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**LIQUID-METAL FLOW THROUGH A THIN-WALLED ELBOW IN A
PLANE PERPENDICULAR TO A UNIFORM MAGNETIC FIELD**

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ABSTRACT

This paper presents analytical solutions for the liquid-metal flow through two straight pipes connected by a smooth elbow with the same inside radius. The pipes and the elbow lie in a plane which is perpendicular to a uniform, applied magnetic field. The strength of the magnetic field is assumed to be sufficiently strong that inertial and viscous effects are negligible. This assumption is appropriate for the liquid-lithium flow in the blanket of a magnetic confinement fusion reactor, such as a tokamak. The pipes and the elbow have thin metal walls.

The flow tends toward the inside surface of the elbow, approaching a vortex about the center of curvature of the elbow. This flow migration away from the uniform fully developed flow in the pipes leads to voltage variations along the pipes. These voltage variations drive four electric current circulations in planes perpendicular to the magnetic field. These current circulations produce significant pressure variations in the cross sections of the pipes and elbow. A long length of pipe is required on both sides of the elbow for the completion of the circuits for these electric current circulations and for the decay of the disturbances to the fully developed flow in the straight pipes. All pressure drops and rises due to the three-dimensional electric current circulations cancel. The total pressure drop is the same as that for fully developed flow in a single straight pipe with the same length. While the analysis treats pipes and elbows with circular cross sections, the absence of a pressure drop in addition to that for fully developed flow is true for any smooth elbow.

1.0 INTRODUCTION

In a fusion reactor, the energy from the plasma is collected by a blanket around the plasma and is delivered to an external heat exchanger by some circulating coolant. For the fusion of deuterium and tritium, approximately eight-tenths of the energy is in the kinetic energy of high-velocity neutrons, while the rest is radiation. Tritium must be bred in the blanket to fuel the plasma. The collision of a neutron with a lithium atom produces a tritium atom. If most of the blanket is occupied by lithium with some neutron multiplier, then the reactor can breed at least replacement amounts of tritium. If liquid lithium or a lithium-lead mixture is used, it can also be the coolant and can be circulated to an external heat exchanger and tritium separator. The lead serves as a neutron multiplier. If lead is excluded because of its effects on stainless steel, then the lithium-seven isotope can serve as a multiplier; the collision of a neutron with Li_7 produces a tritium atom and another neutron. Such a self-cooled blanket is certainly the simplest possible blanket design.

In a magnetic confinement fusion reactor, such as a tokamak or mirror, the liquid metal must be pumped through the strong magnetic field which is produced by superconducting magnet coils and which confines the plasma in a vacuum. The pressure drops for liquid-metal magnetohydrodynamic (MHD) flows are generally much larger than those for flows without a magnetic field. As a result, the pressure in the blanket near the flow entrance is large. This pressure produces a large stress in the first wall which separates the blanket from the vacuum around the plasma. The thickness of the first wall is restricted by thermal stresses and by the need to minimize the temperature drop through this wall. In the Blanket Comparison and Selection Study [1], the best existing predictions for MHD pressure drops led to an optimal design with a predicted maximum first-wall stress which is 29% below the permissible maximum stress. We conclude that a self-cooled blanket is acceptable; but this is a tentative conclusion because the predicted stress does not allow a large factor of safety and because there are uncertainties in the pressure drop calculations.

In a tokamak, the primary magnetic field is in the toroidal direction, i.e., around the larger circumference of the torus. MHD pressure drops are

associated with flow across a magnetic field; the pressure gradients for flows parallel to a magnetic field are the same as those for ordinary, laminar flows. Therefore, if the blanket piping could go completely around the larger circumference of the torus, then the pressure drops would be relatively small. Unfortunately, the reactor must be composed of separate wedge-shaped pieces because of the need for access for maintenance. The coolant flow in each piece must be separate, so that the liquid-metal flows must be essentially poloidal, i.e., around the smaller circumference of the torus. Therefore, the flow is across the toroidal magnetic field, and the MHD pressure drops are large.

The flow of a liquid metal across a magnetic field produces a voltage gradient perpendicular to both the magnetic field and the fluid velocity. The pipe wall represents an electrical resistor in series with this generator. Electric currents flowing through this circuit interact with the magnetic field to produce a body force opposed to the fluid motion. This body force produces the large MHD pressure drops. If the pipes were electrical insulators, the electric currents could only circulate through very thin boundary layers with large electrical resistances, so that the currents and pressure drops would be small. However, suitable electrical insulators compatible with hot liquid lithium or with the high neutron flux have not been developed yet. Therefore, for now, the pipe walls must be metal. The pressure drops are proportional to the wall thickness, so that we make the metal walls as thin as possible.

For the simplest blanket design, a single flow path consists of two straight, parallel pipes, which are perpendicular to the axis of the torus, and each of which is connected by a 90° elbow to one end of a straight pipe, which is parallel to this axis. This U-shaped pipe is repeated in the toroidal direction to complete the inboard blanket (between the plasma and the axis of the torus) or the outboard blanket. The fully developed flows in the straight pipes are well understood [2]. The elbows produce three-dimensional disturbances in the adjacent straight pipes. An important question is whether these disturbances produce a significant pressure drop in addition to that for fully developed flow through an equivalent length of straight pipe. Our understanding of three-dimensional MHD flows in ducts and pipes is much less complete than our understanding of fully developed flows. Walker and Ludford

[3] present analytical solutions for an expansion or contraction between two straight pipes with different diameters. Holroyd and Walker [4] present analytical solutions for a straight pipe with a transverse magnetic field whose strength varies from one uniform value upstream to a different uniform value downstream. Walker [5] presents analytical solutions for a straight pipe with a transverse magnetic field whose strength varies from a uniform value to zero, i.e., the flow near the end of a magnet.

