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A Study of Possible Separable Solutions for Plasma Transport in Ohmically Heated Tokamaks

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TRANSPORT IN OHMICALLY HEATED TOKAMAKS

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ABSTRACT

The transport problem for an ohmically heated tokamak plasma is investigated analytically with the separation of the variables time, t , and $\rho = r^2 h(t)$, where r is the space coordinate and $h(t)$ is only a function of time. This investigation examines the circumstances under which these analytical solutions are possible. The time evolution of the current density and temperature profiles is then obtained for a simple plasma model which includes the electric field diffusion and heat conduction losses. Expressions are also given for plasma discharge parameters such as the time evolution of the safety factor, the plasma current, the poloidal flux function, the skin time, the poloidal beta, the transformer flux, and the electron energy lifetime. The results are then applied to a typical size tokamak device.

1. INTRODUCTION

The objective of this work is to investigate the plasma transport problem for an ohmically heated tokamak by invoking the separation of variables.¹ This leads to analytical solutions for the temperature and current density profiles as a function of time and $\rho = r^2 h(t)$, where $h(t)$ is a function of time only. These solutions are possible under special conditions which are discussed later. The separation of variables technique used here eventually provides a quick, easy solution to the problem without the need to solve a more sophisticated transport problem with a large computer code. Also, this method is capable of evaluating the time behavior of the plasma discharge parameters in the early stage of the heating process.

This paper is organized as follows: Sect. 2 summarizes the plasma model and the basic equations, Sect. 3 outlines the separation of variables technique and the means of obtaining the resulting equations, and Sect. 4 presents the time evolution of the discharge parameters in terms of the system parameters. General discussions and an application of the results are included in Sect. 5.

2. PHYSICS MODEL AND BASIC EQUATIONS

The transport problem for a toroidally confined plasma possesses many mathematical complexities. In order to avoid these difficulties, we assume a cylindrical plasma which is a reasonable approximation of the large aspect ratio devices. Furthermore, for the sake of simplicity, the following assumptions are adopted for the model.

- (1) The plasma density is uniform and constant.
- (2) The plasma ion density n_i is equal to the electron density n_e and $n \equiv n_e = n_i$.
- (3) Z_{eff} is assumed to be unity.
- (4) The electron and ion temperatures are equal, i.e.,
 $T_e = T_i = T$.
- (5) The charge exchange, ionization, radiation, and convection losses are neglected. The ion heat conduction losses are ignored.
- (6) The electrical conductivity σ is classical.

The basic equations are as follows. (The mks system of units is used.)

$$\frac{\partial E}{\partial r} = \frac{\partial B_p}{\partial t}, \quad (1)$$

$$\mu J = \frac{1}{r} \frac{\partial}{\partial r} (r B_p), \quad (2)$$

$$J = \sigma E, \quad (3)$$

$$\sigma = kT^{3/2}, \quad (4)$$

$$3n \frac{\partial T}{\partial t} = \frac{J^2}{\sigma} - \frac{1}{r} \frac{\partial}{\partial r} (r q_r), \quad (5)$$

where

$$q_r = -n\chi \frac{\partial T}{\partial r} \quad (6)$$

is the electron heat conduction flux, E denotes the toroidal electric field,

$$B_p(r, t) = \mu I(r, t)/2\pi r \quad (7)$$

is the induced poloidal magnetic field ($\mu = 4\pi \times 10^{-7}$ H/m), and

$$I(r, t) = 2\pi \int_0^r dr' r' J(r', t) \quad (8)$$

is the plasma current. For the thermal conductivity coefficient χ , we introduce a general form

$$\chi(r, t) = C n^\gamma r^{2s} / I_T^{p,q/2} \quad (9)$$

Here, C is a dimensional numerical factor and the values of γ , s , p , and q are determined later.

As in Ref. 1, the analytical solutions of the problem may be possible in terms of two separable variables, t and $\rho = r^2 h(t)$, where $h(t)$ is a function of t only and its explicit form will be determined. In Sect. 3, we investigate the conditions under which separable solutions are possible for T and J . The set of necessary equations for the problem then follows.

