On Bunch Lengthening Using the Fourth Harmonic Cavity in the NSLS VUV Ring

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MASTER

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1. SUMMARY

A fourth harmonic cavity has been installed in the VUV ring. It is supposed to create a flat region in the acceleration field waveform at the synchronous voltage allowing a wider range for the synchrotron phase of electrons executing incoherent synchrotron oscillations. The longer bunch thus formed would have a higher threshold for any disturbance that scales with the density of the beam.

Forming an inflection point in the acceleration field waveform to give a broad quartic potential well shape has proved to be difficult in practice. The problem is the excitation of a strong signal in the harmonic cavity by the beam. The harmonic field shifts in phase to escape the formation of the inflection point. Simultaneously the phase of the bunch shifts to a new synchronous phase with respect to the fundamental cavity signal. This equilibrium phase instability has been observed in the VUV ring.¹ It has already been studied elsewhere.²

It has been suggested³ that the phase of the beam excited voltage in the harmonic cavity can be controlled by detuning its resonant frequency from the beam current harmonic. Unfortunately the detuning needed to flatten the acceleration waveform also corresponds to the region of Robinson instability for the harmonic cavity. Therefore, lengthening the bunch may be followed by large amplitude synchrotron oscillation of the bunch center of mass.

Bunch lengthening is discussed in this note from several points of view. There follows a simple review of single electron oscillations in a quartic potential. Then equations are developed for the coupled oscillations of a cavity and a rigid bunch as a fully nonlinear, time dependent initial value problem. Next, a computer program that solves these equations for one, two or more cavities, with and without externally driven fields, is described and some simulations of the harmonic cavity interaction are shown. Finally, the fully nonlinear equations are linearized to derive a dispersion relation for the case of beam excitation in the harmonic cavity.

2. SINGLE ELECTRON EQUATION OF MOTION

The motion of a single electron by itself has negligible effect on the cavity fields. Assuming that the amplitudes and phases of the fields are controlled by generators connected to the cavities and that acceleration occurs in short gaps, the accelerating voltage may be written.

$$V(\tau) = \sum_m V_m \cos(m\omega_0 \tau + \eta_m).$$

The phase $\omega_0 r$ is the longitudinal coordinate of the electron with respect to a reference position. The ring revolution period is $T_0 = N(2\pi/\omega_0)$ where N is the number of buckets around the ring, E_0 is the reference orbit energy and U_0 is the synchrotron radiation loss per revolution.

Following Sands⁴ the energy loss due to synchrotron radiation is written

$$U_{\text{rad}} = U_0 + D\epsilon.$$

The deviation ϵ from energy E_0 also leads to a slip in the longitudinal position given by the momentum compaction factor according to:

$$\frac{dr}{dt} = -\alpha \frac{\epsilon}{E_0}.$$

Adding the energy loss due to radiation to that gained crossing the cavity gaps and averaging over many revolution periods, the rate of electron energy change is written

$$\frac{d\epsilon}{dt} = \frac{eV(r) - U_0}{T_0} - \frac{D\epsilon}{T_0}.$$

These two equations are combined to form the equation of motion for the longitudinal synchrotron phase

$$\frac{d^2 r}{dt^2} = -\frac{\alpha}{E_0 T_0} [eV(r) - U_0] - \frac{D}{T_0} \frac{dr}{dt}. \quad (1)$$

It is relatively simple to analyze when the amplitudes and phases of the cavity fields are independent of the electron motion.

Neglecting the damping term on the right (it is usually quite small) a first integral is obtained

$$\frac{1}{2} \left(\frac{dr}{dt} \right)^2 = \Phi_0 - \Phi$$

where

$$\Phi(r) = \frac{\alpha}{E_0 T_0} \int_0^r [eV(r) - U_0] dr$$

is an effective potential that describes the shape of the bucket confining the electron motion.

Considering a ring with a fundamental cavity at ω_0 and a cavity at its m th harmonic, the conditions for a quartic bucket potential in the neighborhood of the synchronous phase at $r = 0$ are

$$V_1 \cos \eta_1 + V_m \cos \eta_m = 1 ,$$

$$V_1 \sin \eta_1 + m V_m \sin \eta_m = 0 ,$$

$$V_1 \cos \eta_1 + m^2 V_m \cos \eta_m = 0 ,$$

The first expression requires that the accelerating voltage at $\tau = 0$ equal the energy loss per revolution. The second and third call for the first and second derivatives to vanish giving an inflection point at the synchronous phase position. Cavity voltages have been normalized to U_0 .