The present paper presents analytical solutions for an elbow connecting two straight pipes in a plane perpendicular to a uniform, strong magnetic field. The results indicate that there is a three-dimensional disturbance which persists for long distances in both adjacent pipes, but that there is no net pressure drop in addition to that for fully developed flow. While the present paper treats circular pipes, the conclusion about the net pressure drop applies to smooth elbows in ducts with any cross sectional shape that does not change along the duct. For fully developed flows, electric currents circulate entirely in cross sections of the pipe or duct. For any three-dimensional flow, there is an additional electric current circulation in planes perpendicular to the magnetic field. The total transverse currents in the upstream and downstream parts of this three-dimensional circuit must be equal and opposite, because of conservation of current. For an expansion, the transverse current upstream in the smaller pipe produces a pressure drop (in addition to that for fully developed flow). The return transverse current downstream in the larger pipe produces a pressure rise. The pressure gradient is equal to the electric current density times the magnetic field strength. In the larger pipe, the current density is smaller because the same total transverse current is spread over a larger area. Therefore, the pressure rise downstream is smaller than the pressure drop upstream, and there is a net pressure drop. For a contraction, there is a pressure rise in the larger upstream pipe, followed by a bigger pressure drop in the smaller downstream pipe, again giving a net pressure drop. For a straight pipe in a magnetic field whose strength varies axially between two different uniform values, the transverse current density for the three-dimensional circulation is the same upstream and downstream because the areas are the same, but now the magnetic field strengths are different. The pressure drop in the stronger field is larger than the pressure rise in the weaker field, leading to a net pressure

drop. For any smooth elbow with constant cross sectional shape, the areas and field strengths are the same for the upstream and downstream parts of any electric current circulation in planes perpendicular to the magnetic field. Therefore, the pressure drop and rise associated with each such circulation exactly cancel. There is an energy loss due to the Joule heating in these three-dimensional current circulations, but this loss is negligible. Holroyd [6] presents experimental results for an elbow in a plane perpendicular to a uniform magnetic field, and these results are compared to the present analytical predictions in the concluding section.

2.0 BASIC FORMULATION

The ratio of the induced magnetic field, which is produced by the electric currents in the liquid metal and in the pipe wall, to the applied magnetic field, $B_{\hat{y}}$, is given by cR_m . Here

$$c = \sigma_w t / \sigma a, \quad R_m = \mu \sigma U a$$

are the wall conductance ratio and the magnetic Reynolds number, respectively; B is the strength of the uniform magnetic field; $\hat{y}$ is a unit vector; σ and μ are the electrical conductivity and magnetic permeability of the liquid metal; σ_w , t and a are the electrical conductivity, wall thickness and inside radius of the pipe; U is the average axial velocity in the pipe. Both c and R_m are small for a fusion blanket, so that it is appropriate to neglect the induced magnetic field.

The ratio of the electromagnetic (EM) body force to the inertial "force" is characterized by the interaction parameter

$$N = \sigma B^2 a / \rho U$$

where ρ is the liquid metal's density. For a self-cooled blanket, $N = 10^4 - 10^5$, so that inertial effects are indeed small. It turns out that the EM body force in the present problem is comparable to c times the characteristic quantity used to define N . Therefore, inertial effects are negligible if $Nc \gg 1$. Since $c = 0.01$ is a typical value for a liquid lithium blanket with stainless steel pipes, Nc is at least 100, and we neglect inertial effects.

The Hartmann number

$$M = Ba(\sigma/\rho\nu)^{1/2}$$

is the square root of the ratio of the EM body force to the viscous force, where ν is the liquid metal's kinematic viscosity. Typically $M = 10^5$ in a fusion blanket, so that viscous effects are confined to Hartmann layers with $O(M^{-1})$ thickness adjacent to the pipe wall. We can neglect viscous effects as long as we relax the no-slip condition on the fluid velocity at the wall. The jumps in dimensionless normal electric current density and dimensionless voltage across a Hartmann layer are $O(M^{-1})$ and $O(M^{-2})$, respectively [2].

The inductionless, inertialess, inviscid, dimensionless governing equations are

$$\nabla p = \underline{j} \times \hat{\underline{y}}, \quad \nabla \cdot \underline{v} = 0 \quad (1a,b)$$

$$\underline{j} = -\nabla\phi + \underline{v} \times \hat{\underline{y}}, \quad \nabla \cdot \underline{j} = 0 \quad (1c,d)$$

Here $p, \underline{j}, \underline{v}$ and ϕ are the pressure, electric current density, fluid velocity and electric potential (voltage), normalized by $\sigma UB^2 a$, σUB , U and UBa , respectively [4]. The boundary conditions are

$$\underline{v} \cdot \hat{\underline{n}} = 0, \quad \underline{j} \cdot \hat{\underline{n}} = c\nabla_2^2\phi \quad (2a,b)$$

at the inside surface of the pipe wall. Here $\hat{\underline{n}}$ is a unit vector normal to the surface into the fluid, and $\nabla_2^2\phi$ is the two-dimensional Laplacian of the electric potential, evaluated at the wall. The thin conducting wall condition (2b) assumes that the pipe does not exchange electric currents with adjacent pipes or structure, that $Mc \gg 1$ and that $t \ll a$ [7]. Typical values for a fusion blanket are $t = 1$ mm. and $a = 4$ cm., so that this condition is appropriate.

