 is the surface charge density (assumed constant, except at the arc site). By using this current in Eq. (19), and making use of the relation

$$e^{kz} \int_{z-1}^z y^n e^{-ky} dy = (n!/k^{n+1}) \sum_{\mu=0}^n (k^\mu/\mu!) [e^k(z-1)^\mu - z^\mu], \quad (27)$$

we find the discharge-cutoff relation

$$z_c^N \sum_{\mu=0}^N B_{N\mu} [kz_c]^{-\mu} \{e^k[1-(1/z_c)]^{N-\mu} - 1\} = (N\Delta V_c / \rho_0 s_n) k,$$

$$\text{where: } N = n+1, \quad B_{N\mu} = N!/(N-\mu)! \quad (28)$$

In this equation, the cutoff time $z_c = T_c/T_e > 1$, and if we assume the cascade factor k is large enough so that $e^k \gg 1$, then an approximate solution is

$$T_c/T_e \approx [S_n]^{1/(n+1)} + [1 - (1/k)],$$

$$\text{where: } S_n = [(n+1)C\Delta V_c / \rho_o s_n] k e^{-k}. \quad (29)$$

This solution holds when both k and $z_c \rightarrow$ large, which in turn means $S_n \rightarrow$ large. Since the (initial) surface charge density: $\rho_o = CV_o/A_I$, where V_o is the voltage at which the arc fires, and A_I is the irradiated area, we can write

$$S_n = [(n+1)(\Delta V_c/V_o) k e^{-k}] (A_I/s_n), \quad (30)$$

so that S_n is directly proportional to A_I , and is large because $A_I \gg s_n$. Then it is evident from Eq. (29) that the term in S_n is dominant, and if we choose $n = 1$, i.e., a cathode feed current linear in time, the cutoff time $T_c \propto (A_I)^{1/2}$. The result reproduces the first of the arcological laws in Eq. (17).

The cathode charge release for a power-law current is

$$\begin{aligned} q(T_c) &= \int_0^{T_c} i(\tau) d\tau = [\rho_o s_n / (n+1)] z_c^{n+1} \\ &= [\rho_o s_n / (n+1)] S_n \{ 1 + [1 - (1/k)] (n+1) S_n^{-1/(n+1)} \}. \end{aligned} \quad (31)$$

For $S_n \rightarrow$ large, the second term in curly brackets is small, and the charge release scales as $S_n \propto A_I$ for all n . This is the second of the arcological laws in Eq. (17). As for the third law, we find the peak arc current

$$\begin{aligned} I(T_c) &= (\rho_o s_n / T_e) \exp(k z_c) \int_{z_c-1}^{z_c} y^n e^{-ky} dy \\ &= (\rho_o s_n e^k / k T_e) [S_n]^{n/(n+1)} \{ 1 + \theta(1/k^2) \}. \end{aligned} \quad (32)$$

Since the charge density ρ_0 is independent of the irradiated area A_I , then the peak current scales with A_I in the same way as S_n , namely: $I(T_c) \propto (A_I)^{n/(n+1)}$. For a linear current, $n=1$, we have the desired square root dependence required by the last law in Eq. (17).

Thus the choice of a cathode current linear in time [i.e., $n=1$ in Eq. (26)], which feeds the arc for a finite time $\sim T_c$, fits the known arcological laws quite well. We summarize the situation in Table II, where the scaling laws predicted by the current theory are compared with observation. We have added a fourth law for which some measurements have been made, namely the area scaling of the arc discharge energy ($\propto \int I^2 dt$). For calculations similar to Eqs. (26)-(32) above, it is easy to show that this energy goes as A_I^p , with $p = (2n+1)/(n+1)$. Again, for $n = 1$, the agreement between theory and experiment is acceptable.

We close this section with two remarks regarding the choice of linear cathode feed current. For $n = 1$, the discharge-cutoff equation, Eq. (28), can be solved exactly. The result is

$$z_c = \{S_1 - [(1/k^2) - M(M-1)]\}^{1/2} + [M - (1/k)],$$

TABLE II. Arc Scaling Rules for Power-Law Cathode Currents^a

Arc Quantity	Scales as $(A_I)^p$	p for Linear Current ($n=1$)	p, Observed ^b
Arc duration T_c (to cutoff)	$p=1/(n+1)$	≈ 0.50	0.46-0.59
Cathode charge release $q(T_c)$	$p=1$	≈ 1.00	1.00-1.06
Arc peak current $I(T_c)$	$p=n/(n+1)$	≈ 0.50	0.51-0.59
Discharge energy $\int I^2 dt$	$p=(2n+1)/(n+1)$	≈ 1.50	1.44-1.64

^aCurrent is: $i(\tau) = (\rho s_n/T_e)(\tau/T_e)^n$, from Eq. (26). The p-values are taken from Eqs. (29)-(32).

^bObserved p-values are from Balmain and Dubois (1979), Figs. 3-6.

where: $S_1 = (\Delta V_c/V_o)[A_I/\pi(uT_e)^2] k/(e^k-1)$,

and: $M = e^k/(e^k-1) \approx 1 + e^{-k}$, for $k \rightarrow \text{large}$. (33)

We have put in the parameter $s_1 = 2\pi(uT_e)^2$ from Eq. (9). u is the uniform velocity of the surface charges that move into the arc site. The parameter S_1 scales directly with the irradiated area A_I , and it dominates the expression for the cutoff time z_c when the cascade parameter $k \rightarrow \text{large}$. Still, z_c does not scale exactly as the square root of A_I ; in fact

$$z_c \propto (A_I)^p, \quad p \approx \frac{1}{2} + \{ [1-(1/k)] \cdot S_1^{1/2} \ln S_1 \}. \quad (34)$$

The correction term (i.e., $p-1/2$) is significant only when $S_1 \rightarrow \text{small}$, i.e., for small arcs from confined surfaces.

Our second remark is to note the arc currents resulting from the linear cathode feed [Eq. (26) with $n = 1$]. By Eq. (20), with $z = t/T_e$, they are

A. $0 \leq t < T_e$ (initial arc traversing gap):

$$I(t) = I_o [e^{kz} - (1+kz)], \quad I_o = \rho_o s_1 / k^2 T_e; \quad (35A)$$

B. $T_e < t < T_c$ (gap populated to cutoff time):

$$I(t) = I(T_e) + I_o k(e^k - 1)(z - 1); \quad (35B)$$

C. $T_c < t < T_c + T_e$ (gap empties after cutoff):

$$I(t) = I_o e^{kz} \{ [1+k(z-1)] - [1+kz_c] \exp[k(z-z_c-1)] \}. \quad (35C)$$

A graph of $I(t)$ vs t is sketched in Fig. 6. Notice that the arc current stops at the anode, in accordance with the remark below Eq. (13).

D. Arc Start and Stop Voltages - the Average Arc

To gain perspective on the calculation at this point, we briefly discuss some general features of the EMP radiated from the arc. The observed EMP of course depends on the observer's coordinates R and θ in Fig. 1; we assume these coordinates are fixed for now. As for the arc itself, the radiation signal is mainly determined by the absolute square of the second time derivative of the arc's dipole moment, i.e. $|\ddot{Id}|^2$, where I is the arc current and d the arc gap-length (Landau and Lifshitz 1951, Sec. 9.2). From Fig. 6, we see that for our model the maximum in $\ddot{I}$ occurs during: $T_c < t < T_c + T_e$, when the gap empties after the cathode current cuts off. Furthermore,

$$|\ddot{Id}| \approx (d/T_e)I(T_c) , \quad (36)$$

Fig. 6. Time-dependence of arc discharge current: a sketch of the arc current $I(t)$ according to Eq. (35). The cathode feed current is linear in time [Eq. (9)]. After the transit time T_e , the arc current increases linearly up to a cutoff time T_c when the cathode current ceases. $I(t)$ then falls exponentially to zero at $(T_c + T_e)$, when the electron avalanche stops. Afterwards, $I(t)$ consists of a slow positive ion current. Notice that the electron current stops at the anode.

in that interval. We shall estimate how this quantity depends on various arc parameters to see what additional arc dynamics are needed to specify the EMP radiation signal.

We calculate the peak current $I(T_c)$ from Eq. (20B), using the power-law cathode current $i(t)$ from Eq. (10), and assuming that the cutoff time $z_c \gg 1$. If also the cascade multiplication factor $e^k \gg 1$, then the dominant contribution is

$$I(T_c) = (CV_o/T_e) [(n+1)\Delta V_c/V_o]^{n/(n+1)} [s_n e^k/A_I k]^{1/(n+1)}. \quad (37)$$

The various symbols are defined elsewhere (see List of Symbols), and we have put in the (initial) surface charge density: $\rho_o = CV_o/A_I$. We next specialize to the $n = 1$ case (linear cathode feed current), and insert the gap capacitance: $C = \kappa A_I/4\pi d$ (cgs units). The arc's radiation intensity (power/area) then scales as

$$|\dot{I}_d|^2 \approx g^2 A_I [(e^k/k^3)V_o \Delta V_c], \quad \text{where: } g = \kappa u v_e \alpha_1 / \sqrt{4\pi}. \quad (38)$$

The parameter g depends only on intrinsic constants of the medium (surface and cascade velocities u and v_e , ionization coefficient α_1 , etc.). The extrinsic factors for the radiation intensity are: A_I (NPB-irradiated area), $k = \alpha_1 d$ (d is the gap length), V_o (voltage at which arc fires), and $\Delta V_c = V_o - V_c$ (discharge voltage of gap capacitance).

The factors in g in Eq. (38) are known, or can be inferred with reasonable accuracy from laboratory data. Also, the variation with A_I and k for the intensity of Eq. (38) is straightforward. What is not known as yet is: at what initial voltage V_o does the arc start, and after what discharge voltage ΔV_c does the arc stop? Experiments show that both the start and stop voltages V_o and $V_c = (V_o - \Delta V_c)$ are statistically distributed (Frederickson 1982), probably because the main discharge mechanism--formation of a cascade via ionization by random collisions--is a chance phenomenon. In this section, we develop a simple probabilistic picture of how the arc starts and stops, and we estimate the peak arc currents for both the most probable and average arcs.

The question of when an arc discharge begins in a spark gap containing a low pressure gas has been investigated by R. A. Wijsman (1949). He calculated the probability P_s that a breakdown occurs by the time the gap voltage reaches a value $V > V_s$, where V_s is the nominal breakdown voltage. His results are

$$P_s = \begin{cases} 1 - (1/w), & w > 1; \\ 0 & , w < 1; \end{cases} \quad w = \gamma [\exp(\eta V) - 1] . \quad (39)$$

Here $\eta = \alpha_1/E \approx E_s$, with α_1 the ionization coefficient and $E \approx E_s$ the electric field, and γ is the probability of liberating an electron at the cathode by any secondary process. The breakdown threshold condition is: $w = 1$, at $V = V_s$; this is a standard result (Loeb 1956). We can write w in a useful form for a parallel plate geometry by noting: $\eta V_s = \alpha_1 d = k$, the cascade factor of Eq. (15) (at threshold), and by defining the arc starting voltage V_o as

$$V_o = V = (1+r)V_s, \quad r = (V_o - V_s)/V_s > 0 . \quad (40)$$

Then the probability factor in Eq. (39) is

$$w = \gamma \{ \exp[k(1+r)] - 1 \} , \quad (41)$$

and the threshold condition becomes: $\gamma [e^k - 1] = 1$.

The distribution of arcs that start in the range V_o to $V_o + dV_o$, or r to $r + dr$, can be found from Eq. (39) as follows. The probability that no arc has occurred up to voltage V_o is $(1-P_s)$, so the probability of an arc occurring in V_o to $V_o + dV_o$ is the amount by which $(1-P_s)$ changes in that interval, namely: $d(1-P_s) = d\bar{P}_s$, or

$$d\bar{P}_s = (-)k \exp(-kr)dr, \quad \text{and:} \quad \bar{P}_s(r) = \exp(-kr) . \quad (42)$$

We have eliminated the parameter γ by use of the threshold condition, and we have assumed the cascade factor k is large enough ($k > 3$, say) so that $e^{-k} \ll 1$. The distribution $\bar{P}_s$ in Eq. (42), for an arc starting at some overvoltage larger than r (with $r > 0$), is analogous to that of the mean free path problem in statistical mechanics: the overvoltage r plays the role of the distance traveled without "collision" (i.e., initiation of a cascade breakdown), while the inverse cascade factor ($1/k$) is the mean free path ("distance" between ionizing collisions). Because $\bar{P}_s$ has this universal character, and because it is independent of parameters specifically tied to low pressure gases, we shall use it to describe the distribution of arc starting voltages V_0 for quantities such as the peak current in Eq. (37).

The stopping or cutoff voltage V_c for the arc is less direct. At least two factors can contribute to this cutoff: (1) some inhibitive condition that develops on the cathode surface and impedes the currents feeding the arc, (2) a diminution of the cascade factor k as the gap voltage drops; the discharge loses its arc-like character when the moving charges no longer have enough energy to multiply by means of ionizing collisions. Factor (1) is less likely to control the arc cutoff than factor (2), for reasons we now discuss.

In Appendix C, we describe a model for the movement of cathode surface currents feeding the arc, as postulated in Eq. (9). Assuming that the surface charge-density is uniform in space but not time, there is little difficulty in using conservation of charge and Gauss' law to show that the required feed currents are plausible; they are driven into the arc site (see Fig. 4) by the large surface fields formed near this charge-depleted region. If the arc site has an effective radius r_0 , then r_0 increases as the arc develops in time. Also, there is a boundary radius $b > r_0$, such that inside b the currents move in toward the arc site (accelerated inward by the surface fields), while outside b the currents move away from the arc site (pushed outward by the interior charge). The schematic motion is shown in Fig. 7. With no dissipation in the model, this motion would continue "forever," i.e., until the entire cathode surface is emptied of charge.

