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**Surface Density Representation
of Critical Configurations of
 $U(5)O_2$ Planar and Linear Arrays**

P. J. Maudlin
J. T. Thomas

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**SURFACE DENSITY REPRESENTATION OF CRITICAL CONFIGURATIONS
OF $U(5)O_2$ PLANAR AND LINEAR ARRAYS**

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SURFACE DENSITY REPRESENTATION OF CRITICAL CONFIGURATIONS OF U(5)O PLANAR AND LINEAR ARRAYS

P. J. Maudlin and J. T. Thomas

ABSTRACT

A criticality data base for planar and linear arrays of low enriched damp uranium dioxide powder contained in cylindrical units is generated with the Monte Carlo criticality program KENO-IV. This information is correlated with the Limiting Surface Density model, $\sigma(m) = c_2(m_0 - m)$, refining the bulky KENO data base into a simple table of correlation constants c_2 and m_0 .

A solid angle representation of the KENO data base is provided. The solid angles were analytically determined by the SNAKE code and these are compared to an approximate formulation of the total solid angle associated with a critical array.

INTRODUCTION

The ability to accurately predict criticality safety information with simple and practical analytical tools is still an area of some concern. Various models have been introduced and used over the years (e.g., see the solid angle approach in Ref. 1) with some degree of success. In particular, the semi-empirical limiting surface density model introduced and developed by Thomas²⁻⁵ is observed to be not only successful in its correlation of criticality data, but also has a wide range of applicability whose limitations are still yet-to-be completely determined. This report extends the range of applicability by investigating planar and linear arrays of low enriched uranium dioxide powder contained in cylindrical units.

A criticality data base for the planar and linear arrays is initially generated with the computer program KENO.⁶ This information is then correlated to obtain the coefficients needed in the limiting surface density model, which presents a refined characterization of the same data base. Also included in this report is an alternate characterization of the KENO data base using the solid angle approach; this includes a comparison of two different solid angle definitions. Some comments are included evaluating the limiting surface density versus the solid angle representations of criticality data.

STORAGE ARRAY DESCRIPTION AND MODELING

The critical configuration of plane and line storage arrays of uranium dioxide is modeled using the Monte Carlo criticality program KENO IV⁶ and the Hansen and Koach sixteen group cross-section set.⁷ Five percent ²³⁵U enriched uranium dioxide (UO₂) powder containing some water is assumed stored in identical cylindrical units, stacked 32 x 32 (total number of units, N, equals 1024) for the planar array cases and 128 units linearly stacked for the linear array cases. All arrays considered are surrounded on six sides with a concrete reflector 20 cm (~8 in.) in thickness. The water content of the UO₂ corresponds to a H:U ratio of three. A complete material description in terms of number densities is given in Table 1; note the absence of container materials to hold the UO₂.

Each unit in an array is evenly spaced with respect to the other units, the center-to-center distance being $2a_n$ where a_n is the "half-cell spacing." For units having a height-to-diameter ratio (H/D) equal to two

Table 1. Material Number Densities

U-235	5.891×10^{-4} atom/barn-cm
U-238	1.105×10^{-4} atom/barn-cm
O	4.074×10^{-2} atom/barn-cm
H	3.492×10^{-2} atom/barn-cm
Concrete	(see Table III-3 of Ref. 1)

or less, the inner face of the concrete reflector is located at a distance of a_n from the center of the peripheral array units. For units having a H/D greater than two, the reflector faces are located in direct contact with the cylinder ends for the planar array cases, and with the cylinder ends and sides for the linear array cases.

Some inherent bias can be expected in the calculated k_{eff} when comparing with true experimental criticality data due to the use of the sixteen group Hansen and Roach cross sections. A more accurate representation would use ENDF/B-IV cross sections collapsed with the "correct" neutron energy spectrum. Therefore, the magnitude of the Hansen and Roach bias was investigated by comparing k_{eff} for a 32 x 32 array of $U(5)O_2$ with a H:U=3 contained in cylinders of H/D=1 using Hansen and Roach and a correctly averaged twenty-seven group ENDF/B-IV set. This comparison indicates that Hansen and Roach estimates k_{eff} $2.69 \pm 0.66\%$ higher than ENDF/B-IV. This result is consistent with another investigation performed by Bucholz⁸ in the analysis of the Rocky Flats Bulk-Oxide Critical Experiments.⁹ These experiments consist of low enriched (in the 5 wt % ^{235}U range) damp uranium oxide powder in thin-walled aluminum cubic cans

having a side dimension of 15 cm. These cans are stacked in various critical arrays, moderated and reflected by Plexiglas.