This paper treats two identical straight pipes connected by a smooth elbow (1) with the same inside radius as the pipes, (2) with a radius of curvature for the centerline equal to αa , and (3) with a total elbow angle of $2\theta_0$. The centerlines of the pipes and the elbow lie in the $y = 0$ plane, which is perpendicular to the magnetic field. Symmetry indicates that v_y and j_y are antisymmetric about $y = 0$, while all other variables are symmetric. Without

inertial effects or induced magnetic fields, the governing equations are linear. The solution for flow in one direction is given by the solution for flow in the opposite direction, simply by changing the signs of all variables. Since the piping system is identical for flow in either direction, this implies a symmetry about the plane of the cross section in the middle of the elbow. The pressure, the electric current density perpendicular to this plane and the velocities parallel to it are antisymmetric about this plane; the electric potential, the electric current densities parallel to the plane and the velocity perpendicular to it are symmetric about this plane. Therefore, we need only treat the flow in one pipe and in half of the elbow, as shown in Fig. 1.

For the straight pipe, we use Cartesian coordinates, where x is the distance along the pipe centerline from the junction with the elbow, and the inside surface of the pipe is at $y = \pm (1 - z^2)^{1/2}$, for $-1 < z < 1$. For the elbow, we use cylindrical coordinates (r, θ, y) , where (1) r is the distance from the magnetic field line through the center of curvature of the elbow, (2) $r = \alpha$ and $y = 0$ is the centerline of the elbow, (3) $\theta = 0$ is the cross section which is a plane of symmetry, (4) $\theta = \theta_0$ is the junction with the pipe, and (5) the inside surface of the elbow is at

$$y = \pm f(r) = \pm [1 - (r - \alpha)^2]^{1/2}, \text{ for } -1 < (r - \alpha) < 1$$

Our objective is an asymptotic solution of the boundary value problem (1,2) for this geometry and for $c \ll 1$. This solution consists of matched asymptotic expansions in certain subregions of the flow. One of these subregions is an interior layer with $O(c^{1/6})$ thickness at the surface at $r = \alpha$ in the elbow and at $z = 0$ in the straight pipe near the elbow. This layer is new and is unusual because all previously treated interior layers lie along magnetic field lines through discontinuities in the geometry or electrical conductivity of the walls. This new layer lies along magnetic field lines, but the pipe is perfectly smooth at $y = \pm 1$ and $r = \alpha$ or $z = 0$. To illustrate the mathematical and physical origins of this new interior layer and of the other subregions, we begin by deriving the governing problems for an arbitrary value of c , and then we consider the asymptotic behavior $c \rightarrow 0$.

FIGURE 1. Section of one pipe and half of the elbow in the $y=0$ plane, showing coordinates.

In the straight pipe, the solutions of the equations (1), with the correct symmetry about $y = 0$, are

$$j_x = \frac{\partial p}{\partial z}, \quad j_y = 0, \quad j_z = -\frac{\partial p}{\partial x} \quad (3a-c)$$

$$v_x = \frac{\partial \phi}{\partial z} - \frac{\partial p}{\partial x}, \quad v_z = -\frac{\partial \phi}{\partial x} - \frac{\partial p}{\partial z} \quad (3d,e)$$

$$v_y = y \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \right) \quad (3f)$$

The boundary conditions (2) give elliptic equations governing the two unknown integration functions $p(x,z)$ and $\phi(x,z)$:

$$(1 - z^2) \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \right) = z \frac{\partial p}{\partial z} + z \frac{\partial \phi}{\partial x} \quad (4a)$$

$$z \frac{\partial p}{\partial x} = c \left[\frac{\partial^2 \phi}{\partial x^2} + (1 - z^2) \frac{\partial^2 \phi}{\partial z^2} - z \frac{\partial \phi}{\partial z} \right] \quad (4b)$$

for $0 < x < \infty$, $-1 < z < 1$. The flow must approach fully developed flow in a straight, thin-walled pipe far from the elbow [8], so that

$$\phi \rightarrow z(1 + c)^{-1}, \quad p \rightarrow P - cx(1 + c)^{-1}, \quad \text{as } x \rightarrow \infty \quad (5a,b)$$

where P is an unknown constant pressure. The regularity conditions

$$(1 - z^2)^{1/2} \frac{\partial p}{\partial z} \rightarrow 0, \quad (1 - z^2)^{1/2} \frac{\partial \phi}{\partial z} \rightarrow 0, \quad \text{as } z \rightarrow \pm 1 \quad (6)$$

insure that there are no sources or sinks of fluid or electric current at $y = 0$, $z = \pm 1$.

In the elbow, the solutions of the equations (1), with the correct symmetry about $y = 0$, are

$$j_r = -\frac{1}{r} \frac{\partial p}{\partial \theta}, \quad j_\theta = \frac{\partial p}{\partial r}, \quad j_y = 0 \quad (7a-c)$$

$$v_r = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} - \frac{\partial p}{\partial r}, \quad v_\theta = \frac{\partial \phi}{\partial r} - \frac{1}{r} \frac{\partial p}{\partial \theta} \quad (7d,e)$$

$$v_y = y \left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} \right) \quad (7f)$$