Dissipation could be put in the model by invoking Joule heating or constructing some ad hoc equation of motion on the surface. Then we could define a cathode current cutoff as that time when the heating reached some fraction of the total surface energy or when the velocity field dropped to some fraction of its initial value. We shall not pursue these ideas, however,

because they (a) require more phenomenology (e.g., putting in an effective surface conductivity), (b) do not identify a definite cutoff (the fractional heating or velocity drop at cutoff is problematic), (c) give no simple picture of what statistical distribution the cutoff voltages might have. Also, in an arc discharge, if we claim the arc is "hot" but the surface is not, then surface dissipation cannot dominate the arc cutoff. Thus, to seek a cutoff criterion, we turn to factor (2) above: decline of the arc cascade factor k as the gap voltage decreases during the discharge process.

As far back as Eq. (7), we assumed that $k = \alpha_1 d$ is constant over the arc's voltage range. This approximation is useful and reasonably accurate if the arc

Fig. 7. Schematic of motion of cathode surface currents. r is the radial distance from the arc site (see Fig. 4), which has a radius $r_0(t)$ that grows in time. The charge depletion inside the arc site induces a strong surface electric field $F(r,t)$ which accelerates electrons into the arc site, thus feeding the ongoing arc; this acceleration occurs in a range $r_0(t) < r < b(t)$. At $r > b(t)$, the motion reverses because of the fields due to the interior charge. Details appear in Appendix C.

gap voltage V lies above, or not too far below, the dielectric breakdown voltage V_g . For example, in the case of a gas (Papoular 1965).

$$k(V) = k_{\infty} \exp(-\phi/V) , \quad (43)$$

where ϕ is a reference voltage on the order of V_g , and $k_{\infty} \sim 10$ is the maximum attainable cascade factor (Jones 1956). When V exceeds or is comparable to $\phi \sim V_g$, the cascade factor k does not change rapidly, and it is a large enough fraction of k_{∞} to maintain a respectable avalanche, as required by the gap discharge equations [Eqs. (15) and (16)]. However, when V is significantly less than ϕ , k decreases rapidly. By the time $k \rightarrow 1$, which corresponds to $V \rightarrow \phi / \ln k_{\infty}$, the avalanche is quite weak and it is in danger of statistical extinction, because the average avalanche electron has enough energy for only one ionizing collision during its trip across the gap.

These remarks suggest that the arc which starts at voltage $V_0 > V_g$, with the distribution $\bar{P}_s$ of Eq. (42), will stop at a voltage V_c which is at or slightly below the threshold V_g . Arcs stopping at voltages V_c significantly less than V_g are possible but are not probable because of the concomitant large reduction in the cascade factor k . If we write the cutoff voltage in a way which parallels Eq. (40)

$$V_c = (1 - s)V_g, \quad s = (V_g - V_c)/V_g > 0 , \quad (44)$$

then a reasonable distribution of cutoff voltages is given by

$$P_c(s) \approx \exp(-\beta s), \quad \beta \approx k. \quad (45)$$

We justify this choice in Appendix D by an argument based on a mean free path analysis, using the analogies discussed below Eq. (42). We set $\beta \approx k$ (when $V_c \lesssim V_g$) because that is the initial size of the cascade factor to be reduced as the stopping "distance" s increases. The arc-stop distribution $P_c(s)$ agrees with the above intuitive notions that most arcs will stop at small s -values

(i.e., stopping voltages $V_c \lesssim V_s$) before their cascade factors become seriously diminished (i.e., before $k \rightarrow 1$).

With the arc start and stop distributions, $\bar{P}_s(r)$ and $P_c(s)$ of Eqs. (42) and (45), we can analyze some statistical features of the discharge. We shall work with the square of the arc peak current, because--according to Eq. (36)--that quantity is proportional to the maximum power developed by the arc, and so it determines the arc's radiation output. From Eq. (37), with $n = 1$, $V_0 = (1+r)V_s$, $\Delta V_c = (r+s)V_s$, etc., we find

$$I_c^2(r,s) = I_s^2(1+r)(r+s),$$

$$\text{where: } I_s = (2CV_s/T_e) [(e^k/k)\pi(uT_e)^2/A_I]^{1/2}. \quad (46)$$

Here $I_c = I(T_c)$ is the peak current at the cutoff time T_c , and it depends on the arc start and stop voltages through r and s of Eqs. (40) and (44). I_s is a scale current that depends on the properties of the medium. If we assume r and s are uncorrelated, with the distributions $\bar{P}_s(r)$ and $P_c(s)$ as above, then the distribution of I_c^2 values, starting from r and ending on s , obeys

$$\partial^2 [I_c^2] / \partial r \partial s = [k\beta I_s^2] G(r,s),$$

$$\text{where: } G(r,s) = (1+r)(r+s)e^{-kr}e^{-\beta s}. \quad (47)$$

$G(r,s)$ is the relative probability of finding an arc at power $I_c^2(r,s)$.

In Fig. 8, we plot $G(r,s)$ vs the fractional discharge voltage: $(r+s) = \Delta V_c/V_s$, for various stop voltage ratios s , and with the cascade parameters $\beta = k = 5$. From the plot, it is clear that most of the arcs occur with relatively small values of r and s . In Appendix E, we calculate that (for $k = 5$) at least 50% of the arcs fire with a combined start-stop total: $(r+s) < 1$, so long as neither r nor s is too small. This is made clear in Fig. 9, which shows the fractional number of arcs firing at power levels $I_c^2(\xi, \eta)$, with ξ and η lying in the ranges: $0 \rightarrow r = 0.7$, $0 \rightarrow s = 0.7$. These ranges correspond to a fractional voltage drop: $\delta = \Delta V_c/V_0 =$

$(r+s)/(1+r) = 0.824$. For such large arcs, according to Appendix B, hitherto neglected arc inductive effects may become important.*

Finally, in Fig. 10, we show how the arc firing fraction (power levels in $0 \rightarrow r = s$) depends on the cascade factor k . If we relax the inductive limit* to accommodate ranges: $0 \rightarrow r = 1$, $0 \rightarrow s = 1$, then--from Fig. 10--the percentage of arcs firing at: $(r+s) \leq 2$, is

k	3	4	5	6	7	8
% firing	82.3	90.7	95.9	98.2	99.3	99.7

(48)

Two points are noteworthy: (1) the variation in % firing is not rapid above $k = 3$, (2) many more "small" arcs become possible as k increases. The fact that the % firing falls off as k decreases does not mean that the theory fails to describe most low- k arcs. On the contrary, the inductive limit disappears completely as k decreases,* the $(r+s)$ range is unlimited, and all arcs are covered. It is reasonable to claim that >90% of all arcs occurring under the present conditions can be accurately described by the current version of the theory. On the other hand, the power levels of very large and infrequent arcs ($r, s > 1$) are likely to be overestimated.

From Fig. 8, it is clear that the arc power distribution $G(r, s)$ has a maximum at small values of r and s . This occurs when

*The estimate in Eq. (B4) of Appendix B for an upper limit on δ (beyond which inductive effects dominate) probably should be raised, because that estimate was done for a worst-case scenario, at the cutoff time T_c , when the inductive energy is maximum and the capacitive energy minimum. If, instead, we calculate the average values of these energies over the interval $0 \rightarrow T_c$, form the ratio $L(\text{energy})/C(\text{energy})$, and insist that ratio be $\leq$ some fraction ℓ , then the upper limit on δ is: $\delta \leq \frac{3}{2} / [1 + (\chi/\ell)]$, where: $\chi = (2\pi m_e / m_L)(u/c)^2(e^k - 1)/k$, and the notation is the same as in Eq. (B4). This revised upper limit does not fall below unity, and thus does not seriously affect applicability of the noninductive version of our arc discharge theory, except in an extreme case such as: $\ell = 10\%$, $u \approx v_e = 10^8$ cm/sec, and $k \approx 8$. Even "large" arcs are probably well accounted for in the present theory.

$$\partial G / \partial r = 0, \text{ or: } r + s = \frac{1}{k} \{1 + [1/(1-s)k]\} . \quad (49)$$

The approximation is good for large k (i.e., $k > 5$) and for small s (i.e., $s < 1/k$). By Eq. (46), the current for this choice of r and s is

$$\bar{I}_c(r,s) \approx I_s [(1-s)/k]^{1/2} \{1 + [1/(1-s)k]\} . \quad (50)$$

To estimate magnitudes, we calculate the scale current I_s of Eq. (46). In convenient units: u = surface current velocity in 10^7 cm/sec, $\langle kE_s \rangle$ = effective breakdown field in MV/cm, A_I = irradiated area in m^2 , we find

Fig. 8. Distributions of arc power $I_c^2(r,s)$ vs total discharge voltage $(r+s)$ at various cutoff voltages s . Plotted is the distribution $G(r,s)$ of Eq. (47) vs the discharge voltage fraction: $\Delta V_c / V_o = r+s$, at several values of the stop voltage ratio: $s = (V_s - V_c) / V_s$. The ordinate gives the relative probability of observing an arc at particular values of r and s . Few arcs occur when $(r+s)$ exceeds unity.

$$I_s = 314 \mu < \kappa E_s > [A_I (e^k - 1)/k]^{1/2}, \text{ amps.} \quad (51)$$

Here we have replaced the avalanche factor (e^k/k) , for $k \rightarrow \text{large}$, by the form $(e^k-1)/k$, appropriate for all k .^{*} Then I_s is finite even as $k \rightarrow 0$. For typical values: $m = 1.5$, $u = 1$, $< \kappa E_s > \sim 0.69$, $A_I \sim 0.5$, $k = 5$, we find $I_s \sim 1250$ A. The arc current of Eq. (50) is then $I_c \sim 600$ A. Such large currents have actually been observed in surface and substrate discharges on board operating spacecraft, and also during laboratory simulations of arcing in charged dielectrics.^{**}

A putatively more useful measure of typical arc behavior is the average power level w.r.t. the distribution $G(r,s)$ in Eq. (47). If we normalize the cutoff distribution $P_c(s)$ to unity over the maximum range $0 \rightarrow s = 1$, then the average peak power level is

$$< I_c^2 > = \int_0^\infty dr [k e^{-kr}] \int_0^1 ds [\beta e^{-\beta s} / (1 - e^{-\beta})] I_c^2(r,s) = I_s^2 \bar{f}(k,\beta),$$

$$\text{where: } \bar{f}(k,\beta) = \frac{1}{k} \left(1 + \frac{2}{k} \right) + \frac{1}{\beta} \left(1 + \frac{1}{k} \right) \{ 1 - [\beta / (e^\beta - 1)] \}. \quad (52)$$

For simplicity, we take $\beta = k$ [see Eq. (D6) in Appendix D], and for illustration, we calculate the following numbers:

k	3	4	5	6	7	8
$\bar{f}(k,k)$	0.930	0.664	0.512	0.414	0.346	0.296
$[\bar{f}(k,k)(e^k-1)/k]^{1/2}$	2.433	2.983	3.885	5.268	7.358	10.51
$I_{rms}, \text{ amps}$	323	396	515	699	976	1394

(53)

*The factor $(e^k-1)/k$ follows from Eq. (35B), at cutoff: $t = T_c$. In later equations [e.g., Eqs. (37), (38), (46)] we approximate this factor by (e^k/k) , when $k \rightarrow \text{large}$. The approximation is good to 5% when $k > 3$.

**Balmain (1986). Substrate discharge currents may also be on the order of hundreds of amps; see Frederickson (1982) and Flanagan et al. (1979), Table I.

The function in the third line gives the k -dependence of $\langle I_C^2 \rangle^{1/2}$; it consists of $\bar{f}(k,k)$ from Eq. (52), and the factor $(e^k - 1)/k$ in Eq. (51). In the last line, we have calculated: $I_{rms} = [\langle I_C^2 \rangle / 3]^{1/2}$, which is the root-mean-square arc current averaged over the interval $0 < t < T_C$. We have used typical values in calculating I_s of Eq. (51): $m = 1.5$, $u = 1$, $\langle kE_s \rangle \sim 0.69$, $A_I \sim 0.5$. Again, the average arc currents are on the order of hundreds of amps.

The calculations in this section provide both the average and distributional power levels developed by bulk discharge arcs in electrically stressed dielectrics. The present theory describes more than 90% of the arc

Fig. 9. Fraction of arc power levels $I_C^2(\xi, \eta)$ with start and stop ratios in ranges: $0 < \xi < r$, $0 < \eta < s$. Plotted is the fraction $\mathcal{F}(r,s)$ of Eq. (E3) in Appendix E vs the start voltage ratio: $r = (V_0 - V_s)/V_s$, at several values of the stop voltage ratio: $s = (V_s - V_c)/V_s$. The assumed cascade factor is $k = 5$; 50% of the arcs fire in the range: $0 \rightarrow r = 0.45$, $0 \rightarrow s = 0.45$ and over 70% in the range: $0 \rightarrow r = 0.6$, $0 \rightarrow s = 0.6$.

events (very large, infrequent, inductance-limited arcs are excluded), and it is consistent with arc currents of the order of hundreds of amps for discharge areas of the order of m^2 (and for cascade factors: $3 < k < 7$). We can now proceed to estimate the radiation signals from such arcs.