For any amplitude V_1 in the fundamental cavity, the amplitude V_m in the harmonic cavity and the phases η_1 and η_m are uniquely determined:

$$\begin{aligned} V_1^2 &= m^2 V_m^2 + \frac{m^2}{m^2 - 1} , \\ V_1 \cos \eta_1 &= \frac{m^2}{m^2 - 1} , \\ V_1 \cos \eta_m &= - \frac{1}{m^2 - 1} . \end{aligned} \quad (2)$$

Figures 1a, 1b, and 1c show the combined voltage waveform and the potential with a fourth harmonic cavity ($m = 4$) and VUV ring parameters $U_0 = 14$ keV and $V_1 = 120$ keV. Figures 1d and 1e show how the potential deforms under a 41% error in V_4 and Figure 1f shows the effect of a 10 degree error in the phase.

To appreciate the significance of these errors it is necessary to estimate the amplitude of synchrotron oscillations excited by quantum recoil during the radiation process. An approximation⁴ for the statistical energy spread is given by

$$\sigma_\epsilon \approx (E_0 u_c)^{1/2}$$

where u_c is the critical synchrotron radiation photon energy. For the VUV ring $E_0 = 0.75$ GeV, $\alpha = 0.02$ and $100 \text{ eV} < u_c < 1 \text{ keV}$. Therefore $2.7 \times 10^5 < \sigma_\epsilon < 8.7 \times 10^5 \text{ eV}$. Taking the upper estimate

$$\Phi_0 = \frac{1}{2} \alpha^2 \sigma_\epsilon^2 / E_0^2 < 2.7 \times 10^{-10} .$$

Comparing this value with the height 10^{-9} of the bump in Figure 1d or 1e shows that the power in the harmonic cavity should be controlled to within about 10% and the phase to one degree. These constraints may vary with V_1 and the level of quantum excitation.

The synchrotron frequency in a normal system (fundamental frequency RF drive) is obtained by expanding $V(\tau)$ in Equation (1) around the synchronous phase at $\tau = 0$. Small oscillations of τ and ϵ occur in a quadratic potential bucket with angular frequency

$$\Omega_s = \left(\frac{\alpha \omega_o e V_1 \sin \eta_1}{E_o T_o} \right)^{1/2}.$$

For the VUV ring $U_o = 14$ keV, $\omega_o = 2\pi \times 52.88$ MHz. With $e V_1 \cos \eta_1 = U_o$, the synchrotron frequency $\Omega_s/2\pi = 12.5$ kHz when $V_1 = 120$ keV and 10.0 kHz when $V_1 = 77$ keV. Therefore, in the normal system the bunch phase length

$$2\omega_o \sigma_\tau = 2\omega_o \frac{\alpha \sigma_\epsilon}{\Omega_s E_o}$$

is expected to be in the range between 0.06 and 0.25 radians at 52.88 MHz.

A third order expansion of $V(\tau)$ is required to estimate the bunch length in a quartic potential. Integrating the expansion, calculating the constant of motion Φ_o and using Equations (2) relates the phase amplitude spread in the bucket to the energy spread:

$$\frac{-4}{\sigma_\tau} = \frac{-12 T_o \alpha \sigma_\epsilon^2}{\omega_o^3 E_o e V_1 \sin \eta_1 (m^2 - 1)}.$$

In the normal quadratic potential bucket

$$\sigma_\tau^2 = \frac{-T_o \alpha \sigma_\epsilon^2}{\omega_o E_o e V_1 \sin \eta_1}.$$

Note that $\cos \eta_1 > 0$, $\sin \eta_1 < 0$, $\cos \eta_m < 0$, and $\sin \eta_m > 0$. The angle η_1 is slightly different in the two cases because the energy that the harmonic cavity extracts from the beam must be compensated by a new synchronous phase at the fundamental cavity. Assuming that η_1 is approximately unchanged a comparison of the bunch lengths gives

$$\frac{\sigma_\tau}{\sigma_e} = \left[\frac{-12 E_o e V_1 \sin \eta_1}{\omega_o T_o \alpha (m^2 - 1)} \right]^{1/4} \sigma_e^{-1/2}.$$

For $V_1 = 77$ keV the ratio is about 3.3.

