Distribution Category:
LMFBR—Physics: Base
Technology (UC-79d)

ANL-81-72

ANL-81-72

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

ADND 015564

AN EXPERIMENTAL METHOD FOR
REACTOR-NOISE MEASUREMENTS OF EFFECTIVE BETA

by

E. F. Bennett

Applied Physics Division

September 1981

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION	1
II. EXPERIMENTAL TECHNIQUE	2
III. EXPERIMENTAL RESULTS	3
IV. ERRORS IN THE MEASUREMENT OF BETA	8
A. Measured Noise	8
B. Subcriticality	8
C. Dispersion Factor D	9
D. Fission Rate	9
E. Summary	10
V. CONCLUSIONS	10
VI. ACKNOWLEDGEMENT	11
VII. REFERENCES	11
APPENDICES:	
A Sensitivity of Noise Measurements to Power Drift	13
B Experimental Method for Noise Measurement of Beta	19
C Conditional Probability and the Point Kinetics Model	37
D Large Area Leakage Neutron Detectors for ZPR-9 Kinetics Measurements	43
E Data Acquisition and Analysis Codes for Noise Measurements	55

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1	Noise vs Subcriticality for Carbide Benchmark	4
2	Noise vs Subcriticality for RSR Reference	6
3	Voltage Dependence of Detector Response	51

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
I	Preliminary Beta Measurement Results.	7
II	Errors in the Noise Determination of β^2	10
III	Variance vs Sampling Time for an α -n Source	50
IV	Data for the ^{252}Cf Source Strength Measurement.	54
V	Data Acquisition for Noise Measurement of Beta.	58
VI	Reduce Data to Provide Effective Beta from Noise.	62
VII	Second Run on Leaky STF1/1, 6/26/80	66
VIII	Second Run on Leaky STF1/1, 6/26/80	67

AN EXPERIMENTAL METHOD FOR REACTOR-NOISE MEASUREMENTS OF EFFECTIVE BETA

by

E. F. Bennett

ABSTRACT

A variance-to-mean noise technique, modified to eliminate systematic errors from drifting of reactor power, has been used to infer integral values of effective beta for uranium and plutonium fueled fast reactor mock-ups. The measurement technique, including corrections for a finite detector-electrometer time response, is described together with preliminary beta measurement results.

I. INTRODUCTION

Measurement programs on critical facilities provide data relating to the reactivity effects of materials. These data must be scaled from units employed in the measurement (typically inhours) to provide values of $\Delta k/k$ needed to compare with computations. For all practical purposes the scale factor is the product of the effective beta, β , and a characteristic delayed-neutron time scale, $\bar{\ell}$, defined as

$$\bar{\ell} = \sum_i \frac{a_i}{\lambda_i}$$

where a_i are the relative precursor yields ($\sum a_i = 1$) and λ_i are the corresponding decay constants. The accuracy of the scale factor is inevitably questioned whenever systematic biases arise in the comparison of experimental results with theory. It is of considerable interest, therefore, to subject the scale factor to observation.¹

β and $\bar{\ell}$ can be measured independently (which is the reason for separating them explicitly in the scale factor). $\bar{\ell}$ is about 12 sec and can be deduced through use of mechanical oscillations of rods or by observing the flux response of reactors following transient reactivity-step changes.

Integral measurements of beta are sometimes made by comparing the measured worth of a quantity (in inhours) to its calculated worth in $\Delta k/k$. The ratio of measured to calculated results is essentially the scale factor $\beta\bar{\ell}$. Examples of the use of this procedure are contained in Ref. 2 for a central material void. The technique of Ref. 2 would be difficult to apply to LMFBR-prototypical mock-ups fabricated with a heterogeneous mix of materials in the form of plates. This technique is predicated upon the assumption that central small-sample (or void) worths are calculable with negligible uncertainty. While this assumption may hold for some special reactor types (homogeneous with highly-enriched uranium or plutonium fuel), there is no a-priori basis for assuming that the computation of central worths is an error-free procedure in general.

Effective beta can be measured by noise techniques. One particular implementation of the noise method is described here. The time (frequency) regime emphasized in noise measurements of beta has important implications for the result. Since the Rossi alpha for a typical fast-spectrum reactor is $\sim 10^4/\text{sec}$ and since the largest delayed-neutron precursor decay constant is $\sim 3/\text{sec}$, a broad intermediate-frequency band exists which is well-removed from either extreme. The result of measurements performed within this band will not be sensitive to either prompt or delayed effects. The only factor contributing to neutron kinetic behavior in this intermediate frequency range for an at-critical reactor is the quantity $D/\beta^2 F$ where F is the absolute fission rate and D is a spatially-averaged factor of magnitude near unity and known to high accuracy. Since β enters as the second power relative to D and F , uncertainty in the determination of β is reduced by half relative to uncertainty in the other parameters involved. The noise technique has, therefore, the potential for results of good accuracy.

II. EXPERIMENTAL TECHNIQUE

Noise measurements of effective beta have been performed using a variety of experimental techniques reflecting state-of-the-art in electronics and the preferences of individual experimenters. The approach followed here was dictated by:

1. the availability of accurate, reliable, and inexpensive voltage-frequency converters together with access to on-line digital processing hardware, and
2. the intent that the method should be of maximum simplicity and reliability to permit its use routinely whenever needed.

Analog signal averaging is achieved in a very straightforward way using voltage frequency converters. Non-linear operations, such as multiplication or squaring can take advantage of the high accuracy inherent in digital processors.

Mean square (and cross products) of average signal levels from efficient detectors exposed to reactor neutrons are good measures of noise, and absolute values of these quantities without accuracy restrictions or supplemental calibration measurements are readily determined. The method is not essentially different from the variance-to-mean technique used originally at Los Alamos³ although there is one important difference dictated by the tendency of reactors operated near critical to undergo a slow power drift exacerbated by temperature instabilities occurring during the course of the measurement. A procedure for reducing the effect of this drift to negligible levels was used in work described here. A detailed discussion may be found in Appendix A.

Detector performance is an important consideration for noise (and other) kinetics measurements and a pair of efficient fast-responding BF_3 ion chambers were fabricated and installed on the top of each half of the matrix of the ZPR-9 critical facility used for mock-up studies at Argonne. A detailed description of these detectors and a discussion of their performance

is provided in Appendix D; these detectors are used routinely for worth and subcriticality measurements as well as for noise.

When the experimental noise method is designed to emphasize the intermediate-frequency region of reactor noise, a measured quantity designated by $\tau\sigma^2$ is determined which, according to interpretation by the point kinetics model, bears the relationship

$$\tau\sigma^2 = \frac{(3/2)D}{F\beta^2(1+\beta)^2} \quad (1)$$

to fission rate, F ; subcriticality, β ; and dispersion, D . This result is derived, including the effects of detector response and delayed and prompt neutrons, in Appendix B. Appendix C contains a derivation of the fundamental mode conditional probability for neutron detection required to obtain Eq. 1 from the noise analysis in Appendix B.

Appendix E contains a pair of codes used to acquire data and to make corrections for detector frequency response and for prompt and delayed neutrons. These corrections are held to the few-percent level by optimal choice of the sampling interval range. Also included in Appendix E is a complete set of data, and the analysis results, for a noise measurement on a ZPR-9 mock-up of a fast-spectrum reactor.

III. EXPERIMENTAL RESULTS

If we assume that the absolute fission rate of the mock-up is proportional to the current level, I , from the detectors, we may write Eq. 1 in the form

$$I(\tau\sigma^2)(1+\beta)^2 = \text{constant} \quad (2)$$

where the constant indicated on the right-hand side of Eq. 2 contains terms not expected to vary significantly with subcriticality if the point model is valid. To the extent that the quantity on the left-hand side of Eq. 2, which contains only measurable terms, is observed to be independent of subcriticality, we retain confidence that the point model remains valid. If, for example, harmonic modes higher than the fundamental become increasingly effective as subcriticality increases, the scaling law of Eq. 2 would be violated. It is, consequently, of considerable interest to extend measurements as far subcritical as possible to discover if Eq. 2 ceases to hold. Some additional comments on this and on other issues relating to the validity of the point kinetics model for interpreting noise measurements are contained in Appendix C.

Figure 1 contains a plot of the quantity $I\tau\sigma^2(1+\beta)^2$ as a function of subcriticality for a plutonium-fueled mock-up referred to as the Carbide Benchmark. The reactor contained spontaneous-fission neutron sources from the ^{240}Pu isotope in the fuel and a steady flux level was maintained by this source at each of the subcritical configurations for which noise measurements were done. Measurements were made up to about 5β subcritical (by introducing boron safety rods) at which point uncertainty in the rod-drop technique for

Fig. 1. Noise vs Subcriticality for Carbide Benchmark

subcriticality determination commences to introduce errors of a significant magnitude. Closer to critical than 5% , the rod-drop measurement technique becomes less uncertain and influences the error in $\beta^2(1+\beta)^2$ correspondingly less. As can be seen from Fig. 1, there is no evidence for dependence of $\beta^2(1+\beta)^2$ upon subcriticality and, consequently, no evidence for a breakdown of the point model description of the noise process.

Figure 2 contains the results of a similar experiment performed on the STF core and also extended to about 5% subcritical. This core was fueled with highly enriched ^{235}U and contained a californium spontaneous-fission source placed near the center. Subcriticality was measured by a source-jerk technique. As can be seen from the data plotted in Fig. 2, a statistically significant increase in $\beta^2(1+\beta)^2$ begins to appear at more than about 2% subcritical. This probably is an indication of breakdown of the point-kinetics model as applied to this core. Since the reactor in its subcritical configurations was driven by a point neutron source, one might expect that the onset of higher flux harmonic contamination of the fundamental mode will begin to appear closer to critical than would be the case for plutonium-fueled reactors where the spontaneous source is uniformly distributed with the fissile loading. Also, since effective beta for the uranium core is almost twice as great as for the plutonium core, the $\Delta k/k$ range covered in Fig. 2 is about twice as great as in Fig. 1. It is possible that the trend seen in Fig. 2 would also appear in Fig. 1 if both contained measured results over the same $\Delta k/k$ range.

The pair of leakage detectors used for the noise measurements was mounted in close proximity to each other on the top of the split-table ZPR-9 facility. Detectors mounted close together would be expected to exhibit an enhanced joint noise response relative to detectors placed apart as the reactor becomes increasingly subcritical and higher spatial flux harmonics become more significant relative to the fundamental.

Plots in Figs. 1 and 2 indicate that little, if any, dependence of noise results upon subcriticality exists at least for subcriticalities less than 1.5% . Further subcritical than 1.5% , for cores of the type considered here having k_∞ in the range 1.2 to 1.4, it would appear that the predictions of the point model are adequate to interpret noise measurements. Use of the point model to infer beta from near critical reactors using low-frequency measurement techniques would appear to be a justifiable procedure.

Although the purpose of this report is to provide a detailed description of the noise measurement technique, measured results with a preliminary analysis are available for numerous mock-ups on ZPR-9. Data relevant to noise measurements of beta are listed in Table I for seven fast-reactor mock-ups, three fueled by enriched uranium and four fueled by plutonium. Two of the uranium mock-ups were similar apart from the extent of peripheral shielding used. They are designated as STF and "leaky" STF; the "leaky" version, with minimal radial shielding, was built to test for possible room-return effects. The plutonium-fueled cores were the GCFR-I (Gas-Cooled Fast Reactor), the Carbide Reference, the RSR (Reactor Safety Research) core, and a small all-plutonium-fueled core (PuSSC) and were more or less prototypical of FBR's, at least in regard to spectrum and $^{238}\text{U}/^{239}\text{Pu}$ fission ratios. Additional descriptive material is given in Refs. 4, 5, 6, 15, 16 and 17. The data in Table I are preliminary and may change to the extent

Fig. 2. Noise vs Subcriticality for RSR Reference

TABLE I. Preliminary Beta Measurement Results

Experimental	GCFR-I	RSR Reference	Carbide Benchmark	STF	Leaky STF	U9	PuSSC
Isotopic Dispersion D	0.805	0.805	0.805	0.795	0.795	0.795	0.805
Spatial Dispersion D _g	1.154	1.138	1.156	1.138	1.138	1.1725	1.0967
Measured $I_{T0}^2(1+\$)^2$ (Units of 10 ⁻³)	6.71	3.177	3.76	1.168	3.806	0.8164	4.256
Power Calibration (Units of 10 ¹⁷)	1.70	3.848	2.554	2.615	0.7927	3.261	6.3325
Calculated Beta (ENDF IV)	0.00324	0.00321	0.00370	0.00672	0.00671	0.00713	0.002188 ^c
Experimental Beta ^{a,b}	0.00350	0.00335	0.00381	0.00667	0.00673	0.00725	0.002217
C/E	0.926	0.957	0.971	1.007	0.997	0.983	0.987

^aExperimental beta derived from
$$\text{Power Calibration } [I_{T0}^2(1+\$)^2] = \frac{3}{2} \frac{D_v D_s}{\beta^2}$$

^bThe error in the derived values for beta is estimated to be $\pm 2\%$, except for GCFR-I where the error is $\pm 3\%$.

^cENDF V data used for the calculated beta.

of a few percent depending upon a more detailed analysis of some of the computed factors. These cores were all of simple (Benchmark) construction amenable to two-dimensional analysis. This allows the absolute fission rate to be determined to the best possible accuracy.

The various quantities in Table I refer to those of Eq. 1. The power factor is defined as the ratio of total fission rate, F , to chamber current, I . The Diven factor, D , is the product of a spatial factor, D_S , and an isotopic factor, D_U . These preliminary results are compared with appropriate ENDF calculations. It would seem that measured and calculated values for uranium and for the all-plutonium PuSSC core agree well but measured results for Pu/ ^{238}U cores are from 3 to 7% higher than calculated. This does not necessarily reflect upon basic ENDF data files for delayed neutrons since a large contribution to beta from plutonium-fueled cores with substantial amounts of ^{238}U comes from fission in ^{238}U , and the relative amount of fission from ^{238}U to fission in plutonium affects beta sensitively.

IV. ERRORS IN THE MEASUREMENT OF BETA

Each of the factors in Eq. 1 is subject to uncertainty. The uncertainty in beta inferred from a measurement is the consequence of errors in the measured fluctuation σ^2 as well as in the quantities F , I , and D . These quantities are not subject to any strong correlation in their determination and errors can be added in quadrature. A brief discussion of the accuracy of each of these terms and of that for the resultant value of beta will be given next.

A. Measured Noise

The measured noise, σ^2 , in Eq. 1 is a mean variance computed over long intervals (typically an hour) at a sampling interval of between 0.02 and 0.08 seconds. Statistics in the measurement are proportional to the inverse root of the total samples (about 100,000 for an hour's run) and are typically only about a half of a percent. However, limitations in the time response of the electrometers employed necessitate a correction of from 1 to 4%, depending upon sampling interval. At the longest sampling times, delayed-neutron effects contribute a few percent which must also be corrected for. These corrections are discussed in Appendices B and D. We have assigned an error of $\pm 1.5\%$ to the measured noise to include all of these sources; this is expected to be more than adequate to cover the uncertainties arising from statistics and from the above-mentioned corrections.

B. Subcriticality

Noise measurements can be made at critical but it is also useful to extend measurements to states of fairly high subcriticality to test the method. Measurements for β determination would ordinarily be made at a fraction of a dollar (or at critical) where the accuracy to which the subcritical configuration determined by a rod-drop experiment is high. An error in $(1 + \beta)^2$ of less than $\pm 1\%$ is more than sufficient to bound any uncertainty in the effect of subcriticality upon the measurement for subcriticalities of less than a half of a dollar. For large values of subcriticality which would not ordinarily be of interest for β measurement interpretation but which are of interest for

validation of the simple point model, the error in $(1 + \beta)^2$ increases to the extent of $\pm 10\%$ or so at about 5β subcritical.

C. Dispersion Factor D

The dispersion factor, D, in Eq. 1 is derived in Appendix B and is given by the product of a spatial dispersion factor

$$D_s = \frac{\int F(\vec{r}) d\vec{r} \int (\bar{\nu}^2) F(\vec{r}) I^2(\vec{r}) d\vec{r}}{[\int \bar{\nu} F(\vec{r}) I(\vec{r}) d\vec{r}]^2} \quad (3-a)$$

and an isotopic dispersion

$$D_v = \langle \bar{\nu}(\bar{\nu}-1)/\bar{\nu}^2 \rangle \quad (3-b)$$

where $F(\vec{r})$ and $I(\vec{r})$ are the spatial distributions of fission rate and fission-spectrum weighted adjoint. Integration in Eq. 3-a and brackets $\langle \rangle$ in Eq. 3-b also imply summation over fissile species. The quantity $\bar{\nu}(\bar{\nu}-1)$ is the value of the product of $\bar{\nu}$ and $\bar{\nu}-1$ (where $\bar{\nu}$ is the neutron emission per fission) averaged over the multiplicity distribution of neutrons emitted per fission. This distribution has been measured for the fissile isotopes of interest (but not for ^{238}U). The quantity $D_v = \bar{\nu}(\bar{\nu}-1)/\bar{\nu}^2$ changes only slightly from isotope to isotope; it has the value 0.795 ± 0.007 and 0.805 ± 0.017 for induced fission in ^{235}U and ^{239}Pu , respectively. A theoretical analysis of all of the relevant data has provided a simple expression containing only $\bar{\nu}$ as an adjustable parameter⁷; from the expression it may be concluded that $\bar{\nu}(\bar{\nu}-1)/\bar{\nu}^2$ for ^{238}U fission is essentially the same as for ^{235}U fission (since $\bar{\nu}$ is essentially the same for these isotopes). For each fissile specimen we may assign an error of about $\pm 2\%$ to the isotopic dispersion factor D_v .

Error to an extent less than $\pm 0.5\%$ can be expected from uncertainties in the spatial averaging indicated in the integral in Eq. 3-a. This arises from uncertainties in the spatial distribution of fission and adjoint; since both of these quantities appear in numerator and denominator of Eq. 3-a the uncertainty in the ratio is less than any spatial point-by-point uncertainty in $F(\vec{r})$ or $I(\vec{r})$ however. The conclusion is that uncertainty in the dispersion factor $D = D_s \cdot D_v$ can be set at about $\pm 2.0\%$ with essentially all of this uncertainty arising from $\bar{\nu}(\bar{\nu}-1)/\bar{\nu}^2$ for fissionable isotopes.