E. AGEEMP: Arc-Generated Electromagnetic Pulse

The calculations of Secs. III-C and III-D give the form, frequency-of-occurrence, and size of arc currents expected during dielectric bulk discharge breakdowns. The form of the currents is given by Eq. (35) and Fig. 6; the

Fig. 10. Fraction of arc power levels $I_c^2(\xi, \eta)$ with start and stop ratios in: $0 < \xi < r$, $0 < \eta < s$, vs cascade factor k . Here the k -dependence of $\mathcal{F}(r, s)$ [Eq. (E3) of Appendix E] is shown for various values of the total discharge parameter $(r+s)$. The firing fraction $\mathcal{F}$ increases with k , and many more "small" arcs can occur. For example, when k increases from 5 to 7, the fraction of sparks with $(r+s) < 0.7$ increases from 40% to 60%.

frequency-of-occurrence is detailed in Eqs. (47)-(50), Appendix E, and Figs. 8-10; the size is some fraction of I_s in Eq. (51) [see also Eq. (53)]. Our arc discharge model is consistent with known--if somewhat scanty--data on laboratory arcs (laws of arcology, size of currents, etc.). Thus, to assess arc detection during NPB-target interactions, we shall use the model as is to estimate the AGEMP (arc-generated electromagnetic pulse) for a distant observer (see Fig. 1). In this section, we calculate the AGEMP at points far distant from the arc, assuming that the arc electron current stops at the anode, in accordance with Eq. (35) and Fig. 6. In effect, the arc strikes a metal surface at the end of its travel.

With reference to Fig. 1, let the arc axis be the z' -axis of an $x'y'z'$ coordinate system. The arc current density along z' is then

$$\vec{J}(x', y', z'; t') = \hat{z} I(t', z') \delta(x') \delta(y'), \quad (54)$$

where $\hat{z}$ is a unit vector along z' , I is the arc current, and the delta functions in x' and y' imply localization in those coordinates. Since the radiating arc has dimensions $d \sim 1$ cm, much less than the broadcast wavelength $\lambda \sim cT_e \sim 300$ cm, in turn much less than the observation distance $R \sim 100$ km, we calculate the vector potential $\vec{A}$ (observer) for the above $\vec{J}$ in the radiation zone approximation (Jackson 1975, Chap. 9, Sec. 1). Thus, in cgs units

$$\vec{A}(\vec{R}, t) = (1/cR) \int \vec{J}(\vec{r}'; t_R) d^3r' = (\hat{z}/cR) \int I(t_R, z') dz', \quad (55)$$

where $t_R = t - (R/c)$ is the retarded time. The radiation fields for the observer are then (Landau and Lifshitz 1951, Sec. 14.1).

$$\vec{cB}_R = (\partial \vec{A} / \partial t) \times \hat{n}, \quad \vec{cE}_R = \vec{B}_R \times \hat{n}, \quad (56)$$

where $\hat{n}$ is a unit vector along $\vec{R}$ (i.e., from arc to observer). In this formulation, the observation distance R must remain constant over time

intervals characteristic of radiation from the source; this condition is well met in the case of arcing from a target irradiated by an NPB.

The integral over the arc current I in Eq. (55) requires the spatial distribution of that current. In the present formulation, we should begin with: $I = v_e \lambda(z', t)$, where $\lambda(z', t)$ is the linear charge density of Eq. (12), and then evaluate $\int I dz'$ over the arc gap spacing: $0 < z' < d$. But this is just what we did in Eqs. (13) and (14) of Sec. III-B, when we defined the average arc current: $I(t) = Q(t)/T_e$. Thus we can interpret $\int I dz'$ in Eq. (55) as equivalent to $I(t)d$, where $I(t)$ is the appropriate average current from Eq. (35), and d is the arc gap spacing. Also, we can write

$$\partial \vec{A} / \partial t = \hat{z}(d/cR) \dot{I}(t_R) , \quad (57)$$

which determines the arc radiation fields through Eq. (56). For example, the electric field at the observer is

$$\vec{E}_R = [(d/c^2 R) \dot{I}(t_R)] \{ \hat{n} \cos \theta - \hat{z} \} , \quad (58)$$

where θ is the azimuthal angle shown in Fig. 1. This field vanishes on the arc axis ($\theta = 0$).

For assessing detectability of the arc radiation, we calculate the Poynting vector, which gives the observed power per unit area

$$\vec{N}_R = (c/4\pi) \vec{E}_R \times \vec{B}_R = \hat{n} (\sin^2 \theta / 4\pi c R^2) [(d/c) \dot{I}(t_R)]^2 . \quad (59)$$

We can keep track of the radiated electric field magnitude E_R by observing that: $E_R^2 = (4\pi/c) N_R$.^{*} As was noted and used in Sec. III-D, Eq. (38), the arc's radiation intensity scales as $(\dot{I}d)^2$.

The current derivatives, for the three time regimes in Eq. (35), are

^{*}Here N_R , the magnitude of the Poynting vector, should not be confused with the index N , which occurs in Sec. III-C, Eq. (28).

A. $0 < t < T_e$ (initial arc traversing gap):

$$\dot{I} = (kI_0/T_e)[e^{kz} - 1], \quad z = t/T_e; \quad (60A)$$

B. $T_e < t < T_c$ (gap populated to cutoff time):

$$\dot{I} = (kI_0/T_e)[e^k - 1]; \quad (60B)$$

C. $T_c < t < T_c + T_e$ (gap empties after cutoff):

$$\dot{I} = -(kI_0/T_e)[(1+kz_c)e^{k(z-z_c)} - e^k], \quad z_c = T_c/T_e. \quad (60C)$$

The current I_0 is defined in Eq. (35A), and we can write

$$kI_0/T_e = \frac{1}{2}\mu u^2 \langle \kappa E_s \rangle (1+r)/k, \quad (61)$$

after inserting the initial charge density $\rho \rightarrow \rho_0 = CV_0/A_I$, the gap capacitance $C = \pi \kappa A_I / 4\pi d$, the arc starting voltage $V_0 = (1+r)V_s$, and the area parameter $s_1 = 2\pi(uT_e)^2$. The power radiated by the arc, viz. $N_R \propto \dot{I}^2$, is sketched as a function of time in Fig. 11; the result is the radiation rate for the arc current of Fig. 6 (corresponding to a linear cathode feed).

Several features of the arc radiation are worth noting. First, and most important: we attribute all arc accelerations and decelerations comprising $\dot{I}^2$ to the natural motion of the arc current I itself, i.e., the electron current I of Eq. (35). The fact [noted in Eq. (13)] that we have ignored current flow past the anode means that the electrons stop there; in effect, the target anode is a thick metal surface where the arc current spreads out (perpendicular to the arc axis) at the end of its trip across the gap. At this endpoint, when the electron current changes direction, there will be corrections to the arc radiation calculated from Eqs. (59) and (60). However, if the electrons spread out at all symmetrically on the anode surface, these corrections are relatively small because the resultant radiation fields tend to interfere destructively; if the main arc radiation fields are dipolar, the endpoint fields are

quadrupolar. Thus, we ignore endpoint corrections and calculate the radiation from Eqs. (59) and (60): this radiation is for a bulk discharge arc that

Fig. 11. History of arc radiation rate. The arc radiates energy at a rate proportional to i^2 . Here that rate is plotted vs time, for the i of Eq. (60); the result is the radiation rate for the arc current of Fig. 6. As the arc begins, over $0 \rightarrow T_e$ (transit), the rate builds up to a constant value, which persists over $T_e \rightarrow T_c$ (cutoff), when the arc gap is filled with charge. This is followed by a large transient, over $T_c \rightarrow T_c + T_e$, when the gap empties and the arc current stops. The discontinuity in i^2 at $t = T_c$ can be removed by assuming the arc cutoff occurs during a finite time interval.

terminates on a metal surface. To be conservative, we shall not include radiation from the motion of image charges on the other side of the metal surface; the "image arc" can only increase our radiation signal (see comment 9 in Sec. III-G).

A second noteworthy feature of Fig. 11 is that the discontinuity in $\dot{I}$ at $t = T_c$ is factitious: it results from assuming the arc current feed cuts off instantaneously at T_c . If the cutoff occurs over a finite time, then $\dot{I}$ will be continuous at T_c . Finally, it is worth mentioning that the arc duration depicted in Fig. 11 (and Fig. 6), i.e., $0 \rightarrow (T_c + T_e)$, depends on the arc size. This follows from Eqs. (29) and (33), which give the cutoff time

$$z_c = T_c/T_e \approx \{[(r+s)/(1+r)]^{1/2} \{[A_I/\pi(uT_e)^2]k/(e^k-1)\}^{1/2} \}. \quad (62)$$

Here r and s are the start and stop voltage ratios of Eqs. (40) and (44), respectively. Clearly, large arcs last longer than small ones and correspondingly broadcast more energy.

As suggested by Fig. 11, the arc radiates most strongly during $T_c \rightarrow (T_c + T_e)$, when the gap empties after cutoff. For a quantitative measure of this, we compare the energy W_R radiated before cutoff

$$W_R(t < T_c) \propto z_c \times [\dot{I}(t < T_c)]^2 \sim z_c [(kI_0/T_e)(e^k-1)]^2, \quad (63)$$

with the energy radiated between T_c and $(T_c + T_e)$

$$W_R(t > T_c) \propto 1 \times [I(t > T_c)]^2 \sim z_c W_R(t < T_c). \quad (64)$$

Here we have used the average of $\dot{I}$ from Eq. (60C), and we have assumed k is large enough so e^{-k} is negligible. We see that the radiation energy after cutoff is z_c times larger than before. From Eq. (62), $z_c \sim 50$ for typical arcs, so the post- T_c radiation certainly dominates the pre- T_c signal. This is because the charge increment in the gap after T_c is much larger than before T_c . In fact, we shall ignore the latter, i.e., we shall fix attention on the AGEMP produced during $T_c \rightarrow (T_c + T_e)$, the period when the gap empties.

Before proceeding to calculate the Poynting vector N_R for the arc radiation, we make one more approximation: we adjust I of Eq. (60C) so there is no discontinuity at $t = T_c$. Removing the discontinuity can be justified on grounds of a finite interval for the cutoff time and can most easily be done by subtracting kz_c from the square bracket in Eq. (60C). This subtraction is a conservative step, because it reduces the apparent arc power. Then, during $T_c \rightarrow (T_c + T_e)$

$$\dot{I} = -(kI_0/T_e) \{ (1+kz_c) [e^{k(z-z_c)} - 1] - [e^k - 1] \} . \quad (65)$$

The values at the endpoints of the interval are

$$\dot{I}(T_c) = +(kI_0/T_e)(e^k - 1), \quad \dot{I}(T_c + T_e) = -kz_c \dot{I}(T_c). \quad (66)$$

$\dot{I}$ is now continuous at $t = T_c$, and changes sign in the interval.

To estimate the arc radiation signal, we put $\dot{I}$ of Eq. (65) into Eq. (59). The magnitude of the Poynting vector is then

$$N_R = (\sin^2 \theta / R^2) \mathcal{P} [j(z; k; r, s)]^2, \quad \text{for } T_c \rightarrow (T_c + T_e),$$

where: $\mathcal{P} = (1/16\pi c^3) [(1+r) \langle \epsilon E_s \rangle \mu^2 d (e^k - 1) / k]^2$,

and: $j(z; k; r, s) = \{ (1+kz_c) [e^{k(z-z_c)} - 1] - (e^k - 1) \} / (e^k - 1).$ (67)

$\mathcal{P}$ has the dimensions of power; in convenient units

$$\mathcal{P} = 8.19 \times [(1+r) \langle \epsilon E_s \rangle \mu^2 d (e^k - 1) / k]^2, \quad \mu W, \quad (68)$$

where $\langle \epsilon E_s \rangle$ is the target dielectric breakdown field in MV/cm, μ is the cathode current velocity in units of 10^7 cm/sec, and d is the arc gap length in cm. The other parameters are dimensionless: r = arc start voltage ratio [Eq. (40)], m = capacitance charge factor [Eq. (2)], and k = arc cascade factor [Eq. (15)]. The current function in Eq. (67), namely $j(z; k; r, s)$, ranges from -1 to $+kz_c$

over the interval, so the energy radiated from the arc will exceed $\mathcal{P}T_e$, by a factor $\sim(kz_c)^2$ --which can be large.

In detail, the AGEMP energy radiated between T_c and $(T_c + T_e)$, per unit area at the observer, is (for $kz_c \rightarrow \text{large}$)

$$\int N_R dt = (\sin^2 \theta / R^2) W_0 [(1+r)(r+s)] ,$$

$$\text{where: } W_0 = (A_I d / 32\pi^2) (v_e / c) [(\mu / c) \langle \kappa E_s \rangle]^2 e^k . \quad (69)$$

W_0 has the dimensions of an energy, and these arc energies obey the same start-stop distribution as the power levels of Eq. (47), et seq. Since the arc radiation electric field is related to N_R via: $E_R^2 = (4\pi/c) N_R$, we can define a time-averaged field over the peak radiation interval $T_c \rightarrow (T_c + T_e)$ by

$$\begin{aligned} \overline{E}_R &= [(4\pi/c) \frac{1}{T_e} \int N_R dt]^{1/2} \\ &= \frac{\sin \theta}{R} [(uv_e/c^2)_m \langle \kappa E_s \rangle] [(A_I e^k / 8\pi) (1+r)(r+s)]^{1/2}. \end{aligned} \quad (70)$$

If the observation distance R is in units of 1000 km, u and v_e are in units of 10^7 and 10^8 cm/sec, respectively, $\langle \kappa E_s \rangle$ is in MV/cm, and the target irradiation area A_I is in m^2 , then the arc radiation field of Eq. (70) is

$$\overline{E}_R = 22.2 \times \frac{\sin \theta}{R} [(1+r)(r+s) A_I e^k]^{1/2} \mu v_e \langle \kappa E_s \rangle , \text{ } \mu\text{V/m}. \quad (71)$$

Typically: $r \sim 0.3 \sim s$, $A_I \sim 0.5$, $k = 5$, $m = 1.5$, $u \sim 1 \sim v_e$, and $\langle \kappa E_s \rangle = 0.69$. These values give: $\overline{E}_R \sim (0.175 \text{ mV/m}) \times (\sin \theta / R)$, which is easily detectable (at distances $R \sim$ several hundred km) by a standard radio receiver, at least in CW operation (Reference Data for Radio Engineers 1968, Chap. 28). However, this arc signal only lasts for a time interval $T_e \sim 10$ ns, so it is relevant to look at the power spectrum of the arc radiation.