3. FULLY NON-LINEAR COUPLED EQUATIONS

In any cavity in the ring the field is a superposition due to a coupled generator and excitation by the beam. Equations governing the beam-excited field are now derived treating the cavity as a lossy, tunable resonator. Using a vector potential A in the radiation gauge with boundary conditions appropriate to a cavity mode near frequency ω_0

$$E = - \frac{\partial A}{\partial t} ,$$

$$\nabla \times \nabla \times A + \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = \mu_0 J$$

where

$$\nabla \times \nabla \times A = \left[\frac{\omega_0 + \Delta\omega}{c} \right]^2 A .$$

A frequency shift $\Delta\omega$ allows the resonant frequency of the empty cavity to be tuned around the beam excitation at ω_0 . Assuming that the spatial mode is unperturbed by the beam and that the field develops slowly in time, the field may be written in terms of a slowly varying amplitude $a(t)$ and phase $\eta(t)$ and the spatial mode function $u(r)$,

$$A(r, t) = a(t) \sin(\omega_0 t + \eta) u(r) .$$

Slowly varying means that $\dot{a} \ll \omega_0 a$ and $\dot{\eta} \ll \omega_0$.

Substituting into the wave equation above and separating sinusoidal quadrature components.

$$\ddot{a} - 2a\omega_0(\dot{\eta} - \Delta\omega) - a(\dot{\eta} - \Delta\omega)^2 = 2\mu_0 c^2 \langle J_s \rangle ,$$

$$2\dot{a}(\omega_0 + \dot{\eta} - \Delta\omega) + a\ddot{\eta} = 2\mu_0 c^2 \langle J_c \rangle$$

where

$$\langle J_{s,c} \rangle = [\int u^2 d^3 r]^{-1} \frac{1}{2T} \int_{-T}^T dt \frac{\sin}{\cos} (\omega_0 t + \eta) \int d^3 r u(r) J(r, t) .$$

The volume integration includes the part of the wall penetrated by currents and fields. The time interval $T \gg \omega_0^{-1}$ but short enough so that $a(t)$ and $\eta(t)$ do not change.

In a typical storage ring cavity the beam traverses a short gap where the electric field is dominant. Considering a cavity with a gap of length Δz at $r = 0$, the electric field acting on an electron in the gap is

$$E(0, t) = - \omega_0 a(t) u(0) \cos(\omega_0 t + \eta)$$

and the corresponding voltage drop is

$$v(t)\cos(\omega_o t + \eta) = -E(o,t)\Delta z$$

where

$$v(t) = \omega_o u(o)\Delta z a(t).$$

The rf energy stored in the cavity is

$$\frac{1}{2} \epsilon_o \omega_o^2 a^2 \int d^3 r u^2(r)$$

and writing it as $\frac{1}{2} C v^2$ defines a capacitance

$$C = \frac{\epsilon_o}{u^2(o)(\Delta z)^2} \int d^3 r u^2(r) .$$

For a cavity with well defined capacitive loading at the gap, C is close to the capacitance between the adjacent boundaries.

The current density includes the beam current $i_b(t)$ in the gap and the current induced in the walls. Using

$$J(r,t) = i_b(t)\delta(x)\delta(y) + \text{wall current}$$

and retaining terms to lowest order in ω_o^{-1} ,

$$\dot{v}(\eta - \Delta\omega) = -\frac{1}{C} \frac{1}{2T} \int_{-T}^T i_b(t) \sin(\omega_o t + \eta) dt ,$$

$$\dot{v} + \frac{\omega_o}{2Q_L} v = \frac{1}{C} \frac{1}{2T} \int_{-T}^T i_b(t) \cos(\omega_o t + \eta) dt .$$

The term involving Q_L takes care of the resistive loss in the walls as well as the resistive part of the coupled generator impedance. The reactance is included in the detuning term $\Delta\omega$.