3. CALCULATIONAL MODEL AND SEPARATION OF VARIABLES

The current density may be written in the form

$$J(r, t) = J_f \left(\frac{t}{\tau} \right)^\alpha F(\rho), \quad (10)$$

where $J_f \equiv J(r = 0, t = \tau)$, $F(\rho = 0) = 1$, and the value of τ (which is a constant) will be determined. Equation (8) then yields

$$I(r, t) = \frac{\pi J_f}{h(t)} \left(\frac{t}{\tau} \right)^\alpha Z(\rho), \quad (11)$$

where

$$F(\rho) = \frac{dZ(\rho)}{d\rho}. \quad (12)$$

In turn Eq. (7) becomes

$$rB_p(r, t) = \mu \frac{J_f}{2h(t)} \left(\frac{t}{\tau} \right)^\alpha Z(\rho). \quad (13)$$

Making use of the variable transformation, combining Eqs. (1) and (2), and applying the separable solutions to the resulting equation, we get

$$4\rho \frac{d^2}{d\rho^2} \left(\frac{F}{\sigma_2} \right) + 4 \frac{d}{d\rho} \left(\frac{F}{\sigma_2} \right) = F \left(C_1 + C_2 \rho \frac{dF}{d\rho} \right), \quad (14)$$

where

$$\sigma(r, t) = \sigma_1(t)\sigma_2(\rho) = kT^{3/2}(r, t), \quad (15)$$

$$C_1 \equiv \frac{\mu \sigma_1}{h J_1} \frac{dJ_1}{dt} = \text{constant}, \quad (16)$$

$$C_2 \equiv \frac{\mu}{h^2} \frac{dh}{dL} \sigma_1 = \text{constant} , \quad (17)$$

$$J_1 \equiv J_f \left(\frac{t}{\tau} \right)^\alpha .$$

We now go back to the energy balance equation, Eq. (5). After the variable transformation, it reads

$$3n \left(\frac{dT}{d\tau} + \frac{\rho}{h} \frac{dh}{d\tau} \frac{dT}{d\rho} \right) = \frac{J^2}{\sigma} + 4nh\chi \left(\frac{dT}{d\rho} + \frac{1}{\chi} \frac{d\chi}{d\rho} \frac{dT}{d\rho} + \frac{d^2T}{d\rho^2} \right) .$$

Using Eq. (9) in the above relation and imposing the separability requirement, one finds

$$\begin{aligned} \rho \frac{dT_2}{d\rho} = \frac{F^2}{3nkT_2^{3/2}} C_3 - C_4 T_2 + \frac{4Cn^\gamma}{3(\pi J_f)^p} h_f^{(p-s+1)} \\ \times \left(\frac{\ell}{\tau} \right) T_f^{-q/2} \left(\frac{t}{\tau} \right)^\delta \frac{d}{d\rho} (\rho T_2 P) \end{aligned} \quad (18)$$

where

$$P(\rho) = \rho^s / Z P_{T_2}^{q/2} , \quad (19)$$

$$C_3 \equiv J_1^2 / T_1^{5/2} \ln h = \text{constant} , \quad (20)$$

$$C_4 \equiv \frac{\partial}{\partial t} \ln T_1 / \frac{\partial}{\partial t} \ln h = \text{constant} , \quad (21)$$

$$\delta \equiv \ell(p - s + 1) + 1 - mq/2 - p\alpha , \quad (22)$$

and

$$T(r, t) = T_1(t) T_2(\rho) .$$

We may now set

$$h(t) = h_f \left(\frac{t}{\tau} \right)^{\ell}, \quad (23)$$

and

$$T_1(t) = T_f \left(\frac{t}{\tau} \right)^m. \quad (24)$$

From Eqs. (20), (21), and (24) and $J_1 = J_f (t/\tau)^\alpha$, we find

$$C_4/C_3 = (m/\tau) (T_f^{5/2}/J_f^2) (t/\tau)^{(5m-2)/2-2\alpha},$$

and since C_4/C_3 is a constant,

$$m = 2(1 + 2\alpha)/5. \quad (25)$$

Similarly, Eqs. (16) and (17) yield

$$\ell = -2(1 - 3\alpha)/5, \quad (26)$$

$$\tau = \mu \sigma_f^{3/2} \alpha / h_f, \quad (27)$$

where $\sigma_f = k T_f^{3/2}$ and $C_1 = 1$. This in turn leads us to

$$C_4 = -(1 + 2\alpha)/(1 - 3\alpha), \quad (28)$$

and

$$C_4/C_3 = 2(1 + 2\alpha) T_f^{5/2} / 5 \tau J_f^2. \quad (29)$$

Now, let us go back to Eq. (18). Requiring this to be only a function of ρ yields $\delta = 0$, or

$$\alpha(6 - 6s + p - 2q) + 3 - 2p + 2s - q = 0. \quad (30)$$

Therefore, in order to have separable solutions in t and ρ , the relation in Eq. (30) must be satisfied. For an arbitrary value of α , this relation reduces to

$$\begin{aligned} 5q + 10s &= 15, \\ 6s + 2q - 6 &= p. \end{aligned} \quad (31)$$