Abbreviate the current function in Eq. (67) by $j(t)$ and denote its Fourier transform by

$$\zeta(\nu) = (1/\sqrt{2\pi}) \int_{-\infty}^{\infty} j(t) \exp(-i\nu t) dt . \quad (72)$$

Parseval's theorem states that

$$\int_{-\infty}^{\infty} dt |j(t)|^2 = \int_{-\infty}^{\infty} d\nu |\zeta(\nu)|^2 . \quad (73)$$

This allows converting the arc radiation energy $\int N_R dt$ from a time to a frequency domain, i.e., $\int \tilde{N}_R d\nu$, provided that

$$\tilde{N}_R(\nu) = (\sin^2 \theta / R^2) \mathcal{P} |\zeta(\nu)|^2 . \quad (74)$$

$\tilde{N}_R$ is the radiation energy per unit area and unit frequency range at the observation point (R, θ) . For the current function $j(t)$ of Eq. (67) the Fourier transform is

$$|\zeta(\nu)|^2 = \frac{1}{2\pi} (T_e z_c)^2 / [1 + (\nu/\Delta\nu)^2] ,$$

$$\text{where: } \Delta\nu = k/T_e = \alpha_i \nu_e . \quad (75)$$

Here we have assumed kz_c and e^k are both $\gg 1$; this is true for large arcs. The transform $|\zeta(\nu)|^2$ is Lorentzian, which follows from the dominant exponential time dependence of $j(t)$. The halfwidth Δ of $|\zeta(\nu)|^2$, converted to the product $\alpha_i \nu_e$ by use of Eq. (15), is a constant (except for field variation) for all arcs in a given material. In any case, using $|\zeta(\nu)|^2$ of Eq. (75) in Eq. (74), we get an approximate power spectrum for the arc radiation.

Assume that at the observation point (R, θ) in Fig. 1, there is a radio receiver tuned to frequency f , with a bandwidth δf about f . The energy at the receiver per unit area and per arc (of duration $T_e \sim 10$ ns) is

$$\tilde{N}_R(\nu) \delta\nu = \frac{\sin^2 \theta}{R^2} (k \mathcal{P} T_e z_c^2 / 2\pi) [1 + (f/\Delta f)^2]^{-1} (\delta f / \Delta f) . \quad (76)$$

$\Delta f = k/2\pi T_e$ is the frequency HWHM of the arc power spectrum in Eq. (75); in a typical case ($k = 5$, $T_e = 10$ ns): $\Delta f \sim 80$ MHz. The average power/area at the receiver, during the arc broadcast period T_e and in the bandwidth δf at frequency f , is then

$$\begin{aligned} \mathcal{S}(f, \delta f) &= (1/T_e) \mathcal{N}_R(v) \delta v \\ &= [k \mathcal{P} z_c^{2/2\pi}] \frac{\sin^2 \theta}{R^2} [1 + (f/\Delta f)^2]^{-1} (\delta f / \Delta f) . \end{aligned} \quad (77)$$

$\mathcal{P}$ is the power parameter of Eq. (67), and z_c is the (dimensionless) arc cutoff time of Eq. (62). We take $\mathcal{S}$ to be the useful radio signal strength available to the observer.

The scale of arc signal strengths is set by the coefficient

$$\begin{aligned} k \mathcal{P} z_c^{2/2\pi} &= \frac{c A_I}{32\pi^3} [m \langle \kappa E_s \rangle (u v_e / c^2)]^2 (e^k - 1) (1+r) (r+s) \\ &= 0.415 \times A_I [m \langle \kappa E_s \rangle u v_e]^2 (e^k - 1) (1+r) (r+s), \text{ watts} . \end{aligned} \quad (78)$$

The numerical factor results from conversion of units; we have made no assumptions yet about the size of the physical variables. Factors with units are: A_I = NPB-irradiated target area in m^2 , $\langle \kappa E_s \rangle$ = target dielectric effective breakdown field in MV/cm (Table I), u = cathode current velocity in 10^7 cm/sec, v_e = arc cascade velocity in 10^8 cm/sec. Dimensionless factors are: m = capacitance charge multiplier [Eq. (2)], k = arc cascade factor [Eq. (15)], r and s = start and stop voltage ratios [Eqs. (40) and (44)].

A numerical value for the arc signal strength is obtained by putting Eq. (78) into Eq. (77). We set the irradiated area: $A_I = \pi (D \Delta \phi / 2)^2 \sec \alpha$, as in Eq. (2), and we measure the accelerator-target distance D and target-observer distance R in units of 1000 km. The beam divergence $\Delta \phi$ is in μrad . Then the signal strength observed in bandwidth δf at frequency f is

$$\mathcal{S}(f, \delta f) = N \mathcal{S}_0 (D \Delta \phi \sin \theta / R)^2 \sec \alpha [1 + (f/\Delta f)^2]^{-1} (\delta f / \Delta f),$$

$$\text{where: } N = [m \langle \kappa E_g \rangle_{uv_e}]^2 (e^k - 1) (1+r) (r+s),$$

$$\text{and: } \mathcal{S}_0 = 3.26 \times 10^{-14} \text{ mW/cm}^2.$$

(79)

This result is the main point of this calculation. For specific numerical work, we use "typical" values: $m = 1.5$, $u \sim 1 \sim v_e$, $\langle \kappa E_g \rangle \sim 0.69$, $k = 5$, $\Delta \phi = 1$ (in units defined above), we measure the detector bandwidth δf in units of 100 kHz, and we set $\Delta f = 80$ MHz [per remark below Eq. (76)]. Also, we take the average orientation angles: $\theta \sim 45^\circ \sim \alpha$ (see Fig. 1). Then

$$\mathcal{S} \sim 4.55 \times 10^{-15} \times (1+r)(r+s)(D/R)^2 [1 + (f/\Delta f)^2]^{-1} \delta f, \text{ mW/cm}^2. \quad (80)$$

Notice that this signal [and the more general form of Eq. (79)] has the same statistical distribution over values of r and s as the peak power I_c^2 in Eqs. (46)-(53). In particular, the distribution of broadcast $\mathcal{S}$ -values is described by Figs. 8-10.

As for detectability of the arc broadcast, let the observer's receiver be tuned to $f \sim \Delta f = 80$ MHz, and consider an "average" arc, with start and stop voltage ratios: $r \sim 0.5 \sim s$. Then Eq. (80) gives

$$\mathcal{S} \sim 3.41 \times 10^{-15} \times (D/R)^2 \delta f, \text{ mW/cm}^2, \quad (81)$$

for the arc signal power density. Suppose the target and observer distances are comparable: $D \sim R \sim 1000$ km, and the receiver bandwidth $\delta f \sim 1(\times 100 \text{ kHz})$. Then the nominal arc signal at the observer is: $\mathcal{S}(\text{nominal}) \sim 3.41 \times 10^{-15} \text{ mW/cm}^2$. We note the following:

- 1) $\mathcal{S}(\text{nominal})$ corresponds to a peak radiation electric field $E_R \sim 5 \text{ } \mu\text{V/m}$ at the receiver,* located 1000 km from the target and operating at 80 MHz with

*For a plane EM wave, the peak electric field E_R (in $\mu\text{V/m}$) and average energy flux (in mW/cm^2) are related by: $= 1.33 \times 10^{-16} E_R^2$ (Panofsky and Phillips 1955).

bandwidth 100 kHz. This signal arrives as a short pulse (~ 10 -ns width) at wavelength 3.75 m.

- 2) If $\mathcal{S}(\text{nominal})$ is collected by a parabolic antenna of aperture diameter Δ (in m), with effective area: $A_{\text{eff}} = 0.54(\pi\Delta^2/4)$ (Reference Data for Radio Engineers 1968, Chap. 26), then the power to the receiver is: $\mathcal{S}(\text{nom.}) \times A_{\text{eff}} \sim 1.45 \times 10^{-11} \Delta^2$ mW. If $\Delta \sim 1$, this power level amounts to a receiver input of ~ 0.85 μ V across a (matched) 50- Ω load. Also, this arc signal is ~ 16 dB above the ambient thermal noise power (at 290 K) over the same 100-kHz bandwidth.
- 3) For external noise, we note that the power density of galactic noise in a bandwidth δf (in 100 kHz) at frequency f (in 100 MHz) is (Clodius 1986a)

$$A\delta f = (5.8 \text{ to } 48) \times 10^{-18} [\delta f/f^{0.7}], (\text{mW/cm}^2)/\text{steradian} . \quad (82)$$

For $\delta f = 1$, $f = 0.8$, and a nondirectional detector: $4\pi A\delta f = (8.5 \text{ to } 70) \times 10^{-17}$ mW/cm², which is only a factor of 5 to 40 times smaller than the arc signal $\mathcal{S}(\text{nominal})$. However, if the antenna is directional to just ~ 0.1 steradian, $\mathcal{S}(\text{nominal})$ will be ~ 17 to 26 dB above galactic noise.

- 4) At frequencies ~ 100 MHz in the exosphere, noise sources other than galactic (quiet sun, earth-produced, or artificial) are probably not significantly larger than galactic noise (Clodius 1986a). Then the arc signal $\mathcal{S}(\text{nominal})$, received by a moderately directional antenna at distance $R \sim 1000$ km from the target, appears to be ~ 15 dB above internal and external noise.

On these grounds, we pronounce the nominal arc signal to be detectable, at distances up to 1000 km, by radio receivers (with suitable antennas) capable of operating at a detection threshold of $\sim (-)105$ dBm.

Several remarks are in order regarding our estimate of arc signal strength. First, we have assumed a rather large arc cascade factor, $k = 5$. This is reasonable for a thick target dielectric, but k will decrease for thin targets. According to Eq. (79), if $k = 5 \rightarrow 2.5$, then the AGEMP signal decreases by a factor of 13.2 (11.2 dB), and $\mathcal{S}(\text{nominal})$ of Eq. (81) drops to only a few dB above irreducible detector noise. Second, we have assumed a

large target-observer separation, $R \sim 1000$ km. If this could be halved, by accident or design, $S(\text{nominal})$ would increase by 6 dB and the receiver detection threshold of $(-)\text{105 dBm}$ could be relaxed. Alternatively, reduction in R can compensate for a reduction in k : if $k = 5 \rightarrow 2.5$ and $R = 1000 \rightarrow 275$ km, the same arc signal strength $S(\text{nominal})$ is maintained. Finally, we remark on the somewhat peculiar distance scaling for S of Eq. (81): S decreases with the target-observer distance R as expected, but it increases with the accelerator-target distance D . This results from the facts that the arc signal strength [Eqs. (77)-(78)] scales as the target area A_I irradiated (charged) by the NPB, and A_I in turn goes as D^2 because of the beam divergence at the target. Reducing D will reduce the target area struck, and --in our arc discharge model--the resultant arcs will be more frequent [Eq. (2)] but smaller in size and signal strength.

In this section, we have estimated the AGEMP signal strengths arriving at a radio receiver situated ~ 1000 km from an NPB-irradiated target. We assume the arc signal is due to the "natural" motion of an arc electron current, which terminates on an underlying metal surface. For nominal strong arcs, as described in Sec. III-D, signal strengths appear favorable--at least 15 dB about ambient noise levels--and can be detected (at ~ 1000 km) by a receiver capable of operating at or near a threshold of $(-)\text{105 dBm}$. We shall now proceed to look at the question of hit assessment and target discrimination on the basis of such AGEMPs.

F. Target Discrimination by Arc Signals

Using the target dielectric arc signal calculations of Sec. III-E, we consider the possibility of distinguishing between various target configurations by analyzing the AGEMP. In this section, we compare the AGEMPs from "large" arcs (generated in "thick" targets) and "small" arcs (from "thin" targets). Apart from signal strength, the AGEMP frequency spectrum changes significantly from thick to thin targets; if the observer is able to scan over an interval of arc signal frequencies, the character of the AGEMP spectrum shows whether the arcs originate from a thick or thin target. Our treatment is still for the case of arcs terminating at an anode, which is effectively a metal surface. Further target discrimination may follow when the radiation is sampled from an arc current (and the current of stripped NPB nuclei), which

passes entirely through the anode. This problem has been discussed in a recent report (Clodius 1986b) and will not be treated further here.

The difference between arc signals from thick and thin targets is determined mainly by the k-dependence of the observed signal $\mathcal{S}$ in Eq. (79). Here: $k = \alpha_1 d$, is the arc cascade factor, with α_1 the ionization coefficient of the target material, and d the arc discharge path length. We expect that α_1 does not change much from one target dielectric to another (just as the breakdown field does not change markedly; see Table I). Thus, roughly speaking, k should scale as the target thickness d : $k \rightarrow$ large for thick targets, and $k \rightarrow$ small for thin targets. A value of $k = 5$ appears reasonable for thick ($d \sim 1$ cm) targets, since $k = 5$ gives arc currents of the size observed experimentally--see Eqs. (51)-(53) in Sec. III-D. k -values of interest may then range downward to $k = 0.5$ for thin ($d \sim 1$ mm) targets.