The boundary conditions (2) give elliptic equations governing the two unknown integration functions $p(r, \theta)$ and $\phi(r, \theta)$:

$$f^2 \left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} \right) = (r - \alpha) \left(\frac{1}{r} \frac{\partial \phi}{\partial \theta} + \frac{\partial p}{\partial r} \right) \quad (3a)$$

$$\frac{(r - \alpha)}{r} \frac{\partial p}{\partial \theta} = c \left[f^2 \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} (2f^2 + \alpha^2 - \alpha r - 1) \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} \right] \quad (3b)$$

for $-1 < (r - \alpha) < 1$, $0 < \theta < \theta_0$. For the condition (2b), we use toroidal coordinates in order to get the two-dimensional Laplacian of ϕ on the inside surface of the elbow, and then we change to the cylindrical coordinates to get the equation (8b). As $\alpha \rightarrow \infty$, $r - \alpha$ becomes z , $\alpha\theta$ becomes x , and the equations (8) become the equations (4). The regularity conditions

$$f \frac{\partial \phi}{\partial r} \rightarrow 0, \quad f \frac{\partial p}{\partial r} \rightarrow 0, \quad \text{as } r \rightarrow \alpha \pm 1 \quad (9a, b)$$

again exclude sources and sinks of fluid and current at $y = 0$, $r = \alpha \pm 1$. Symmetry about the middle cross section of the elbow gives the boundary conditions

$$p = 0, \quad \frac{\partial \phi}{\partial \theta} = 0, \quad \text{at } \theta = 0 \quad (10a, b)$$

The boundary value problems in the straight pipe and elbow are linked by the conditions that p , ϕ and their axial derivatives are continuous at the junction of the pipe and elbow.

If θ_0 is large, then the solution near $\theta = 0$ approaches the solution for fully developed flow in an infinitely long helical pipe with a radius of curvature $= \alpha a$ and in which the effects of the pitch of the helix are negligible. This fully developed flow solution is given by the solution of the equations (8) with the regularity conditions (9), with $\partial \phi / \partial \theta = 0$, and with the average $v_\theta = 1$:

$$\phi = \phi + A(1 + c)^{-1} \ln r, \quad p = -cA(1 + c)^{-1}\theta \quad (11a,b)$$

$$v_\theta = A/r, \quad A = \frac{1}{2} [\alpha - (\alpha^2 - 1)^{1/2}]^{-1} \quad (11c,d)$$

where ϕ is an unknown constant voltage. It turns out that $\alpha\theta_0$ must be at least $O(c^{-1/2})$ before the flow approaches the fully developed solution (11) near $\theta = 0$. For $c = 0.01$ and $\alpha = 2$, at least two complete turns of the helix are needed before the flow approaches fully developed helical flow near the middle. The flow does not approach fully developed helical flow in a 90° elbow ($\theta_0 = 0.785$ radians). Nevertheless, the solution (11) indicates the tendency of the flow in an elbow.

As the flow in the upstream straight pipe approaches the elbow, it leaves the uniform velocity for fully developed flow in a straight pipe, and it migrates in the direction it will eventually flow in the other pipe. The flow tends toward a vortex about the center of curvature in the elbow, and then migrates back to uniform flow far downstream in the second straight pipe. It turns out that the electric current density is $O(c)$ because the current circulates through the thin pipe wall. Neglecting the $O(c)$ term in the equation (3d), $\partial\phi/\partial z = v_x$. At $x = \infty$, $v_x = 1$ and $\phi = z$. At $x = 0$, $\partial\phi/\partial z = v_x < 1$ for $0 < z < 1$ and $\partial\phi/\partial z = v_x > 1$ for $-1 < z < 0$, because of the shift toward a vortex. If the voltage at $z = 0$ was still zero at $x = 0$, then the voltages at $x = 0$ would be less than those at $x = \infty$ for all values of z . These axial voltage differences would drive electric currents from $x = \infty$ to $x = 0$ for all z . However, conservation of current indicates that there is no net axial current at any cross section of the pipe, so that the voltage level at $x = 0$ rises until there is no net axial current. The sketches in Fig. 2 indicate that the voltages near $z = 0$ are greater at $x = 0$ than at $x = \infty$, so electric current flows in the $+x$ direction near the $z = 0$ plane. On the other hand, the voltages near $z = \pm 1$ are greater at $x = \infty$ than at $x = 0$, so electric current flows in the $-x$ direction in the regions near $z = \pm 1$. No current crosses the $\theta = 0$ cross section, so that the electrical circuits are completed in the elbow. Figure 3 presents a sketch of the electric current lines for the current circulations in planes perpendicular to the magnetic field. These currents are superimposed on the transverse current for fully developed flow. The three-dimensional current circulations sketched in Fig. 3 produce a body force directed away from or toward the center of each

FIGURE 2. Sketch of the voltage distribution for fully developed flow ($x = \infty$) and at the junction of the straight pipe and elbow ($x=0$).

FIGURE 3. Sketch of electric current lines in the $y=0$ plane for the three-dimensional disturbance.

circulation. This body force produces a negative or positive disturbance pressure at the center of the current circulation. At $x = 0$, the pressure at the current center for $z > 0$ is positive because the body force is inward, while the pressure at the current center for $z < 0$ is negative because the force is outward. The three-dimensional disturbance involves transverse pressure differences.