D. Fission Rate

The absolute fission rate, F, at the power level setting used in the noise measurement is, by a significant margin, the least accurate of the various quantities defined in Eq. 1. F is ordinarily determined by making an absolute fission rate measurement at some spatial location in the reactor and multiplying this result by a calculated ratio of total fission rate to fission rate at the point of absolute measurement. Any uncertainty in modeling the reactor core and its cell structure will appear in F. On occasion, extensive experimental programs for reaction-rate measurements are available to help to reduce the uncertainty, but lacking spatially-detailed

measurements, uncertainty in F at a level of about $\pm 3\%$ appears unavoidable. In some instances, where unit cell structure is such that large local flux and spectrum changes occur, uncertainty in F is probably greater than 3% . There may occasionally be other difficulties such as inadequate core reflection which would permit a significant amount of room-return neutron flux not properly accounted for in the synthesis of total fission rate. The GCFR-I results in Table I are subject to additional error from this source. An error in F of $\pm 3\%$ is realistic for simple cores having a non-complex cell structure and an adequate isolation from the environment of the critical facility.

E. Summary

Table II below lists all of the quantities in Eq. 1 (except β , which is to be determined) and estimates of rms errors.

Table II. Errors in the Noise Determination of β^2

Parameter	Approximate % Error
Measured Noise	1.5
$(1 + \beta)^2$	1.0 (or zero if $\beta=0$)
D	2.0
F	3.0
Total error (quadrature addition)	4.0

Since β in Eq. 1 occurs to second power, the uncertainty from all other parameters is reduced by $1/2$. The result is that the error in effective beta extracted from a noise measurement is $4.0/2 = 2.0\%$ where errors are those listed in Table II, and this error is dominated by uncertainty in the absolute fission rate. The four quantities in Table II are determined by methods having little in common with one another, and the quadrature addition employed in arriving at a total error should be well justified.

V. CONCLUSIONS

Noise measurements using the modified variance technique here described are not subject to systematic error from reactor drift and provide a value for effective beta with less uncertainty than is quoted for basic data used in the construction of ENDF data files. The technique is insensitive to delayed-neutron effects while emphasizing the low frequency region of reactor kinetic response where the point model has been well validated. Results of use of the method, although preliminary as reported here, do not indicate any statistically significant discrepancy between measured and calculated values for beta

for uranium or for all-plutonium mock-ups but do indicate an underprediction of beta by 3 to 7% for FBR-prototypical plutonium-fueled mock-ups containing fertile ^{238}U where ENDF-IV basic data files are used for the calculation of beta.

The purpose of this report is to provide a description of the noise measurement technique and not to enter into a discussion of the methods used for calculation of the Diven factor or for the measurement of absolute power, although results for these parameters are essential to the interpretation of noise data. A comprehensive description of all of this relevant material and a final reporting of the results of beta measurements compared with calculated values using the most recent ENDF data will be published later.

VI. ACKNOWLEDGMENT

Appreciation is here expressed to F. Tsang and J. Paul for their assistance in the development of the detector system. Computations of the spatial dispersion factor were provided by D. Wade, B. Yarlagadda, and R. Schaefer. J. Morman and G. DiIorio provided results of their absolute fission rate determinations; and D. Wade, R. Schaefer, and R. McKnight performed the computational analyses required to derive beta from ENDF files and to synthesize total absolute fission rates. The author also wishes to acknowledge many valuable discussions concerning reactor kinetics measurements with S. Carpenter and C. Cohn.

VII. REFERENCES

1. S. G. Carpenter, J. M. Gasidlo, J. M. Stevenson, "Beta Measurements in Two Fast Reactor Critical Assemblies," in Applied Physics Division Annual Report, June 1969 to June 1970, ANL-7710, 202 (1971).
2. G. E. Hansen and H. C. Paxton, A Critical Assembly of Uranium Enriched to 10% in Uranium-235, Nucl. Sci. Eng., 72, 230 (1979).
3. R. Feynmann, F. DeHoffmann, R. Serber, J. Nucl. Energy, 3, 64 (1956).
4. G. J. DiIorio, D. M. Smith, and W. R. Robinson, "The Uranium/Iron Benchmark Assembly: Operational Parameters," in Reactor Development Program Progress Report, ANL-RDP-85, 4.3 (July 1979).
5. S. K. Bhattacharyya, D. C. Wade, R. G. Bucher, R. D. McKnight, and L. G. LeSage, A Critical Experimental Study of Integral Physics Parameters in Simulated LMFBR Meltdown Cores, International Meeting on Nuclear Power Reactor Safety, Brussels, Belgium (October 1978).
6. R. B. Pond, W. R. Robinson, D. M. Smith, "The Advanced-Fuels-Program Carbide Benchmark Assembly," in Reactor Development Program Progress Report, ANL-RDP-56, 5.21 (December 1976).
7. E. K. Hyde, The Nuclear Properties of the Heavy Elements Vol. III: Fission Phenomena, Prentice Hall, Englewood Cliffs, New Jersey (1964).

8. E. F. Bennett, The Rice Formulation of Pile Noise, Nucl. Sci. Eng., 8, 53 (1960).
9. W. Seifritz, D. Stegemann, and W. Vath, Two-Detector Crosscorrelation Experiments in the Fast Thermal Argonaut Reactor STARK, Proc. Intern. Symp. on Neutron Noise, Waves, and Pulse Propagation, Gainesville, Florida, AEC-Symp., Ser. No. 9, 195-213 and KFK 413 (1966).
10. T. Nomura, S. Gotoh, and K. Yamaki, Reactivity Measurements by the Two-Detector Crosscorrelation Method and Subcritical Reactor Noise Analysis, Proc. Intern. Symp. on Neutron Noise, Waves, and Pulse Propagation, Gainesville, Florida, AEC-Symp., Ser. No. 9, 217-246 (1966).
11. W. Seifritz, The Polarity Correlation of Reactor Noise in the Frequency Domain, Nuclear Applications and Technology, 7, 513 (1969).
12. M. Otsuka and T. Iijima, Space-Dependent Formula for Rossi-Alpha Measurements, Nukleonik, 7, 488 (1965).
13. M. M. R. Williams, Random Processes in Nuclear Reactors, Pergamon Press, Oxford (1974).
14. J. T. Mihalczo, Nucl. Sci. Eng. 60 262 (1976).
15. S. K. Bhattacharyya, W. R. Robinson, R. D. McKnight, and E. M. Bohn, Evaluation of the Critical Mass of the ZPR-GCFR Phase I Assembly, ZPR-TM-198 (June 16, 1975).
16. R. B. Pond, ZPR-9 Assembly No. 36, An Enriched Uranium/Depleted Uranium Assembly, ZPR-TM-380 (October 10, 1980).
17. R. B. Pond and R. Scharping, Operational Measurements in ZPR-6 Assembly No. 10, a Pu/C/SST Assembly, ZPR-TM (to be issued).

APPENDIX A

Sensitivity of Noise Measurements to Power DriftA. Drift Sensitivity and Variance Definition

Detectors exposed to reactor neutrons exhibit fluctuations in output which are in excess of counting statistics; the excess fluctuations are referred to as pile noise. Where the measure of noise is a mean square fluctuation in short-term average levels, the technique is labelled a variance technique. To develop the variance methods, we focus upon a variable of primary interest which is a dimensionless function $U(t)$ representing the instantaneous fractional variation of electrometer output voltage about its mean value

$$U(t) = \frac{v(t) - \bar{v}}{\bar{v}} . \quad (A.1)$$

$v(t)$ is the instantaneous output level at time t and $\bar{v}$ is the long-term average of $v(t)$. Instantaneous output values are of little physical interest. Averages over fixed time intervals are readily measured with the use of voltage-frequency converters and scalars, and fluctuations in these averages provide a convenient quantitative measure of reactor noise. Let U_k be the average of $U(t)$ over the time interval $\tau = t_{k+1} - t_k$,

$$U_k = \frac{1}{\tau} \int_{t_k}^{t_{k+1}} U(t) dt . \quad (A.2)$$

By definition, the long term average of U_k is zero.

In the absence of long-term drifting of flux level in response to effects such as temperature change, the normal variance here defined as

$$\left[\begin{array}{c} \text{normal} \\ \text{variance} \end{array} \right] = \frac{1}{K} \sum_{k=1}^K U_k^2 \quad (A.3)$$

would serve as an adequate measure of noise. However, for practical measurement conditions on zero power mock-up facilities, long-term drifting will introduce large systematic errors into the measurement of variance by Eq. A.3 and this approach must be abandoned.

A simple alternative measure of noise can be introduced which is essentially unbiased by any level of drift encountered in practice. The basic unit of this measure is the quantity δU_k defined in terms of the U_k of Eq. A.2 by

$$\delta U_k = U_k - \frac{1}{2} [U_{k+1} + U_{k-1}] . \quad (A.4)$$

From the δU_k we form the long-term average

$$\langle \delta U_k^2 \rangle = \frac{1}{K-2} \sum_{k=2}^{K-1} \delta U_k^2 \quad (\text{A.5})$$

and refer to $\langle \delta U_k^2 \rangle$ as a "local" variance; it is the mean square of the fluctuation relative to an average over points adjacent to the center point. Also, it is of interest to note that the grouping of flux averages according to Eq. A.4 is equivalent (in the limit of vanishingly-short averaging times) to the taking of a second derivative of the noise profile

$$\delta U_k = U_k - \frac{1}{2} [U_{k+1} + U_{k-1}] = -\frac{1}{2} \frac{d^2 U}{dk^2} \quad (\text{A.6})$$

As a practical matter, it is no more difficult to do on-line data processing of measured averages through use of Eqs. A.4 and A.5 than by use of the normally-defined variance grouping of Eq. A.3; both are easily handled with digital processors.

The use of a locally-defined variance to determine reactor noise fluctuations leads to a very large reduction in sensitivity of measured noise to spurious drift. To illustrate this drift-sensitivity effect in a quantitative way, estimates of the effect of drift upon measurement of both the usually-defined variance and the local variance will be made here.

The local variance is equivalent to use of the second derivative of the flux in noise estimates. If $U(t)$ denotes the fractional variation of flux about its mean, then, for stationary noise

$$U(t) = \sum_{n=1}^{\infty} a_n \cos w_n t + b_n \sin w_n t \quad (\text{A.7})$$

and the second derivative of $U(t)$, denoted by $U^{11}(t)$, becomes

$$-\frac{1}{2} U^{11}(t) = \frac{1}{2} \sum a_n w_n^2 \cos w_n t + b_n w_n^2 \sin w_n t \quad (\text{A.8})$$

The random variables U and U^{11} are defined over an ensemble record of length, T , where $w_n = 2\pi n/T$. a_n and b_n are Fourier amplitudes whose average square value is the power spectrum, W , where

$$\langle a_n^2 \rangle = \langle b_n^2 \rangle = W(w_n)/T \quad (\text{A.9})$$

a_n and b_n obey the ensemble-averaging conditions

$$\langle a_j b_k \rangle = \langle a_j a_k \rangle = \langle b_j b_k \rangle = 0 \quad (j \neq k) \quad (\text{A.10})$$

where brackets $\langle \rangle$ denote an average over an ensemble of records each of length T . If Eqs. A.7 and A.8 are squared and an ensemble average taken with the help of Eqs. A.9 and A.10, and the result is integrated over frequency, we find that

$$\langle U^2 \rangle = \frac{1}{\pi} \int W(\omega) d\omega \quad (\text{A.11})$$

$$\langle (U^{11})^2 \rangle = \frac{1}{4\pi} \int \omega^4 W(\omega) d\omega \quad (\text{A.12})$$

In the intermediate-frequency region, between β/λ_0 (the ratio of beta to prompt lifetime) and the delayed-neutron precursor decay constants λ_1 emphasized in beta measurements, the pile-noise power spectrum near critical has a value

$$W(\omega) \sim \frac{2}{F} \frac{1}{\beta^2} \quad (\text{A.13})$$

apart from a constant close to unity.

From Eqs. A.11 and A.12 evaluated using Eq. A.13 over a frequency range from ω_1 to ω

$$\langle U^2 \rangle = \frac{2}{\pi F \beta^2} (\omega - \omega_1) \sim \frac{2}{\pi F \beta^2} \omega \quad (\text{A.14})$$

$$\langle (U^{11})^2 \rangle = \frac{1}{10\pi F \beta^2} (\omega^5 - \omega_1^5) \sim \frac{1}{10\pi F \beta^2} \omega^5 \quad (\text{A.15})$$

where in Eqs. A.14 and A.15 we have retained only the upper frequency range of frequencies included in the fluctuation measurement. In practice, this corresponds to a variance sampling time, τ , of about 0.02 sec or an equivalent frequency of $\omega \sim 1/\tau = 50/\text{sec}$ in Eqs. A.14 and A.15.

Equations A.14 and A.15 tell us how the variance and local variance depend, approximately at least, upon reactor parameters where only normal pile noise constitutes the fluctuation source. Let us now suppose that a "drift function" $D(t)$ also exists which describes the long-term fractional variation of flux level about its value at $t = 0$ in response to effects such as a slow temperature change. If the reactivity change in consequence of the temperature change is linear with time, we may express it as

$$\delta(t) = R t \quad (\text{A.16})$$

where R is the rate of drift (dollars/sec) in the time interval T and the reactor was assumed to be critical at time zero. Equation A.16, when used in the approximate expression

$$\frac{dn}{dt} = n \frac{\delta(t)}{\lambda} \quad (\text{A.17})$$

relating slow changes in neutron flux n to reactivity excess, $\rho(t)$, gives the simple drift function

$$D(t) = \exp\left[\frac{Rt^2}{2\bar{\lambda}}\right] - 1 \sim \frac{Rt^2}{2\bar{\lambda}} \quad (\text{A.18})$$

where we have also assumed that the total extent of the drift over the full interval is small, i.e.,

$$\frac{RT^2}{2\bar{\lambda}} \ll 1 \quad . \quad (\text{A.19})$$

$\bar{\lambda}$ is the mean delayed-neutron reciprocal decay constant and has a value of about 12 sec. We may now form average square values for both $D(t)$ and its second derivative with the result

$$\langle D^2 \rangle = \frac{1}{20} \left(\frac{R}{\bar{\lambda}}\right)^2 T^4 \quad (\text{A.20})$$

$$\left\langle \frac{1}{4} (D^{(2)})^2 \right\rangle = \frac{1}{4} \left(\frac{R}{\bar{\lambda}}\right)^2 \quad . \quad (\text{A.21})$$

The ratio of Eq. A.20 to Eq. A.14 is the relative contribution of drift to pile noise for normal variance and the ratio of Eq. A.21 to Eq. A.15 is the same quantity for local variance.

$$\frac{\text{drift}}{\text{noise}} \left(\frac{\text{normal}}{\text{variance}} \right) = \frac{\pi}{40} F\beta^2 \left(\frac{R}{\bar{\lambda}}\right)^2 \frac{T^4}{w} \quad (\text{A.22})$$

$$\frac{\text{drift}}{\text{noise}} \left(\frac{\text{local}}{\text{variance}} \right) = \frac{5\pi}{2} F\beta^2 \left(\frac{R}{\bar{\lambda}}\right)^2 \frac{1}{w^5} \quad (\text{A.23})$$

The ratio of Eq. A.22 to Eq. A.23 is a measure of the relative sensitivity to drift of the normally-defined variance to that for local variance

$$\frac{\text{Relative Drift}}{\text{Sensitivity}} = \frac{(wT)^4}{100} \quad . \quad (\text{A.24})$$

With w at 50/sec over an interval $T = 5$ min, the sensitivity of normal variance to drift is about 10^{14} greater than is the sensitivity of local variance to drift.

A number of simplifying assumptions have been made here to obtain quantitative estimates of the effect of reactor power drift upon noise measurements but for any kind of drift assumed, the reduction in drift-sensitivity using the local-variance treatment is very large. As an example, fast reactor mock-ups usually have about an inhour per $^{\circ}\text{C}$ temperature coefficient. Due to heating from alpha decay, plutonium-fueled criticals are subject to thermal imbalances amounting to as much as a tenth of a degree per hour or so.

Uranium-fueled mock-ups tend to drift less since heating of the core is negligible, but not by more than about an order of magnitude at which point ambient room air conditions become a limiting factor in determining drift (for ZPR facilities at Argonne). Spontaneous fission neutron sources of magnitude $\sim 2 \times 10^7$ are inherent in the use of plutonium fuel with the consequence that total fission rates of at least $10^9/\text{sec}$ are required if subcriticality is not to exceed about one dollar. As may be seen from Eqs. A.22 and A.23, the effect of drift upon measured noise worsens with increasing fission rate. If we assume the following set of values for parameters in Eq. A.22; $F = 10^9/\text{sec}$, $\beta = 0.0035$, $\bar{t} = 12 \text{ sec}$, $w = 50/\text{sec}$, $T = 5 \text{ min}$, $R = 10^{-7} \text{ dollars/sec}$, then Eq. A.22 indicates that drift error will exceed pile noise by almost two orders of magnitude where normal variance is used. This large drift-induced systematic error will occur even though the maximum extent of drift will only be $10^{-7}(300)^2/(2.12) = 0.04\%$ of average level during the measurement interval. Clearly, meaningful results cannot be achieved under these circumstances. The situation is better for uranium-fueled reactors with very small spontaneous sources present but, in general, only under the most ideal circumstances or where measurements are carried out with the reactor substantially subcritical with only a weak source present will normal variance measurements be feasible. The very large reduction in drift sensitivity inherent in the use of a local variance treatment permits drift-free measurement of noise regardless of reactor stability.

Other groupings of data than by second derivative can be used. Higher order derivatives will exhibit even less sensitivity to drift. However, statistical measurement error can be shown to increase progressively with the use of higher derivatives and since second derivative is more than adequate for drift elimination, motivation for use of any other grouping does not exist.

APPENDIX B

Experimental Method for Noise Measurement of BetaA. The Pile Noise Power Spectrum

The observable quantities in noise measurements by variance techniques are the average detector levels (jointly and individually for detectors in pairs) over specified time intervals. For peripherally-mounted $^{10}\text{BF}_3$ detectors essentially no sensitivity other than to neutrons occurs -- the blanket surrounding the core will reduce fission-product gamma rays to negligible levels compared to neutrons. Some sensitivity to gammas from neutrons captured in the detector may exist but this is not objectionable and only serves to enhance efficiency slightly. The large amount of ionization created by absorption of neutrons in ^{10}B will ensure that little contribution to chamber currents will occur from gamma rays whatever their origin.