The full k -dependence of the arc signal is not included in $\mathcal{S}$ of Eq. (79), because in Eq. (75) we approximated the Fourier transform $|\zeta(v)|^2$ for $k \rightarrow$ large and $kz_c \gg 1$. A more accurate treatment, assuming only that $k > 0.1$ (but kz_c still large), shows that $\mathcal{S}$ is multiplied by a correction factor which differs from unity as $k \rightarrow$ small. The corrected version of $\mathcal{S}$ of Eq. (79) is

$$\mathcal{S}(f, \delta f) = \mathcal{N}_0 \mathcal{S}_0 (D \Delta \phi \sin \theta / R)^2 \sec \alpha [\mathcal{L}(k, x)] (\delta f / \Delta f) ,$$

$$\text{where: } \mathcal{N}_0 = [m \langle \kappa E_s \rangle_{uv} e]^2 (1+r)(r+s), \quad \mathcal{S}_0 = 3.26 \times 10^{-14} \text{ mW/cm}^2 ,$$

$$\text{and: } \mathcal{L}(k, x) = \frac{e^k - 1}{1+x^2} \left\{ \left[1 - Z_k \frac{\sin kx}{kx} \right]^2 + \left[Z_k \frac{1 - \cos kx}{kx} \right]^2 \right\} ,$$

$$\text{with: } x = f / \Delta f = v / \Delta v, \quad Z_k = k / (e^k - 1) . \quad (83)$$

The various symbols and units are defined in Eqs. (76)-(79). The only thing new here is the correction factor in curly brackets in the expression for $\mathcal{L}(k, x)$; that factor is ~ 1 for $k \rightarrow$ large [as in Eq. (79)], but it changes significantly as $k \rightarrow$ small. In any case the Lorentzian-like frequency spectrum function $\mathcal{L}(k, x)$ contains the entire k -dependence of the arc signal $\mathcal{S}$, for $k > 0.1$, provided that $kz_c \gg 1$.

In Fig. 12, we plot the arc signal spectrum $\mathcal{L}(k, x)$ vs the dimensionless frequency variable $x = f / \Delta f$ for several values of k , in the range: 5 (thick

target) $> k > 0.5$ (thin target). Here f is the receiver frequency, and $\Delta f = \alpha_1 v_e / 2\pi$ is the spectrum halfwidth for large k [see Eq. (76)]. $\Delta f \sim 80$ MHz for typical dielectrics and probably does not change much with target thickness or composition. Three features are apparent from Fig. 12 for small k : (1) $\mathcal{L}$ rapidly diminishes in overall strength, because of the factor $(e^k - 1)$ in Eq. (83); (2) $\mathcal{L}(k, 0) \rightarrow 0$; the signal vanishes at low frequencies; (3) the arc signal frequency spectrum becomes very broad. The difference in shape between the spectra for large and small k is related to their time scales: the former are long in duration compared to the latter and therefore

Fig. 12. Arc spectrum function $\mathcal{L}(k, x)$ vs frequency $x = f / \Delta f$ for several k -values. Plotted is the spectrum function $\mathcal{L}$ of Eq. (83), with f the receiver frequency and $\Delta f = \alpha_1 v_e / 2\pi \sim 80$ MHz. Values of the cascade factor range over: 5 (thick target) $> k > 0.5$ (thin target). As $k \rightarrow$ small, the spectrum weakens and broadens; in principle, this allows discrimination between arc signals from thick and thin targets.

much more localized in the frequency domain. The limiting forms for arc signals from thick and thin targets are

large k (thick target)

$$\mathcal{L}(k, x) \approx e^{k/(1+x^2)};$$

small k (thin target)

$$\mathcal{L}(k, x) \approx \frac{k}{1+x^2} \{ [1 - 2\sigma_1(kx) + \sigma_2^2(kx)] + \frac{k}{2} [1 - \sigma_2^2(kx)] \},$$

$$\text{where: } \sigma_n(\mu) = (n/\mu) \sin(\mu/n). \quad (84)$$

The difference in these functional forms provides a way of discriminating between arc signals originating from thick and thin target dielectrics.

We assume that the observer's radio receiver can be scanned over a nominal frequency range: $0.4 \Delta f \sim 30 \text{ MHz} \leq f \leq 2.5 \Delta f \sim 200 \text{ MHz}$. If, upon recording several AGEMPs in this range, the signal strength is found to decrease with frequency, then the arcs are very probably from a thick target. If, on the other hand, the signal strength increases with frequency, the arcs are of thin-target origin. Two comments are relevant to this target discrimination procedure. First, it requires that several AGEMP frequencies be sampled, so the target must be observed by a single sensor during several NPB pulses, or by several sensors during a single NPB pulse. Second, the procedure requires detectable signal levels; here, there is little problem with large (thick-target) arcs [per Eqs. (80)-(81)], but the much weaker signals from small (thin-target) arcs may be undetectable under nominal operating conditions. Nevertheless, the thick-target arcs can be identified readily by their signature: their frequency spectrum falls off rapidly with increasing receiver frequency f . Thin-target arcs do not show this behavior for frequencies below a few GHz.

The above frequency sampling procedure provides NPB hit assessment for thick targets and target discrimination when both thick- and thin-target arc signals are detectable. However, the arcs treated all terminate on metal

surfaces--these arcs "punch through" a dielectric layer covering an underlying metal anode, and the arc charge stops at the anode. Other types of arcs are possible, e.g., arcs that strike through an unsupported dielectric layer and "blow off" charges at the anode (Flanagan et al. 1979). The radiation from these blowoff arcs will occur together with a signal from the stripped NPB nuclei (Clodius 1986b). There is little doubt that the frequency spectra of blowoff and punch-through arc signals will differ sufficiently to provide further target discrimination. We shall consider the problem of blowoff arcs in a later report.

G. Improvements and Extensions to the Theory

As a final note, we comment on how various aspects of our calculation might be extended or improved. Although we have made reasonable estimates of rf signals generated by bulk discharge arcs in NPB-irradiated target dielectrics, we have done so by assuming rather idealized conditions. Some non-ideal aspects of the problem and features of possible future concern are the following.

1. Throughout this calculation, we assume precise tracking of the target by both the NPB accelerator vehicle and the remote observer. At target distances ~ 1000 km, the irradiated target area is about 1 m in size, so the NPB vehicle must be able to "lock" onto the target to an angular tracking precision of $\sim 1 \mu\text{rad}$ over at least one 10-ms-long NPB pulse. If the target is in sub-orbital flight, then during this time it may move relative to the NPB vehicle by a few tens of meters. Thus the NPB vehicle must be able to track the target over an angular range of a few tens of μrad , to a precision of $1 \mu\text{rad}$. The tracking requirement for the observer depends on the directionality of his rf antenna, but it certainly will be less stringent than for the NPB vehicle. In any case, target-tracking requirements at 1000 km are nontrivial, and tracking errors--when folded into our calculation--will reduce the estimates of arc signal strengths.
2. Early in Sec. III-A, we opted to study bulk discharge arcs rather than surface flashovers. The justification for this choice is that at an NPB energy of 250 MeV (for neutral hydrogen), the electrons deposited arrive at an energy of 136 keV and are embedded deeply enough in the target

material so as to inhibit formation of surface flashover arcs. However, at electron energies ≤ 25 keV, it appears that surface flashovers are at least as numerous as bulk discharges (Frederickson 1982; Flanagan et al. 1979). While the surface arcs probably generate weaker EM signals than the bulk arcs (because the latter exhibit current amplification by charge cascade), their number and possible influence on the electrical stress in the target material (Frederickson 1983) may affect our bulk arc signal estimates. The role of surface flashover arcs should be evaluated more thoroughly.

3. In Eq. (3), we introduced an average dielectric breakdown field $\langle E_s \rangle \approx 0.690$ MV/cm. We did not deal with questions such as how this field may vary with target thickness or how it may change with radiation dosage (see remarks at end of Appendix A, however). Since this field enters quadratically in the arc signal of Eq. (79), it may bear closer scrutiny. Also, it would be interesting--in itself--to explain why $\langle E_s \rangle$ is very nearly constant for a wide variety of insulating materials. See Table I.
4. Early in Sec. III-B, we assume the arc ionization coefficient α_i and average transport velocity v_e are not strongly field-dependent, in order to arrive at a tractable arc discharge equation. Later, in Eq. (14), we ignored arc inductive effects. These restrictions rule out an accurate treatment of very large (but infrequent) bulk discharge arcs. Removal of these restrictions leads to a much more difficult, nonlinear problem--which might be tractable only by means of a computer code.
5. In Sec. III-C, Eq. (17), we should emphasize that the arcological laws have been observed for combinations of flashover and bulk discharge arcs, at relatively low incident electron energies (~ 25 keV), and with the flashovers dominating in number of events (Flanagan et al. 1979). Although there is no immediate reason to believe that flashover and bulk arcs obey significantly different scaling laws, or that these laws should depend strongly on the incident electron energy, one could entertain the notion that bulk arcs (induced by high-energy NPB irradiation) depart from the scaling laws of Eq. (17). Such a departure would change the cathode

current feed scheme of Sec. III-C and Appendix C, and--later--the arc signal strength estimates of Sec. III-E.

6. In Appendix C, we constructed a model of cathode surface currents that feed the bulk discharge arc, but we did not use that model to impose any important restrictions on the arc duration or intensity. Later, we found that the assumption of an abrupt cutoff in the cathode feed, occurring when the arc cascade factor falls to its cutoff value, led to a discontinuity in the arc current derivative $\dot{I}$. In Eq. (65), we removed the $\dot{I}$ discontinuity by a somewhat ad hoc argument, based on the idea that the cathode feed cutoff is really gradual. Clearly the transition from abrupt to gradual cutoff for the cathode currents should be considered in more detail, because it affects the value of $\dot{I}$ during the main portion of the arc radiation period (the interval $T_c + T_c + T_e$ in Fig. 11). Also, if the pre-cutoff surface current model were augmented (e.g., by adding dissipation), it might serve as a starting point for studying the surface flashover arcs discussed in comments 2 and 5 above.
7. In Eqs. (43)-(45) and Appendix D, we devised an arc-stop distribution by claiming that the arc ceases when its effective cascade factor becomes small enough. The analysis was based on the field dependence of the arc ionization coefficient $\alpha_1(E)$ for a gas. If the functional form of $\alpha_1(E)$ for a solid is markedly different, then our arc statistics will be changed. Unless these statistics change radically, however, we do not expect significant changes in our arc signal strength estimates. A more important effect of a strong field dependence for $\alpha_1(E)$, particularly near the end of the arc, might be to alter the arc signal spectrum [since in Eq. (83): $k = \alpha_1 d = \text{fcn}(E)$, and $E = \text{fcn}(t)$]. This effect could be treated as part of the nonlinear problem mentioned in comment 4 above.
8. Throughout the paper we have treated the cascade factor $k = \alpha_1 d$ as a phenomenological arc parameter. In Eq. (53) we found that "large" arcs--with average currents on the order of hundreds of amps, which correspond to laboratory events observed in large surfaces ($\sim 1 \text{ m}^2$)--require k -values in the range $3 < k < 7$. It was in this way that we fixed the scale: k (thick target) $\sim 5 \rightarrow k$ (thin target) ~ 0.5 ; see Sec. III-F

and Fig. 12. It would be much more satisfying to find an a priori reason to fix the k-scale this way.

9. We began Sec. III-E for AGEMPS by remarking that the present treatment applies only to the radiation signals from arc electron currents that stop at the anode; in effect, the anode is a metal surface where the electron current spreads out at the end of its journey. This has several consequences for the arc signal we have calculated. First, if the anode is truly metallic, then we should account for the motion of image charges in the metal as the arc strikes. At distances large compared to the length of the arc, the charge motions of the image arc and true arc are indistinguishable, so the effect of the image arc is to increase the signals we have calculated; this increase should be by a factor of 4 in the signal of Eq. (79), since the arc radiated power goes as $[\dot{I}(\text{arc})]^2$, by Eq. (59). A second consequence of our model is that radiation signals from any currents that burst through the anode must be treated separately from the signals we have calculated. Examples of currents that may burst through a sufficiently thin metallic anode (e.g., a foil) are the blowoff arcs and penetrating NPB nuclei discussed at the end of Sec. III-F. The radiation from the former (latter) currents tends to increase (decrease) our calculated signals. Evidently, it would be desirable to catalog the signals expected from various target configurations and arc combinations.
10. In Eqs. (63)-(64), after noting that most of the arc radiation occurs near the end of the arc's lifetime, we calculated the arc signals broadcast during the interval T_c to $(T_c + T_e)$, i.e., cathode current cutoff to arc gap depletion. For completeness, the pre- T_c contribution to the arc radiation should be included. The pre- T_c radiation will probably make the thin-target arcs appear brighter and also will broaden the frequency spectrum of the thick-target arcs somewhat.
11. The specifications for the observer's rf detector in Sec. III-F are fairly stringent: operation at frequency 80 MHz (nominal), bandwidth 100 kHz, detection threshold at $(-)$ 105 dBm, frequency scanning capability over 30 to 200 MHz (for a single observer), and a directional antenna capable of picking up signals in the wavelength range 1.5 to 10 m. Operation of such

a detector in a space environment may require state-of-the-art engineering. If such a detector is available, however, it can provide not only target hit assessment for NPB irradiation of thick targets at long distances (~1000 km) but also hit discrimination between thick and thin targets at shorter distances (~100 km).

Of the comments above, the ones deserving the most immediate attention are number 2 (role of flashover arcs) and number 6 (details of the bulk arc cutoff). Classification of signals expected from various target configurations and possible arc combinations would also be useful.

ACKNOWLEDGMENTS

The author is grateful to W. B. Maier for valuable discussions and suggestions concerning this calculation. Also, helpful comments and information were supplied by J. Buchenauer and W. Clodius.