3.0 ELBOW AND ADJACENT PART OF PIPE FOR $c \ll 1$

For $c \ll 1$ and $r \neq \alpha$, the equations (7a,b,8b) and the boundary condition (10a) indicate that the pressure and electric current density in the elbow are $O(c)$. The elbow creates an $O(c)$ current circulation in $y = \text{constant}$ planes which involves a non-zero $O(c)$ j_x at the junction with the pipe. The circuits for this axial current at the junction must be completed in a finite length of pipe because $j_x = 0$ for fully developed flow. Walker and Ludford [3] treat a semi-infinite pipe with a uniform transverse magnetic field and with an $O(c^{1/2})$ axial electric current density prescribed at one end. The completion of the circuit for this current and the evolution of the disturbed flow to fully developed flow require a long, $O(c^{-1/2})$ length of pipe. The deviations from the fully developed ϕ and p are $O(1)$ and $O(c^{-1/2})$ respectively. Here the j_x at $x = 0$ is $O(c)$ instead of $O(c^{1/2})$. The completion of the electrical circuits and the evolution to fully developed flow again require an $O(c^{-1/2})$ length of pipe, but here the deviations from the fully developed ϕ and p are $O(c^{1/2})$ and $O(c)$, respectively. Throughout the pipe

$$\phi = z + c^{1/2} \phi_p(x, z) + O(c) \quad (12)$$

For $c \ll 1$ and $r \neq \alpha$, the solutions in the elbow which satisfy the equations (8) and the conditions (9,10) and match the ϕ in the pipe (12) are

$$\phi = (r - \alpha) + c^{1/2} \phi_p(0, r - \alpha) + O(c) \quad (13a)$$

$$p = c\theta[(r - \alpha)^{-1} - 2(r - \alpha) - \alpha] + c^{3/2}\theta(r - \alpha)^{-1}f \frac{\partial \phi_p}{\partial r} [rf \frac{\partial \phi_p}{\partial z}(0, r - \alpha)] + O(c^2) \quad (13b)$$

for $0 < |r - \alpha| < 1$, $0 < \theta < \theta_0$. The pressure for fully developed flow in an equivalent length of straight pipe is $p = -c\theta\alpha$. The other two terms in the $O(c)$ pressure are antisymmetric about $r = \alpha$, corresponding to the electric current circulations in opposite directions on either side of the $r = \alpha$ surface in the elbow, as sketched in Fig. 3. The $O(c)$ $p \rightarrow \pm \infty$, as $r \rightarrow \alpha$ from above or below, respectively. The boundary value problem (8-10), with boundary conditions at $\theta = \theta_0$, is a well-posed, fourth-order, elliptic problem, and its solutions are bounded everywhere. As $c \rightarrow 0$, the equations (8) cease to be elliptic and simply give the values of $\partial\phi/\partial\theta$ and $\partial p/\partial\theta$, for $r \neq \alpha$. Near $r = \alpha$, the balances between the left and right sides of the equations (8) change, and the terms with $\partial^2/\partial r^2$ become comparable to those with $(r - \alpha) \partial/\partial\theta$.

To treat the interior layer at $r = \alpha$ which matches the singularity in the pressure (13b), we introduce

$$r = \alpha + c^{1/6}\zeta, \quad p = c^{5/6} p_i(\zeta, \theta) \quad (14a,b)$$

$$\phi = c^{1/6}\zeta + c^{1/3} \phi_i(\zeta, \theta) \quad (14c)$$

The equations (8) become

$$\frac{\partial^2 p_i}{\partial \zeta^2} - \frac{\zeta}{\alpha} \frac{\partial \phi_i}{\partial \theta}, \quad \frac{\zeta}{\alpha} \frac{\partial p_i}{\partial \theta} = \frac{\partial^2 \phi_i}{\partial \zeta^2} + \frac{1}{\alpha} \quad (15a,b)$$

for $-\infty < \zeta < \infty$, $0 < \theta < \theta_0$. The boundary conditions (10) apply for ϕ_i and p_i . Matching the solutions (13) gives the boundary conditions

$$\phi_i \rightarrow 0, \quad p_i \rightarrow \theta\zeta^{-1}, \quad \text{as } \zeta \rightarrow \pm \infty \quad (16a,b)$$

The boundary value problem (10,15,16) has a certain symmetry about $\zeta = 0$, so that we need only solve the problem for $0 < \zeta < \infty$ with the boundary conditions

$$p_i = 0, \quad \frac{\partial \phi_i}{\partial \zeta} = 0, \quad \text{at } \zeta = 0 \quad (17a,b)$$

The solutions for p_i and ϕ_i are presented later in this section.

The equation (8b) comes from the boundary condition (2b), which gives the normal component of the $O(c)$ electric current into the fluid, once the voltages in the fluid and therefore in the wall are known. As $c \rightarrow 0$, the equation (8a) indicates that the $O(1)$ ϕ must be independent of θ , so it has the same linear variation as that in the pipe, namely $\phi = z = r - \alpha$. In the pipe, this voltage distribution gives a positive normal current for $-1 < z < 0$, a negative one for $0 < z < 1$, and a zero one at $z = 0$, $y = \pm 1$. However, in the elbow, the same linear voltage distribution gives a positive normal current out of the wall for

$$-1 < r - \alpha < \frac{1}{4} [(\alpha^2 + 8)^{1/2} - \alpha]$$

The right side of this expression equals 0.5 for $\alpha = 1$ and decreases monotonically to zero as α increases to ∞ . At $r = \alpha$, $y = \pm 1$, the normal current is $\mp j_y$. The linear voltage distribution gives a normal current equal to $c\alpha^{-1}$ at these points, but the equation (7c) shows that $j_y = 0$ everywhere. This contradiction leads to the singularity in the solution (13b) at $r = \alpha$. The voltage must deviate from the linear distribution in some neighborhood of $r = \alpha$, so that the normal current is zero at $r = \alpha$, $y = \pm 1$. Inside the interior layer the voltage adjusts by an $O(c^{1/3})$ amount. The same total current leaves the wall in the neighborhood of $r = \alpha$, but this current is shifted symmetrically away from the points $r = \alpha$, $y = \pm 1$ and to the nearby parts of the wall where there is a non-zero component of j_r normal to the wall to accept this current. This interior layer does not occur in the three-dimensional problems previously treated [3-5]. These problems were all symmetric about the $z = 0$ plane. With ϕ as an odd function of z , the normal current out of the wall was automatically zero at the points where it equalled $\mp j_y$.