Now the beam current is composed of electrons that pass through the gap every T_o seconds with some additional delay τ connected with the energy deviation ϵ . The contribution to the current in the gap by the i th electron on its j th revolution is $e\delta(t - \tau_i - jT_o)$. Summing over all electrons and all revolutions past and present,

$$i_b(t) = \sum_{i,j} e\delta(t - \tau_i - jT_o) .$$

It follows that

$$\frac{1}{2T} \int_{-T}^T i_b(t) \frac{\sin(\omega_o t + \eta)}{\cos(\omega_o t + \eta)} dt = \frac{1}{T_o} \sum_i \frac{\sin(\omega_o \tau_i + \eta)}{\cos(\omega_o \tau_i + \eta)}.$$

The field equations for each cavity take the form

$$\dot{v}(\eta - \Delta\omega) = - \frac{e}{CT_o} \sum_i \sin(\omega_o \tau_i + \eta), \quad (3)$$

$$\dot{v} + \frac{\omega_o}{2Q_L} v = \frac{e}{CT_o} \sum_i \cos(\omega_o \tau_i + \eta). \quad (4)$$

The right hand side driving terms can represent individual electrons or rigid bunches. In either case each synchrotron phase coordinate is governed by an equation identical to Equation (1) with $V(t)$ equal to the sum of all the cavity fields. Even for a few bunches a numerical computation can be rather long.

Interesting and useful results are obtained by solving the system for one rigid bunch containing all of the circulating current in the storage ring. Using normalized voltages $V_m = e v_m / U_o$ and phase and time measured in radians at the fundamental frequency the equations for the system consisting of two cavities and one rigid electron bunch are

$$V'_1 = A_1 \cos(\tau + \eta_1) - V_1 / 2Q_1, \quad (5a)$$

$$V_1 \eta'_1 = -A_1 \sin(\tau + \eta_1) + V_1 (\tan \psi_1) / 2Q_1, \quad (5b)$$

$$V'_m = A_m \cos(m\tau + \eta_m) - mV_m / 2Q_m, \quad (5c)$$

$$V_m \eta'_m = -A_m \sin(m\tau + \eta_m) + mV_m (\tan \psi_m) / 2Q_m, \quad (5d)$$

$$\begin{aligned} \tau'' = & -B[V_{1g} \cos(\tau + \eta_{1g}) + V_{mg} \cos(m\tau + \eta_{mg}) + V_1 \cos(\tau + \eta_1) + V_m \cos(m\tau + \eta_m) - 1] \\ & - (DB/\alpha) \tau', \end{aligned} \quad (5e)$$

where constants $A(1,m) = i_b e / C(1,m) U_o$, $B = \alpha U_o / E_o \omega_o T_o$, $\tan \psi_1 = 2Q_1 \Delta\omega_1 / \omega_o$, and $\tan \psi_m = 2Q_m \Delta\omega_m / m\omega_o$.

A short computer program for solving this system as an initial value problem appears in the Appendix. Comments describe the input and output data and the structure of the program. The program provides data for plotting the orbit in synchrotron phase space and it gives the time development of the amplitude and phases of the cavity fields.

4. SIMULATIONS FOR A RIGID BUNCH AND A FOURTH HARMONIC CAVITY

The impedance presented to the beam by the fundamental cavity in the NSLS VUV ring is influenced by a feedback system that stabilizes the amplitude and phase of the rf voltage at the cavity. In effect it compensates for the field excited by the beam. Although there is no analog for the feedback control loops in Equations (5) above, the effect is nearly identically produced by setting the generated voltage V_g and phase η_g and reducing Q_L to a low value. The Q of the unloaded cavity is 20,000. It is reduced to 200 in the computation. This also suppresses the Robinson effect but in any event the fundamental cavity is detuned to operate in the region of Robinson damping.

A synchrotron phase trajectory for the motion of a rigid bunch in the VUV ring without the harmonic cavity is shown in Figure 2a. The phase excursion 0.3 radians is consistent with the initial energy deviation 1.0 MeV. The incoherent motion of individual electrons with equivalent thermal energy would be similar.

Figure 2b shows a phase plot for a rigid bunch of 1.0 MeV deviation acted upon by the fundamental cavity and the fourth harmonic cavity. They are driven by generators to satisfy the voltage and phase relations given in Equations (2). The beam current for the computation was set equal to zero so that the Q of the cavities was of no significance. The bunch is only 2.4 times as long. The bunch lengthening effect varies inversely with the square root of the energy deviation.