LIST OF SYMBOLS USED

<u>Symbol</u>	<u>Definition</u>	<u>Equation</u>
I_B	NPB pulse instantaneous current	1,3, etc.
T_B	NPB pulse temporal length	1
$\Delta\phi$	NPB pulse angular divergence	1,2, etc.
D	NPB accelerator-target distance	Fig. 1
R	Target-observer distance	Fig. 1
θ	Observer orientation angle	Fig. 1
α	NPB angle of incidence on target	2,3, etc.
A_B	NPB cross-sectional area (at target)	-
A_I	Target area irradiated	- 17, etc.
ΔT_B	NPB irradiation time on target	2,3, etc.
C	Capacitance of target dielectric layer	-6, etc.
κ	Relative dielectric constant	2, TI
ϵ_0	Permittivity of free space	-
d	Target capacitance gap spacing	-8, etc.
m	Target capacitance charge factor	2,3, etc.
E	Target interior electric field	2
V	Voltage across target capacitance	-6, etc.
σ_0	Conductivity of unstressed dielectric	Appendix A
R_L	Resistance of "leaky" dielectric	A1,A2
σ	Conductivity of stressed dielectric	A2, etc.
τ	Dielectric relaxation time: $\tau = m\kappa\epsilon_0/\sigma$	A2, etc.
$f(t)$	Conductivity time dependence: $f(t) = \sigma/\sigma_0$	A4,A5
τ_0	Unstressed relaxation time: $\tau_0 = m\kappa\epsilon_0/\sigma_0$	A4, etc.
E_M	Maximum possible dielectric charging field	A5
σ_R	Radiation-induced conductivity	A7,A11
$D, \dot{D}$	Irradiation dose and dose rate	A7,A9
$K(D)$	RIC proportionality factor	A7,A8
J	Current density of irradiating beam	A9
dE/dx	Stopping power for irradiating particles	A9,A10
t_s	Time to onset of breakdown	A12
σ_{crit}	Critical conductivity at breakdown	A12,A13
q_0	Instantaneous charge leaving cathode	4,7, etc.

$I(t)$	Arc current through gap d	4,6, etc.
v_e	Average avalanche electron velocity	4,7, etc.
T_e	Avalanche electron transit time	4,7, etc.
α_i	Ionization coefficient of target medium	4,7, etc.
V_0	Target voltage when arc fires	5,7, etc.
$V(t)$	Instantaneous target voltage (during arc)	5,7, etc.
E_s	Min. breakdown field for arc discharge	-
V_s	Min. breakdown voltage for arc discharge	-
ΔV	Voltage drop due to arc discharge	8
$i(t)$	Instantaneous cathode current feeding arc	9,10, etc.
ρ	Charge per unit area on cathode	9,10, etc.
u	Velocity of cathode current flow	9,33, etc.
s_n	Cathode current area parameter	10,26, etc.
$Q(t)$	Instantaneous electron avalanche charge	11,13, etc.
$\lambda(x,t)$	Arc charge/length at position x , time t	12,13, etc.
k	Electron cascade amplification factor	15,16, etc.
$\Delta V(t)$	Instant. voltage drop during arc discharge	15,16, etc.
T_∞	Time to drain entire cathode charge	-
T_c	Time at which cathode current cuts off	17,18, etc.
$q(T_c)$	Total cathode charge release during arc	17,24, etc.
$I(T_c)$	Arc peak current	17,25, etc.
y	Dimensionless variable: $y = \tau/T_e$	18,19, etc.
z	Dimensionless time: $z = t/T_e$	18,20, etc.
$\tilde{y}$	Time variable: $\tilde{y} = y T_e$	18,20, etc.
$\tilde{z}$	Observation time: $\tilde{z} = z T_e$	18,20, etc.
z_c	Dimensionless cutoff time: $z_c = T_c/T_e$	18,19, etc.
ΔV_c	Capacitor voltage drop at cathode cutoff	19,22, etc.
$\Delta \tau$	Current pulsewidth for Dickey model	21-25
h	Dimensionless transit time: $h = T_e/\Delta \tau$	22-24
S	Dimensionless discharge parameter (Dickey)	23-25
n	Exponent for power-law cathode current	26-32, TII
N	For power-law current: $N = n + 1$	28
S_n	Discharge parameter for power-law current	29-32
M	Cascade correction factor: $M = e^k/(e^k - 1)$	33
p	Exponent for arcological scaling laws	34, TII
I_0	Elementary current for linear cathode feed	35

L	Self-inductance of arc discharge	B1
δ	Fractional discharge voltage: $\delta = \Delta V_c / V_o$	B2
ω_A	Ring frequency of arc: $\omega_A = 1/(LC)^{1/2}$	B3
μ_o	Permeability of free space	--
δ_c	Upper bound on capacitance-limited arcs	B4
m_L	Correction factor for arc self-inductance	B4
$\dot{I}$	Time derivative of arc current	36,38, etc.
g	Intrinsic parameter for arc radiation	38
P_s	Probability of arc breakdown by voltage V	39
w	Wijsman's probability factor	39,41
γ	Cathode current supply factor	39
η	α_1/E , the Stoletow function	39
r	Fractional overvoltage: $r = (V_o - V_s)/V_s$	40,41, etc.
$\bar{P}_s(r)$	Arc breakdown probability at $>r$	42
$\vec{u}$	Radial velocity of cathode surface currents	C1,C2, etc.
r_o	Radius of arc-site charge depletion region	C1,C3, etc.
$\Omega(t)$	Surface current velocity scale function	C2,C3, etc.
b_U	Surface current in-out boundary radius	C3
$\vec{F}$	Electric field in cathode surface	C4,C5, etc.
b_F	Surface field in-out boundary radius	C5
ρ_o	Initial cathode surface charge density ($t=0$)	C6,C9, etc.
ϵ	Ratio: early arc-site area to total charged area	C7,C10, etc.
$b(z)$	Unified field and current in-out boundary	C9
$f(z)$	Normalized surface charge density	C10,C11, etc.
K	Dimensionless ratio: $K = m\kappa(u/v_e)$	C11,C12
a	Dimensionless ratio: $a^2 = (e^{1/2}/k)\epsilon$	C11,C12
k_∞	Maximum attainable cascade factor	43,D3
ϕ	Reference voltage for cascade saturation	43,D3
V_c	Gap voltage at arc cutoff	44,
s	Fractional undervoltage: $s = (V_s - V_c)/V_s$	44,45, etc.
$P_c(s)$	Distribution of arc cutoff voltages	45
β	Stopping voltage scale parameter ($\beta \approx k$)	45,D6
ψ	Ratio: reference to sparking voltage ($\psi = \phi/V_s$)	D3,D4 etc.
I_c	Arc peak current (at cutoff): $I_c = I(T_c)$	46,47, etc.
I_s	Peak current scale: $I_s = 2aCV_s/T_e$	46,47, etc.
$G(r,s)$	Arc power distribution function	47,49, etc.

$\Gamma(r,s)$	Number of arcs firing in range: $0 \rightarrow r, 0 \rightarrow s$	E2, E4, etc.
$F(r,s)$	Fraction of arcs firing in: $0 \rightarrow r, 0 \rightarrow s$	E3, Fig. 9
X	Arc start variable: $X = kr$	E4
Y	Arc stop variable: $Y = \beta s$	E4
$\overline{I_c}$	Peak current at $\partial G / \partial r = 0$	50
$\langle I_c^2 \rangle$	Average arc peak power level	52
ℓ	Inductive/capacitive energy ratio	Footnote p.32
χ	Scale factor: capacitance vs inductance	Footnote p.32
$\overline{F}(k,\beta)$	Averaging factor for arc peak power level	52
I_{rms}	Arc current averaged over size and duration	53
$\vec{J}$	Current density for arc discharge	54,55
$\vec{A}$	Vector potential for arc radiation fields	55,56
t_R	Retarded time: $t_R = t - (R/c)$	55,57
$\vec{B}_R$	Arc radiation magnetic field	56
$\vec{E}_R$	Arc radiation electric field	56,58
$\dot{I}(t_R)$	Time rate-of-change of arc current	57,58
$\hat{n}$	Unit vector: arc to observation point	56,58
$\vec{N}_R$	Poynting vector for arc radiation	59,67
W_R	Energy radiated by arc discharge	63,64
$\mathcal{P}$	Arc radiation power parameter	67
$j(z)$	Dimensionless arc current shutoff function	67,72
W_o	Arc radiation energy parameter	69
$\overline{E}_R$	Time-average arc radiation field	70,71
ν	Arc radiation frequency variable	72,73
$\zeta(\nu)$	Fourier transform of arc current function	72,73
$\tilde{N}_R(\nu)$	Arc radiation energy/area/frequency	74,76
$\Delta\nu$	Arc freq. spectrum halfwidth: $\Delta\nu = k/T_e$	75,76
f	Receiver frequency (at observation point)	76,77
δf	Receiver bandwidth (at observation point)	76,77
$\mathcal{S}$	Power/area broadcast during arc discharge	77,79
$\mathcal{N}$	Dimensionless arc-parameter multiplier	79
$\mathcal{S}_o$	Power/area scale constant for arc broadcast	79
$\mathcal{N}_o$	Dimensionless arc-parameter multiplier	83
$\mathcal{L}(k,x)$	Arc radiation freq. spectrum function	83,84
Z_k	Inverse avalanche factor	83
x	Dimensionless freq. variable: $x = f/\Delta f$	83,84

APPENDIX A

EFFECT OF FINITE CONDUCTIVITY ON TARGET CHARGING

In Eqs. (2) and (3) of Sec. III-A, we calculated target charging under irradiation by an NPB current I_B , assuming for simplicity that the target dielectric formed a nonconducting parallel plate capacitor. Realistically, the dielectric has a small but finite intrinsic conductivity σ_0 , which increases rapidly near the breakdown field. Also, σ_0 can increase significantly due to RIC (radiation-induced conductivity) from ionization of the bulk material upon passage of the stripped NPB nuclei. In this Appendix, we discuss the effects of a finite dielectric conductivity on the NPB target-charging process.

As in Eq. (2), assume the target dielectric forms a parallel-plate capacitor of capacitance $C = m\kappa\epsilon_0 A_I/d$. Assume also that this capacitor is "leaky": the dielectric has a finite conductivity σ . Represent this finite σ by a resistance $R_L = d/\sigma A_I$, which shunts the capacitor. Then, under irradiation by an NPB current I_B , the target dielectric charges according to

$$C\dot{V} + (1/R_L)V = I_B. \quad (A1)$$

$V = V(t)$ is the instantaneous voltage across the dielectric (as in Fig. 4). If I_B , R_L , and C are all time-independent, then the solution with $V(0) = 0$ is

$$V(t) = I_B R_L [1 - \exp(-t/\tau)], \quad \tau = R_L C = m\kappa\epsilon_0/\sigma. \quad (A2)$$

The field in the dielectric layer is $E = V/d$, where d is the layer thickness. Only at early times is this field linear in time [as was assumed in Eq. (2)], since

$$E(t) = (I_B t/Cd)[1 - \frac{1}{2}(t/\tau) + \dots], \quad t \ll \tau. \quad (A3)$$

If the relaxation time τ becomes comparable to the NPB irradiation time t (~ 10 ms), then the charging field is reduced, and it may fall below the breakdown field E_g (~ 0.2 MV/cm); in this case, no bulk discharge arc can occur

during an NPB pulse. Such leakage is not a problem for unstressed dielectrics, since (from Table I) the relaxation time $\tau \sim 100$ sec, and the correction to $E(t)$ in Eq. (A3) is only $t/\tau \sim 10^{-4}$. But if the conductivity σ were to increase by a factor $\sim 10^4$ (so τ is reduced similarly), then leakage would seriously affect our target-arcing scenario.

The above analysis changes when we note that under NPB irradiation the target conductivity σ is time-dependent; this is true for both RIC and for the approach to breakdown (since $\sigma = \text{fcn}[E(t)]$). If we represent the functional dependence by: $\sigma/\sigma_0 = f(t)$, with σ_0 the unstressed conductivity and $f(0) = 1$, then the circuit equation (A1) and its solution (A2) are modified to

$$\begin{aligned} \dot{V} + [f(t)/\tau_0]V &= I_B/C, \\ V(t) &= (I_B/C) \int_0^t \exp \{-[F(t) - F(t')]\} dt', \\ \text{where: } F(t) &= \frac{1}{\tau_0} \int_0^t f(t') dt', \quad \tau_0 = R_0 C = m\kappa\epsilon_0/\sigma_0. \end{aligned} \quad (\text{A4})$$

$\tau_0 \sim 100$ s is the unstressed dielectric relaxation time, and $t < 10$ ms is the NPB irradiation time; we note that $t/\tau_0 < 10^{-4}$ is small. Ignoring RIC for the moment, we also note that $f(t)$ can become large if t extends up to breakdown: $\sigma/\sigma_0 = f(t) \sim 10^3$, when $E(t) \rightarrow E_s$, for typical dielectrics.* In principle, this enhanced conductivity (and accompanying leakage current)--by itself--could reduce the charging field significantly. A large enough reduction would preclude breakdown. For NPB irradiation, however, we claim the reduction during the approach to breakdown simply delays the onset of breakdown.