One may proceed in standard fashion⁶ to express the instantaneous output of detectors as a Fourier series in sine and cosine terms having random amplitudes. All time averages of output will then be related to the dependence of these random amplitudes (power spectra) upon frequency. The power spectrum is derivable from the conditional probability that an event is detected at a time, t_2 , given that an event occurred at t_1 . This probability is discussed in Appendix C and a point-model derivation carried out. The power spectrum will also be influenced by the intrinsic time response of the detector and only if these detector-response phenomena are included specifically in the analysis can one choose optimum conditions for implementing the measurement technique and analyzing the results. It is essential therefore to approach the derivation in a general way and to display detector-response effects at each step.

The variable of primary interest will be taken to be a random function $U(t)$ which represents the instantaneous fractional detector output relative to its mean value.

$$U(t) = (v(t) - \bar{v})/\bar{v}$$

where $v(t)$ is the instantaneous detector level at time t and $\bar{v}$ is the long-term average of $v(t)$. We may represent $U(t)$ as a random Fourier series

$$U(t) = \sum_{n=1}^{\infty} a_n \cos w_n t + b_n \sin w_n t \quad (B.1)$$

where $w_n = 2\pi n/T$ and a_n, b_n are the usual coefficients

$$a_n = \frac{2}{T} \int_0^T U(t) \cos w_n t \, dt \quad (B.2)$$

$$b_n = \frac{2}{T} \int_0^T U(t) \sin w_n t \, dt \quad .$$

The absence of $n = 0$ in the summation of Eq. B.1 assures a null average for $U(t)$.

The a_n, b_n are random amplitudes, with Gaussian distribution, and obey the ensemble-averaging conditions

$$\langle a_j b_k \rangle = \langle a_j a_k \rangle = \langle b_j b_k \rangle = 0 \quad (j \neq k) \quad (B.3)$$

where brackets $\langle \rangle$ are used to denote an average over an infinite ensemble of records. The ensemble-averaged values $\langle a_j^2 \rangle$ and $\langle b_j^2 \rangle$ are equal and are essentially the power spectrum.

Much of the following discussion will treat the joint fluctuation exhibited by a pair of detectors. There is little new in including joint fluctuations in the formalism. Where two detectors are treated, Eqs. B.1 and B.3 apply to each and one may proceed as in Ref. 8 to express the dependence of each spectrum upon pile and detector phenomena. Joint ensemble-averaged coefficients must also be considered -- these may be defined as

$$\langle a_j^{(1)} a_j^{(2)} \rangle = \langle b_j^{(1)} b_j^{(2)} \rangle = \text{Joint power spectrum.}$$

Detectors 1 and 2 are distinguished by superscripts. By virtue of the independence of random variables the off-diagonal terms in the average of joint coefficients will vanish as for Eq. B.3.

To include detector effects we introduce a function $f(t)$ which is the time response of the detector to an ionizing event at time zero. For BF_3 chambers $f(t)$ will rise according to charge-collection properties of the chamber and decay with some appropriate electrometer time constant when all ionization is collected. Not every event will occur at the same location in the chamber nor will track orientation be the same for different events and it will be necessary to subscript $f(t)$ to mark the time of occurrence and to add a superscript to distinguish between detectors. For example, $f_j^{(1)}(t_j)$

will be used to indicate the response of detector 1 to an event at time t_j . With this notation we may represent the instantaneous output $v(t)$ by

$$v_{(1,2)}(t) = \sum_{j=1}^k f_j^{(1,2)}(t - t_j) \quad (B.4)$$

where $f_j^{(1,2)}(t)$ vanishes for negative argument. The explicit dependence of a_n, b_n upon $f_j^{(1,2)}$ may be found using Eqs. B.1 through B.4. The ensemble-averaging procedure may then be evoked to remove dependence upon details of pulse arrival times and individual pulse shapes and the desired results obtained. When this procedure is carried through⁸ the result for individual detectors is

$$\langle a_n^2 \rangle = \langle b_n^2 \rangle = \frac{2}{FT} B_o(w_n) \frac{R(w)}{\epsilon} + \frac{2}{\epsilon} \left[\left\langle \sum_{l>m} \cos w_n(t_l - t_m) \right\rangle \right]. \quad (B.5)$$

$B_o(w_n)$ is given in terms of the Fourier integral, $\bar{f}(w)$, of $f(t)$ according to

$$B_o(w_n) = \frac{\langle \bar{f}(w_n) \rangle \langle \bar{f}^*(w_n) \rangle}{\left\langle \int_0^\infty f(t) dt \right\rangle^2} \quad (B.6)$$

and the quantity $R(w)$ is defined to be

$$R(w) = \frac{\langle \bar{f}(w_n) \bar{f}^*(w_n) \rangle}{\langle \bar{f}(w_n) \rangle \langle \bar{f}^*(w_n) \rangle}. \quad (B.7)$$

F in Eq. B.5 is the fission rate, T is the time extent of the record and ϵ is the detector efficiency defined as the ratio of detection rate to fission rate. The quantity $\left\langle \sum_{l>m} \cos w_n(t_l - t_m) \right\rangle$ in Eq. B.5 is the ensemble average of

the time-ordered cosine of argument $w_n(t_l - t_m)$. It is this term which requires a model of pile kinetic phenomena to evaluate; only phenomena in which a correlation exists between events detected at different times will show a finite value for this average. Brackets occurring in the definitions of $B_o(w)$ and $R(w)$ in Eqs. B.6 and B.7 imply an ensemble average over the intrinsic detector response, $f(t)$, or its Fourier transform $\bar{f}(w)$.

Where detectors are used in pairs, the corresponding cross-power spectrum is deduced in essentially the same manner used to obtain Eq. B.5. The only difference is the non-appearance of a term equivalent to $R(w)$, and a somewhat different expression for $B_o(w)$. The result is

$$\langle a_n^{(1)} a_n^{(2)} \rangle = \langle b_n^{(1)} b_n^{(2)} \rangle = \frac{4}{FT} B_{1.2}(w_n) \cdot \frac{1}{\epsilon} \left\langle \sum_{l>m} \cos w_n(t_l - t_m) \right\rangle \quad (B.8)$$

where $B_{1.2}(w)$ involves both detector response functions and is given by

$$B_{1.2}(w_n) = \frac{\text{Real Part of } \langle \bar{f}_1(w_n) \bar{f}_2^*(w_n) \rangle}{\left\langle \int_0^\infty f_1(t) dt \right\rangle \left\langle \int_0^\infty f_2(t) dt \right\rangle}. \quad (B.9)$$

The time-ordered ensemble average in Eq. B.8 implies that events in one detector occur at times t_2 while events in the other detector occur at times t_m . The efficiency, ϵ , in Eq. B.8 applies to the detector responding at t_2 -- it does not matter how detectors are ordered.

Thus far we have outlined the derivation of values for the ensemble-averaged random Fourier amplitudes in terms of the detector response, the induced pile fission rate, and the detector efficiency. No assumptions concerning the kinetic properties of the pile have been introduced. The random behavior of a pair of detectors viewing any radiation source may be described in this way.

To evaluate the power spectrum explicitly we require the conditional probability $P(t_2, t_m)$ that an event is detected at time t_m given that an event did occur at time t_2 . This probability is derived in Appendix C and if we use this result to evaluate Eqs. B.8 and B.9 we find

$$\langle a_n^2 \rangle = \langle b_n^2 \rangle = \frac{2B_o(w_n)}{FT} \left[\frac{R(w_n)}{\epsilon} + \frac{2Dk^2}{\ell_o^2} \sum_p \frac{A_p T(-S_p)(-S_p)}{w_n^2 + S_p^2} \right] \quad (B.10)$$

$$\langle a_n^{(1)} a_n^{(2)} \rangle = \langle b_n^{(1)} b_n^{(2)} \rangle = \left[\frac{4B_{1.2}(w_n)}{FT} \frac{Dk^2}{\ell_o^2} \sum_p \frac{A_p T(-S_p)(-S_p)}{w_n^2 + S_p^2} \right]. \quad (B.11)$$

$B_o(w)$ and $R(w)$ are given by Eqs. B.6 and B.7 and $B_{1.2}$ is given by Eq. B.9.

Equations B.10 and B.11 provide a complete description of one and two-detector noise consistent with the assumed kinetics model. If only those frequencies well below the reciprocal of the detector response time are emphasized, $B(w)$ will go to unity and $R(w)$ will become a constant, r , and serve only to re-scale the efficiency to produce a "modified" efficiency, ϵ/r , less than ϵ . It is, as a practical matter, rather essential that experiments conform to this low-frequency limit since the basic detector response (and ultimately the kinetic model) will tend to make the problem increasingly intractable as frequency increases.

B. Joint Local Variance for the Detector Pair

All relevant random phenomena for the two detectors can be interpreted through use of the expressions in Eqs. B.10 and B.11 for the individual or joint power spectra. The measure of joint fluctuation with which we shall deal here is the product of a locally-defined fluctuation in average output of the detector pair measured over identical time intervals. We will define a quantity δU_k to be the "local" flux variation according to

$$\delta U_k = \frac{1}{\tau} \int_{(k-1)\tau}^{k\tau} U(t) dt = \frac{1}{2} \left[\frac{1}{\tau} \int_{(k-2)\tau}^{(k-1)\tau} U(t) dt + \frac{1}{\tau} \int_{k\tau}^{(k+1)\tau} U(t) dt \right] \quad (B.12)$$

where $U(t)$ is the instantaneous fractional value of the deviation of detector output from its mean value. The mean value of the product of δU_k for the detector pair will be defined subsequently to be the joint local variance $\sigma_{1,2}^2$ according to

$$\sigma_{1,2}^2 = \frac{1}{N-2} \sum_{k=1}^{N-2} \delta U_k^{(1)} \delta U_k^{(2)} \quad (\text{B.13})$$

where superscripts on δU_k refer to either detector. The index k runs from 1 to N during an ensemble of length T containing N points ($N\tau = T$). Since the first and last intervals are not used, however, only $N-2$ points are included in the average. Ordinarily $N \gg 2$. The difference between the definition of δU_k we make here and the usual one entering variance-to-mean determinations is simply that we choose to use as a measure of fluctuation the difference between the average flux and its locally defined mean value rather than the difference between average flux and a long-term value for the mean. The implication of the use of a locally-defined measure of fluctuation in regards to elimination of reactor drift is discussed in more detail in Appendix A.

Starting with the definition of Eqs. B.12 and B.13 for joint local variance, we may proceed to use Eq. B.11 to express $\sigma_{1,2}^2$ in terms of the joint power spectrum. It will be somewhat more convenient if, before undertaking this derivation, we reformulate the way in which random and pile noise effects are treated. Define a separate quantity U_1 and U_2 for each detector according to

$$U_1(t) = \frac{V_1(t) - \bar{V}_1}{\bar{V}_1} \quad (\text{B.14})$$

$$U_2(t) = \frac{V_2(t) - \bar{V}_2}{\bar{V}_2} .$$

Now show random and correlated effects explicitly by making use of the following definitions

$$U_1(t) = \psi_1(t) + \phi(t) \quad (\text{B.15})$$

$$U_2(t) = \psi_2(t) + \phi(t)$$

where ψ_1 and ψ_2 refer to detector-related effects only and ϕ is the pile-related effect which is common to both detectors. U_1 and U_2 are the previously defined instantaneous fractional variations of detector output relative to the mean value. ψ_1 , ψ_2 , and ϕ are independent random variables which may be represented by the expansions

$$\psi_1 = \sum_{n=1}^{\infty} r_{n,1} \cos w_n t + q_{n,1} \sin w_n t \quad (\text{B.16})$$

$$\psi_2 = \sum_{n=1}^{\infty} r_{n,2} \cos w_n t + q_{n,2} \sin w_n t$$

$$\phi = \sum_{n=1}^{\infty} a_n \cos w_n t + b_n \sin w_n t$$

The random amplitudes $r_{n,1}$; $r_{n,2}$; $q_{n,1}$; $q_{n,2}$ refer to noise phenomena peculiar to detectors 1 and 2 individually whereas a_n and b_n refer to the common pile-noise effect. It is strictly legitimate to separate the pile and detector terms, as has been done in Eq. B.16, when the time response is the same for both detectors. This condition will hold for the pair of leakage detectors here considered and we may use the results of Eq. B.11 to write

$$\langle a_n^{(1)} a_n^{(2)} \rangle = \langle b_n^{(1)} b_n^{(2)} \rangle = B_{1,2}(w_n) \frac{4Dk^2}{FT\epsilon_0^2} \int \frac{A_P T(-S_P)(-S_P)}{w_n^2 + S_P^2} \quad (\text{B.17})$$

while, from Eq. B.10

$$r_{n,1}^2 = q_{n,1}^2 = B_o(w_n) \frac{2R(w_n)}{FT\epsilon_1} \quad (\text{B.18})$$

$$r_{n,2}^2 = q_{n,2}^2 = B_o(w_n) \frac{2R(w_n)}{FT\epsilon_2}$$

become the random-noise spectra for each detector. Equations B.17 and B.18 are nothing more than a re-write of Eqs. B.10 and B.11 to provide power spectra consistent with the notion of Eq. B.16 where detector and pile effects were exhibited explicitly. As usual, all cross ensemble averages of the random amplitudes a , b , r , q vanish.

By making use of Eqs. B.12, B.15, and B.16 it may readily be shown that the k th local variance for each detector is given by

$$\delta U_{k,1} = \delta \psi_{k,1} + \delta \phi_k \quad (\text{B.19})$$

$$\delta U_{k,2} = \delta \psi_{k,2} + \delta \phi_k$$

where

$$\delta\phi_k = \sum \frac{a_n}{w_n} C_{n,k} + \frac{b_n}{w_n} S_{n,k} \quad (B.20)$$

$$\delta\phi_{k,1} = \sum \frac{r_{n,1}}{w_n} C_{n,k} + \frac{q_{n,1}}{w_n} S_{n,k}$$

$$\delta\phi_{k,2} = \sum \frac{r_{n,2}}{w_n} C_{n,k} + \frac{q_{n,2}}{w_n} S_{n,k}.$$

The coefficients $C_{n,k}$, $S_{n,k}$ are generalized cosine and sine functions given by

$$C_{n,k} = \sum_{l=-2}^1 A_l \cos w_n (t_k + l\tau) \quad (B.21)$$

$$S_{n,k} = \sum_{l=-2}^1 A_l \sin w_n (t_k + l\tau)$$

and $A_{-2} = 1/2$, $A_{-1} = -3/2$, $A_0 = 3/2$, $A_1 = -1/2$. Equations B.19, B.20, and B.21 present the local variation in average flux as a random variable. Since we will, henceforth, only be dealing with the local variations in average flux it is much more convenient to structure the problem in this way.

C. Infinite-Record Joint Local Variance

We will require infinite-record values for $\sigma_{1.2}^2$ as defined in Eq. B.13 and as calculated using results of the point-reactor model. Where finite-record values, as are derived in practice, are necessary these will be denoted by subscript T, i.e., $\sigma_{T,1.2}^2$ will denote the result for a single measurement over an ensemble of duration T while $\sigma_{1.2}^2$ is the result in the limit $T \rightarrow \infty$ (or, equivalently, the result of an average over an infinite ensemble of records each of length T). The finite joint local variance is, using Eqs. B.13 and B.19

$$\sigma_{T,1.2}^2 = \frac{1}{N} \sum_{k=1}^N (\delta\psi_{k,1} + \delta\phi_k)(\delta\psi_{k,2} + \delta\phi_k). \quad (B.22)$$

The ensemble average of Eq. B.22, which is the same as the infinite-record value, is

$$\sigma_{1.2}^2 = \langle (\delta\psi_{k,1} + \delta\phi_k)(\delta\psi_{k,2} + \delta\phi_k) \rangle = \langle \delta\phi_k^2 \rangle \quad (B.23)$$

since all cross-averages vanish. Introducing Eqs. B.20 and B.21 which define $\delta \hat{a}_k$, we can readily show that

$$\sigma_{1.2}^2 = \sum_{n=1}^{\infty} \frac{\langle a_n^2 \rangle}{w_n^2} (C_{n,k}^2 + S_{n,k}^2) \quad (B.24)$$

The generalized sine and cosine terms in Eq. B.24 may be combined, using the definitions in Eq. B.21, to give

$$\sigma_{1.2}^2 = \frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{\langle a_n^2 \rangle}{w_n^2} \left[\sum_{q=0}^3 E_q \cos w_n q \tau \right] \quad (B.25)$$

where $E_0 = 5$, $E_1 = -15/2$, $E_2 = 3$, $E_3 = -1/2$.

If we now introduce the expression for power spectrum from Eq. B.17 and pass from the sum of discrete frequencies to the limit of an integral over a frequency continuum ($dw \rightarrow \frac{2\pi}{T}$) we find that

$$\sigma_{1.2}^2 = \frac{2Dk^2}{F\pi\tau^2k_0^2} \int_0^{\infty} \sum_{q=0}^3 \frac{E_q \cos q\omega\tau}{\omega^2} B_{1.2}(\omega) \sum_p \frac{A T(-S_p)(-S_p)}{\omega^2 + S_p^2} d\omega \quad (B.26)$$

The integral over frequency in Eq. B.26 leads to the required relationships needed to interpret the measurement and it is essential that the effect of detector response be retained. It is important not to have to choose the sampling interval unduly long in order to retain good measurement statistics. At the same time, one does not wish to have results sensitive at all to prompt neutron roll-off or dependent in other than an easily managed fashion upon detector response.

The expression for $B_{1.2}(\omega)$ in Eq. B.9 may be re-written making use of the Fourier integral transform as

$$B_{1.2}(\omega) = \frac{\int_0^{\infty} \int_0^{\infty} \cos \omega(t_1 - t_2) \langle f(t_1) \rangle \langle f(t_2) \rangle dt_1 dt_2}{\left\langle \int_0^{\infty} f(t) dt \right\rangle^2} \quad (B.27)$$

and we have assumed an identical response for both detectors. Introducing Eq. B.27 into Eq. B.26 and changing the order of integration over t_1 , t_2 and ω gives

$$\sigma_{1.2}^2 = \frac{2Dk^2}{\pi F \tau^2 k_o^2 \left(\int_0^\infty \langle f(t) \rangle dt \right)^2} \sum_p A_p T(-S_p)(-S_p) \int_0^\infty \int_0^\infty \langle f(t_1) \rangle \langle f(t_2) \rangle dt_1 dt_2 \left[\int_0^\infty \cos w(t_1 - t_2) \frac{\sum_q E_q \cos qw\tau}{w^2} \frac{dw}{w^2 + S_p^2} \right]. \quad (B.28)$$

The integral over frequency in brackets in Eq. B.28 may be readily established by contour methods and has the value

$$\int_0^\infty \cos wDt \frac{\sum_q E_q \cos qw\tau}{w^2} \frac{dw}{w^2 + S_p^2} = -\frac{\pi}{4} \frac{\sum_q E_q (|q\tau + Dt| + |q\tau - Dt|)}{S_p^2} + \frac{\pi}{4} \frac{\sum_q E_q \left(e^{\frac{S_p}{q} |q\tau + Dt|} + e^{\frac{S_p}{q} |q\tau - Dt|} \right)}{S_p^3} \quad (B.29)$$

where $Dt = t_2 - t_1$.