A CRITICALITY DATA BASE

Critical configurations calculated via KENO for the planar and linear arrays described above are presented in Tables 2 through 13. Information provided in these tables includes the cylinder H:D ratio, the concrete reflector location, the half-cell dimension (a_n), the unit cylinder radius (R), the mass of uranium contained in the cylinder (m), and the calculated k_{eff} for the given array geometry. A different geometry is indicated by each row of the tables and is the result of a dimension-search by the KENO code for the critical radius and height, conserving H/D. Use of 30,000 neutron histories for each k-search yielded a standard deviation in k_{eff} on the order of 0.5 percent, also given in the tables.

The results in Tables 2 through 13 are codified by H/D (each table varies the unit spacing for a specified H/D), with the range of unit cylinder H/D varying from 0.2 to 8. Note that the optimum H/D in terms of neutron conservation, i.e., $H/D \approx 1$, has the smallest critical mass for a given array of half cell dimension a_n .

The limit as a_n becomes large or small is also of interest: As a_n becomes large, the unit mass approaches that of a single bare cylinder for the cases $H/D=0.2, 0.5, 1, 2$, and approaches that of a single cylinder with reflected ends for the cases $H/D=5, 8$. As a_n becomes small, the cylinder dimensions and a_n reach a physical lower bound below which

Table 2. Critical Configuration for Planar Arrays of Cylindrical Units in a 32 X 32 X 1 Arrangement.

Cylindrical Unit Height/Diameter = 1			
Concrete Reflector Location: $\begin{matrix} \pm x, \pm y = \pm 32 a_n \\ \pm z = \pm a_n \end{matrix}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
20	14.00	79.2	0.9967 $\pm$ 0.0049
30	17.09	144.1	0.9995 $\pm$ 0.0041
40	19.47	213.1	0.9972 $\pm$ 0.0041
50	21.16	273.6	0.9978 $\pm$ 0.0049
60	22.56	331.5	1.0038 $\pm$ 0.0044
70	24.00	399.2	0.9997 $\pm$ 0.0049
90	24.47	423.1	0.9970 $\pm$ 0.0045
100	24.60	429.9	0.9974 $\pm$ 0.0044
120	25.38	472.1	1.0041 $\pm$ 0.0048
150	25.74	492.4	0.9984 $\pm$ 0.0041
200	25.76	493.6	0.9959 $\pm$ 0.0044
400	26.58	542.2	1.0044 $\pm$ 0.0046

Table 3. Critical Configuration for Planar Arrays of Cylindrical Units in a 32 X 32 X 1 Arrangement.

Cylindrical Unit Height/Diameter = 2			
Concrete Reflector Location: $\begin{matrix} \pm x, \pm y = \pm 32 a_n \\ \pm z = \pm a_n \end{matrix}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
30	14.63	180.7	1.0007 $\pm$ 0.0045
40	15.77	226.3	1.0036 $\pm$ 0.0045
50	17.40	303.9	0.9999 $\pm$ 0.0050
60	18.50	365.7	1.0000 $\pm$ 0.0048
80	20.00	462.0	0.9985 $\pm$ 0.0040
100	21.20	550.3	1.0033 $\pm$ 0.0041
120	21.54	577.2	1.0013 $\pm$ 0.0044
150	21.88	604.9	0.9971 $\pm$ 0.0045

Table 4. Critical Configuration for Planar Arrays of Cylindrical Units in a 32 X 32 X 1 Arrangement.