Under the separability condition, Eq. (30), we get

$$\chi(r, t) = \frac{C n^\gamma}{(\pi J_f)^p} \frac{h_f^{(p-s)}}{T_f^{q/2}} \left(\frac{t}{\tau} \right)^{-3(1+2\alpha)/5} P(\rho), \quad (32)$$

where

$$\begin{aligned} P(\rho) &= \rho^s / Z^p G^{q/2}, \\ T(r, t) &= T_f \left(\frac{t}{\tau} \right)^{2(1+2\alpha)/5} G(\rho), \end{aligned} \quad (33)$$

and here $T_f \equiv T(r = 0, t = \tau)$.

Equation (17) may now be rewritten as

$$A \frac{dS}{d\rho} = DG - F^2 G^{-3/2} - D \frac{(1 - 3\alpha)}{(1 + 2\alpha)} \rho \frac{dG}{d\rho}, \quad (34)$$

where

$$D = \frac{6}{5} \frac{(1 + 2\alpha)}{\alpha} n T_f h_f / \mu J_f^2, \quad (35)$$

$$A = (4C \sigma_f T_f n^\gamma / J_f^2) \frac{h_f^{(p-s+1)}}{(\pi J_f)^p T_f^{q/2}}, \quad (36)$$

and $S(\rho)$ is defined by

$$\chi r \frac{\partial T}{\partial r} = 2C [n^\gamma h_f^{(p-s)} T_f^{(q/2-1)} / (\pi J_f)^p] (t/\tau)^{-(1+2\alpha)/5} S(\rho). \quad (37)$$

We also have

$$\frac{dG}{d\rho} = SG^{q/2} Z^p / \rho^{s+1} . \quad (38)$$

In order to satisfy the self-consistency, one should include the electric field diffusion equation, Eq. (1), which can be expressed in terms of t and ρ as

$$\frac{1}{J} \frac{\partial J}{\partial \rho} - \frac{1}{\sigma} \frac{\partial \sigma}{\partial \rho} = \frac{\sigma}{2\rho J} \left[\frac{\partial}{\partial t} (rB_p) + \frac{\rho}{h} \frac{\partial h}{\partial t} \frac{\partial}{\partial \rho} (rB_p) \right] . \quad (39)$$

Using Eqs. (10), (11), (15), (23), and (33) in Eq. (39), we find

$$\rho \frac{dF}{d\rho} = \frac{3}{2} FSG^{(q/2-1)} Z^p / \rho^{s+1} + \frac{G^{3/2}}{2} \left[\left(\frac{2-\alpha}{10\alpha} \right) Z / \rho - \left(\frac{1-3\alpha}{5\alpha} \right) F \right] . \quad (40)$$

Before leaving this section we may briefly conclude that having invoked the separation of variables, we have reduced Eqs. (1)-(5) to a new set of equations in terms of one variable, ρ ; the new set is Eqs. (12), (34), (38), and (40). This nonlinear set of equations can easily be solved by numerical techniques.²

In order to complete the scheme, the boundary and the initial conditions must be added to the set. From Eqs. (10), (11), (33), and (37) $\rho = 0$ (i.e., $r = 0$), $F(0) = 1$, $Z(0) = 0$, $G(0) = 1$, and $S(0) = 0$ since the heat flux has to vanish on the major axis. At the boundary of the torus, $r = a$, we assume that $T(r = a, t) = T_0$, which implies that

$$G(\rho_a) = \left(\frac{T_0}{T_f} \right) \left(\frac{t}{\tau} \right)^{-2(1+2\alpha)/5} , \quad (41)$$

where $\rho_a = a^2 h(t)$. At an initial time, say at time $t = t_0$, we begin the discharge and assume $T(t = t_0, r = 0) = T_0$. Thus, from Eq. (33), we get

$$t_0 = \tau \left(\frac{T_0}{T_f} \right)^{5/2(1+2\alpha)} , \quad (42)$$

and therefore the solutions are valid for $t > t_0$.