We cannot proceed to complete the evaluation of Eq. B.29 without the explicit details of the detector time response which we usually do not know. We recognize however that if the electrometer feedback time constant t_{EL} is chosen greater than the duration of the current pulse, $f(t)$ will decay exponentially for $t > t_{EL}$. Consequently, when the variance sampling interval τ is greater than t_{EL} by a factor of 10 or so, only a negligible fraction of detector response will remain. This permits us to make the following substitution

$$1/2 \sum_{q=0}^3 E_q (|q\tau + Dt| + |q\tau - Dt|) + E_o |Dt| + \tau \sum_q q E_q = 5|Dt| - 3\tau \quad (B.30)$$

without introducing any significant error. In addition,

$$1/2 \sum_q E_q \left(e^{\frac{S_p}{q} |q\tau + Dt|} + e^{\frac{S_p}{q} |q\tau - Dt|} \right) = E_{oe}^{\frac{S_p}{q} |Dt|} + 1/2 \sum_{q>1} E_q \left(e^{\frac{S_p}{q} |q\tau + Dt|} + e^{\frac{S_p}{q} |q\tau - Dt|} \right). \quad (B.31)$$

Equations B.30 and B.31 may be considered exact where sampling interval exceeds time constant by a magnitude or so (and electrometer time constant exceeds ion collection time). Introducing these results into Eqs. B.28 and B.29 allows us to write

$$F\tau\sigma_{1.2}^2 = \frac{2Dk^2}{k_o^2} \sum_p \frac{A_p T(-S_p)}{-S_p} \left[\frac{3}{2} - \frac{5|\overline{Dt}|}{2\tau} + \frac{5}{2S_p\tau} \right. \\ \left. \frac{\int_0^\infty \int_0^\infty e^{S_p|Dt|} \langle f(t_1) \rangle \langle f(t_2) \rangle dt_1 dt_2}{\left(\int_0^\infty \langle f(t) \rangle dt \right)^2} \right. \\ \left. + \frac{\sum_{q>1} E_q \int_0^\infty \int_0^\infty \left(e^{S_p|q\tau+Dt|} + e^{S_p|q\tau-Dt|} \right) \langle f(t_1) \rangle \langle f(t_2) \rangle dt_1 dt_2}{4S_p\tau \left(\int_0^\infty \langle f(t) \rangle dt \right)^2} \right] \quad (B.32)$$

where we define the quantity $|\overline{Dt}|$ to be the detector response time according to

$$\overline{Dt} = \frac{\int_0^\infty \int_0^\infty t_1 - t_2 \langle f(t_1) \rangle \langle f(t_2) \rangle dt_1 dt_2}{\left(\int_0^\infty \langle f(t) \rangle dt \right)^2} \quad (B.33)$$

Since the quantity $Dt = t_1 - t_2$ in the last term on the right-hand side of Eq. B.32 is negligible compared to τ (at least when τ is chosen sufficiently large) we can neglect it relative to τ . Also, for singularities of the transfer function other than S_o , the quantity $e^{S_p|Dt|}$ in Eq. B.32 can be given its first order value $1 + S_p|Dt|$. In consequence of these assumptions, we may re-write Eq. B.32 in the form

$$F\tau\sigma_{1.2}^2 = \frac{2Dk^2}{k_o^2} \sum_p \frac{A_p T(-S_p)}{-S_p} \left[\frac{3}{2} + \frac{\frac{5}{2} - \frac{15}{4} e^{S_p\tau} + \frac{3}{2} e^{2S_p\tau} - \frac{1}{4} e^{3S_p\tau}}{S_p\tau} \right] \\ - \frac{2Dk^2}{k^2} \frac{A_o T(-S_o)}{-S_o} \left[\frac{5}{2} \frac{|\overline{Dt}|}{\tau} - \frac{5}{2S_o\tau} \left(e^{S_o|\overline{Dt}|} - 1 \right) \right] \quad (B.34)$$

where

$$\overline{e^{S_0}} = \frac{\int_0^\infty \int_0^\infty e^{S_0 |D\tau|} \langle f(t_1) \rangle \langle f(t_2) \rangle dt_1 dt_2}{\left(\int_0^\infty \langle f(t) \rangle dt \right)^2} . \quad (B.35)$$

The second term on the right of Eq. B.34 depends upon the prompt singularity, S_0 , and the detector response $f(t)$ through Eqs. B.33 and B.35. It accounts for finite detector time response effects and constitutes a correction to the measured value of $F\tau\sigma_{1,2}^2$. This correction term can be evaluated in terms of reactor parameters from the transfer function $T(S)$ defined in Appendix C. The singularity, S_0 , together with its residue, A_0 , and the value for $T(-S_0)$ may readily be shown to be

$$S_0 = -\frac{k\beta}{\ell_0} (1 + \beta) \quad (B.36)$$

$$A_0 = 1$$

$$T(-S_0) = \frac{\ell_0}{2k\beta(1 + \beta)} .$$

Equation B.36 may be deduced from the transfer function by virtue of the fact that the singularity S_0 is always much greater than other singularities of $T(S)$ which are on the time scale of delayed neutron behavior. This assumption is very good for fast reactors where $\ell_0 \sim 0.5$ μ sec -- it would fail for thermal reactors where ℓ_0 may be as great as 100 μ sec or longer. If the detector response correction term in Eq. B.34 is denoted by X , and if the prompt-neutron parameters defined by Eq. B.36 are introduced, the result may be expressed as

$$X = \frac{D}{\beta^2(1 + \beta)^2} \left[\frac{5}{2} \frac{|D\tau|}{\tau} + \frac{5/2}{\alpha\tau(1 + \beta)} (e^{-\alpha(1+\beta)|D\tau|} - 1) \right] \quad (B.37)$$

and α in Eq. B.37 is the at-critical Rossi alpha.

We cannot proceed further in the evaluation of the correction term, Eq. B.37, lacking full detail regarding the detector response function $\langle f(t) \rangle$, information which is difficult to obtain, to any high degree of accuracy at least. We can, however, make the approximation that

$$e^{-\alpha(1+\beta)|D\tau|} \simeq e^{-\alpha(1+\beta)|\overline{D\tau}|} \quad (B.38)$$

which allows us to express the correction factor in terms of only a single characteristic time $|\overline{D\tau}|$ defined by Eq. B.33. With this approximation, we may write

$$X = \frac{D}{\beta^2(1+\beta)^2} \left[\frac{5}{2} \frac{|\overline{D\tau}|}{\tau} + \frac{5/2}{\alpha\tau(1+\beta)} (e^{-\alpha(1+\beta)|\overline{D\tau}|} - 1) \right] . \quad (B.39)$$

The approximation in Eq. B.38 is exact when the argument $\alpha(1+\beta) |\overline{Dt}|$ is small. It fails when this argument is large but if this is the case the term $e^{-\alpha(1+\beta) |\overline{Dt}|}$ is negligible compared to unity anyway and it is irrelevant whether or not Eq. B.38 holds. The characteristic detector response time $|\overline{Dt}|$ is the only parameter required to allow for the effects of finite detector response time upon the noise measurement when the sampling time, τ , is larger, by an order of magnitude or so, than detector response. $|\overline{Dt}|$ can be observed experimentally for the detector-electrometer system employed for noise measurements and it is not necessary to define the detector time response in further detail. Information relevant to the evaluation of $|\overline{Dt}|$ from experimental data is provided in Appendix D.

Assuming that $|\overline{Dt}|$ is known, measured results are corrected for detector and for delayed and prompt neutron effects by use of Eq. B.34 (and the result of Eq. B.39). This correction is implemented in the data reduction code of Appendix E.

D. Statistical Precision in Joint-Variance Measurements

In Section III we derived an infinite-record expression for $\sigma_{1.2}^2$. It remains to also derive the statistical precision in a single measurement over a record of duration T which we denote by $\sigma_{T,1.2}^2$ according to

$$e_{1.2}^2 = \frac{\langle (\sigma_{T,1.2}^2 - \sigma_{1.2}^2)^2 \rangle}{(\sigma_{1.2}^2)^2} \quad (B.40)$$

We begin by substituting Eqs. B.13 and B.19 into Eq. B.40 and recall that $\sigma_{1.2}^2 = \langle \delta\phi_j^2 \rangle$. The result is

$$\begin{aligned} \langle \delta\phi_j^2 \rangle^2 N^2 e_{1.2}^2 = & \sum_{j,k} \langle \delta\phi_j^2 \delta\phi_k^2 \rangle - N^2 \langle \delta\phi_j^2 \rangle + \sum_{j,k} \langle \delta\psi_{j,1} \delta\psi_{k,1} \rangle \langle \delta\psi_{j,2} \delta\psi_{k,2} \rangle \\ & + \sum_{j,k} \langle (\delta\psi_{j,1} \delta\psi_{k,1} + \delta\psi_{j,2} \delta\psi_{k,2}) \rangle \langle \delta\phi_j \delta\phi_k \rangle \end{aligned} \quad (B.41)$$

where various cross-product ensemble averages have been set to zero. First we consider the term

$$T_1 = \sum_{j,k} \langle \delta\phi_j^2 \delta\phi_k^2 \rangle - N^2 \langle \delta\phi_j^2 \rangle^2 \quad (B.42)$$

We can show by expanding the $\delta\phi$ using Eqs. B.20 and B.21 that

$$T_1 = \frac{2}{\tau^4} \sum_{j,k} \left\{ \sum_{n=1}^{\infty} \frac{\langle a_n^2 \rangle}{w_n^2} \sum_{q=-3}^{+3} Q_q \cos w_n (\tau_j - \tau_k + \tau_q)^2 \right\} \quad (B.43)$$

where we have neglected a term of order $1/N$ which will be insignificant since the total number of points, N , will always be large. The coefficients Q_q and the times τ_q are listed below.

q	$-Q_q$	t_q
-3	1/4	-3 τ
-2	-3/2	-2 τ
-1	15/4	- τ
0	-5	0
1	15/4	τ
2	-3/2	2 τ
3	1/4	3 τ

If we now introduce the power spectrum (Eq. B.17) into Eq. B.43 and integrate over w , the result will be

$$T_1 = \frac{2D^2k^4}{\tau^4 F^2 \epsilon_0} \sum_{j,k} \left[\sum_p \frac{A T(-S_p)}{-S_p} \left\{ \sum_q -Q_q |t_j - t_k + t_q| \right. \right. \\ \left. \left. + \frac{\sum_q Q_q e^{S_p |t_j - t_k + t_q|}}{S_p} \right\} \right]^2 \quad (B.44)$$

where $|t_j - t_k + t_q|$ is the absolute value of $t_j - t_k + t_q$. We will not carry any terms involving detector response since they will modify the final error values only slightly for the experimental conditions adhered to during the measurement. This is equivalent to setting the $B(w)$ terms to unity and the $R(w)$ terms to a constant which can then be absorbed in the definition of detector efficiency.

Now treat the second term in Eq. B.41

$$T_2 = \sum_{j,k} \langle \delta\psi_{j,1} \delta\psi_{k,1} \rangle \langle \delta\psi_{j,2} \delta\psi_{k,2} \rangle. \quad (B.45)$$

By again using the previously defined relationships we can show that

$$\langle \delta\psi_{j,1} \delta\psi_{k,1} \rangle = \frac{1}{\tau^2} \sum_{n=1}^{\infty} \frac{\langle x_{n,1}^2 \rangle}{w_n^2} \sum_q Q_q \cos w_n (t_j - t_k + t_q) \quad (B.46)$$

and a similar result for detector 2. Introducing Eq. B.18 for the power spectrum and performing the indicated integration over w gives

$$\langle \delta\psi_{j,2} \delta\psi_{k,1} \rangle = \frac{1}{2F\tau^2 \epsilon_1} \sum_q -Q_q |t_j - t_k + t_q| \quad (B.47)$$

and a similar result for detector 2. The value for T_2 in Eq. B.45 will therefore be

$$T_2 = \frac{1}{4F^2\tau^4\epsilon_1\epsilon_2} \sum_{j,k} \left(\sum_q Q_q |t_j - t_k + t_q| \right)^2. \quad (B.48)$$

We are still left with the term $\delta\phi_j\delta\phi_k$ in Eq. B.41 to evaluate, which we can do in the same way as for the others with the result

$$\langle \delta\phi_j\delta\phi_k \rangle = \frac{Dk^2}{\tau^2 F \ell_o^2} \sum_p \frac{A T(-S_p)}{-S_p} \left\{ \sum_q Q_q |t_j - t_k + t_q| + \frac{\sum_q Q_q e^{S_p |t_j - t_k + t_q|}}{S_p} \right\}. \quad (B.49)$$

We have now obtained values for each of the terms occurring in Eq. B.41. The range of experimental interest will be for sampling times where the effects of delayed-neutron time dependence can be ignored and this has the effect of limiting the p summation to only the prompt-neutron singularity while rendering negligible the quantity

$$\sum_q Q_q e^{S_o |t_j - t_k + t_q|} / S_o.$$

We may also make use of the substitution

$$\sum_p \frac{A T(-S_p)}{-S_p} + \frac{A T(-S_o)}{-S_o} = \frac{\ell_o^2}{2k^2\beta^2(1+\$)^2} \quad (B.50)$$

and, making use of the Q_q and t_q values derived previously,

$$\sum_{j,k=1}^N \left(\sum_q Q_q |t_j - t_k + t_q| \right)^2 = 35N\tau^2/2. \quad (B.51)$$

If we now make use of results of Eqs. B.50 and B.51 to reduce the terms entering Eq. B.41, we will get finally that

$$Ne_{1.2}^2 = \frac{35}{9} \left[1 + \frac{\beta^4(1+\$)^4}{2D^2\epsilon_1\epsilon_2} + \frac{\beta^2(1+\$)^2}{2D} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) \right]. \quad (B.52)$$

Equation B.52 allows the error in a single measurement of $\sigma_{1.2}^2$ to be inferred at any degree of subcriticality and for any combination of detector efficiencies.

For example, where $\epsilon_1 = \epsilon_2 = 1 \times 10^{-3}$ and for $\beta = 0.003$ and $D = 0.8$, a measurement of $\sigma_{1.2}^2$ near critical for 10 minutes with a sampling time of $\tau = 0.01$ sec will give an error of

$$\sqrt{e_{1.2}^2} \sim \sqrt{\frac{35}{9} \frac{1}{60000}} \sim 0.8\% \text{ in } \sigma_{1.2}^2.$$

At 30\$ subcritical the error will increase to ~6%. The desirability of detectors of high efficiency is made clear from the results in Eq. B.52.

E. Mean Square Difference Fluctuation and Error

In addition to the joint local variance $\sigma_{1,2}^2$, the mean square difference σ_{1-2}^2 for the detector pair will also be formed in the measurement and may be used to determine detector efficiency and statistical error. The finite-record result for difference fluctuations may be readily constructed using definitions in Eqs. B.22 and B.19 to give

$$\begin{aligned}\sigma_{1-2}^2 &= \frac{1}{N} \sum_{k=1}^N [(\delta\psi_{k,1} + \delta\phi_k) - (\delta\psi_{k,2} + \delta\phi_k)]^2 \\ &= \frac{1}{N} \sum_{k=1}^N [\delta\psi_{k,1} - \delta\psi_{k,2}]^2.\end{aligned}\quad (\text{B.53})$$

The common pile noise term, $\delta\phi_k$, has cancelled identically in the formulation of each difference of local variance. The infinite record value is seen to be

$$\sigma_{1-2}^2 = \frac{1}{N} \sum_{k=1}^N [\delta\psi_{k,1} + \delta\psi_{k,2}]^2 = \langle \psi_{k,1}^2 \rangle + \langle \psi_{k,2}^2 \rangle \quad (\text{B.54})$$

since all ensemble averages involving cross terms vanish. Introducing the expansions in Eqs. B.20 and B.21 and the individual detector power spectra (Eq. B.18) will reduce Eq. B.54 to the form

$$F\tau\sigma_{1-2}^2 = \frac{1}{\pi\tau} \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right) \int_0^\infty B_0(w)R(w) \frac{\sum_q E_q \cos qw\tau}{w^2} dw \quad (\text{B.55})$$

where we assume identical response for each detector. From Eqs. B.6 and B.7 we see that

$$B_0(w)R(w) = \frac{\langle \bar{f}(w)\bar{f}^*(w) \rangle}{\left(\int_0^\infty \langle f(t) \rangle dt \right)^2} = \frac{\int_0^\infty \int_0^\infty \cos w(t_1 - t_2) \langle f(t_1)f(t_2) \rangle dt_1 dt_2}{\left(\int_0^\infty \langle f(t) \rangle dt \right)^2} \quad (\text{B.56})$$

again using the Fourier expansion (and neglecting the $\sin w(t_1 - t_2)$ term which is odd and will not contribute to the integral in Eq. B.55). If we now introduce Eq. B.56 into Eq. B.55 and use the result

$$\int_0^\infty \cos wD\tau \frac{\sum_q E_q \cos qw\tau}{w^2} dw = -\frac{\pi}{4} \sum_q E_q (|q\tau + D\tau| + |q\tau - D\tau|) \quad (\text{B.57})$$

we find that

$$F\tau\sigma_{1-2}^2 = -\frac{1}{4\tau} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) \frac{\int_0^\infty \int_0^\infty \langle f(t_1)f(t_2) \rangle dt_1 dt_2}{\left(\int_0^\infty \langle f(t) \rangle dt \right)^2} E_q(|q\tau + Dt| + |q\tau - Dt|) \quad (B.58)$$

We may, subject to the usual limitation that detector response time is much less than τ , use the result previously set down in Eq. B.30 to reduce Eq. B.58 to read

$$F\tau\sigma_{1-2}^2 = \frac{3}{2} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) \left[\frac{\int_0^\infty \int_0^\infty \langle f(t_1)f(t_2) \rangle dt_1 dt_2}{\int_0^\infty \langle f(t) \rangle dt^2} - \frac{5}{3\tau} \frac{\int_0^\infty \int_0^\infty |t_2 - t_1| \langle f(t_1)f(t_2) \rangle dt_1 dt_2}{\int_0^\infty \langle f(t) \rangle dt^2} \right] \quad (B.59)$$

It is convenient to re-define detector efficiency, ϵ , to be the ratio of its usual value (detector rate/induced fission rate) to the constant

$$r = \frac{\int_0^\infty \int_0^\infty \langle f(t_1)f(t_2) \rangle dt_1 dt_2}{\left(\int_0^\infty \langle f(t) \rangle dt \right)^2} \quad (B.60)$$

These re-normalized efficiencies were, in fact, those used in the error analysis leading to Eq. B.55. If, further, we define $|\overline{Dt}|'$ to be

$$|\overline{Dt}|' = \frac{\int_0^\infty \int_0^\infty |t_2 - t_1| \langle f(t_1)f(t_2) \rangle dt_1 dt_2}{\int_0^\infty \int_0^\infty \langle f(t_1)f(t_2) \rangle dt_1 dt_2} \quad (B.61)$$

we arrive at the result

$$F\tau\sigma_{1-2}^2 = \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) \left[\frac{3}{2} - \frac{5|\overline{Dt}|'}{2\tau} \right] \quad (B.62)$$

for the mean square local variance difference.