Cylindrical Unit Height/Diameter = 5			
Concrete Reflector Location: $\frac{+x}{+z}, \frac{+y}{+H} = \frac{+32}{+H} a_n$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
25	9.38	119.2	0.9998 $\pm$ 0.0046
35	11.91	243.9	0.9991 $\pm$ 0.0046
45	13.50	355.2	1.0011 $\pm$ 0.0039
55	14.62	451.2	0.9979 $\pm$ 0.0045
70	15.99	590.3	1.0039 $\pm$ 0.0043
80	16.86	692.0	0.9996 $\pm$ 0.0049
100	18.00	842.0	0.9991 $\pm$ 0.0042
120	18.91	976.3	0.9986 $\pm$ 0.0041
150	20.00	1155.0	1.0021 $\pm$ 0.0040
200	20.80	1299.0	1.0046 $\pm$ 0.0049

Table 5. Critical Configuration for Planar Arrays of Cylindrical Units in a 32 X 32 X 1 Arrangement.

Cylindrical Unit Height/Diameter = 8			
Concrete Reflector Location: $\frac{+x}{+z}, \frac{+y}{+H} = \frac{+32}{+H} a_n$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
20	7.67	104.4	0.9989 $\pm$ 0.0044
30	9.19	179.3	0.9999 $\pm$ 0.0045
40	10.85	295.1	1.0008 $\pm$ 0.0043
45	11.60	360.6	0.9998 $\pm$ 0.0040
50	12.20	419.5	1.0001 $\pm$ 0.0045
55	12.91	497.1	1.0021 $\pm$ 0.0048
60	13.45	562.1	1.0009 $\pm$ 0.0041
80	15.26	820.9	0.9975 $\pm$ 0.0043
100	16.52	1041.0	1.0006 $\pm$ 0.0050
120	17.63	1266.0	0.9965 $\pm$ 0.0039
150	18.73	1518.0	1.0006 $\pm$ 0.0039
200	20.00	1848.0	1.0018 $\pm$ 0.0041
400	21.04	2152.0	0.9963 $\pm$ 0.0040

Table 6. Critical Configuration for Planar Arrays of Cylindrical Units in a 32 X 32 X 1 Arrangement.

Cylindrical Unit Height/Diameter = 1/2			
Concrete Reflector Location: $\pm x, \pm y = \pm 32 a_n$ $\pm z = \pm a_n$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
25	20.25	119.9	1.0005 + 0.0039
30	22.58	166.2	1.0015 + 0.0042
40	25.64	243.3	1.0002 + 0.0045
50	28.28	326.5	0.9983 + 0.0040
60	30.37	404.3	1.0025 + 0.0045
80	32.05	475.3	0.9999 + 0.0045
100	34.00	567.5	1.0006 + 0.0044
120	34.06	570.5	0.9984 + 0.0048
150	35.11	624.9	0.9999 + 0.0046

Table 7. Critical Configuration for Planar Arrays of Cylindrical Units in a 32 X 32 X 1 Arrangement.

Cylindrical Unit Height/Diameter = 1/5			
Concrete Reflector Location: $\pm x, \pm y = \pm 32 a_n$ $\pm z = \pm a_n$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
45	42.00	427.9	0.9999 + 0.0044
50	43.99	491.4	1.0037 + 0.0047
60	48.48	657.9	1.0023 + 0.0042
70	51.25	777.9	1.0008 + 0.0048
80	54.42	930.8	0.9961 + 0.0044
100	60.00	1247.0	1.0040 + 0.0044
120	62.37	1401.0	1.0006 + 0.0042
150	66.10	1668.0	1.0011 + 0.0044
200	67.88	1806.0	0.9982 + 0.0044
400	70.47	2021.0	1.0037 + 0.0045

Table 8. Critical Configuration for Linear Arrays of Cylindrical Units with 128 Units

Cylindrical Unit Height/Diameter = 1			
Concrete Reflector Location: $\frac{+x}{+y}, \frac{+z}{+z} = \frac{+128}{+a_n}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
25	19.66	219.4	0.9986 $\pm$ 0.0042
30	21.03	268.6	0.9975 $\pm$ 0.0049
40	22.68	336.9	1.0000 $\pm$ 0.0052
50	24.00	399.2	0.9981 $\pm$ 0.0044
60	24.80	440.4	1.0015 $\pm$ 0.0044
70	25.26	465.4	1.0008 $\pm$ 0.0040
80	25.53	480.5	1.0008 $\pm$ 0.0043
100	25.72	491.3	0.9977 $\pm$ 0.0050
150	26.10	513.4	1.0023 $\pm$ 0.0044
200	26.22	520.5	0.9968 $\pm$ 0.0041