4. CALCULATION OF PLASMA PARAMETERS

The plasma parameters can now be evaluated in terms of the characteristic parameters of the system, e.g., α , h_f , and D . First, it is convenient for the calculation to define B_f and T_B as

$$B_f \equiv B_p(r = a, t = \tau) = \mu I_f / 2\pi a, \quad (43)$$

$$T_B \equiv B_f^2 / 4\mu n. \quad (44)$$

The plasma discharge parameters may then be computed.

(1) The initial poloidal beta, β_{p_0} , is

$$\beta_{p_0} \equiv \frac{2nT_o}{B_f^2/2\mu} = \frac{16nT_f a^2 h_f^2 G_f}{\mu J_f^2 Z_f^2} = \frac{T_o}{T_B} \quad (45)$$

where $Z_f \equiv Z(\rho_f)$, $G_f \equiv G(\rho_f)$, and $\rho_f \equiv \rho(r = a, t = \tau) = a^2 h_f$.

(2) The averaged poloidal beta, $\bar{\beta}_p$, is

$$\bar{\beta}_p \equiv \frac{2n \int_0^a 2\pi r dr T(r, t = \tau)}{(B_f^2/2\mu)\pi a^2}$$

or

$$\bar{\beta}_p / \beta_{p_0} = \int_0^{\rho_f} d\rho G(\rho) / \rho_f G_f. \quad (46)$$

(3) The safety factor, q_s , is

$$q_s = (r/R_o)(B_T/B_p) = (2\pi r^2/R_o)B_T/\mu I,$$

where B_T is the toroidal magnetic field and R_o is the major radius of the torus. Hence we find from Eq. (11)

$$q_s(r, t)/q_s(r = 0, t) = \frac{\rho}{Z(\rho)} \left(\frac{t}{\tau} \right)^{-\alpha}, \quad (47)$$

and therefore

$$q_b \equiv q_s(r = a)/q_s(r = 0) = \rho_f/Z_f. \quad (48)$$

(4) The poloidal flux function, ψ , is

$$\psi = R_o \int_0^r dr' B_p,$$

and because

$$\frac{\partial \psi}{\partial r} \frac{q_s}{r} \sim B_T,$$

we find from Eqs. (7) and (11)

$$\psi(r, t) = \mu R_o \frac{J_f}{4h_f} \left(\frac{t}{\tau} \right)^{(2-\alpha)/5} \int_0^\rho d\rho' Z(\rho')/\rho'. \quad (49)$$

(5) The skin time, τ_s , is

$$\frac{1}{\tau_s} \equiv \frac{1}{\psi} \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t} \ln \psi.$$

Thus we get

$$\frac{1}{\tau_s} = \frac{2-\alpha}{5t} + \rho \frac{\partial}{\partial t} \ln h \frac{d/d\rho \int_0^\rho d\rho' Z(\rho')/\rho'}{\int_0^\rho d\rho' Z(\rho')/\rho'}.$$

Substituting Eq. (23) into the above gives

$$\tau/\tau_s = \frac{\tau}{5t} \left[(2 - \alpha) - \frac{2(1 - 3\alpha)Z(\rho)}{\int_0^\rho d\rho' Z(\rho')/\rho'} \right] . \quad (50)$$

(6) The electron energy lifetime, τ_E , is

$$\tau_E(r) = \frac{\int_0^r \frac{3}{2} nT(r', t = \tau) r' dr'}{\int_0^r \frac{J^2(r', t = \tau)}{\sigma(r', t = \tau)} r' dr'} . \quad (51)$$

Using Eqs. (10) and (33) in Eq. (51), for $t = \tau$ we get

$$\tau_E(r)/\tau_{E_f} = \frac{\int_0^\rho d\rho' G(\rho')}{\int_0^\rho d\rho' F^2(\rho')/G^{3/2}(\rho')} . \quad (52)$$

Here, we have $\tau_{E_f} \equiv W_f/P_\Omega$, where $W_f \equiv \frac{3}{2} nT_f$ and $P_\Omega \equiv J_f^2/\sigma_f$.