The quantity r in Eq. B.60 will be somewhat in excess of unity depending upon the variation in shape and amplitude of individual pulses. For instance if shape and amplitude are not too strongly correlated, which will be true for the BF_3 detectors and electrometer here considered, r will just be the ratio of mean square to square mean event amplitude. The fact that only the ratio of efficiency to r appears in statistical considerations is in consequence of the fact that current collection rather than pulse counting is done. The efficiency defined as detection rate over fission rate could be a very large number where numerous weakly ionizing events are detected together with some of large ionization. r would also be large in this case, however, and the re-normalized efficiency would be reduced correspondingly over the detection efficiency.

Also note that $|Dt|'$ as defined in Eq. B.61 is not quite the same as $|Dt|$ defined in Eq. B.33. The difference is related to the fact that shape and amplitude of events are not strictly independent of each other but the practical implications of this can be ignored here and it will not be necessary to make a distinction subsequently.

The derivation of the statistical error in a measurement of σ_{1-2}^2 over a finite measurement time may be arrived at following the same procedure as was used in the derivation of the statistical error in σ_{1-2}^2 . Neglecting details of detector response, which are of little influence upon the error, we may readily show that the mean square measurement error, e_{1-2}^2 , in σ_{1-2}^2 is

$$e_{1-2}^2 = \frac{35}{9N} \quad . \quad (B.63)$$

•

APPENDIX C

Conditional Probability and the Point Kinetics Model

We require the probability that a neutron be recorded in detector 2 a time t after a neutron was known to have registered in detector 1. The order of detectors is irrelevant and the same considerations apply to pairs of events recorded by a single detector. Neutrons are assumed to be destroyed in the detection event.

Although we follow the standard derivation (Ref. 3) a finite spatial extension of the reactor is permitted.⁹⁻¹³ Consider the conditional probability distribution $P(t_1, t_2)dt_1dt_2$ that a count will occur in the interval dt_1 and dt_2 ($t_2 > t_1$) in response to the effect of all processes prior to t_1 . This probability will consist of a contribution from randomly-detected events, $C^2dt_1dt_2$ where C is the detection rate, in addition to a contribution from events which had a common fission ancestor. To determine the common-ancestor contribution, it will become necessary to introduce a kinetics model of the reactor. The simplest model assumes that space-and-time-related effects are separable and that only the fundamental mode is important. This allows us to define the time-dependent flux as the product of a spatial neutron flux amplitude which we can interpret as the number of neutrons in the reactor at position $\vec{r}$ and a time-dependent shape function $N(t)$. $N(t)$ is interpreted as the die-away of neutrons injected into the reactor fundamental mode at time zero. It is readily obtained from the point kinetics model including delayed-neutron effects.

The correlated contribution to the conditional probability is the product of three separate terms:

1. $\int_{\vec{r}} F(\vec{r})d\vec{r} dt_0$ = the probability that a fission occurs at location $\vec{r}$ at time t_0 which is the closest common ancestor of neutrons detected later at times t_1 and t_2 .
2. $P_\nu \nu I(\vec{r}) N(t_1 - t_0) dt_1 / \lambda_d$ = the probability that neutrons released in the fission at t_0 initiate chains leading to events detected at t_1 . P_ν is the probability for emitting ν neutrons in the fission. $I(\vec{r})$ is the spectrum weighted adjoint which we assume to be independent of multiplicity. $N(t)$ is the number of neutrons in the reactor at time t relative to the population at time 0. λ_d is the lifetime of neutrons for capture by the detector.
3. $(\nu - 1) I(\vec{r}) N(t_2 - t_0) dt_2 / \lambda_d$ = the probability that the $\nu - 1$ neutrons remaining (after excluding the neutron responsible for the chain detected at t_1) initiate chains leading to a neutron detected at t_2 .

If the random counting term is added to the product of expressions 1 through 3 above and the result is integrated over all fissions prior to t_1 , and also integrated over the spatial extent of the reactor and averaged over the neutron emission multiplicity distribution, P_ν , we find that

$$P(t_1, t_2) dt_1 dt_2 = dt_1 dt_2 \left[C^2 + \frac{\int \bar{\nu}(\nu-1) F(\vec{r}) I^2(\vec{r}) d\vec{r}}{\ell_d^2} \int_{-\infty}^{t_1} N(t_1 - t_0) N(t_2 - t_0) dt_0 \right] \quad (C.1)$$

where $\bar{\nu}(\nu-1) = \Sigma P_{\nu} \nu(\nu-1)$.

If we now divide everything in Eq. C.1 by $C dt_1$, the probability that an event did occur in the interval dt_1 , we find for the conditional probability $\chi(t)$ that an event is detected at time t after an event occurred the result:

$$\chi(t) = C + \frac{\int \bar{\nu}(\nu-1) F(\vec{r}) I^2(\vec{r}) d\vec{r}}{C \ell_d^2} \int_0^{\infty} N(t_0) N(t_0 + t) dt_0. \quad (C.2)$$

In order to eliminate detector lifetime, ℓ_d , from Eq. C.2 we note that the effective multiplication, k , is just the ratio of neutron production rate to neutron loss rate. Since n is the number of neutrons in the reactor, n/ℓ_0 is the loss rate where ℓ_0 is the prompt-neutron lifetime. The neutron production rate is just $\int \bar{\nu} F(\vec{r}) I(\vec{r}) d\vec{r}$ and, consequently,

$$k = \frac{\int \bar{\nu} F(\vec{r}) I(\vec{r}) d\vec{r}}{n/\ell_0}. \quad (C.3)$$

Since the detection rate, C , is given by the ratio n/ℓ_d , n can be eliminated from Eq. C.3 to give

$$k C \ell_d = \ell_0 \int \bar{\nu} F(\vec{r}) I(\vec{r}) d\vec{r}. \quad (C.4)$$

It is also convenient to introduce a detection efficiency, ϵ , defined as the ratio of detection rate to fission rate

$$\epsilon = C / \int F(\vec{r}) d\vec{r}. \quad (C.5)$$

Equations C.3, .4 and .5 when introduced into Eq. C.2 provide the result

$$\chi(t) = C + \epsilon D \frac{k^2}{\ell_d^2} \int_0^{\infty} N(t_0) N(t + t_0) dt_0 \quad (C.6)$$

where the "Diven Factor", D , is a dimensionless quantity

$$D = \frac{\int F(\vec{r}) d\vec{r} \cdot \int \bar{\nu}(\nu-1) F(\vec{r}) I^2(\vec{r}) d\vec{r}}{[\int \bar{\nu} F(\vec{r}) I(\vec{r}) d\vec{r}]^2}. \quad (C.7)$$

Integration in Eq. C.7 defining D also implies summation over all fissile isotopes.

As a practical matter, the quantity $\overline{v(v-1)}/\overline{v}^2$ varies hardly at all (with a value of about 0.80) from one fissile species to another. $\overline{v(v-1)}$ can, therefore, be set proportional to $\overline{v}^2$; the proportionality constant can be taken as an average of $\overline{v(v-1)}/\overline{v}^2$ for different fissile species weighted by the total fission rate for each. When this is done, D in Eq. C.7 can be written as the product of a spatially-integrated quantity D_s defined as

$$D_s = \frac{\int F(\vec{r}) d\vec{r} \int \overline{v}^2 F(\vec{r}) I^2(\vec{r}) d\vec{r}}{[\int \overline{v} F(\vec{r}) I(\vec{r}) d\vec{r}]^2} \quad (C.8)$$

and the average value for $\overline{v(v-1)}/\overline{v}^2$ for the core which we refer to as the isotropic dispersion factor D_0 . D_s in Eq. C.8 contains only those quantities routinely generated in reactor neutronics computations and can be evaluated from these computations for any core of interest.

In order to complete the evaluation of Eq. C.6 for conditional probability, we must derive the time behavior of the shape function $N(t)$ from the point kinetics model. The point-model equations for $N(t)$ are

$$\frac{dN}{dt} = (k - 1 - k\beta) \frac{N}{l_0} + \lambda_1 C_1 + W \quad (C.9)$$

$$\frac{dC_1}{dt} + \lambda_1 C_1 = \beta_1 k N / l_0 .$$

β_1 and λ_1 are the delayed-neutron precursor fractions and decay constants. C_1 are the precursor concentration shape functions. W is the strength of any fixed neutron source. k is the effective multiplication and l_0 is the prompt neutron lifetime. The set of Eqs. C.9 can be readily solved in response to an instantaneous neutron pulse at $t=0$ and the result is

$$N(t) = \sum_p A_p e^{S_p t} \quad (C.10)$$

and S_p and A_p are the poles (of negative magnitude) and residues of the transfer function $T(s)$ given by

$$T(s) = \frac{l_0}{S \left(l_0 + k \left[\frac{\beta_1}{S + \lambda_1} \right] \right) + 1 - k} \quad (C.11)$$

Introducing the die-away function, Eq. C.10, into Eq. C.6 for the conditional probability gives the result

$$\chi(t) = C + \frac{\epsilon D k^2}{l_0^2} \sum_p A_p T(-S_p) e^{S_p t} \quad (C.12)$$

for conditional probability according to the point kinetics model -- it is just

a sum of decaying exponentials superimposed upon a constant background from events detected at random.

Although only the fundamental mode has been considered here, all higher harmonics contribute to the conditional probability. Therefore, $\chi(t)$ as given in Eq. C.12 is only an approximation valid for times t sufficiently long to have allowed the contribution from higher modes to have decayed away. For near-critical reactors having k_{∞} in the range 1.2 to 1.5 (which includes the cores of interest here) the decay constant of these higher modes is large and their amplitude is small compared with the fundamental. Their effect could only be detected by measurements at much shorter times than the typical minimum sampling time of 1/60 sec used for β measurements by the noise technique. This will change as subcriticality increases however and, ultimately, one would expect to observe breakdown of the simple fundamental-mode description on any measurement time scale. This suggests that measurements should be extended as far subcritical as instrumental limitations allow to permit validation of the simple model near critical where data are used to infer effective beta.

The simple fundamental mode model has been tested in this way and results are included in this report for different cores. No indication of failure of the fundamental mode description has been observed within about 1.5% $\Delta k/k$ of critical. Measurements further subcritical than this may indicate the onset of significant deviations from fundamental mode behavior; this is the probable explanation of Fig. 2 of the text. On near-critical reactors for phenomena acting on a relatively long time scale (typically greater than 1/60 sec) the effects of higher spatial harmonics are sufficiently depressed, relative to the fundamental, to allow them to be ignored. This carries the implication that placement of detectors is irrelevant; the pair of leakage detectors used in the measurements reported here were located on top of the fixed and movable halves of the split-table assembly. Failure of the point model, manifested by the increased importance of higher spatial flux harmonics as subcriticality increases, could cause results to become sensitive to detector placement.^{12,13} For far subcritical reactors, one would expect that widely separated detectors would show less correlation and detectors placed close together would show an increased correlation. As criticality is approached, the fundamental mode dominates and all locations in or around the reactor become equivalent, at least for measurements where long time intervals (or low frequencies) are emphasized.

Additional experimental information on fast-reactor mock-ups exists which confirms the validity of the point kinetics model at frequencies lower by an order of magnitude than the Rossi-alpha limit β/λ_0 . In these experiments, a pair of neutron detectors was located at various positions around the periphery of large mock-ups on ZPPR and joint fluctuation spectra were measured.* The magnitude of the joint fluctuations at frequencies below about 10% of β/λ_0 was observed to be detector-placement independent to within $\pm 0.5\%$. The values for β/λ_0 inferred from the measured data varied by $\pm 6\%$, however, depending upon whether detectors were placed close together or far apart. Experiments of this type, as well as those in which the amplitude of noise is observed as a function of subcriticality, provide ample support for the point kinetics model at frequencies well below β/λ_0 , but there are clear indications that difficulties appear at the β/λ_0 limit.

*S. Carpenter, private communication.

Essentially the same conclusions were reached in noise experiments by Mihalcz¹⁴, where spatial effects in Rossi-alpha measurements were suggested as the likely source of a large discrepancy between measured and calculated kinetics results.

The conditional probability is fundamental to the interpretation of reactor noise since it is used directly to evaluate the ensemble average of the time-ordered cosine term

$$\langle \cos w(t_i - t_m) \rangle = \int_0^{\infty} \cos w \tau \chi(\tau) d\tau \quad (C.13)$$

The constant detection rate, C , in Eq. C.12 will not contribute to the average and the result of use of Eq. C.12 in Eq. C.13 is

$$\langle \cos w(t_i - t_m) \rangle = \frac{\epsilon D k^2}{\lambda_0^2} \int \frac{A(-S) T(-S)}{P} \frac{P}{w^2 + S_p^2} \quad (C.14)$$

APPENDIX D

Large Area Leakage Neutron Detectors for ZPR-9 Kinetics MeasurementsA. General Considerations Relating to Detector Design

Subcriticality and small-sample worth measurements are more precise where neutron detectors of high efficiency are used. High detector efficiency, together with fast response of detectors and electrometers, is also needed for noise measurements of reactor parameters including effective beta. A pair of leakage detectors for ZPR-9 is described which are conventional BF_3 ion chambers with homogeneous moderator. The detectors are each a factor of 3 more efficient than the most efficient unit previously used and have a maximum response time of about one millisecond over the range of currents which is encountered in ZPR experimental work. This appendix will consider the detectors and electrometers and will provide results of various tests intended to demonstrate essential operational characteristics.

Conventional BF_3 chambers imbedded in polyethylene are commonly utilized for high efficiency monitoring applications around fast reactor critical experiments. BF_3 ion chambers are very stable for long periods, sufficiently fast for most purposes, and relatively inexpensive. The tendency in fast reactor mock-up work is to include only a minimum amount of outer blanket, reflector, or shield since matrix size and material inventory limitations are stringent. The upshot is that a significant fraction of all neutrons generated leak into the external environment and are available for monitoring without penalties associated with in-core detector placement. If 4% of all neutrons leak (this is about the extent of leakage for ZPR-9 mock-ups) and, of these, 1% are detected (which is consistent with the area intercepted and albedo for neutrons striking a large externally mounted detector), the efficiency will be about 0.4×10^{-3} neutrons detected per fission. A reactivity penalty and flux tilt would be associated with any detector placed in-core which absorbed so large a fraction of the overall neutron population

Gamma compensation of chambers was not considered to be necessary. Gammas accompanying fission would, if detected, enhance to a small extent the overall detection efficiency. No evidence exists that fission product gammas which accumulate during power operation are a measurable influence upon leakage detectors. Highly precise observations of leakage detector response over periods of up to 5 hours after reaching a configuration about \$1 subcritical indicated no evidence of the monotonic rise in detector level characteristic of sensitivity to fission product gammas. Temperature-related reactivity drift obscures any effect relating to fission-product gamma rays. There are, of course, unusual situations which could alter this conclusion. In particular, subcritical operation immediately following an extended high power run would raise the ratio of fission-product-to-neutron sensitivity. This is not a routine procedure, however, and it is doubtful that it would ever be necessary to do this in practice.

Commercially available BF_3 chambers are expensive and sold only as a special order in the sizes and types here required. Since we needed a large number of tubes for the pair of detector arrays, and since costs for commercial units would have been high, we fabricated tubes inhouse and filled them with BF_3 obtained from a commercial fluorine chemist who reduced the enriched $\text{BF}_3 \cdot \text{CaF}_2$ salt provided him.

It is possible to build an arbitrarily large detector array, with correspondingly higher efficiency, but this is not very practical beyond a certain size. Operational reactor control instruments compete for the same space on top of the matrices and some compromise is necessary to assure that all requirements for space are adequately met. The leakage detectors here described cover about half of the useful top surface of the ZPR-9 matrix structure and do not interfere excessively with other essential instrumentation. The area intercepted by the leakage detectors here described is about double the area required for units in use previously on ZPR-9 but the efficiency is three times higher.

Individual tubes in each detector array were about 40 cm long and about 4 cm O.D. (1 mm steel wall) and were filled with about 1 atmosphere of BF_3 . Twelve of these tubes were imbedded in a block of polyethylene for each detector. The macroscopic thermal cross section in the BF_3 gas was several times in excess of its value in the equivalent surrounding polyethylene which assures a high ratio for neutrons detected per neutron incident onto the detector. Since the neutrons leaking from a reactor blanket or shield are highly degraded in energy, the age of these neutrons to thermal is only a few centimeters squared and the likelihood for capture in boron of a neutron entering the detector array is large.

Noise applications require fast response time for the detector-electrometer system. Slowing down time in polyethylene is fairly rapid, less than about 100 microseconds for leakage-spectrum neutrons. Since detectors are outside of the core and blanket they will be exposed to room-return neutrons as well as neutrons from the reactor directly, but the same 100 microsecond time scale applies. This inherent response limit will be less than ion collection time in the BF_3 gas. Drift velocity of charge in the BF_3 gas is long, several tenths of a millisecond according to some rough estimates which were made. Charge collection time in the ion chambers will be inversely proportional to voltage and at the maximum voltage chosen for routine operation (2000 volts), it appears that charge-collection is less than about 0.2 milliseconds. This characteristic of the detector was checked experimentally and results will be included later. It should be pointed out that, although electron collection times are very short, most electrons created in the gas will probably be quickly attached to impurities. In consequence, mobilities of both positive and negative charge in the gas are those for heavy ions. No analysis of the purity of the BF_3 gas used was made, and the lack of any special effort to obtain high purity probably implies that substantial contamination by electro-negative impurities exists. For the modest time response required for the intended application, full electron attachment does not introduce any problems and impurity requirements are not stringent. The overall time response of these detectors is adequate to permit noise measurements in the intermediate frequency region without much sensitivity to either delayed-neutron effects or to prompt lifetime.