Table 9. Critical Configuration for Linear Arrays of Cylindrical Units with 128 Units

Cylindrical Unit Height/Diameter = 2			
Concrete Reflector Location: $\frac{+x}{+y}, \frac{+z}{+z} = \frac{+128}{+a_n}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
40	19.16	406.2	0.9972 $\pm$ 0.0042
45	19.84	451.0	1.0031 $\pm$ 0.0042
50	20.18	474.6	1.0020 $\pm$ 0.0046
60	20.75	516.0	0.9988 $\pm$ 0.0039
70	21.35	562.0	1.0022 $\pm$ 0.0046
80	22.00	614.9	1.0037 $\pm$ 0.0047
90	22.19	631.0	0.9965 $\pm$ 0.0043
100	22.19	631.0	1.0023 $\pm$ 0.0042
120	22.34	643.9	0.9976 $\pm$ 0.0051
150	22.73	678.2	1.0012 $\pm$ 0.0043
200	23.12	713.7	0.9973 $\pm$ 0.0042

Table 10. Critical Configuration for Linear Arrays of Cylindrical Units with 128 Units

Cylindrical Unit Height/Diameter = 5			
Concrete Reflector Location: $\begin{matrix} \underline{+x}_n = \underline{+128} \ a_n \\ \underline{+y}_n = \underline{+a}_n, \ \underline{+z}_n = \underline{+H} \end{matrix}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
35	15.99	590.3	0.9958 $\pm$ 0.0042
45	17.54	779.1	1.0012 $\pm$ 0.0043
55	18.28	881.9	1.0038 $\pm$ 0.0044
65	19.00	990.3	0.9960 $\pm$ 0.0045
75	19.61	1088.0	1.0042 $\pm$ 0.0043
90	20.20	1190.0	1.0023 $\pm$ 0.0042
120	20.56	1255.0	0.9990 $\pm$ 0.0054
150	21.11	1358.0	1.0018 $\pm$ 0.0044
200	21.25	1385.0	1.0001 $\pm$ 0.0045

Table 11. Critical Configuration for Linear Arrays of Cylindrical Units with 128 Units

Cylindrical Unit Height/Diameter = 8			
Concrete Reflector Location: $\begin{matrix} \underline{+x}_n = \underline{+128} \ a_n \\ \underline{+y}_n = \underline{+a}_n, \ \underline{+z}_n = \underline{+H} \end{matrix}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
40	16.11	965.9	1.0019 $\pm$ 0.0043
45	16.83	1101.0	1.0012 $\pm$ 0.0045
50	17.20	1175.0	1.0004 $\pm$ 0.0038
55	17.50	1238.0	1.0023 $\pm$ 0.0043
60	18.00	1347.0	0.9981 $\pm$ 0.0032
65	18.45	1451.0	0.9987 $\pm$ 0.0047
70	18.43	1446.0	0.9976 $\pm$ 0.0048
80	19.03	1592.0	1.0024 $\pm$ 0.0041
90	19.19	1632.0	1.0013 $\pm$ 0.0041
100	19.87	1812.0	1.0000 $\pm$ 0.0041
150	20.41	1964.0	0.9975 $\pm$ 0.0045

Table 12. Critical Configuration for Linear Arrays of Cylindrical Units with 128 Units

Cylindrical Unit Height/Diameter = 1/2			
Concrete Reflector Location: $\begin{matrix} +x, = +128 a_n \\ +y, +z = +a_n \end{matrix}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
35	29.29	362.8	0.9961 ± 0.0046
40	30.57	412.5	0.9972 ± 0.0043
50	31.68	459.0	1.0026 ± 0.0046
60	33.00	518.9	0.9995 ± 0.0044
70	33.84	559.5	0.9966 ± 0.0042
80	34.00	567.5	0.9998 ± 0.0042
100	35.00	619.0	1.0035 ± 0.0045
200	35.76	660.2	0.9976 ± 0.0043

Table 13. Critical Configuration for Linear Arrays of Cylindrical Units with 128 Units