In each case the cavities were tuned to their resonant frequencies. The low Q or the low beam current assured that the fields excited by the beam were small. The weak synchrotron radiation damping term controlled the Robinson instability.

Figures 3, 4, and 5 show trajectories when the harmonic cavity field is not controlled. The generator voltage in the harmonic cavity was set to zero and its Q_L value was set at 4000. Figure 3 is for detuning angle in the harmonic cavity equal to -60 degrees. Strong Robinson damping diminishes the amplitude of the synchrotron oscillation. The phase oscillations of the harmonic cavity voltage are also damped as they follow the phase of the bunch. The amplitude settles down to a steady state determined by the beam current and the cavity impedance.

The detuning angle for the trajectory shown in Figure 4 is 0 degrees. In other words the cavity was tuned to resonance. The Robinson effect should be absent and the residual damping is due to the synchrotron damping term in the equation of motion.

Figure 5 shows the catastrophic result of detuning the cavity to +60 degrees. Anti-damping causes the rigid bunch oscillation amplitude to increase until a truly unstable point is reached and the bunch escapes. Positive detuning near 90 degrees is necessary to lengthen the bunch. Whether or not a rigid bunch instability exists near 90 degrees depends upon the cavity Q , the beam current and the detuning angle in a complicated manner.

These computations apply to a rigid bunch and therefore indicate a low threshold for Robinson instability. In a thermalized electron bunch the initial energy spread and the quantum excitation scattering of the synchrotron phase would elevate the threshold to higher values of the beam current or cavity Q. Only the coherent damping term governed by the factor D stabilizes the Robinson effect in this computation. With some generalization of Equations (5) thermal effects and multibunch interactions could be computed.

5. LINEAR STABILITY WITH A HARMONIC CAVITY

Linear stability is now examined for the special case where the harmonic cavity is excited only by the beam and where the fundamental cavity, driven by an external generator is stabilized at a fixed amplitude and phase. Equations (5) are linearized to derive a dispersion relation. As for any stability analysis, a well defined equilibrium state must be specified and small quantities expressed as departures from their values in the equilibrium state.

Consider the gap voltage in the fundamental cavity to be $V_1 \cos(\omega_0 t + \eta_1)$ with constant amplitude V_1 and phase η_1 and a harmonic cavity gap voltage $V_m \cos(m\omega_0 t + \eta_m)$ where the amplitude and phase are variable. There follows from Equations (5) (dropping unnecessary subscripts)

$$\begin{aligned} V' &= A \cos(m\tau + \eta) - mV/2Q, \\ \eta' &= \Delta\omega/\omega_0 - (A/V) \sin(m\tau + \eta), \\ \tau'' &= -B[V_1 \cos(\tau + \eta_1) + V \cos(m\tau + \eta) - 1] - d\tau'. \end{aligned} \quad (6)$$

An equilibrium state with constant amplitude and phase in the harmonic cavity and no synchrotron motion exists with

$$V' = \eta' = \tau'' = \tau' = \tau = 0,$$

$V = V_m$ and $\eta = \eta_m$ provided that

$$V_1 \cos \eta_1 + V_m \cos \eta_m = 1,$$

$$V_m = 2AQ \cos \eta_m \quad \text{and}$$

$$V_m / (\Delta\omega / m\omega_0) = A \sin \eta_m.$$

The last two relations may be combined to give

$$\tan \eta_m = 2Q(\Delta\omega / m\omega_0)$$

showing that η_m is just the off resonance detuning angle for the harmonic cavity being driven at frequency $m\omega_0$ by the mth harmonic component of the beam current. The mth harmonic current amplitude is twice the average beam current times a shape factor which is close to 1 for $m = 4$.

Defining small amplitude variables

$V = V_m + \delta V$, $\eta = \eta_m + \delta \eta$, and with $\Delta\omega/m\omega_o \ll 1$, keeping only linear terms gives

$$(\delta V)' = -A \sin \eta_m (m\tau + \delta \eta) - m\delta V/2Q ,$$

$$(\delta \eta)' = (A/V_m) \cos \eta_m (m\tau + \delta \eta) \quad \text{and}$$

$$\tau'' = -B[\cos \eta_m (\delta V) - V_1 \sin \eta_m (\tau) - V_m \sin \eta_m (m\tau + \delta \eta)] - d\tau' .$$

Assuming that δV , $\delta \eta$, and τ vary as $\exp(s\omega_o t)$ a dispersion relation follows directly:

$$(s + m/2Q)^2 (s^2 + sd - BV_1 \sin \eta_m - BmV_m \sin \eta_m) + BmV_m \sin \eta_m / 4Q^2 = 0 ,$$

where $V_m = (2AQ \cos \eta_m)/m$.