(7) The transformer flux, ψ_{tr} , is a flux function related to the electric field at the transformer coils, $r = b$, by the relation

$$2\pi R_o E(r = b, t) = \frac{\partial}{\partial t} \psi_{tr} .$$

We compute ψ_{tr} for $t = \tau$, that is,

$$\begin{aligned} \psi_{tr}(r = b, t = \tau) &= 2\pi R_o \int_{t_o}^\tau dt \left[E(0, t) + \int_r^b \frac{dr'}{r'} \frac{\partial}{\partial t} (r' B) \right] \\ &\equiv 2\pi R_o (\psi_1 + \psi_2) . \end{aligned} \quad (53)$$

The first time integration in Eq. (53) becomes

$$\psi_1 = \frac{J_f}{\sigma_f} \frac{5\tau}{(2-\alpha)} [1 - (t_0/\tau)^{(2-\alpha)/5}] ,$$

and the second integration is

$$\psi_2 = \mu \frac{J_f}{4h_f} \left[2Z_f \ln (b/a) + \int_0^{\rho_f} d\rho \, Z(\rho)/\rho \right] .$$

Finally, making use of Eqs. (27) and (42) in ψ_1 and ψ_2 , we obtain

$$\begin{aligned} \frac{\psi_{tr}}{\mu R_o I_f} = \frac{10\alpha}{(2-\alpha)Z_f} \left[1 - G_f^{(1-\alpha/2)/(1+2\alpha)} \right] + \left[\ln (b/a) \right. \\ \left. + \frac{1}{2Z_f} \int_0^{\rho_f} d\rho \, Z(\rho)/\rho \right] . \quad (54) \end{aligned}$$

Here $I_f \equiv I(r = a, t = \tau) = \pi J_f Z_f / h_f$.

5. DISCUSSIONS AND AN APPLICATION OF THE RESULTS

Some well-known scaling laws for χ may be obtained by inspection of Eqs. (9) and (30). For various values of s , Table 1 is obtained when $\chi = \chi(s, p, q, \gamma = 0)$. As we see from the table, the pseudoclassical scaling is

$$\chi(s = 1, q = 1, p = 2) \equiv \chi^{PS} \sim \frac{1}{T^{1/2} B_p^2} \sim \nu_{ei} \rho_{pe}^2,$$

where ρ_{pe} is the Larmor radius and ν_{ei} is the collision frequency of the electrons. As r approaches zero, χ^{PS} diverges. That can be avoided by introducing

$$\chi^M \equiv \left(\frac{r}{a}\right)^2 \chi^{PS} = \chi(s = 2, q = 1, p = 2),$$

which is known as the Mercier³ scaling. In this case α turns out to be 1/3, which has also been suggested by Jensen.⁴

Table 1. The heat conductivity coefficient, $\chi = C n_Y r^{2s} / I^p T^q / 2$

α	s	q	p	χ
Arbitrary	0	3	0	$1/T^{3/2}$ (neoclassical)
Arbitrary	1	1	2	$r^2/I^2 T^{1/2}$ (pseudoclassical)
1/3	2	1	2	$r^4/I^2 T^{1/2}$ (Mercier-like)

We now discuss the coefficient C which is defined by Eq. (9). As we see, C is a dimensional quantity. We may compute this for two cases, pseudoclassical and Mercier scaling.

With pseudoclassical scaling, $s = 1$, $q = 1$, and $p = 2$. Thus, for $\gamma = 0$, we have

$$\chi^{\text{PS}} = C^{\text{PS}} \frac{r^2}{I^2 T^{1/2}} = C_o v_{ei} \rho_e^2 .$$

Using

$$\sigma = 2 \frac{ne^2}{mv_{ei}} = kT^{3/2} ,$$

we get

$$C^{\text{PS}} = C_o \left(\frac{4\pi}{\mu} \right)^2 \frac{n}{k} ,$$

where C_o is a constant. For this case, Eqs. (35) and (36) become

$$D = \left(\frac{6n}{\mu a^2} \right) \frac{T_f \rho_f}{5J_f^2} \left(\frac{1 + 2\alpha}{\alpha} \right) ,$$

$$A = C_o \left(\frac{10}{3} D \frac{\alpha}{1 + 2\alpha} \right)^2 .$$

For Mercier scaling, since

$$\chi^{\text{M}} = \left(\frac{r}{a} \right)^2 \chi^{\text{PS}} ,$$

we get $C^{\text{M}} \equiv C^{\text{PS}}/a^2$ and

$$A = C_o \left(\frac{2}{3} D \right)^2 / \rho_f .$$

The conventional values for C_o in the case of pseudoclassical scaling are⁵

$$C_o = \begin{cases} 10 , & \text{Artsimovich} \\ 10 , & \text{Oak Ridge National Laboratory} \\ 1-4 , & \text{Princeton Plasma Physics Laboratory} . \end{cases}$$