Cylindrical Unit Height/Diameter = 1/5			
Concrete Reflector Location: $\begin{matrix} +x, = +128 a_n \\ +y, +z = +a_n \end{matrix}$			
Half-cell Dimension (cm)	Cylinder Radius (cm)	Mass (kgU)	Calculated k_{eff}
70	59.58	1221.0	0.9990 ± 0.0042
80	61.00	1311.0	0.9962 ± 0.0043
90	62.96	1441.0	1.0013 ± 0.0040
100	64.74	1567.0	0.9983 ± 0.0043
110	65.57	1628.0	0.9977 ± 0.0047
120	66.31	1684.0	1.0018 ± 0.0049
400	71.23	2087.0	1.0035 ± 0.0039

array criticality is no longer possible since the number of units in the array is held constant.

LIMITING SURFACE DENSITY REPRESENTATION

The array criticality data base presented in Tables 2 through 13 can be correlated with the surface density approach introduced by Thomas.^{2,3} This approach defines a "limiting surface density" having the form:

$$\sigma(m) \equiv \frac{n_z m}{(2a_n)^2} (1 - c/N)^2 \quad ; \quad (1)$$

the quantity n_z is the number of array units in the direction of the least number of units (n_z is one for planar and linear arrays), and c is an array-independent correlation constant² having dependence only on the neutron energy spectrum. For a fast neutron spectrum c is equal to 0.55 ± 0.18 , decreasing toward zero as the spectrum shifts to the thermal end. In this work the water content of the uranium oxide corresponds to an H:U=3, justifying the assumption that c is about zero. Other quantities appearing in Eq. (1) have been defined previously.

The limiting surface density defined by Eq. (1) was found to be linearly correlated to the mass of uranium contained in an array unit, m , for a wide variety of materials, unit geometries and array geometries.³ The form of this correlation is:

$$\sigma(m) = c_2(m_0 - m) \quad , \quad (2)$$

where the correlation constants, i.e., c_2 and m_0 , are typically obtained by a linear regression analysis of the data points $[m, \sigma(m)]$.

Figures 1 and 2 show the KENO data of Tables 2 through 13 presented as limiting surface density versus the uranium mass of an individual unit; each row in a given table corresponds to a data point shown in the figures. These data points are seen to statistically correlate into lines of the form of Eq. (2), having a negative slope c_2 and horizontal axis intercept m_0 for each specific value of unit H/D. Regression analyses values for c_2 and m_0 , complete with standard deviations, are provided in Table 14.

Table 14. Isolated Cylindrical Critical Masses and Characteristic Array Constants for Concrete-Reflected Arrays

Array	Unit H/D	m_0 (kg)	c_2 (cm ⁻²) x 10 ⁴
32x32 planar	1	548.3 ± 26.5	1.028 ± 0.041
32x32 planar	2	721.8 ± 31.7	0.7052 ± 0.0265
32x32 planar	5	1465.0 ± 96.0*	0.3719 ± 0.0216
32x32 planar	8	2276.0 ± 85.0*	0.2289 ± 0.0075
32x32 planar	1/2	715.4 ± 31.8	0.8246 ± 0.0315
32x32 planar	1/5	2218.0 ± 100.0	0.2905 ± 0.0114
128 linear	1	541.7 ± 19.2	2.725 ± 0.075
128 linear	2	719.6 ± 44.0	1.968 ± 0.094
128 linear	5	1439.0 ± 62.0*	1.396 ± 0.048
128 linear	8	2095.0 ± 148.0*	1.283 ± 0.075
128 linear	1/2	675.7 ± 38.9	2.333 ± 0.106
128 linear	1/5	2138.0 ± 112.0	0.6543 ± 0.0276

*Reflector located in contact with unit cylinders.

The quantity m_0 is more than just a correlation constant, it has physical interpretation: The constant m_0 is the isolated unit mass in the limit as the half cell spacing, a_n , becomes very large, and thus m_0 represents an isolated critical mass of uranium. This critical mass may be unreflected as in the work of Ref. 3, or may be reflected in one or two dimensions as is the case for the planar and linear arrays, respectively, reported here.

Fig. 1. Limiting surface density correlations for 32 x 32 x 1 planar arrays of cylinders containing U(5)O₂ and H/3=3, with concrete reflection on six sides.