This fourth order equation for the mode eigenvalues s may be treated numerically using the following parameters appropriate for the NSLS VUV ring and the 211 MHz harmonic cavity:

$$A = 5.66 \times 10^{-3} \text{ ib}$$

$$B = 6.6 \times 10^{-9}$$

$$d = 6.6 \times 10^{-7}$$

$$\omega_o/2\pi = 52.88 \text{ MHz}$$

$$T_o = 1.7 \times 10^{-7} \text{ sec}$$

$$U_o = 14 \times 10^3 \text{ eV}$$

$$E_o = 0.75 \times 10^9 \text{ eV}$$

$$\alpha = 0.02$$

$$C = 38 \times 10^{-12} \text{ fd}$$

$$Q_L = 4000 .$$

Comparing the synchrotron frequency

$$\Omega_s/\omega_o = (BV_1 \sin \eta_1)^{1/2} = 2 \times 10^{-4}$$

with the response time $m/Q_L = 10^{-3}$ shows that the time scales for the cavity response and for the synchrotron motion are too nearly equal. Nevertheless it is instructive to examine two trivial limits.

For $Q_L \gg \Omega_s/\omega_0$ the dispersion relation has two roots near $s = 0$ and the other two are given by

$$s^2 + sd - B(V_1 \sin \eta_1 + mAQ \sin 2\eta_m) = 0.$$

There is a threshold for instability that depends upon the detuning angle and upon the stored beam current.

When Q_L is very small the coupling to the harmonic cavity vanishes to lowest order and the dispersion relation that results simply describes single particle synchrotron oscillations in the potential due to the fundamental cavity. For $Q_L \ll \Omega_s/\omega_0$ but not negligible the phase of the harmonic field is dragged along by the synchrotron oscillations which it follows adiabatically.

The parameters for the VUV ring require the solution of the fourth order dispersion relation. Numerical computations have shown the persistence of anti-damping for a rigid bunch at a current as low as several milliamperes. Also, artificially increasing the Q of the harmonic cavity to separate time scales does lead to an instability threshold current.

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Fig. 1

Fig. 2a

Fig. 2b

Fig. 3

Fig. 4

Fig. 5

C PROGRAM SNCH SOLVES FOR THE SYNCHROTRON OSCILLATION PHASE OF A
 C BUNCH IN THE POTENTIAL OF TWO CAVITIES AT HARMONIC RELATED
 C FREQUENCIES. OPTIONS INCLUDE INFLECTION POINT AT SYNCHRONOUS
 C PHASE AND ALSO OFF-RESONANCE CAVITY DETUNING.

C INPUT DATA:
 C U0 (EV) RADIATION LOSS PER REVOLUTION
 C F0 (HZ) SYNCHRONOUS ROTATION FREQUENCY
 C NBKT NUMBER OF BUCKETS
 C M HARMONIC NUMBER OF CAVITY 2
 C V1G (EV) EXTERNAL DRIVE GAP VOLTAGE AMPLITUDE IN CAVITY 1
 C C1 (FD) CAVITY 1 CAPACITANCE
 C C2 (FD) CAVITY 2 CAPACITANCE
 C Q1 CAVITY 1 LOADED Q WITHOUT BEAM LOADING
 C Q2 CAVITY 2 LOADED Q WITHOUT BEAM LOADING
 C ALPHA MOMENTUM COMPACTION OF STORAGE RING
 C D RADIATION DAMPING FACTOR (APPROXIMATELY 2.0)
 C E0 (EV) SYNCHRONOUS ENERGY
 C EPS (EV) INITIAL ENERGY DEVIATION OF BUNCH