However, for Mercier's scaling, the value of C_0 depends on the aspect ratio, $\epsilon = a/R_0$, by the relation⁶

$$C_0 = 2000 , \quad \epsilon \leq 0.2$$

$$C_0 = 2000 \left(\frac{0.2}{\epsilon} \right)^2 , \quad \epsilon > 0.2 .$$

It should be noted that elementary tokamak MHD stability theory requires $q_s(r=a) \gtrsim 2.5$ at the equilibrium ($t = \tau$) and $\bar{\beta}_p \lesssim (R_0/a)$. These constraints must be satisfied in order to overcome the unstable operations due to the kink and ballooning modes.

Before applying the results of the separation of variables technique to an example, we should mention that it is convenient for the numerical calculations to work with q_b , β_{p_0} , and η , which is given by

$$\eta = (\tau - t_0)/\tau_B ,$$

where

$$\tau_B = \tau (T_B/T_f)^{3/2} \rho_f / \alpha .$$

Assuming $t_0/\tau \ll 1$, the system parameters then become

$$\rho_f = \left[\frac{\alpha}{\eta} \left(\frac{40}{3} \frac{\alpha}{1+2\alpha} D \right)^{3/2} q_b^3 \right]^{2/5} , \quad (55)$$

$$G_f = \frac{3}{40} \beta_{p_0} \left(\frac{1+2\alpha}{\alpha} \right) \frac{\rho_f}{D q_b^2} , \quad (56)$$

and

$$\beta_{p_0} / G_f = T_f / T_B . \quad (57)$$

The last expression yields

$$T_f = \frac{40}{3} \frac{\alpha}{(1 + 2\alpha)} \frac{D}{\rho_f} q_b^2 T_B.$$

To illustrate the results, a plasma sized for The Next Step (TNS) is chosen. The typical values for the device are: $R_o = 6$ m, $a = 1.25$ m, $b = 1.5$ m, $q_s(r = a, t = \tau) = 4$, $n = 6 \times 10^{19} \text{ m}^{-3}$, and $B_T = 8\text{T}$. The computation is carried out for pseudoclassical and Mercier's scaling. The system parameters used for the calculations are $\beta_{p_o} = 5 \times 10^{-3}$ and $\eta = 5 \times 10^{-3}$. Figures 1-7 present the results of the computation for $J(r, t)/J_f$, $T(r, t)/T_f$, $q_s(r, t)/q_s(r = 0, t)$, $\chi(r, t)$, $I_p(t)/I_f$ (where I_p is the plasma current), $\tau/\tau_s(t)$, and $\psi(r = a, t)$. Each figure shows the results for both pseudoclassical (a) and Mercier (b) scaling. Other plasma parameters from the computation are summarized in Table 2.

Table 2. The results of the discharge parameters for TNS-size plasma

Parameters	Scaling	
	Pseudoclassical	Mercier
α	10^{-2}	1/3
T_f (keV)	3.4	2.79
T_o (eV)	18	18
I_f (kA)	2600	2600
τ (sec)	2.38	2.38
τ_s (sec)	10.48	7.14
$\tau_E(r = a)$ (sec)	1.46	0.364
$\overline{\chi}$ (m^2/sec)	0.138	26.95
$\overline{\beta}_p$	0.226	0.122
ψ_{tr} (V-sec)	27.82	31.15
C_o	10	2500

Fig. 1. Time evolution of the radial profiles of the current density, $J(r, t)/J_f$.

Fig. 2. Time evolution of the temperature profiles, $T(r, t)/T_f$.

Fig. 3. Time evolution of the normalized safety factor, $q_s(r)/q_s(0)$.

 Fig. 4. Evolution of the heat conductivity coefficient, $\chi(r, t)$.

Fig. 5. Evolution of the plasma current, $I_P(t)/I_f$.

Fig. 6. Evolution of the normalized skin time at the edge, $\tau/\tau_s(t)$.

Fig. 7. Evolution of the poloidal flux function at the edge, $\psi(a, t)$.

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