The equations (7b,13b) give

$$j_\theta = -c\theta[2 + (r - \alpha)^{-2}] + O(c^{3/2})$$

for $r \neq \alpha$. Therefore, j_θ is negative everywhere in this half of the elbow, except inside the interior layer. The $O(c)$ pressure (13b) increases monotonically as r decreases from $\alpha + 1$ toward α . As $r \rightarrow \alpha$ from above, we enter the interior layer, where p_i matches this algebraically growing pressure

and $p_i = 0$ at $r = \alpha$. For a composite solution and for a specific value of c , the graph of pressure versus r for $\alpha < r < \alpha + 1$ would look like the curve for $X = 0.02$ in Fig. 4. The pressure reaches some maximum value inside the interior layer. The electric current is in the $- \theta$ direction between this maximum pressure point and $r = \alpha + 1$, while all the return current in the $+ \theta$ direction is concentrated between $r = \alpha$ and the maximum pressure point. The current in the $+ \theta$ direction in this half of the elbow in Fig. 3 is inside the interior layer at $r = \alpha$. However, this sheet of electric current at $r = \alpha$ may be more an artifact of the asymptotic analysis for $c \ll 1$ than a physical reality. For $c = 0.01$, $c^{1/6} = 0.46$, so that we can expect the "thin" interior layer to actually occupy about half of the elbow's cross section. Experimentally, one would probably observe a rather smooth pressure variation, again comparable to the $X = 0.02$ curve in Fig. 4.

In the straight pipe for $c \ll 1$, $x = O(1)$ and $z \neq 0$, ϕ and p must satisfy the reduced form of the equations (4) and the regularity conditions (6), must match the solutions (13) in the elbow and must match the form of the voltage (12) in the pipe far from the elbow, namely

$$\phi = z + c^{1/2} \phi_f(X, z) + O(c), \quad X = c^{-1/2}x \quad (18a, b)$$

where X is a compressed axial coordinate. The solutions are

$$\phi = z + c^{1/2} \phi_f(0, z) + O(c) \quad (19a)$$

$$p = c\theta_0(z^{-1} - 2z - \alpha) - cx + c^{3/2}z^{-1}(1 - z^2)^{1/2*}$$

$$+ \frac{\partial}{\partial z} \{ (1 - z^2)^{1/2} [(z + \alpha)\theta_0 + x] \frac{\partial \phi_f}{\partial z}(0, z) \} + O(c^2) \quad (19b)$$

for $x = O(1)$ and $0 < |z| < 1$. The $\phi_p(0, r - \alpha)$ in the solutions (13) is now replaced by $\phi_f(0, r - \alpha)$. The pressure for fully developed flow in a straight pipe with the same length is $p = -c(\alpha\theta_0 + x)$. In the elbow, electric currents which cannot complete their circuits in cross sections accumulate into axial currents. The magnitudes of these axial currents are proportional to θ . In the part of the straight pipe adjacent to the elbow, no additional axial currents accumulate because the normal currents from and to the pipe wall can now complete their circuits in the cross sections. The axial

FIGURE 4. Disturbance pressure in the straight pipe.

currents which accumulate in the elbow pass unchanged through this $O(1)$ length of straight pipe.

The equations (3d,7e,13,19) indicate that the $O(c^{1/2})$ voltages and axial velocities are independent of θ and x , in the elbow and in the adjacent $O(1)$ length of pipe. The migration of the flow from the uniform fully developed flow toward a vortex in the elbow, which was described in the previous section, consists of an $O(c^{1/2})$ perturbation velocity superimposed on the $O(1)$ $v_x = v_\theta = 1$ everywhere. All of the flow migration occurs in $O(c^{-1/2})$ lengths of the straight pipes. The distorted velocity profile passes unchanged through the elbow and adjacent $O(1)$ lengths of the pipes.

There is a singularity in the pressure (19b) at $z = 0$. The strength of this singularity is now constant, while in the elbow its strength is proportional to θ . The interior layer persists into the $O(1)$ length of pipe. We introduce $z = c^{1/2}\zeta$ and the expressions (14b,c) with θ replaced by x . The equations (4) become

$$\frac{\partial^2 p_i}{\partial \zeta^2} = \zeta \frac{\partial \phi_i}{\partial x}, \quad \zeta \frac{\partial p_i}{\partial x} = \frac{\partial^2 \phi_i}{\partial \zeta^2} \quad (20a,b)$$

for $-\infty < \zeta < \infty$, $0 < x < \infty$. The boundary conditions (16,17), with θ replaced by θ_0 , again apply. At the junction of the elbow and the straight pipe, p_i and ϕ_i are continuous. If we define the coordinate λ along the centerline from the plane of symmetry $\theta = 0$, then $\lambda = \alpha\theta$, for $0 < \theta < \theta_0$, and $\lambda = x + \alpha\theta_0$, for $0 < x < \infty$. The equations (15,20) combine to give

$$\frac{\partial^2 p_i}{\partial \zeta^2} = \zeta \frac{\partial \phi_i}{\partial \lambda} \quad (21a)$$

$$\zeta \frac{\partial p_i}{\partial \lambda} = \frac{\partial^2 \phi_i}{\partial \zeta^2} + \frac{1}{\alpha} H(\alpha\theta_0 - \lambda) \quad (21b)$$

for $0 < \lambda < \infty$, where H is the Heaviside function. The boundary conditions (10,16,17), with θ replaced by λ/α , apply.