The electrometer used was remote ranging (using CMOS switches) and was adjusted by feedback capacitance to a uniform 0.5 millisecond time response on the 1×10^{-6} through 3×10^{-9} current ranges where noise measurements might be done. A voltage-frequency converter was attached directly to the operational amplifier at the detector and both analog and digital information are available at a console panel.

Although the detector/electrometer system was of conventional design, a substantial calibration effort was made to demonstrate those characteristics such as time response which are crucial to practical use. At the termination of this effort a very thorough understanding of system operation was obtained. This information together with detector design details will be here described.

B. Detector Fabrication Procedures

Twelve detectors were placed in a polyethylene block which was rigidly attached to the base of a 1/32-in.-thick steel containment box. The polyethylene served as moderator and as electrical insulation. The outer cathodes of detectors were connected in parallel and input into the electrometer. The center rod of each detector was attached to the voltage bias supply bus which was filtered by a 2 megohm resistor and a pair of 0.1 microfarad capacitors. Five centimeters of additional top-mounted polyethylene reflector was added prior to installation to increase the overall efficiency to leakage neutrons.

Each detector is made up of two stainless steel coaxial cylinders. The inner cylinder is a solid rod 0.64 cm diameter and 40 cm long. The outer cylinder had an O.D. of 4 cm and 1-mm-thick wall. The coaxial seals were from a commercial manufacturer. Seals were machined to fit just inside the outer cylinder. Anode rods were then soldered to the central electrode of the seals using high melting soft solder. The choice of dimensions for individual detectors was restricted to availability of triaxial seals. In order to fit the seals selected into the cathode case, a case diameter of about 4 cm was indicated. A rigid anode is desirable to raise microphonic frequency response to high values and an anode of fairly large diameter improves ion collection time. Each detector is, effectively, placed inside a 5-cm-square polyethylene block. The macroscopic cross section for absorption of thermal neutrons in the block is several times less than for absorption by BF_3 at 1 atmosphere in the 4-cm-diameter chambers. The upshot is that detector-environment competition for neutrons is favorable and efficiency is, therefore, not compromised. Age of leakage neutrons to thermal is only a few square centimeters in polyethylene, implying that only a minimum amount of moderator should be used between leakage source and detectors. Some material for electrical insulation is required however.

Prior to final welding of the seal to the outer cylinder, extensive cleaning of seals and structures using distilled water, alcohol, and benzene was done. Electrical checks were made by attaching a voltage supply to the guard ring structure of each seal and measuring leakage current to both the anode and cathode. Final welding of the unit was not done until seal resistance was better than about 10^{12} ohms. Welding of seals to cathode tubes was accomplished using chill blocks to minimize heating loads on the ceramic of the

insulators. After welding, additional solvents were introduced through the pump out tube and cleaning was continued until an acceptably high seal leakage resistance was obtained.

Although several tubes were broken or were not acceptable after final filling and testing, the procedures employed provided a workable detector about 80% of the time.

A valve was attached to each detector through a short section of 1/4-in-O.D. soft-copper tubing to be pinched off after final testing. Initial roughing was done using a dirty pump, which was also used to evacuate residual BF_3 gas in manifold lines. BF_3 is a moderately toxic substance and the manifold was contained in a hood which was evacuated continuously through a filter to outside air. Tanks of high pressure BF_3 with boron enriched to 92% in the number 10 isotope could not be obtained directly from any of the commercial gas suppliers. One thousand grams of the $\text{BF}_3 \cdot \text{CaF}_2$ salt (92% ^{10}B) was obtained from Eagle-Picher (Miami, Oklahoma). The salt was sent to fluorine chemists at Ozark Mahoney (Tulsa, Oklahoma) where the evolved BF_3 was placed at high pressure into two small research type bottles of about 1 liter capacity. No analysis of impurity levels was undertaken since high purity was not a requirement for the ion chamber application.

After final pumping and outgassing, BF_3 was introduced at a pressure of 1 atmosphere. The detector valve was closed and tests were made of the electrical resistance after which a pinch-off tube was used to seal the detector permanently.

To the author's knowledge, there are no phenomena apart from extremely high radiation exposure (not possible in the reactor environment at ZPR-9) that will cause the detectors to degrade in the course of time. These detectors have been observed over about four years with no evidence of change. It is possible that very small cracks in the detectors may exist that were not discovered prior to filling. Over a long period of time, interchange of BF_3 and air could corrode insulation and ruin individual detectors.

C. Remote Ranging Electrometers

A pair of identical remote ranging electrometers was constructed for placement immediately adjacent to the detectors on the top of the ZPR-9 matrix. The CMOS switches used to implement remote ranging were adequate for this application since current ranges less than 1×10^{-9} amps were not required. Intermediate ($\times 3$) ranges were provided, and a pair of 10^6 full scale voltage-frequency converters (one for each polarity) was attached directly to the electrometer.

A current generator was used to check the absolute accuracy of the electrometers. Each electrometer input was loaded with a 700 pf capacitance (to simulate the detector) during the range check. Over all ranges from 3×10^{-5} to 1×10^{-9} the electrometers agreed within $\pm 0.5\%$ with the current source. The 1×10^{-9} and 3×10^{-10} ranges were within $\pm 1.5\%$ of the current generator reading but these very low ranges are seldom used with the leakage neutron detectors.

An understanding of the time response of the detector-electrometer system is essential to the interpretation of any type of kinetic experiment, rod drop as well as noise. Neutron chains in fast reactors decay with a time constant ℓ_0/β of about 0.5 microseconds/0.003 = 0.15 milliseconds. Those neutrons which leak from the core and enter the polyethylene moderator surrounding the detector will thermalize and be captured on a time scale less than ℓ_0/β . Ionization created by a neutron reaction in the BF_3 gas will not be collected until all electrons and positive ions have drifted to electrodes. The purity of the filling gas used is not sufficiently high (in all probability) to allow electrons to migrate to electrodes without attachment. Ion collection times of a few tenths of a millisecond must be allowed as an additional degradation of detector response. All of the effects mentioned place fundamental limitations on the ability of the detectors themselves to define the decay of a neutron chain. There is little advantage in constructing an electrometer whose response is faster than other limiting processes inherent in the reactor and detector. Since these processes occur on a time scale of 0.5 milliseconds, detector response need only be comparable or faster than this value.

For noise measurements there are advantages in maintaining a fixed system time response across all current ranges where measurements are to be done. This avoids use of range-dependent calibrations and simplifies operational requirements for this type of measurement. Using a detector having capacitance of 700 pf with the operational amplifier used in the electrometer, it was possible to maintain response time at about 0.5 milliseconds on current ranges from 1×10^{-6} to 3×10^{-9} by adjusting feedback capacitance. For ranges lower than 3×10^{-9} , parasitic shunt capacitances in resistors and switches caused response time to increase beyond 0.5 milliseconds. For ranges of 3×10^{-6} and higher (currents of 3×10^{-6} amps and above) the amount of feedback capacitance becomes so large that phase-shift instability occurs when a 0.5 millisecond response time is attempted and the electrometer oscillates. On these ranges response must be allowed to assume values shorter than 0.5 milliseconds.

Electrometer response was measured and adjusted using an audio oscillator feeding through a 700 pf capacitor (equal to the leakage detector capacitance) to the electrometer input. At each range setting, frequency was varied from 1 to 10,000 Hz and the rms output of the electrometer plotted and fit to the function

$$\text{rms output} = \frac{w}{\sqrt{w^2 + \alpha^2}}$$

where w is the frequency and α is the reciprocal of the electrometer time response. α was adjusted by changing feedback capacitance until a value of about 0.5 msec was achieved. The range of variation of response time ($1/\alpha$) is within about 10% over all ranges from 3×10^{-9} to 1×10^{-6} for both electrometers. Response time for currents of 3×10^{-6} amps and above and for currents of 1×10^{-9} amps and below could not be adjusted to a value of 0.5 milliseconds for reasons stated previously.

D. Data Acquisition Using a Minicomputer

Voltage-frequency converter pulses were routed to a pair of 17 bit binary counters interfaced to the minicomputer. A readout initiate pulse generated by a stable time mark generator terminates counting and raises logic levels which are sensed, in a loop, by the computer. Data from the scaler pair are then read, sequentially, into operational computer registers and counting is reenabled until the next time mark pulse appears. Because of the time delay inherent in a sense loop sequence for data input, a timing error of about 7 microseconds maximum value is made in the absolute time mark interval. Since the interval between the time marks is not required to be less than about 10 milliseconds, the effect of this error can be neglected in all the applications of interest here. Also, since scalars are read into computer registers sequentially (followed by a reset which reenables counting) the second scaler of the pair will have a systematic timing interval deficit of about 2 microseconds relative to the first which is time required to input data to an operational computer register. The effect of this additional timing error is also negligible for the range of sampling intervals of interest for the application here described. This timing uncertainty effect is easily measured by providing parallel input to both scalars from the same V-F converter driven by a constant (battery) voltage supply. When this was done at a sampling interval of $\tau = 0.01$ sec, a finite value for σ_1^2 , σ_2^2 and $\sigma_{1,2}^2$ of about 4×10^{-8} was observed. This corresponds to an rms timing uncertainty of 2×10^{-4} of the base sampling interval or 2 microseconds. This agrees with the time required to input data from one scaler and to reset the scaler. This degree of inherent timing jitter is of no consequence for reactor measurements or for any of the test measurements here reported. The effect of this jitter is about 3 orders of magnitude below the coherent effect observed in a reactor near critical and in tests using Californium sources. It could be eliminated by use of some relatively simple hardware modifications to the I/O interface and to the scalars but is not troublesome in the applications here described. The effect was not present when the SEL-840 system was used as described further in Appendix E.

Although joint fluctuations in a pair of detectors not subject to a correlated source are low (or of magnitude set by scaler-computer effects) finite, random values of correlation will be observed in any measurement taken over finite time intervals. This residual fluctuation will be influenced by noise contributions from the electrometer itself as well as noise from a radiation source. On the lower-current electrometer ranges where electrometer gain is high, more inherent noise will appear and the measurement of a joint fluctuation will be accompanied by an inherent dispersion in finite-time records which will ultimately be set by the electrometer. For the electrometer-detector system here described, subject to sources including reactors and low-strength spontaneous fission, residual joint fluctuations will be set by the efficiency of the detector array and not by inherent electrometer noise.

Sixty cycle line pickup is an effect which is a potential error source of large magnitude in any correlation measurement on reactors. This comes about since the frequency emphasized in reactor noise measurements coincides, by chance, with AC line frequency. In this respect, the sampling technique

here described is at advantage relative to other noise techniques since sampling interval can be picked to be a multiple of basic line frequency. The fundamental 60 cycle and all of the higher harmonics integrate to 0 in each sample. A sampling rate of 60 per second is nearly ideal for the reactor measurement since 1/60 seconds is much longer than electrometer response time but not sufficiently long that delayed-neutron effects are of much significance.

E. Tests of the Detector System Using Random and Non-Random Neutron Sources

For noise measurements, the detector pair is exposed to leakage neutrons from fast reactors and the joint fluctuations determined. This is a common experimental physics procedure and is not, in any essential way, different from any other measurement of source strength by coincidences between a pair of detectors. If radiation emitted by the sources is correlated in time (such as is true for gamma-gamma or beta-gamma de-excitation of nuclei, or for neutron chains in a reactor initiated by fission) a pair of detectors will show coincidences. Where radiation is emitted at random, such as for alpha-n neutron sources, no coincidences will appear. Californium spontaneous fission is an excellent and widely available source of non-random neutrons and provides a direct indication of the performance of neutron detectors for coincidence measurement on reactors. In the following we will describe tests of the detector system using alpha-n and ^{252}Cf neutron sources; these tests lead to a good understanding of the inherent characteristics of the detection process without involving the reactor directly.

1. Tests with an Alpha-n Source

An alpha-n source (Am-Be) emitting about 2×10^7 neutrons per second was centered between the pair of detectors to produce maximum efficiency. Mean square values for joint fluctuation and for the fluctuation in each detector output individually were recorded as a function of sampling interval τ . A typical set of data is shown in Table I for sampling intervals ranging from 0.05 to 1.6 seconds. As expected, for the longer values of τ , $\tau\sigma_1^2$, $\tau\sigma_2^2$, and $\tau\sigma_{1,2}^2$ change very little. Statistical precision at the longest value of τ is about $\pm 2\%$ for $\tau\sigma_1^2$, $\tau\sigma_2^2$, and $\tau\sigma_{1,2}^2$ and error decreases as τ decreases since more samples contribute for a given total measurement time. Since the alpha-n source is random (an alpha particle emitted from americium decay cannot create more than one neutron) the joint correlation $\sigma_{1,2}^2$ is a reflection of the finite time span of the record and of a small timing error described previously and shows a residual value, of either sign, with magnitude depending upon record length. Table III also shows the same variables as measured using sampling intervals from 0.005 to 0.16 seconds (with the detector bias at 1000 volts). It is observed that $\tau\sigma^2$ values increase as τ increases over the sampling time range and approaches an asymptotic value. Again, $\tau\sigma_{1,2}^2$, the joint correlation, exhibits only a small residual fluctuation by virtue of the random nature of the neutron source.

TABLE III. Variance vs Sampling Time for an α -n Source

Sampling Interval τ (sec)	$\tau\sigma_1^2$ ($\times 10^{-5}$)	$\tau\sigma_2^2$ ($\times 10^{-5}$)	$\tau\sigma_{1,2}^2$ ($\times 10^{-7}$)
0.005 ^a	0.284	0.263	0.151
0.010 ^a	0.367	0.343	0.207
0.020 ^a	0.409	0.385	0.224
0.040 ^a	0.431	0.409	-0.169
0.080 ^a	0.441	0.421	0.411
0.160 ^a	0.448	0.428	-0.224
0.05 ^b	0.436	0.414	0.013
0.10 ^b	0.442	0.419	0.242
0.20 ^b	0.440	0.422	-0.133
0.40 ^b	0.446	0.433	0.150
0.80 ^b	0.442	0.417	-0.167
1.60 ^b	0.459	0.427	0.220

^aData measured with a 0.005 sec base sampling time.

^bData measured with a 0.050 sec base sampling time.

According to the analysis developed in Appendix B, the increase in $\tau\sigma_1^2$, $\tau\sigma_2^2$, and $\tau\sigma_{1,2}^2$ with τ is in consequence of time response limitations of the electrometer and of the charge collection properties of BF₃. Since charge created by fast particles from the $n(B^{10}, \alpha)^{12}C$ reaction will be collected more quickly as the detector bias rises, one would expect to observe a change in the rate at which variance values approach asymptotic as bias changes. The shape of the variation of $\tau\sigma^2$ with τ is given by

$$\tau\sigma^2 = A \left(1 - \frac{B}{\tau}\right) \quad (D.1)$$

when τ is greater than the characteristic time scale B which is 5/3 of the overall detector response time $|Dt|$. It is of interest to vary detector bias systematically from low to high voltages and to observe the effect upon the parameters A and B . As voltage becomes very high, charge collection time will become shorter than the inherent electrometer response time of 0.5 milliseconds on the electrometer range (1×10^{-8}) used for these measurements. With an asymptotically high detector voltage the quantity B in Eq. D.1 will only reflect the finite time response of the electrometers. In Fig. 3 we show the results of a least squares fitting procedure for A and B for the quantity $\tau\sigma_{1,2}^2$ derived from data similar to that shown in Table I and plotted against detector bias from 250 volts to 2500 volts. Below 500 volts the asymptotic value, A , is found to be well below its value observed at all higher voltages. The explanation probably is that recombination of ionization is occurring in the weak electric field and the net charge collection is reduced. Above 500 volts, essentially all charge is collected and A ceases to change as bias increases.

Fig. 3. Voltage Dependence of Detector Response

The characteristic time response, B , is seen to be much longer at the lower voltages but has dropped to about 1.7 milliseconds at 2500 volts. Detector response, $|D\tau|$, as defined in Appendix B is $3/5$ of B or 1.0 milliseconds. To first approximation, $|D\tau|$ is the sum of the electrometer response time (0.5 milliseconds) and mean ion drift time, which must be about 0.5 milliseconds at 2500 volts. This value for ion drift time is consistent with estimates based upon mobility of heavy ions in BF_3 under the influence of an applied field.

Measurements of the variation of τ^2 with τ at any operating bias can be made with high accuracy and the ratio of τ^2 at finite τ to its value at asymptotically large values of τ can be ascertained. For example, with the detector operated at 2000 volts, τ^2 values at a sampling rate of 60 per second ($\tau = 0.0167$ seconds) must be scaled by the factor $1/(1-1.7/16.7) = 1.10$ to become consistent with a detector system not limited in time response. The correction factor is accurately known however and is fixed for the detector-electrometer system. Since the statistical precision of measurements is improved as sampling interval decreases, it is desirable to operate at the shortest possible values of τ and to correct results to asymptotic.

2. Tests with a Californium Source

A ^{252}Cf spontaneous-fission neutron source of known strength was available to serve as a non-random neutron source for detector testing. Whereas $\tau_{1.2}^2$ is null (apart from finite-record statistical errors) for an alpha-n source, $\tau_{1.2}^2$ is finite for spontaneous fission since neutrons are emitted in bunches and neutrons from a single fission can arrive in time coincidence in both detectors. The underlying theory is simple and in no essential way different from that outlined in Appendices B and C for reactors as a source of non-random events in detectors. Let F be the spontaneous fission rate and λ_0 and λ_d be lifetimes for neutron loss by all processes and by capture in the BF_3 gas, respectively. Detection rates are assumed proportional to the neutron strength νF via an efficiency $\epsilon = \lambda_0/\lambda_d$. The probability that a fission at time t_0 will lead to counts at t_1 and t_2 is the product of three independent probabilities:

1. $F dt_0$.

The probability that fission occurs at t_0 .

2. $\nu \exp - \frac{t_1 - t_0}{\lambda_0} dt_1 / \lambda_d$.

The probability that a neutron is detected at time t_1 . ν is the neutron multiplicity.