C PHI (DEG) PHASE ERROR OF DRIVE VOLTAGE IN CAVITY 2
 C PSI1 (DEG) RESONANT FREQUENCY DETUNING ANGLE OF CAVITY 1
 C PSI2 (DEG) RESONANT FREQUENCY DETUNING ANGLE OF CAVITY 2
 C IB (AMP) AVERAGE BEAM CURRENT
 C TIB (SEC) BEAM CURRENT RISE TIME
 C TMULT INTERVAL BETWEEN RECORDED DATA. REAL TIME
 C STARTS AT 0 SEC, ENDS AT NPLT*TMULT/(2.*PI*F0)
 C NPLT NUMBER OF INTEGRATION STEPS, PLOT POINTS (300)
 C OPTION A,B,C (SEE COMMENTS BELOW)

C NOTE: TO ELIMINATE BEAM GENERATED VOLTAGE IN EITHER CAVITY
 C LOWER THE Q. THE EXTERNALLY GENERATED VOLTAGE DOES NOT
 C CHANGE.

C CALCULATED CONSTANTS:
 C ETA1G EXTERNAL DRIVE GAP VOLTAGE PHASE IN CAVITY 1
 C V2G EXTERNAL DRIVE GAP VOLTAGE AMPLITUDE IN CAVITY 2
 C ETA2G EXTERNAL DRIVE GAP VOLTAGE PHASE IN CAVITY 2
 C A1,A2 FIELD EQUATION COEFFICIENTS
 C B SYNCHROTRON EQUATION OF MOTION COEFFICIENT

C VARIABLES:

C Y(1) NORMALIZED BEAM GENERATED GAP VOLTAGE AMPLITUDE
 C IN CAVITY 1
 C Y(2) BEAM GENERATED GAP VOLTAGE PHASE IN CAVITY 1
 C Y(3) NORMALIZED BEAM GENERATED GAP VOLTAGE AMPLITUDE
 C IN CAVITY 2
 C Y(4) BEAM GENERATED GAP VOLTAGE PHASE IN CAVITY 2
 C Y(5) SYNCHROTRON 'VELOCITY' $d(\tau)/dt$
 C Y(6) SYNCHROTRON PHASE τ

C IMPLICIT REAL*8 (A-H,O-Z)
 C REAL*8 IB,Y(6),WK(128)
 C INTEGER IWK(6)
 C CHARACTER*1 OPTION
 C EXTERNAL FCN, FCNJ
 C COMMON ALPHA,A1,A2,B,D,Q1,Q2,V1G,V2G,ETA1G,ETA2G,PSI1,PSI2,M
 C NAMELIST/SNCDAT/U0,F0,NBKT,M,V1G,C1,Q1,C2,Q2,
 1 ALPHA,D,E0,EPS,IB,PHI,PSI1,PSI2,TIB,TMULT,NPLT,OPTION

C
 C READ INPUT DATA
 C READ(10,SNCDAT)
 C PI = 3.14159
 C NORMALIZE GAP VOLTAGES TO U0

V1G = V1G/U0

```

      IF (OPTION.EQ.'A') GOTO 101
      IF (OPTION.EQ.'B') GOTO 102
      IF (OPTION.EQ.'C') GOTO 103

C
C   OPTION A: FUNDAMENTAL CAVITY ONLY.
C       ETA1G SET FOR SYNCHRONOUS PHASE AT GAP VOLTAGE V1G.
C
101  ETA1G = -DACOS(1./V1G)
      V2G = 1.E-5
      ETA2G = 0.0
      A1 = IB/(2.*PI*C1*U0*F0)
      A2 = 0.0
      Q2 = 1.E5
      B = ALPHA*U0/(2.*PI*NBKT*EO)
      GOTO 110

C
C   OPTION B: FUNDAMENTAL CAVITY AND HARMONIC CAVITY WITH EXTERNAL
C       DRIVE. ETA1G,ETA2G,V2G SET FOR INFLECTION POINT VOLTAGE
C       WAVEFORM.
C
102  ETA1G = -DACOS(M*M*(1./V1G)/(M*M-1.))

      V2G = DSQRT((V1G/M)**2 - 1./(M*M-1.))
      ETA2G = +DACOS(-1./((M*M-1.)*V2G))
      A1 = IB/(2.*PI*C1*U0*F0)
      A2 = (C1/C2)*A1
      B = ALPHA*U0/(2.*PI*NBKT*EO)
      GOTO 110

C
C   OPTION C: BEAM EXCITATION OF HARMONIC CAVITY.
C       V1G AND ETA1G SET FOR SYNCHRONOUS PHASE.
C
103  ETA1G = -DACOS(1./V1G)
      V2G = 1.E-5
      ETA2G = 0.0
      A1 = IB/(2.*PI*C1*U0*F0)
      A2 = (C1/C2)*A1
      B = ALPHA*U0/(2.*PI*NBKT*EO)
      GOTO 110

C
C   SET CONTROL PARAMETERS FOR IMSL ROUTINE DGEAR
C
110  H=1.0E-3
      INDEX=1

      METH=1
      MITER=0
      NEQN=6
      TOL=1.0E-7

C
C   CONVERT ANGLES FROM DEGREES TO RADIANs
C
      PHI = PHI*PI/180.
      PSI1 = PSI1*PI/180.
      PSI2 = PSI2*PI/180.
      ETA2G = ETA2G + PHI

C
C   SET INITIAL VALUES FOR DGEAR
C
      Y(1) = 1.0E-5
      Y(2) = 0.0
      Y(3) = 1.0E-5
      Y(4) = 0.0
      Y(5) = (-ALPHA*EPS/EO)
      Y(6) = 0.0