We support this claim by showing that under typical NPB irradiation, the maximum field attainable in the target dielectric exceeds the breakdown field. We estimate the maximum field as follows. If $f(t)$ increases with time, then $V(t)$ in Eq. (A4) first increases with time t , later passes through a maximum at $t = t_M$ such that $\dot{V}(t_M) = 0$, and thereafter decreases with t . The maximum voltage is: $V_M = I_B R_0 / f(t_M)$; this corresponds to a maximum attainable charging field $E_M = V_M/d$, or

* See Adamec and Calderwood (1975). Judging from their Fig. 2a (Mylar, $E_s \sim 1.6$ MV/cm) or Fig. 4 (polyethylene, $E_s \sim 0.5$ MV/cm), the conductivity $\sigma/\sigma_0 \sim 10^3$ (approximately), as E (applied field) $\rightarrow E_s$ (breakdown field).

$$E_M = 10^5 (I_B / \sigma_o A_I) / f(t_M), \text{ MV/cm} , \quad (\text{A5})$$

where: I_B = NPB current (units: 100 mA), σ_o = target conductivity (units: 10^{-16} mho/cm), A_I = target area irradiated (units: m^2). We choose conservative values: $I_B = 1$, $\sigma_o = 1$, $A_I = 1/2$, breakdown field $E_s = 0.200$ MV/cm, and we assume (Adamec and Calderwood 1975) that f scales with field as: $\log f \sim E^{1/2}$. Then a numerical solution to Eq. (A5) shows that $E_M = 3.33 E_s$. This means that the NPB charging rate (during the approach to breakdown) is typically high enough to achieve breakdown, even with relatively large leakage currents present near breakdown. It also means that during the approach to breakdown, we can approximate the solution for $V(t)$ in Eq. (A4) by its leading terms. The resultant field is

$$E(t) = (I_B t / Cd) [1 - \frac{1}{2} (t/\tau_o) f(t) + \dots] . \quad (\text{A6})$$

This is similar to Eq. (A3), but now incorporates a time-dependent conductivity: $\sigma / \sigma_o = f(t)$. With $t/\tau_o \sim 10^{-4}$ and $f(t) \sim 10^3$ (at most), the correction term in square brackets is only a few percent; likewise, the time to reach breakdown is increased by only a few percent. Thus, as promised, we have shown that under NPB irradiation, the conductivity increase near breakdown delays--but does not prevent--actual breakdown.

We now turn to RIC, the conductivity induced in the target material by passage of the stripped NPB nuclei, which we assume to be protons. If the passing protons create ions at a rate v_R per unit time and volume, then (in equilibrium) the density of charge carriers in the material increases by v_R / Γ_R , where Γ_R is an ion recombination rate. This scaling with the ion creation rate v_R suggests an empirical law for RIC

$$\sigma_R = \dot{D} \times K(D) , \quad (\text{A7})$$

where $\dot{D}$ is the dose rate of ionizing radiation (in rads/sec, with 1 rad = absorption of 100 ergs per gram of material), and $K(D)$ is a material-

dependent factor, which may vary with the accumulated dose D . $K(D)$ has been measured for high-energy electrons (3 MeV), at high dose rates ($\dot{D} \sim 10^{11} - 10^{14}$ rads/sec), in a variety of insulators (Riddell et al. 1982; Chervenak and van Lint 1985). For total doses $D \sim 1^{\circ} - 100$ krad, K varies over

$$2 \times 10^{-19} \text{ (Kapton)} < K < 3 \times 10^{-18} \text{ (Teflon)}, \left(\frac{\text{mho}}{\text{cm}}\right) / \left(\frac{\text{rad}}{\text{sec}}\right). \quad (\text{A8})$$

Most common insulators (Mylar, polyethylene, etc.) lie in this range.

The target material dose rate under NPB irradiation is much smaller than for the existing electron experiments above, mainly because the proton flux is relatively small. If the irradiating beam has a current density J (in A/cm^2) of particles of charge ze , and the material struck consists of atoms of atomic number Z and atomic weight A , then the dose rate is

$$\dot{D} = 6 \times 10^{10} (zZ/A)(dE/dx)_0 J, \text{ rads/sec}, \quad (\text{A9})$$

where--for fast, heavy particles such as the NPB protons--the normalized stopping power is (Jackson 1975, Chap. 13, Sec. 3)

$$(dE/dx)_0 = \frac{5.88}{\beta^2} \left\{ 1 - 0.0867 \left[\beta^2 + \ln \left(\frac{I}{10} \cdot \frac{1-\beta^2}{\beta^2} \right) \right] \right\}, \frac{\text{MeV}}{\text{cm}}. \quad (\text{A10})$$

Here the particle speed is βc , and I (in eV) is the energy required to create an ion pair in the target material. The target density drops out of $\dot{D}$ in Eq. (A9) because it appears in both the numerator (as a multiplicative factor in the unnormalized stopping power) and denominator (as a normalization factor in the definition of a rad).

For light materials: $Z/A = 1/2$, and for NPB protons at 250 MeV: $z = 1$, $\beta = 0.614$, and $(dE/dx)_0 = 14.4 \text{ MeV/cm}$ (for $I = 10 \text{ eV}$). Then Eq. (A9) yields a dose rate: $\dot{D} = 4.3 \times 10^{11} J, \text{ rads/sec}$. The nominal NPB current density is 100 mA/m^2 ; corrected for the incident angle α , we have: $J = 10^{-5} \cos \alpha, \text{ A/cm}^2$. If we take $\alpha = 45^\circ$, on the average, then: $\dot{D} = 3 \times 10^6 \text{ rads/sec}$, is a nominal

dose rate for NPB irradiation. In turn, the nominal NPB induced conductivity is--from Eqs. (A7) and (A8)--

$$0.6 \times 10^{-12} < \sigma_R < 9 \times 10^{-12}, \text{ mho/cm} . \quad (\text{A11})$$

Depending on the target material, the conductivity increases by a factor in the range: $\sigma_R/\sigma_0 = 10$ (Kapton), to 10^6 (Teflon). The larger increases can obviously affect the approach to breakdown.

To see whether breakdown can occur when $\sigma_0 + \sigma_R$ under NPB irradiation, we solve Eq. (A2) for the time t_s necessary to reach the breakdown field E_s . The result is

$$t_s = t_{s0} \{ (1/y) \ln[1/(1-y)] \} \approx t_{s0} \{ 1 + \frac{1}{2}y + \dots \} ,$$

$$\text{where: } t_{s0} = E_s C_d / I_B ,$$

$$\text{and: } y = \sigma_R / \sigma_{\text{crit}}, \quad \sigma_{\text{crit}} = I_B / E_s A_I . \quad (\text{A12})$$

t_{s0} is the breakdown time for a perfect insulator, as calculated in Eq. (3); now, however, t_{s0} is corrected to $t_s > t_{s0}$ by leakage currents due to σ_R . The critical (intrinsic) conductivity is

$$\sigma_{\text{crit}} = 10^{-11} (I_B / A_B E_s) \cos \alpha, \text{ mho/cm}, \quad (\text{A13})$$

where I_B = NPB current (in 100 mA), A_B = NPB area at target (in m^2), and E_s is the target breakdown field (in MV/cm). For nominal values: $I_B \sim 1$, $A_B \sim 1$, $E_s = 0.200$, and $\alpha = 45^\circ$ we find: $\sigma_{\text{crit}} \sim 35 \times 10^{-12}$ mho/cm. Then, from Eq. (A11), the RIC ratio is

$$0.017 < y = \sigma_R / \sigma_{\text{crit}} < 0.257 . \quad (\text{A14})$$

By Eq. (A12), these γ -values increase the breakdown time t_{s0} by $\sim 1\%$ to 16% , depending on the material irradiated. Such an increase due to RIC does not change the conclusion in Sec. III-A that a breakdown is likely to occur during a single NPB pulse. Again, the finite conductivity only delays the breakdown.

Finally, we remark that irradiation of a material may alter its breakdown field E_s , and thus further change the breakdown time t_s of Eq. (A12); note that t_s is an increasing function of E_s . The ionizing radiation has at least two effects on breakdown: (1) creation of new charge carriers, which improves conduction and decreases E_s ; (2) creation of heating and/or defects in the material, which increases electron-phonon collisions, and increases E_s . The relative size of these two opposing effects appears to depend strongly on the temperature of the target material (Frohlich 1947), and it is not easy to say whether E_s will decrease or increase due to irradiation of a given material at moderate temperatures. Some experimental evidence shows the change in E_s is not dramatic: for Kapton and Mylar (at room temperature) at doses $D \sim 100$ krads, $E_s(\text{Kapton})$ decreased by $\sim 20\%$, while $E_s(\text{Mylar})$ showed no detectable change (Riddell et al. 1982). In lieu of information to the contrary, we shall assume that under NPB irradiation (at doses $D < 30$ krads) the target material breakdown field E_s does not change significantly. Clearly, more experimental data and a better theoretical treatment are desirable.

APPENDIX B

ESTIMATE OF ARC INDUCTIVE EFFECTS

In writing the circuit equation of Eq. (14), we neglect the arc self-inductance L on the grounds that the stored magnetic field energy is negligible compared to the stored electric field energy, i.e.,

$$\frac{1}{2}LI^2 \ll \frac{1}{2}CV^2. \quad (B1)$$

That this is a self-consistent procedure can be seen as follows. From Eqs. (33) and (35), the peak arc current [for a cathode feed current linear in time, per Eq. (9)] is

$$I(T_c) = (CV_o/T_e)[2\delta(s_1 e^k/A_I k)]^{1/2} \quad (B2)$$

where: $\delta = \Delta V_c/V_o$, is the fractional voltage drop across the gap at time T_c (when the cathode current cuts off), and $k > 1$ is the cascade amplification factor of Eq. (15). At time T_c , the magnetic energy in Eq. (B1) is highest, while the electric energy is lowest, since the gap voltage is $V = V_o(1-\delta)$. Then, in order for the inequality in Eq. (B1) to be satisfied

$$\omega_A T_e \gg \{ [2\delta/(1-\delta)^2] (s_1 e^k/A_I k) \}^{1/2}, \quad (B3)$$

where: $\omega_A = 1/(LC)^{1/2}$, is the arc's ringing frequency.

The capacitance used in Eq. (2), et seq., is $C \sim 5 \times 10^{-9}F$, under nominal operating conditions. For the arc's self-inductance per unit length, we take: $L/d \sim \mu_o \sim 1.26 \mu h/m$. Then for an arc of length $d \sim 1$ cm, and transit time $T_e \sim 10$ nsec, we find: $\omega_A \sim 2\pi \times (20 \text{ MHz})$, and: $\omega_A T_e \sim 1$. So long as the fractional voltage discharge δ is not too large ($\delta < 0.7$), the inequality of Eq. (B3) holds because [with $s_1 = 2\pi(UT_e)^2$, from Eq. (9)] the area ratio is so small: $s_1/A_I \sim 10^{-4}$.

Large arcs, those with values of $\delta \rightarrow 1$, may show inductive effects. To find where the inductive and capacitive energy storage are comparable, we set

the energies of Eq. (B1) equal, use the arc peak current of Eq. (B2), and solve for the fractional discharge voltage for such an arc. The result is

$$\delta_c \approx 1 - [4\pi(u/c)^2(m\kappa/m_L)e^k/k]^{1/2}. \quad (B4)$$

u is the velocity of the surface currents feeding the arc, c = velocity of light, m = capacitance correction factor [see Eq. (2)], κ = relative dielectric constant of the arcing medium, m_L = inductance correction factor (here: $L = u_0 d/m_L$), and k is the cascade factor. δ_c in Eq. (B4) is independent of the arc geometry; it is determined by the nature of the medium. We expect that arcs with $0 < \delta < \delta_c$ are capacitance limited, while arcs with $\delta \approx \delta_c$, and larger, are inductance limited [a precise statement requires corrections to the peak current of Eq. (B2)]. Typically: $u/c \approx 1/300$, $m \sim m_L \sim 2$, $\kappa = 3$, and $k = 5$; this gives: $\delta_c = 0.89$. Our capacitive discharge theory, which neglects the arc self-inductance, thus applies for discharge fractions $\delta < 0.89$. In fact, the inductive/capacitive energy ratio is less than 1/10 up to $\delta = 0.7$.

The estimate here of a limiting fractional voltage drop δ beyond which inductive effects predominate is quite conservative, since we have based it on that time T_c when the inductive energy is largest. If we average the inductive and capacitive energies over $0 \rightarrow T_c$, then the upper limit on δ is relaxed. See remarks in footnote on p. 32.

APPENDIX C

CATHODE SURFACE CURRENTS

In Sec. III-C, we showed that certain areal scaling laws for arc discharges in dielectrics are consistent with cathode surface currents which feed into the arc site and grow linearly with time [$n = 1$ in Eq. (26)]. In this Appendix, we construct a model of such currents to show they are plausible. In Sec. III-D [after Eq. (42)], we argue that these surface currents have little effect on the arc beyond supplying charge to it. Thus, the following model only justifies the arc supply; it does not control the arc dynamics.

As in Fig. 4, we consider a surface current $i(t)$ flowing radially in toward the arc site. We imagine the arc site has a boundary of radius $r_0(t)$ which increases with time, and that the charge which crosses this boundary--at constant velocity u --flows into the arc and disappears from the surface. The current is provided by depleting the surface charge density ρ ; for simplicity, we suppress the spatial dependence of ρ [for $r > r_0(t)$] and assume $\rho(t)$ is simply a (decreasing) function of time. Charge conservation then demands

$$\partial \rho / \partial t + \rho \operatorname{div} \vec{U} = -2\pi r_0 \rho u / (A_I - \pi r_0^2) , \quad (C1)$$

for radial distances $r > r_0$ outside the arc site. Here $\vec{U}$ is the (radial) velocity of charges on the surface, and the quantity on the RHS is the current "lost" at the boundary, per area of charge supplying that current (A_I is the initial charged area, and πr_0^2 is the area discharged by the arc).

By assumption, all the terms in Eq. (C1) depend on time only, including $\operatorname{div} \vec{U}$. Then, with $\Omega(t)$ some function of time

$$\operatorname{div} \vec{U} = \frac{1}{r} \partial (rU) / \partial r = \Omega(t) . \quad (C2)$$

The solution of Eq. (C2) with boundary condition $U = (-)u$ at $r = r_0$ is

$$v(r, t) = [\Omega(t)/2r](r^2 - b_U^2) , \quad r > r_0 ,$$

$$\text{with: } b_U^2 = r_0^2 + [2u/\Omega(t)]r_0 . \quad (C3)$$

If $\Omega(t)$ is positive, this velocity field is radially in toward the arc site for all positions r in the range $r_0 < r < b_U$.