3. $(\nu-1) \exp - \frac{t_2 - t_0}{\lambda_0} dt_2 / \lambda_d$.

The probability that a neutron from the same fission was detected at time t_2 . Since a neutron was assumed to have been captured at t_1 , this neutron cannot contribute to the event at t_2 , hence the factor $\nu-1$.

It is assumed that all neutrons from the source are detected with uniform efficiency in either detector. It is not essential that the efficiency of both the detectors be identical although they were, very nearly, for the tests run. Detectors employing moderator to slow neutrons to near-thermal energies where they are captured are sensitive to changes in energy spectrum. In the MeV region, where neutron age is a rapidly increasing function of energy, significant dependence of detector efficiency upon neutron energy should occur. If the energy spectrum of neutrons is a function of multiplicity in neutron emission, the assumptions of the simple analysis will not hold.

Multiplication of the factors 1, 2, 3 above and integration over t_0 will provide an expression for conditional probability that can be used directly in the formulation of Appendices B and C to provide a relationship between the joint local variance $\sigma_{1,2}^2$ and fission rate. The result is

$$F\sigma_{1,2}^2 = 3/2 D \quad (D.2)$$

where $D = \sqrt{v(v-1)}/v^2$ is the Diven factor for californium and has the numerical value $0.85 \pm 1\%$. Equation D.2 may be compared with an analysis of joint variance for a point reactor source neglecting delayed neutrons. From Appendix B the point reactor variance is given by

$$F\sigma_{1,2}^2 = 3/2 D \frac{1}{\beta^2(1+S)^2} \quad (D.3)$$

where F is now the total reactor fission rate, β is the effective beta, and S is the subcriticality. That Eqs. D.2 and D.3 are consistent is apparent since the prompt multiplication of neutrons by the factor $1/\beta(1+S)$ increases variance for a reactor by $1/\beta^2(1+S)^2$ relative to an unmultiplied source (the total length of neutron chains as seen by each detector must be increased by the prompt multiplication factor giving the square dependence). If the effective source is about 5×10^7 per second (approximate for Pu-fueled reactor mock-ups) it may be verified that $\sigma_{1,2}^2$ induced by a 1×10^9 per second californium source

is comparable to that induced by a fast reactor \$1 subcritical. A test of the detection system using a 1×10^6 ^{252}Cf source is, therefore, equivalent to experimental data obtained on a fast reactor mock-up apart from delayed neutrons which only appear at the longer values of sampling intervals. A californium source in a container 2×5 cm was available with a source strength of 1.42×10^9 neutrons per second based upon a several year old manganese bath experiment at a calculated decay loss. The absolute strength of this source is thought to be known to better than 1%. The source was centered between a pair of detectors and a test of the effect of source positioning on detector efficiency was made by moving the source vertically and laterally and observing the change in detector current. Because of the large physical size of the detectors compared to the source, no significant sensitivity of efficiency to source positioning occurred over the physical size of the source when the source was centered between the two detectors. We can therefore assume that detection efficiency is uniform for all regions of the source.

Table IV summarizes a run with the Cf source. The asymptotic $\tau\sigma_{1.2}^2$ was estimated to be 0.322×10^{-5} with a statistical precision of about $\pm 2\%$. If we take τ to be $3.73 \pm 1\%$ and D to be $0.85 \pm 1\%$, we infer the source strength to be

$$S = \frac{3}{2} D \frac{\tau}{\tau\sigma_{1.2}^2} = \frac{1.5(0.85)(3.73)}{0.322 \times 10^{-5}} = 1.48 \times 10^6 \pm 2.5\%. \quad (D.4)$$

The inferred value of $1.48 \times 10^6 \pm 2.5\%$ is to be compared with the known strength of $1.42 \times 10^6 \pm 1\%$. The two values are just outside of RMS error limits.

TABLE IV. Data for the ^{252}Cf Source Strength Measurement

Sampling Interval τ (sec)	$\tau\sigma_1^2$ ($\times 10^{-4}$)	$\tau\sigma_2^2$ ($\times 10^{-4}$)	$\tau\sigma_{1.2}^2$ ($\times 10^{-5}$)
0.005	0.2249	0.2055	0.2204
0.010	0.2639	0.2419	0.2637
0.020	0.2852	0.2637	0.3010
0.040	0.2961	0.2750	0.3098
0.080	0.3026	0.2823	0.3172
0.160	0.3096	0.2898	0.3298

In addition to directly testing the operation of the detector system under circumstances equivalent to those encountered in fast reactor mock-ups, the extent of agreement between the inferred and the known strength of the source may be of some further interest. If the energy spectrum of fission neutrons had any substantial dependence upon multiplicity this would, in light of a presumably strong dependence of efficiency of detectors upon energy for primary fission spectra neutrons, introduce a bias between inferred and known strengths. The fact that no significant bias beyond statistics is observed lends support to the assumption that neutron emission spectra in fission are not multiplicity dependent. The importance of this carries over to reactor measurements since calculation of the spatial dependence of the Diven factor needed to interpret reactor noise measurements does not consider neutron emission energy spectra in fission to be multiplicity dependent either.

The detectors used for the Cf source strength determination were required for installation at ZPR-9 soon after the measurement was done. Had time permitted, it would have been interesting to have extended results to another californium source of higher strength and more recent calibration and to have made a more thorough study of systematic error.

APPENDIX E

Data Acquisition and Analysis Codes for Noise Measurements

The pair of Fortran codes here appended (Tables V and VI) are used for the acquisition of data from the leakage detectors and for the reduction of this data to provide a value for effective beta.

The data acquisition code DACQI uses a machine subroutine INPUT (ISCAL) to read periodically the accumulation of counts from a scaler whose input is a voltage-to-frequency converter attached to the leakage-detector-electrometer system. Sequential use of the subroutine INPUT, with arguments designating the pair of scalers, causes transfer of the contents of the scalers to computer core after receipt of a time mark coincidentally to both scaler logic controllers. On termination of the read-in of both scalers, reset occurs simultaneously for both scalers, and another accumulation interval commences. The time associated with read-in and reset is only a few computer cycles and is negligible compared with the sampling interval of 1/60 sec.

Operation of the data acquisition code is straightforward. INPUT consists of a pair of numbers ISCAL, JSCAL which designate the pair of scalers used followed by the electrometer range settings EL1R, EL2R for both detectors and the base sampling interval TAUB.

Noise records for both detectors are accumulated in an array of 3584×2 points. Individual and joint local variances are formed and adjacent-point averaging carried out during the time between scaler input operations. At the end of the acquisition period ($3584 \times 2/60 \sim 2$ min), acquisition is terminated and the array is processed for increasing equivalent sampling time intervals by averaging adjacent points. With a total number of groupings $NGP = 6$, the longest sampling interval is 0.533 sec when the base rate is 1/60 sec.

At the termination of each run intermediate data are displayed and, depending upon a sense-switch option, additional runs may be made or the measurement terminated. Termination may be desirable for measurements at critical if drifting of power to the extent of 10% or so occurs although drifting to this extent will not introduce measurement error. Termination of the measurement consists of a computation of average values for all of the runs; these results serve as input to the data analysis code DANALZ which requires the numbers from DACQI together with delayed-neutron parameters and subcriticality and a value for the electrometer time response TS defined in Appendix B. The rationale for use of separate codes is to allow for data acquisition in circumstances where the subcriticality of the configuration is to be defined later on. The procedures used to determine subcriticality in dollars are assumed to make use of the same set of delayed neutron parameters as are used in DANALZ.

In addition to correcting for delayed-neutron and detector response effects, the code derives the appropriate values for statistical error and produces a weighted average for the quantities $3/2 D/\beta^2$ and $3/\epsilon$ apart from the power factor. The significance of these quantities has been discussed in Appendix B. The absolute fission rate per electrometer current (power factor) must be found (together with a value for D) before β can be determined.

Data for different sampling times are weighted to provide the final result. The weighting procedure assumes that a $\pm 1.5\%$ systematic error is present in all measurements due to delayed-neutron and prompt-lifetime correction uncertainties, electrometer and voltage-frequency converter calibration errors, and electrometer time response uncertainty. This fixed error is added in quadrature with a purely statistical error to provide weights for averaging of results at the different equivalent sampling times. The effect of statistical uncertainty is, for all practical purposes, negligible and the overall measurement uncertainty is taken to be $\pm 1.5\%$.

Tables VII and VIII contain data from a noise measurement on an enriched uranium fast reactor -- the "leaky STF" core. Table VII provides results monitoring the acquisition of data using the DACQ1 Code of Table V. The full-scale current range for both detectors was set to 1×10^{-7} amps for this measurement and the sampling time was set at 1/60 sec.

Information from each run (of about 2 minutes duration) is printed after the run terminates. Numbers proportional to the average level for each detector (LEVEL1, LEVEL2) and to mean square fluctuations (SIG1, SIG2) and joint fluctuation (SIG12) at the base sampling interval of 1/60 sec are shown. This run was unique only insofar as 25 consecutive runs over a total interval of about 50 minutes were made with the reactor at critical without experiencing a drift in excess of a few percent, as can be seen from the LEVEL1, LEVEL2 changes from run to run. Ordinarily, no more than 10 runs would be processed with drifting occurring to an extent of 10% or less.

After termination of the sequence of runs, average values normalized to agree with the terminology of Appendix B for the individual variances (SIG1, SIG2), joint variance (SIG12) and joint difference (SIG1M2 = SIG1 + SIG2 - 2SIG12) are listed for six groupings of the base sampling time. This information, together with average electrometer levels, is required in the code DANALZ listed in Table VI.

The output of DANALZ for this measurement is listed in Table VIII. The electrometer time response, TS, was 1.00 msec (Appendix D) and the reactor was critical (a default to 0.1% subcriticality is made by the code for at-critical measurements prior to deriving corrections for delayed-neutron effects). For each value of sampling time TAU, the corrected values for joint and difference fluctuations, SIG12 and SIG1M2, are listed. The correction, including detector response and delayed-neutron effects, is listed under the heading KINETIC CORR. Weighted values for $3/2 D/B^2$ and for $3/\epsilon$ (apart from the power calibration) are listed. The numbers provided must be multiplied by the power calibration which is the ratio of absolute fission rate to electrometer current. The power factor for this measurement was determined to be 0.7927×10^{17} fissions/sec per amp of total detector current (sum of currents from the pair of detectors used). Consequently,

$$3/2 D/B^2 = 0.38098 \times 10^{-12} \times 0.7927 \times 10^{17} = 3.02 \times 10^4$$

and

$$3/\epsilon = 0.57193 \times 10^{-13} \times 0.7927 \times 10^{17} = 4.53 \times 10^3.$$

If the factor D is known, β can be derived. The mean efficiency, ϵ , for the detector pair (Appendix B) is given by

$$\epsilon = 3/4.53 \times 10^3 = 0.66 \times 10^{-3}.$$

This value for detection efficiency is somewhat higher than average for ZPR-9 mock-ups due to the absence of additional peripheral shielding.

TABLE V.

```

C DRQ1 DATA ACQUISITION FOR NOISE MEASUREMENT OF BETA,
C
C DIMENSION NA1(3584),NA2(3584),S1(7,25),S2(7,25),S12(7,25),
C X E1(25),E2(25),SG1(7),SG2(7),SG12(7),SG1M2(7),IDENT(18)
C COMMON NA1,NA2,S1,S2,S12
C
C 1 FORMAT(13A4)
C 2 FORMAT(6E12,5)
C 3 FORMAT(/25H INPUT(214) SCALERS USED, )
C 4 FORMAT(/
C X 39H FULL SCALE FOR ELECTROMETERS 1 AND 2 = 2E12,5 /
C X 30H EFFECTIVE SAMPLING INTERVAL = E12,5, 6H SEC, )
C 5 FORMAT(13,5E12,5)
C 6 FORMAT(/4H RUN 3X,6H LEVEL 1 6X,6H LEVEL 2 7X,4H SIG1 8X,4H SIG2
C X 9X,5H SIG12 )
C 7 FORMAT(/6H GROUP 4X,4H SIG1 9X,4H SIG2 6X,5H SIG12 7X,6H SIG1M2 )
C 8 FORMAT(/29H AVERAGE ELECTROMETER LEVELS 2E12,5 )
C 9 FORMAT(14I1)
C 10 FORMAT(214)
C 11 FORMAT(/33H INPUT (E12,5) SAMPLING INTERVAL. )
C 12 FORMAT(/26H INPUT RUN IDENTIFICATION. )
C 13 FORMAT(/47H INPUT (2E12,5) ELECTROMETER FULL-SCALE RANGES. )
C
C USE SEL SCALERS WITH EXTERNAL TIME MARK (POS PULSES WITH
C TERMINATION) SIMULTANECUSLY TO 32TH EXTERNAL READOUT-INITIATE INPUTS,
C CONNECT GATE OUT OF THE SECOND SCALER TO BE READ (JSCAL) TO THE
C EXTERNAL GATE IN OF THE FIRST SCALER TO BE READ IN (ISCAL). USE
C FULL WORD LENGTHS ON 32TH SCALERS,
C INPUT THE PAIR OF SCALERS USED (60,61,62,63). DEFAULT = 60,61).
C WRITE(1,3)
C READ(1,10) ISCAL,JSCAL
C INPUT THE EFFECTIVE BASE SAMPLING TIME INTERVAL (DEFAULT = .01667).
C WRITE(1,11)
C READ(1,2) TAUB
C IDENTIFY THE RUN.
C 43 WRITE(1,12)
C READ(1,1) IDENT
C WRITE(5,9)
C WRITE(5,1) IDENT
C INPUT ELECTROMETER RANGES FOR DETECTORS 1 AND 2.
C WRITE(1,13)
C READ(1,2) EL1R,EL2R
C WRITE(5,4) EL1R,EL2R,TAUB
C ALLZK A TOTAL OF NGP TIME GROUPINGS OF BASE DATA.
C NGP=5
C
C ZERO AREAS CONTAINING ELECTROMETER FLUCTUATIONS
C S1 AND S2 FOR EACH DETECTOR, AND JOINT FLUCTUATION S12,
C
C K=1
C D220 L=1,25
C E1(L)=0.0
C E2(L)=0.0
C D221 J=1,NGP
C S1(J,L)=0.0
C S2(J,L)=0.0
C 21 S12(J,L)=0.0
C 20 CONTINUE
C WRITE(5,6)
C WRITE(1,6)

```

PAUSE

```

C REJECT FIRST THREE POINTS AND THEN BEGIN ANALYSIS.
C THE SET ROUTINE INPUT WHEN USED SEQUENTIALLY PLACES A PAIR OF
C SINGLE PRECISION NUMBERS IN CORE.
C
28 M1=INPUT(ISCAL)
   M2=INPUT(JSCAL)
   M1=INPUT(ISCAL)
   M2=INPUT(JSCAL)
   M1=INPUT(ISCAL)
   M2=INPUT(JSCAL)
C
C ANALYSE FOR AN ARRAY LENGTH OF 7168 POINTS AT BASE RATE.
C AVERAGE ADJACENT POINTS TO FORM AN ARRAY OF LENGTH 3584 POINTS.
C
   D1=0.0
   D2=0.0
   D12=0.0
   N=3584
   I=1
   M1=INPUT(ISCAL)
   M2=INPUT(JSCAL)
   IK1=M1
   IK2=M2
   M1=INPUT(ISCAL)
   M2=INPUT(JSCAL)
   IK3=M1
   IK4=M2
30 M1=INPUT(ISCAL)
   M2=INPUT(JSCAL)
   V1=FLZAT(IK3-(IK1+M1)/2)
   V2=FLZAT(IK4-(IK2+M2)/2)
   D1=D1+V1*V1
   D2=D2+V2*V2
   D12=D12+V1*V2
   NA1(I)=(IK1+IK3)/2
   NA2(I)=(IK2+IK4)/2
   IK1=IK3
   IK2=IK4
   IK3=M1
   IK4=M2
   M1=INPUT(ISCAL)
   M2=INPUT(JSCAL)
   V1=FLZAT(IK3-(IK1+M1)/2)
   V2=FLZAT(IK4-(IK2+M2)/2)
   D1=D1+V1*V1
   D2=D2+V2*V2
   D12=D12+V1*V2
   IK1=IK3
   IK2=IK4
   IK3=M1
   IK4=M2
   I=I+1
   IF(I-3584)30,30,31
31 S1(1,K)=D1
   S2(1,K)=D2
   S12(1,K)=D12
C
C AT END OF SET, ANALYSE FOR FLUCTUATIONS FOR AN ADDITIONAL
C SIX TIME GROUPINGS OF THE BASIC DATA INTERVAL.