To solve this problem, we introduce Fourier cosine transforms with respect to both λ and ζ for ϕ_i , and corresponding Fourier sine transforms for

p_i . These transforms incorporate the boundary conditions and reduce the equations (21) to a pair of first-order ordinary differential equations with respect to ξ , which is the Fourier transform variable for ζ . The double transforms of both ϕ_i and p_i are equal to

$$\frac{1}{2} \pi \alpha^{-1} \tau^{-2} \sin(\alpha \theta_0 \tau) \exp(-\xi^3/3\tau)$$

where τ is the Fourier transform variable for λ . The Fourier inversions involve integrals over the quarter plane $0 < \tau, \xi < \infty$. If we introduce polar coordinates for this quarter plane, the radial integral can be evaluated analytically. The results are

$$\phi_i = S(\lambda, \zeta) + S(\lambda, -\zeta), \quad p_i = S(\lambda, \zeta) - S(\lambda, -\zeta)$$

$$S = \frac{1}{4\alpha} \int_0^{\pi/2} [\operatorname{erf}(q_+) + \operatorname{erf}(q_-)] \sec^2 \Omega \, d\Omega$$

$$q_{\pm} = \frac{1}{2} (3 \cot \Omega)^{1/2} [(\alpha \theta_0 \pm \lambda) \cot \Omega \mp \zeta]$$

While the remaining integral cannot be evaluated analytically, its numerical evaluation by Gauss quadrature for each set of values for λ and ζ is straightforward. The integrand has a square-root singularity at $\Omega = \pi/2$, so that an appropriately modified Gauss or Gauss-Chebyshev quadrature is preferable.

The interior layer in the elbow accumulates an electric current in the $+\theta$ direction. In the $O(c^{-1/2})$ length of pipe, the solution sees a source of electric current at $X = z = 0$. The interior layer in the $O(1)$ length of pipe (1) accepts the accumulated axial current from the interior layer in the elbow, (2) carries this current through the $O(1)$ length of pipe with no exchange of current with the adjacent core regions, and (3) matches the current-source singularity in the solution for the $O(c^{-1/2})$ length of pipe. At every cross section, the net axial current, given by the integral of $2(1 - z^2)^{1/2} j_x$ or $2fj_\theta$ over $-1 < z < 1$ or $-1 < (r - \alpha) < 1$, is zero.

4.0 FLOW MIGRATION OVER $O(c^{-1/2})$ PIPE LENGTH

For $c = O(1)$, the solutions of the boundary value problem (4,6,8-10) approach the fully developed flow solutions (5) in an $O(1)$ distance from the pipe-elbow junction. However, as $c \rightarrow 0$, the length of pipe required to reach fully developed flow increases to $O(c^{-1/2})$. We introduce the expressions (18) and

$$p = -c(\alpha\theta_0 + x) + cp_f(X, z) + O(c^{3/2})$$

The equations (4) become

$$(1 - z^2) \frac{\partial^2 p_f}{\partial z^2} = z \frac{\partial p_f}{\partial z} + z \frac{\partial \phi_f}{\partial X} \quad (22a)$$

$$z \frac{\partial p_f}{\partial X} = (1 - z^2) \frac{\partial^2 \phi_f}{\partial z^2} - z \frac{\partial \phi_f}{\partial z} \quad (22b)$$

and the regularity conditions (6) apply for ϕ_f and p_f . Matching the solution (19b) gives the boundary condition

$$p_f = \theta_0(z^{-1} - 2z), \quad \text{at } X = 0, \quad \text{for } z \neq 0 \quad (23)$$

Matching the fully developed flow (5) gives

$$p_f \rightarrow -P_0, \quad \phi_f \rightarrow 0, \quad \text{as } X \rightarrow \infty$$

where $2P_0$ is the pressure drop due to the elbow, in addition to the pressure drop for fully developed flow in a straight pipe with the same length along the centerline.

We introduce the separation of variable solutions

$$p_f = -P_0 + \theta_0 \sum_{n=1}^{\infty} a_n p_n(\beta) \exp(-\gamma_n X) \quad (24a)$$

$$\phi_f = \theta_0 \sum_{n=1}^{\infty} a_n \phi_n(\beta) \exp(-\gamma_n X) \quad (24b)$$

where $z = \sin\beta$. The equations (22) reduce to the ordinary differential equations

$$\frac{d^2 p_n}{d\beta^2} + \gamma_n \sin\beta \phi_n = \frac{d^2 \phi_n}{d\beta^2} + \gamma_n \sin\beta p_n = 0 \quad (25)$$

for $-\pi/2 < \beta < \pi/2$. The regularity conditions (6) become

$$\frac{dp_n}{d\beta} = \frac{d\phi_n}{d\beta} = 0, \quad \text{at} \quad \beta = \pm \frac{\pi}{2} \quad (26)$$

Holroyd and Walker [4] present the first thirty positive eigenvalues γ_n for the eigenvalue problem (25,26). The corresponding eigenfunctions p_n and ϕ_n can be expressed in terms of Mathieu functions, but it is more practical to generate them by numerically integrating the equations (25) with the various values of γ_n and with the boundary conditions (26). The only difference between the present eigenfunctions and those presented by Holroyd and Walker [4] is that here $\phi_n(\beta)$ and $p_n(\beta)$ are the even and odd parts of $S_n = \phi_n + p_n$, respectively. The coefficients a_n and P_0 are determined by the boundary condition (23). This condition gives $P_0 = 0$, so that there is no net pressure drop in addition to that for fully developed flow in a straight pipe. The coefficients a_n are the same for all smooth circular elbows, i.e., they are independent of the parameters α and θ_0 . The equation (24b) now gives the values of $\phi_f(0,z)$; the equations (3d,7e,13a,19a) give the $O(c^{1/2})$ voltages and axial velocities in the elbow and in the $O(1)$ length of pipe; the equations (13b,19b) give the $O(c^{3/2})$ pressures in the elbow and adjacent $O(1)$ pipe length. The latter represent $O(c^{3/2})$ current circulations which also require an $O(c^{-1/2})$ pipe length to complete the circuits.