To find the surface electric field F that drives the above charge motion, we solve Gauss' equation

$$\text{div} \vec{F} = \frac{1}{r} \partial (rF) / \partial r = -4\pi\rho(t) . \quad (C4)$$

Since ρ has dimensions of charge/area, the dimensions of F are charge/length, characteristic of a purely 2D field. If we assume the cathode potential is $(-)V(t)$ at $r = r_0$, and that it falls uniformly to zero at the center of the arc site, then the boundary condition on the surface field is: $F(r_0, t) = V(t)$. The solution to Eq. (C4) is then

$$F(r, t) = [2\pi\rho(t)/r](b_F^2 - r^2) , \quad r > r_0 ,$$

$$\text{with: } b_F^2 = r_0^2 + [V(t)/2\pi\rho(t)]r_0 . \quad (C5)$$

For $r_0 < r < b_F$, this field is radially outward, and it accelerates electrons into the arc site.

At this point, we have two free parameters: arc-site radius $r_0(t)$ and the velocity scale $\Omega(t)$. But r_0 must be fixed by requiring that the current across the boundary-- $2\pi r_0 \rho u$ in Eq. (C1)--in fact is the linear current $2\pi\rho_0 u^2 t$ proposed in Eq. (9) of the text. This requires

$$r_0(t) = u t \rho_0 / \rho(t) , \quad (C6)$$

where $\rho_0 = \rho(0)$ is the initial surface charge density. Equation (C6) ensures that the power-law current results in Eqs. (26)-(35) hold for a cathode current with exponent $n = 1$. In particular, the arc gap voltage at long times can be found from Eq. (28); it is

$$V(z) \approx V_0 [1 - (e^k/k)\varepsilon z^2], \text{ for } z = t/T_e \gg 1, \\ \text{with: } \varepsilon = \pi(uT_e)^2/A_I = (u/v_e)^2 \pi d^2/A_I \ll 1. \quad (C7)$$

Here V_0 is the voltage at which the arc starts. Since the surface charge velocity $u \sim 10^7$ cm/sec (Flanagan et al. 1979), while $v_e \sim 10^8$ cm/sec for fast arcs (Frederickson 1982) (transit time $T_e \sim 10$ ns over gap lengths $d \sim 1$ cm), then the ratio ε is likely to be very small: $\varepsilon \sim 10^{-5}$, for $A_I \sim 1$ m². If the arc is not otherwise limited, it will entirely discharge the initial voltage V_0 , according to Eq. (C7), at times $z \sim 50$ -100; this is for an assumed cascade factor $k \sim 4$ -5.

The other free parameter, $\Omega(t)$, may be fixed by imposing that the region over which the surface currents flow into the arc site [Eq. (C3)] coincides with the region where these currents are being accelerated inward [Eq. (C5)]. Thus, we set the boundaries equal: $b_U = b_F$, which gives

$$\Omega(t) = 4\pi u \rho(t)/V(t). \quad (C8)$$

The resultant single boundary b , whose time dependence is

$$b^2(z) = r_0^2 \{ 1 + [V(z)/2\pi\rho_0 u T_e z] \}, \quad (C9)$$

now separates regions $r_0 < r < b$ and $b < r$, where the cathode surface currents are moving into and away from the arc site, respectively. A schematic diagram of this motion appears in Fig. 7.

What remains is to solve the continuity equation, Eq. (C1). If we use the condition in Eq. (C6), convert to the dimensionless time variable $z = t/T_e$, and define $f(t) = \rho(t)/\rho_0$, then

$$\partial f / \partial z + (T_e \Omega) f = -2\epsilon z / [1 - (\epsilon z^2 / f^2)] , \quad (C10)$$

where $\epsilon \ll 1$ is defined in Eq. (C7). We insert Ω of Eq. (C8), use the gap voltage $V(z)$ of Eq. (C7), and notice that the RHS of Eq. (C10) is always small: $\epsilon z < 10^{-3}$, even near the voltage cutoff for $V(z)$. If we ignore the RHS of Eq. (C10) entirely, then a solution is

$$f(z) = 1 / \{ 1 + (K/2a) \ln[(1+az)/(1-az)] \} ,$$

$$\text{with: } a^2 = (e^k/k)\epsilon, \quad K = 4\pi u T_e \rho_o / V_o = m\kappa(u/v_e) . \quad (C11)$$

This solution should be reliable at early times, and when $az < 1$, the surface charge density will fall off slowly with time, as

$$\rho(z) = \rho_o / \{ 1 + Kz [1 + \frac{1}{3} (az)^2 + \dots] \} . \quad (C12)$$

At later times, when $f^2 \rightarrow \epsilon z^2$ on the RHS of Eq. (C10), the charge density rapidly vanishes as its time derivative becomes large and negative. The surface currents must stop when $f^2 \sim \epsilon z^2$, which is equivalent to the condition that the arc-site area πr_o^2 is comparable to the initially charged area A_I . This condition gives a cutoff time: $z_c \sim [1/K\epsilon]^{1/2} \sim 25$, somewhat smaller than the time for the voltage in Eq. (C7) to cut off.

At this juncture, our model of cathode currents which feed the arc and grow linearly with time shows that (1) the motion of such currents is plausible; strong fields exist near the arc site that can drive the requisite charge into the arc; (2) the dynamics are reasonable, in that the arc-site area grows with time, the charge density on the remaining cathode area decreases with time, etc.; (3) the cutoff time for the surface current supply to the arc is $z_c = T_c/T_e \sim 25$ to 100 under typical conditions, so--with $T_e \sim 10$ ns (through ~ 1 cm of material)--the arc may last up to ~ 1 μ s. The model is phenomenological in assuming a constant surface charge velocity u at the arc-site boundary, and it is approximate in treating only the time-dependence of the surface charge density. A more important limitation is the lack of

dissipation in the model; surface currents $J_s = \rho U$ continue to flow [per Eq. (C3)] until some extrinsic limit is reached, such as the cathode charge vanishing. In brief, the model predicts no clear cutoff for the cathode current supply, short of extinction.

Dissipation can be put into the model by invoking an effective surface conductivity σ_s , which produces Joule heating $\sim \int (J_s^2 / \sigma_s) dr$ as the currents move. A cutoff may result when this heating becomes comparable to the intrinsic energy of the surface fields, or reaches some fraction of the discharge energy. We shall not pursue this idea, however, because it (a) requires introducing σ_s , another phenomenological constant, (b) is rather arbitrary in naming the fractional heating where cutoff occurs, (c) gives no clear idea of what statistical distribution the cutoff voltages might have. We prefer stopping the arc by arguments related to the diminution of its cascade factor k as the discharge proceeds; these arguments are given in Sec. III-D and Appendix D.

APPENDIX D

DISTRIBUTION OF ARC STOPPING VOLTAGES

In this Appendix, we wish to find a criterion for stopping the arc. In particular, we want to justify the choice made in Eq. (45), namely that the distribution of arc cutoff voltages is: $P_c(s) \approx \exp(-ks)$, where k is the cascade factor at or near the start of the arc, and s is the fractional voltage drop below threshold: $s = (V_s - V_c)/V_s$. To justify Eq. (45), we consider a mean free path (MFP) problem where the MFP λ is a function of distance traveled. The probability of traveling distance x without a collision is then

$$P(x) = \exp\left\{(-)\int_0^x d\xi / \lambda(\xi)\right\} . \quad (D1)$$

In the arc stopping problem, we draw the analogy--as in Sec. III-D [see remarks below Eq. (42)]--that the distance $d\xi$ traveled parallels the voltage change ds , and λ = distance traveled per collision is analogous to the inverse cascade factor $1/k$ = "distance" (voltage) traveled per ionization. Then the probability

$$P(s) = \exp\left\{(-)\int_0^s k(\sigma) d\sigma\right\} , \quad (D2)$$

is the likelihood that the arc has produced no ionizing cascades in the voltage range 0 to s . Since such ionizations are required to sustain the arc, $P(s)$ is then the probability that the arc will stop at some voltage $> s$.

As a model for k , we use Eq. (43). For a stopping voltage: $V = (1-\sigma)V_s$, $0 < \sigma < 1$, k can be written as

$$k(\sigma) = k_\infty \exp[-\psi/(1-\sigma)], \quad \psi = \phi/V_s . \quad (D3)$$

The ratio ψ is of order unity. We put this form of k into Eq. (D2) and change integration variables to: $y = \psi/(1-\sigma)$. Then

$$\ln P(s) = -k_\infty \psi \int_\psi^{\psi_c} (e^{-y}/y^2) dy, \quad \psi_c = \psi/(1-s) ,$$

$$\ln P(s) = -k_{\infty} \psi \left[\left(\frac{e^{-y}}{y} - E_1(y) \right) \right]_{y=\psi_c}^{y=\psi}, \quad (D4)$$

$E_1(y)$ is the exponential integral function (Handbook of Mathematical Functions 1972). If we note that the difference: $(\psi_c - \psi) = [s/(1-s)]\psi$, is small when s is small, then Eq. (D4) can be expanded about ψ . The result, to terms second order in s , is

$$\ln P(s) \approx -(k_{\infty} e^{-\psi}) s [1 - (\psi/2)s]. \quad (D5)$$

The first factor on the RHS is just the nominal cascade factor for arcs that start not too far from the threshold, i.e., we can set: $k_{\infty} \exp(-\psi) \approx k$. Then

$$P(s) \approx \exp(-\beta s), \quad \beta = k[1 - (\psi/2)s]. \quad (D6)$$

This result is the origin of Eq. (45) in the text. The factor $\beta \approx k$ is present because that is the initial size of the cascade factor to be reduced during the stopping process. The distribution of Eq. (D6) is reasonable in stressing the idea that arcs with "large" s -values (i.e., stopping voltages $V_c \ll V_s$) are improbable because of the large reduction in the effective value of k .

APPENDIX E

ARC POWER DISTRIBUTION

This Appendix concerns the distribution of arc power levels in the ranges $0 \rightarrow r$ and $0 \rightarrow s$ of arc start and stop voltages. In Eq. (47) of the text, we write the distribution of power levels $I_C^2(r,s)$ over the start and stop voltage ratios r and s as

$$\partial^2 [I_C^2] / \partial r \partial s = [k\beta I_s^2] G(r,s),$$

$$\text{where: } G(r,s) = (1+r)(r+s)e^{-kr} e^{-\beta s}. \quad (E1)$$

The number of arcs with power I_C^2 in $0 \rightarrow r$ and $0 \rightarrow s$ is proportional to

$$\Gamma(r,s) = \int_0^r d\xi \int_0^s d\eta G(\xi,\eta). \quad (E2)$$

The complete ranges of the start and stop voltage ratios are: $0 \leq r \leq \infty$, $0 \leq s \leq 1$. Then the fraction of arcs that fire at power levels: $I_C^2(\xi,\eta) = I_s^2(1+\xi)(\xi+\eta)$, with: $0 \leq \xi \leq r$, $0 \leq \eta \leq s$, is

$$\mathcal{F}(r,s) = \Gamma(r,s) / \Gamma(\infty,1). \quad (E3)$$

A straightforward integration yields

$$\begin{aligned} k^4 \Gamma(r,s) = & \frac{k}{\beta} (1-e^{-Y}) \{ (k+2) [1-(1+X)e^{-X}] - X^2 e^{-X} \} + \\ & + \frac{k^2}{\beta^2} [1-(1+Y)e^{-Y}] \{ (k+1)(1-e^{-X}) - X e^{-X} \}, \end{aligned} \quad (E4)$$

with: $X = kr$, and: $Y = \beta s$. The norm factor in Eq. (E3) is then

$$k^4 \Gamma(\infty,1) = \frac{k}{\beta} \left\{ \left[k + 2 + \frac{k}{\beta} (k+1) \right] (1-e^{-\beta}) - k(k+1)e^{-\beta} \right\}. \quad (E5)$$

The fraction in Eq. (E3) is now the ratio of Eq. (E4) to Eq. (E5).

In Appendix D, we showed that the stop parameter β is almost equal to the initial cascade parameter k ; we now assume $\beta=k$. Next, we form the fraction in Eq. (E3) and plot it versus the arc start voltage ratio r , for various values of the stop voltage ratio s . The results appear in Fig. 9, for $k=5$. We see from Fig. 9 that when neither r nor s is small, about half of all arcs occur with a combined total: $(r+s) < 1$. More precisely, for $k = 5$ and a fixed value of s , let $\tilde{r}$ be the r -value at which $\mathcal{F}(\tilde{r}, s) = 50\%$; i.e., 50% of the arc power levels lie in the ranges: $0 \rightarrow \tilde{r}$, $0 \rightarrow s$. Then

stop range: $0 \rightarrow s$	0.80	0.70	0.60	0.50	0.40	0.30	0.25
50% start in: $0 \rightarrow r$	0.29	0.31	0.33	0.36	0.43	0.58	0.78
$\tilde{r}+s$	1.09	1.01	0.93	0.86	0.83	0.88	1.03
max. drop: $\Delta V_c/V_o$	0.845	0.771	0.699	0.632	0.580	0.557	0.579

(E6)

Here we have also calculated the maximum voltage drop for the arcs that have fired: $\delta = \Delta V_c/V_o = (\tilde{r}+s)/(1+\tilde{r})$. By Appendix B, for $k=5$, arcs with $\delta \sim 0.8$ can show significant inductive effects. So the present arc discharge theory--which ignores inductive effects--becomes less accurate for "large" arcs, those with a stop voltage ratio $s > 0.7$.

Finally, in Fig. 10, we show the k -dependence of the arc firing fraction $\mathcal{F}(r, s)$ of Eq. (E3). Here $\beta = k$, $r = s$, and we plot $\mathcal{F}(r, s)$ vs k for various choices of $(r+s) = \text{cnst}$. The firing fraction increases with k , and many more "small" arcs [e.g., $(r+s) < 0.7$] become possible for $k > 5$. When $k = 5$ increases to $k = 7$, the fraction of arcs with $(r+s) < 0.7$ increases from 40% to 60%.

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