```

```

C
D025 J=2,NGP
      =71.9/2*(J-1)
      I=J-1
      V1=0.0
      V2=0.0
      S12=0.0
D027 I=2,NM1
      V1=FLDAT(NA1(I))-(NA1(I-1)+NA1(I+1))/2
      V2=FLDAT(NA2(I))-(NA2(I-1)+NA2(I+1))/2
      S1=S1+V1*V1
      S2=S2+V2*V2
22 S12=S12+V1*V2
      S1(I,K)=S1
      S2(I,K)=S2
      S12(I,K)=S12
      I=I+1
      L=L+1
24 NA1(I)=(NA1(I)+NA1(I+1))/2
      NA2(I)=(NA2(I)+NA2(I+1))/2
      I=I+1
      L=L+1
      IF(I-J)24,25,25
25 CONTINUE
C DETERMINE THE TOTAL COUNT OVER BOTH ELECTROMETERS FOR LAST GROUP.
D028 I=1,N
      E1(K)=F1(K)+FLDAT(NA1(I))
23 E2(K)=F2(K)+FLDAT(NA2(I))
C
C LIST RESULTS FOR THE SET AT BASE SAMPLING INTERVAL, TEST TO GO ON.
C WRITE(5,5) K,E1(K),E2(K),S1(1,K),S2(1,K),S12(1,K)
      WRITE(1,5) K,E1(K),E2(K),S1(1,K),S2(1,K),S12(1,K)
      K=K+1
C TERMINATE RUN IF SENSE 3 IS UP OR IF THE SUB-RUN TOTAL EXCEEDS 25,
      IF(K-25)19,19,33
19 CALL SWITCH(3,IS)
      GO TO(33,28),IS
C
C AT THE END OF THE TOTAL NUMBER OF RUNS, SUM AND LIST AVERAGES,
C
33 KRUN=K-1
D034 I=1,NGP
      SG1(I)=0.0
      SG2(I)=0.0
34 SG12(I)=0.0
D027 I=1,NGP
D035 K=1,KRUN
      SG1(I)=SG1(I)+S1(I,K)
      SG2(I)=SG2(I)+S2(I,K)
35 SG12(I)=SG12(I)+S12(I,K)
27 CONTINUE
C ADD UP ALL ELECTROMETER SUMS AT LAST GROUPING.
      AV1=0.0
      AV2=0.0
D036 K=1,KRUN
      AV1=AV1+E1(K)
36 AV2=AV2+E2(K)
C
C NORMALIZE THE AS-COLLECTED DATA COUNTS TO THE PROPER TIME AND
C ELECTROMETER LEVEL.
C

```

```

PTS=7168.*FLOAT(KRUN)
X=PTS*TAUB
V=X/2.*(NGP-1)
AV1=AV1/V
AV2=AV2/V
U=AV1*AV1*X
V=AV2*AV2*X
W=AV1*AV2*X
SG1(1)=SG1(1)/U
SG2(1)=SG2(1)/V
SG12(1)=SG12(1)/W
SG1M2(1)=SG1(1)*SG2(1)*2.*SG12(1)
D037 I=2,NGP
D1=2.*(I-1)
D2=D1*D1*3584./(3584.-D1)
SG1(I)=SG1(I)*D2/U
SG2(I)=SG2(I)*D2/V
SG12(I)=SG12(I)*D2/W
37 SG1M2(I)=SG1(I)*SG2(I)*2.*SG12(I)

C
C   LIST RESULTS.
C
WRITE(5,7)
D041 I=1,NGP
INDEX=2*(I-1)
41 WRITE(5,5) INDEX,SG1(I),SG2(I),SG12(I),SG1M2(I)
AV1=AV1*EL1R/1.E06
AV2=AV2*EL2R/1.E06
WRITE(5,8) AV1,AV2

C
C   SAVE RESULTS FOR POINT MODEL COMPARISON.
C
WRITE(3,1) IDENT
WRITE(3,5) NGP,PTS,TAUB,AV1,AV2
D042 I=1,NGP
42 WRITE(3,5) I ,SG1(I),SG2(I),SG12(I),SG1M2(I)
PAUSE
C   RETURN FOR ANOTHER COMPLETE ANALYSIS.
GO TO 43
END
$0

```


TABLE VI

```

C  ANALZ  REDUCE DATA TO PROVIDE EFFECTIVE BETA FROM NOISE.
C
      DIMENSION SG12(7),SG1M2(7),ERR12(7),ERR1M2(7),IDENT(18)
      COMMON F(6),Y(6),SP(7),A(7),T(7),Q(7),BEFF,PRMPT
C
      1 FORMAT(18A4)
      2 FORMAT(6E12,5)
      3 FORMAT(/16H 3/2D/BEFF*2 = E12,5,23H TIMES FISSIONS/SEC/AMP //
        X 10H 3/EPSI = E12,5,23H TIMES FISSIONS/SEC/AMP //
        X 30H TOTAL ELECTROMETER CURRENT = E12,5, 5H AMPS //
        X 41H RATIO OF DETECTOR ELECTROMETER LEVELS = E12,5 )
      4 FORMAT(53H INPUT(E12,5) THE AMP*DOLLAR VALUE FOR SUBCRITICALITY )
      5 FORMAT(I3,5E12,5)
      6 FORMAT(2X,5E12,5)
      7 FORMAT(/25H ASSUMED SUBCRITICALITY = E12,5, 9H DOLLARS )
      8 FORMAT(/21H BASE SAMPLING TIME = E12,5 //
        X 16H TOTAL POINTS = E12,5 )
      9 FORMAT(1H1,/,/,/,/,/)
     10 FORMAT(F10,5)
     11 FORMAT(/48H TAU KINETIC CORR CORR SIG12 ERR SIG12
        X 27H SIG1M2 ERR SIG1M2 )
     12 FORMAT(/16H RELATIVE YIELDS 6F7,4 /
        X 16H DECAY CONSTANTS 6F7,4 /18H LIFETIME AND BETA 2E12,5 // )
     13 FORMAT(/50H INPUT(F10,5) DETECTOR-ELECTROMETER RESPONSE TIME. )
     14 FORMAT(/36H DETECTOR-ELECTROMETER RESPONSE TIME = F10,6 )
     15 FORMAT( 46H INPUT(I3) THE DEVICE WHICH HAS THE NOISE DATA )
     16 FORMAT( /40H THE AMP*DOLLAR CALIBRATION CONSTANT IS E12,5 )
C
C  READ RELATIVE YIELDS AND DECAY CONSTANTS FOR DELAYED NEUTRONS IN
C  ORDER OF INCREASING VALUES FOR DECAY CONSTANT,
C  ALSO READ THE PROMPT LIFETIME AND EFFECTIVE BETA. THE VALUES FOR
C  PRMPT AND BEFF READ IN ARE USED TO MAKE A CORRECTION OF SMALL
C  MAGNITUDE AND FINAL RESULTS WILL NOT BE VERY SENSITIVE TO THEM.
C
      PAUSE
      READ( 4 ,2) (F(I),I=1,6),(Y(I),I=1,6),PRMPT,BEFF
C
C  HEAD INPUT DEVICE, 4 IF DATA ON CARDS, 3 IF DATA ON PAPER TAPE,
      WRITE(1,15)
      READ(1,5) NUNIT
C
C  INPUT THE DETECTOR-ELECTROMETER TIME RESPONSE TS.
C  TS FOR THE ZPR 6/9 IS .00100
      WRITE(1,13)
      READ(1,10) TS
C
C  INPUT THE AMP*DOLLAR CALIBRATION FOR SUBCRITICALITY. AMPDOL IS
C  INPUT ZERO IF ALL RUNS ARE AT CRITICAL.
      WRITE(1,4)
      READ(1,2) AMPDOL
C
C  43 PAUSE
C
C  INPUT THE BASIC NOISE DATA IN THE SAME FORMAT AS PRODUCED.
C
      READ(NUNIT,1) IDENT
      READ(NUNIT,5) NGP,PTS,TAUB,AV1,AV2
      DO50 I=1,NGP
     50 READ(NUNIT,5) K,X,X,SG12(I),SG1M2(I)

```

C
C DERIVE SUBCRITICALITY FROM THE PRODUCT OF TOTAL (BOTH ELECTROMETERS)
C CURRENT AND THE SUBCRITICALITY IN DOLLARS AT THAT CURRENT,
C DEFAULT TO .1 CENT IF CLOSER TO CRITICAL.

C
C ETOT=AV1+AV2
C DOLLS=AMPDOL/ETOT
C IF(DOLLS=,001)98,99,99
98 DOLAR=.001
C GO TO 97
99 DOLAR=DOLLS
97 RH0=DOLAR*BEFF/(1.+DOLAR*BEFF)

C
C WRITE(5,9)
C WRITE(5,1) IDENT
C WRITE(5,8) TAUB,PTS
C WRITE(5,14) TS
C WRITE(5,16) AMPDOL
C WRITE(5,7) DOLLS
C WRITE(5,12) (F(I),I=1,6),(Y(I),I=1,6),PRMPT,BEFF

C
C GET THE POINT MODEL KINETIC PARAMETERS FOR CORRECTIONS,
C CALL TRANSF(RH0)

C
C CORRECT THE JOINT VARIANCE FOR TIME RESPONSE AND KINETICS EFFECTS.
C DERIVE STATISTICAL ERRORS AND LIST RESULTS,

C
C WRITE(5,11)
C Z=SG1M2(1)/SG12(1)
C DRAT=AV1/AV2
C Z=(1.+.25*Z*Z/(1.+.5*(DRAT+1./DRAT))+.5*Z)*3.89/PTS
C D051 I=1,NGP
C GRPS=2,*(I=1)
C TAU=TAUB*GRPS
C DUM=BEFF*(1.+DOLAR)*TS/PRMPT
C X=BEFF**2*(1.+DOLAR)**2*VARINT(TAU)/1.5
C X=1.667*TS*(1.-(1.+EXP(-DUM))/DUM)/TAU
C W=1.-1.667*TS/TAU
C SG12(I)=SG12(I)/X
C SG1M2(I)=SG1M2(I)/W
C ERR12(I)=SQRT(Z*GRPS)
C ERR1M2(I)=SQRT(3.89*GRPS/PTS)
51 WRITE(5,2) TAU,X, SG12(I),ERR12(I),SG1M2(I),ERR1M2(I)

C
C ASSUME THAT THE SYSTEMATIC UNCERTAINTY IS 1.5PC AND WEIGHT
C RESULTS WITH RANDOM AND SYSTEMATIC ERROR IN QUADRATURE,

C
C X=0,0
C XX=0.0
C Z=0,0
C ZZ=0.0
C D052 I=1,NGP
C WEIT1=.015**2*ERR12(I)**2
C WEIT2=.015**2*ERR1M2(I)**2
C X=X+SG12(I)/WEIT1
C Z=Z+SG1M2(I)/WEIT2
C XX=XX+1./WEIT1
52 ZZ=ZZ+1./WEIT2
C X=X*ETOT*(1.+DOLLS)**2/XX
C Z=Z*ETOT/ZZ

C

```

C FINAL RESULTS FOR POINT MODEL COMPARISON. FRAT IS THE RATIO OF
C TOTAL PILE FISSION RATE TO TOTAL (SUM OF BOTH) ELECTROMETER
C CURRENT. D IS THE GIVEN FACTOR,

```

```

WRITE(5,3) X,Z,ETOT,DRAT

```

```

GO TO 43

```

```

END

```

```

C

```

```

C

```

```

C

```

```

C

```

```

FUNCTION TO COMPUTE THE THEORETICAL INTEGRAL USED IN THE LOCAL
VARIANCE DERIVATION.

```

```

FUNCTION VARINT(TAU)

```

```

COMMON F(6),Y(6),SP(7),A(7),T(7),Q(7),BEFF,PRMPT

```

```

DUM=0.0

```

```

DO3 K=1,7

```

```

V=-SP(K)*TAU

```

```

IF(V=0.1)1,1,2

```

```

2 D=EXP(-V)

```

```

J=1.5-(2.5-D*(3.75-D*(1.5-.25*D)))/V

```

```

GO TO 3

```

```

1 J=V*V*(.25+11.*V*V/RD.)

```

```

3 DUM=DUM+J*Q(K)

```

```

VARINT=2.*DUM

```

```

RETURN

```

```

END

```

```

C

```

```

C

```

```

C

```

```

COMPUTE THE POLES AND RESIDUES OF THE TRANSFER FUNCTION.

```

```

SUBROUTINE TRANSF(RH0)

```

```

COMMON F(6),Y(6),SP(7),A(7),T(7),Q(7),BEFF,PRMPT

```

```

N=6

```

```

NP=7

```

```

I=1

```

```

SING=0.0

```

```

DEL=-Y(1)

```

```

GO TO 30

```

```

16 JCT=5

```

```

5 DEL=0.1*DEL

```

```

S=DEL+SING

```

```

IF(S-SING)3,13,13

```

```

13 SP(I)=S

```

```

GO TO 14

```

```

3 T2=D(S,RH0)

```

```

IF(T2)1,1,2

```

```

2 S=S*DEL

```

```

GO TO 3

```

```

1 SING=S*DEL

```

```

JCT=JCT+1

```

```

IF(JCT)4,4,5

```

```

4 T1=D(SING,RH0)

```

```

SP(I)=S*DEL*(T2/T1)/((1.0-T2/T1))

```

```

14 I=I+1

```

```

IF(NP-I)7,17,8

```

```

17 SING=-Y(N)

```

```

DEL=-Y(6)*BEFF/PRMPT

```

```

GO TO 16

```

```

8 SING=-Y(I-1)

```

```

DEL=Y(I-1)-Y(I)

```

```

30 DEL=0.5*DEL

```

```

S=SING+DEL

```

```

T2=D(S,RH0)

```

```

      IF(T2)30,30,31
31  SING=S
      DEL=-0.5*(S+Y(I))
      S=SING+DEL
      T2=D(S,RH0)
      IF(T2)16,16,31
C  CALCULATE THE A, T AND Q ARRAYS FOR EACH SINGULARITY.
      D09 I=1,NP
      DUM=0.0
      D010 L=1,N
      ARG=SP(I)+Y(L)
10  DUM=DUM+Y(L)*F(L)/(ARG*ARG)
      A(I)=1.0/(PRMPT+BEFF*(1.0-RH0)*DUM)
      D011 I=1,NP
      DUM=0.0
      D012 K=1,NP
12  DUM=DUM+A(K)/(SP(I)*SP(K))
      T(I)=-DUM*(1.-RH0)*(1.-RH0)
11  Q(I)=-A(I)*T(I)/SP(I)
      RETURN
      END
C  DETERMINE VALUES OF THE RECIPROCAL TRANSFER FUNCTION,
      FUNCTION D(S,RH0)
      COMMON F(6),Y(6),SP(7),A(7),T(7),Q(7),BEFF,PRMPT
      V=0.0
      D01 I=1,6
      1 V=V+F(I)/(S+Y(I))
      D=S*(PRMPT+BEFF*V*(1.0-RH0))*RH0
      RETURN
      END
C
C  FUNCTION EXF COMPUTES EXPONENTIALS WITH ARGUMENT LIMITS.
C
C      FUNCTION EXF(V)
C
C      IF(V-69.077553)1,1,2
      2 EXF=1.E+30
      RETURN
      1 IF(V+69.077553)3,3,4
      3 EXF=1.E-30
      RETURN
      4 EXF=EXP(V)
      RETURN
      END
$0

```

TABLE VII. Second Run on Leaky STF1/1, 6/26/80

Full Scale for Electrometers 1 and 2 = 0.10000E-06 0.10000E-06

Effective Sampling Interval = 0.16666E-01 sec

RUN	LEVEL1	LEVEL2	SIG1	SIG2	SIG12
1	0.30682E+07	0.22314E+07	0.20367E+09	0.11036E+09	0.13925E+09
2	0.30711E+07	0.22333E+07	0.20778E+09	0.11207E+09	0.14153E+09
3	0.30744E+07	0.22359E+07	0.21301E+09	0.11634E+09	0.14673E+09
4	0.30704E+07	0.22330E+07	0.20633E+09	0.11258E+09	0.14175E+09
5	0.30599E+07	0.22252E+07	0.20779E+09	0.11348E+09	0.14288E+09
6	0.30595E+07	0.22249E+07	0.20532E+09	0.11082E+09	0.13976E+09
7	0.30490E+07	0.22174E+07	0.21218E+09	0.11210E+09	0.14311E+09
8	0.30376E+07	0.22092E+07	0.21045E+09	0.11175E+09	0.14271E+09
9	0.30316E+07	0.22048E+07	0.20730E+09	0.11196E+09	0.14212E+09
10	0.30285E+07	0.22024E+07	0.19583E+09	0.10599E+09	0.13353E+09
11	0.30215E+07	0.21972E+07	0.20366E+09	0.11130E+09	0.13987E+09
12	0.30147E+07	0.21924E+07	0.19679E+09	0.10811E+09	0.13517E+09
13	0.30103E+07	0.21892E+07	0.20320E+09	0.11048E+09	0.13917E+09
14	0.30122E+07	0.21905E+07	0.19915E+09	0.10962E+09	0.13714E+09
15	0.30055E+07	0.21857E+07	0.20217E+09	0.10847E+09	0.13736E+09
16	0.29984E+07	0.21805E+07	0.20301E+09	0.11140E+09	0.13980E+09
17	0.29929E+07	0.21767E+07	0.20275E+09	0.10919E+09	0.13839E+09
18	0.29877E+07	0.21727E+07	0.19897E+09	0.10652E+09	0.13519E+09
19	0.29866E+07	0.21721E+07	0.20310E+09	0.10825E+09	0.13744E+09
20	0.29872E+07	0.21722E+07	0.20032E+09	0.11029E+09	0.13811E+09
21	0.29879E+07	0.21730E+07	0.20100E+09	0.10730E+09	0.13627E+09
22	0.29950E+07	0.21781E+07	0.20613E+09	0.11124E+09	0.14121E+09
23	0.29947E+07	0.21778E+07	0.20279E+09	0.10896E+09	0.13837E+09
24	0.29821E+07	0.21686E+07	0.20311E+09	0.10800E+09	0.13778E+09
25	0.29815E+07	0.21682E+07	0.20609E+09	0.11078E+09	0.14104E+09

Group	SIG1	SIG2	SIG12	SIG1M2	

1	0.26098E-05	0.26670E-05	0.24518E-05	0.37315E-06	
2	0.27527E-05	0.28225E-05	0.25938E-05	0.38749E-06	
4	0.28295E-05	0.28704E-05	0.26516E-05	0.39671E-06	
8	0.28868E-05	0.29492E-05	0.27157E-05	0.40407E-06	
16	0.28623E-05	0.29680E-05	0.27131E-05	0.40408E-06	
32	0.30216E-05	0.30878E-05	0.28531E-05	0.40317E-06	

Average Electrometer Levels			0.80905E-07	0.58837E-07	

TABLE VIII. Second Run on Leaky STF1/1, 6/26/80

Base Sampling Time = 0.16666E-01
 Total Points = 0.17920E+06
 Detector-Electrometer Response Time = 0.001000
 Assumed Subcriticality = 0.10000E-02 dollars
 Relative Yields 0.0386 0.2132 0.1878 0.4062 0.1281 0.0261
 Decay Constants 0.0127 0.0317 0.1160 0.3110 1.4000 3.8700
 Lifetime and Beta 0.32760E-06 0.67250E-02

TAU	KINETIC CORR	CORR SIG12	ERR SIG12	SIGLM2	ERR SIGLM2
0.16666E-01	0.89999E+00	0.27243E-05	0.48395E-02	0.41462E-06	0.46591E-02
0.33332E-01	0.95016E+00	0.27298E-05	0.68441E-02	0.40789E-06	0.65890E-02
0.55664E-01	0.97581E+00	0.27173E-05	0.96790E-02	0.40688E-06	0.93183E-02
0.13333E+00	0.99075E+00	0.27411E-05	0.13688E-01	0.40969E-06	0.13178E-01
0.26666E+00	0.10055E+01	0.26983E-05	0.19358E-01	0.40662E-06	0.18637E-01
0.53331E+00	0.10337E+01	0.27600E-05	0.27376E-01	0.40443E-06	0.26356E-01

3/2D/BEFF**2 = 0.38098E-12 times fissions/sec/amp

3/EPSI = 0.57193E-13 times fissions/sec/amp

Total Electrometer Current = 0.13974E-06 amps

Ratio of Detector Electrometer Levels = 0.13751E+01