```

```

V1 = V1G*U0

V2 = V2G*U0
WRITE(1,1001)OPTION
WRITE(2,1001)OPTION
1001 FORMAT(A1,' V1G V2G IB PHI EPS TMULT$')
WRITE(1,1002)V1,V2,IB,PHI,EPS,TMULT
WRITE(2,1002)V1,V2,IB,PHI,EPS,TMULT
1002 FORMAT(2(F7.0),1X,F4.3,F6.1,1X,E6.1,1X,E6.1,'$')
NPRBLM=999
NVAR=6
NGRP=1
WRITE(1,1003)NPRBLM,NVAR,NGRP
NVAR=4
WRITE(2,1003)NPRBLM,NVAR,NGRP
1003 FORMAT(3I5)
WRITE(1,1004)
1004 FORMAT(1X,' T IB V1B ETA1B ',
1' V2B ETA2B')
WRITE(2,2004)
2004 FORMAT(1X,' T IB TAUDOT OMEGAOTAU')
IFLG = 1
DO 10 J = 1,NPLT
TEND = TMULT*FLOAT(J)

CALL DGEAR(NEQN,FCN,FCNJ,T,H,Y,TEND,TOL,METH,
1 MITER,INDEX,IWK,WK,IER)
WRITE(1,1005)IFLG,T,IB,Y(1),Y(2),Y(3),Y(4)
WRITE(2,1005)IFLG,T,IB,Y(5),Y(6)
1005 FORMAT(I1,6(E12.5))
10 CONTINUE
IFLG = 0
WRITE(1,1005)IFLG
WRITE(2,1005)IFLG
STOP
END

C
C
SUBROUTINE FCN(NEQN,T,Y,YPRIME)
C ENTER EQUATIONS OF MOTION FOR BUNCH AND FOR FIELDS
C
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 Y(6),YPRIME(6)
COMMON ALPHA,A1,A2,B,D,Q1,Q2,V1G,V2G,ETA1G,ETA2G,PSI1,PSI2,M
C
YPRIME(1) = A1*DCOS(Y(6)+Y(2)) - Y(1)/(2.*Q1)
YPRIME(2) = DTAN(PSI1)/(2.*Q1) - A1*DSIN(Y(6)+Y(2))/Y(1)

YPRIME(3) = A2*DCOS(M*Y(6)+Y(4)) - Y(3)*M/(2.*Q2)
YPRIME(4) = M*DTAN(PSI2)/(2.*Q2) - A2*DSIN(M*Y(6)+Y(4))/Y(3)
YPRIME(5) = -B*(Y(1)*DCOS(Y(6)+Y(2)) + Y(3)*DCOS(M*Y(6)+Y(4))
1 + V1G*DCOS(Y(6)+ETA1G) + V2G*DCOS(M*Y(6)+ETA2G) - 1.)
1 - (D*B/ALPHA)*Y(5)
YPRIME(6) = Y(5)
RETURN
END

C
SUBROUTINE FCNJ(NEQN,T,Y,PD)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 Y(NEQN),PD(NEQN,NEQN)
RETURN
END

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