Fig. 2. Limiting surface density correlations for 128 x 1 x 1 linear arrays of cylinders containing $U(5)O_8$, and $H/3=3$, with concrete reflection on six sides

SOLID ANGLE REPRESENTATION

Recalling that the idea behind the solid angle approach is to somehow represent the neutronic coupling between array units, the critical geometries of the planar and linear storage arrays given in Tables 2 through 13 are characterized here using two different definitions of solid angle, Ω . What solid angle definition to use is really rather arbitrary: exact versus approximate with shadowing versus no-shadowing are the options. Therefore, the definition used should, first, be a matter of convention, and second, be simple to use such as the definition proposed by Thomas:⁵

$$\Omega_{\text{approx}} = F(N) \frac{A}{(2a_n)^2} \quad (3)$$

The quantity A is the area of a second unit as seen from the center of a first unit for a pair of identical array units separated by the center-to-center distance $2a_n$. The array geometry function, $F(N)$, is dependent only on the number of array units extending in three dimensions, i.e., n_x , n_y , and n_z , as seen by the unit nearest the center of the array (no-shadowing is selected):

$$F(N) = \sum_i \sum_j \sum_k \frac{1}{i^2 + j^2 + k^2} \quad (4)$$

where the index i varies from $-n_x/2$ to $n_x/2$ if n_x is odd or from $-n_x/2$ to $n_x/2-1$ if n_x is even. The indices j and k vary in an identical fashion. The reader is referred to Ref. 5 for the details of Eqs. (3) and (4).

Obviously Eq. (3) represents an approximation of the exact solid angle, especially if the area, A , is taken simply as $H \times D$ for a cylinder or πR^2 for a sphere. An exact solid angle, Ω_{exact} , can always be defined; e.g., as that solid angle subtended by the central-most unit in an array of units.^{1,10,11} In this work Ω_{exact} is defined as that angle seen by the center of the central-most unit in the array (no-shadowing is selected), and is calculated using the SNAKE¹² code.

Figure 3 shows comparisons of Ω_{exact} versus Ω_{approx} , Eq. (3), for the planar array results given in Tables 2 through 7. The solid angle is shown plotted as a function of H/D with a_n as parameter; the solid and dashed curves correspond to Ω_{exact} and Ω_{approx} , respectively. Figure 4 shows similar results for the case of linear arrays.

The results in Figs. 3 and 4 show a nonlinear relationship between Ω and H/D having a minimum near the optimum H/D for the reflected cylinders. This minimum corresponds directly to the minimum uranium mass needed unit to achieve array criticality. When the cylindrical units are close together and quite long, e.g., the $a_n = 50$ curves in Fig. 4 for H/D large, Eq. (3) will overestimate Ω_{exact} : the cylinder ends have a decreasing contribution to the solid angle as the cylinder H/D increases. However, the typical 5 percent difference between Ω_{exact} and Ω_{approx} in Figs. 3 and 4 indicates that the use of Eqs. (3) and (4) is justified for characterizing most planar and linear arrays.

Fig. 3. Comparison of an exact solid angle definition calculated with SNAKE (solid curves) and a simple Eq. (3) solid angle definition (dashed curves) for planar arrays.

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Fig. 4. Comparison of an exact solid angle definition calculated with SNAKE (solid curves) and a simple Eq. (3) solid angle definition (dashed curves) for linear arrays.

SUMMARY AND CONCLUSIONS

A criticality safety data base for critical planar and linear storage arrays of cylindrical units containing damp $U(5)O_6$, with H:U equal to three (see Table 1) was generated using the KENO computer code (see Tables 2 through 13). This data base was successfully correlated (see Figs. 1 and 2) using the limiting surface density methodology, $\sigma(m)$. Correlation constants together with their uncertainties are given in Table 14 as a function of array type and unit H/D.

The solid angle methodology was investigated by comparing different solid angle definitions characterizing the critical geometries of the planar and linear array data base. An "exact" definition using the SNAKE code with the assumption of no-shadowing compared well with the simple approach represented by Eqs. (3) and (4) and described in detail in Ref. 5. Thus the use of Eqs. (3) and (4) in lieu of more complicated procedures for obtaining solid angles provides a good estimate for most planar and linear arrays.

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