Figure 4 presents graphs of p_f/θ_0 for several values of X . The disturbance pressure p_f is an odd function of z , so only values for $0 < z < 1$ are given. At $X = 0.02$, there is an electric current in the $+x$ direction for $|z| < 0.385$, and an equal electric current in the $-x$ direction for $0.385 < |z| < 1$. As X increases, the circuits for these axial currents are progressively closed, and the magnitudes of the disturbance pressure decrease. Fully developed flow is reached by $x = 0.4c^{-1/2}$. Figure 5 presents graphs of v_{xf}/θ_0 for several values of X , where

FIGURE 5. Disturbance velocity in the straight pipe.

$$v_x = 1 + c^{1/2} v_{xf}(X, z)$$

is the axial velocity in the $O(c^{1/2})$ length of pipe. Again v_{xf} is an odd function of z . The negative values for $0 < z < 1$ in Fig. 5 represent the decrease in v_x for $z > 0$, as the flow migrates toward the vortex in the elbow. There is an equal increase in v_x for $-1 < z < 0$. The peak velocity at $X = 0.02$ is $v_{xf} = -8.66 \theta_0$. This indicates that the maximum deviation from $v_x = 1$ is ± 0.68 , for $c = 0.01$ and a 90° elbow. While the flow does not actually reach the vortex motion (11) in a 90° elbow, the $O(c^{1/2})$ perturbation velocity is not really small for practical values of c .

Holroyd [6] presents measurements of the voltage differences between points at $z = 1$, $y = 0$ and reference points, which are apparently at $z = 0$, $y = \pm 1$. As the flow migrates across the $z = 0$ plane, the voltage difference between $z = 0$ and $z = 1$ decreases. Holroyd measures a maximum decrease of 15% for $c = 0.118(c^{1/2} = 0.344)$. This result and the lengths of the pipes over which the flow is disturbed are compatible with the present results. Specific comparisons are not possible for two reasons. First, the elbow in Holroyd's experiment is not smooth. A straight pipe is cut at a 45° angle and rejoined to form a sharp 90° elbow. With such an elbow, all the electric current circulations in the present smooth elbow are compressed into an interior layer at the junction of the two straight pipes. Either inertial or viscous effects are significant inside this layer, depending on the relationship between N and M . Since inertial or viscous forces are significant for part of the electrical circuit, we can not longer conclude that the pressure drops and rises exactly cancel, as they do for a smooth elbow. Holroyd measures a net pressure drop and recognizes its possible relationship to the particular construction of his elbow. The second reason that comparisons are not possible is that Holroyd indicates that the flow entering the magnetic field may not reach fully developed flow before the disturbance due to the elbow begins. Therefore, the elbow and non-uniform magnetic field disturbances are coupled. Nevertheless, the magnitude of the decrease of the voltage difference between $z = 0$ and $z = 1$, and the fact that the flow is disturbed for approximately 3.5 diameters on either side of the elbow are compatible with the present analytical results.

5.0 CONCLUSIONS

The present results lead to two important conclusions for tokamak blanket designs. First, a smooth elbow between a pair of pipes or ducts in the poloidal direction involves no net pressure drop in addition to that for fully developed flow through an equal length of straight pipe or duct.

Second, the velocity distribution tends to a vortex about the center of curvature of the elbow. If the plasma is on the same side of the elbow as the center of curvature, this tendency helps the heat transfer because it increases the flow velocity adjacent to the first wall. This benefit is realized throughout the elbow and for at least a one-diameter length of both straight pipes for $c = 0.01$. Actually the return to fully developed flow requires a two-diameter length of pipe for this c , but the flow concentration near the first wall is only significant for half of this length.

On the other hand, if the plasma is on the opposite side of the elbow from the center of curvature, then the tendency to a vortex hurts the heat transfer because it decreases the flow velocity near the first wall. This adverse effect can be prevented by adding guide vanes which are parallel to both the toroidal magnetic field and the centerline of the pipe and elbow. For $c = 0.01$, the guide vanes should extend at least two diameters on either side of the elbow. One guide vane, which is relatively close to the first wall, might be sufficient. Forcing a certain fraction of the flow to remain in its original part of the cross section throughout the elbow makes the voltage deviations different from those in the present study. These differences change the electric current circulations and the disturbance pressures. However, the three-dimensional pressure drops and rises still exactly cancel for the same reason, so that there is no additional pressure drop associated with these guide vanes. The disturbances in the neighborhood of the upstream and downstream ends of the guide vanes are negligible as long as the guide vanes are metal plates. The change in the velocity is simply that associated with the reduction of the flow area by the addition of the vanes. However, if the vanes have a laminated construction with an electrical insulator between two metal plates, then high-velocity sheet jets occur on both sides of each guide vane. These jets significantly reduce the velocity elsewhere. Since there is no benefit to high velocities near the guide vanes, simple metal plates are better than laminated vanes.

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